Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.7-Miscellaneous/139-4.7.5-x^m-trig-a+b-log-c-x^n-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [330]. This is test number [139].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (330)	0.00 (0)
Mathematica	92.42 (305)	7.58 (25)
Fricas	51.82 (171)	48.18 (159)
Mupad	45.15 (149)	54.85 (181)
Maxima	42.73 (141)	57.27 (189)
Maple	32.42 (107)	67.58 (223)
Giac	27.27 (90)	72.73 (240)
Sympy	20.91 (69)	79.09 (261)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

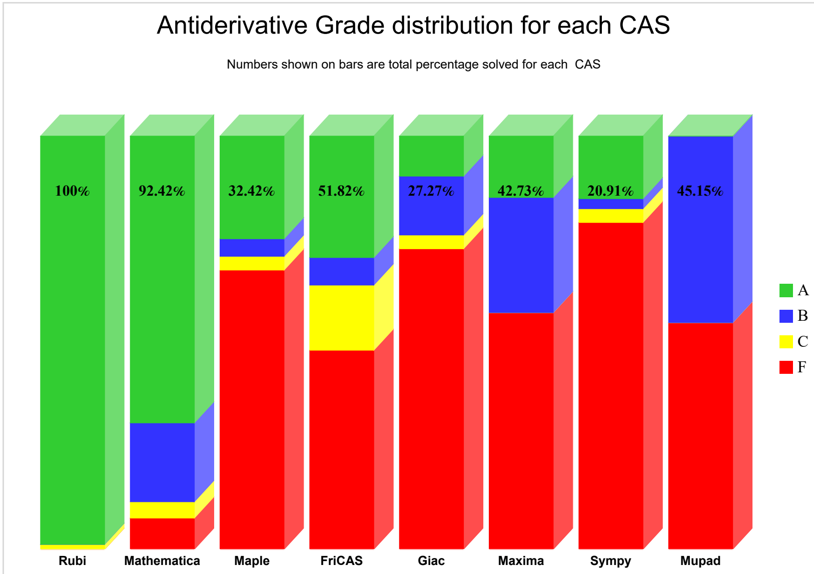
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

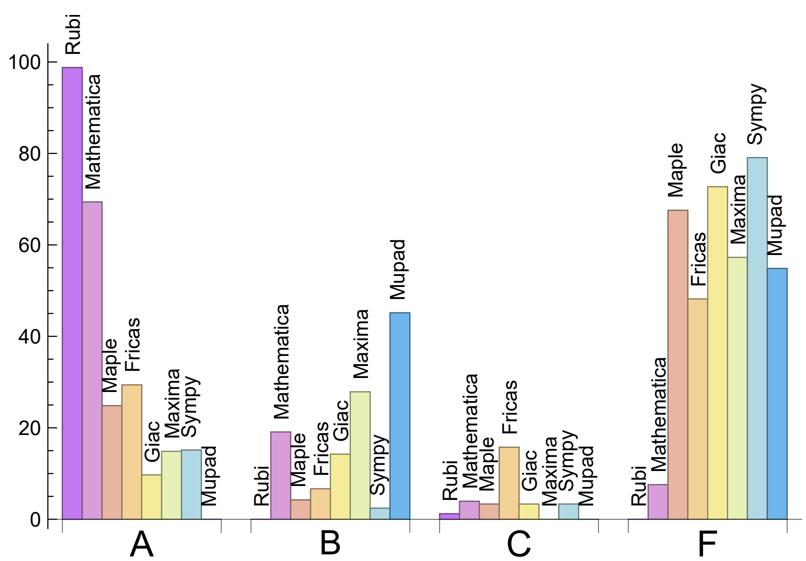
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.79	0.00	1.21	0.00
Mathematica	69.39	19.09	3.94	7.58
Fricas	29.39	6.67	15.76	48.18
Maple	24.85	4.24	3.33	67.58
Sympy	15.15	2.42	3.33	79.09
Maxima	14.85	27.88	0.00	57.27
Giac	9.70	14.24	3.33	72.73
Mupad	N/A	45.15	0.00	54.85

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	25	100.00 %	0.00 %	0.00 %
Maple	223	100.00 %	0.00 %	0.00 %
Fricas	159	61.64 %	0.00 %	38.36 %
Giac	240	67.50 %	30.83 %	1.67 %
Maxima	189	98.94 %	0.00 %	1.06 %
Sympy	261	85.06 %	8.43 %	6.51 %
Mupad	181	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

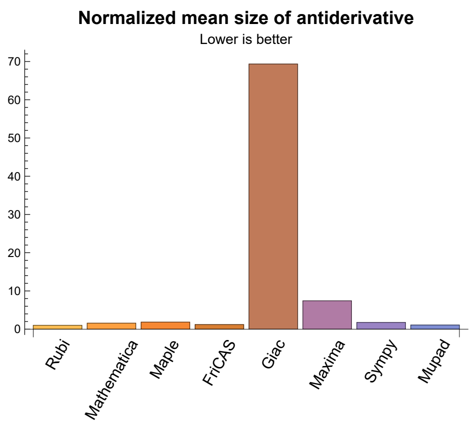
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

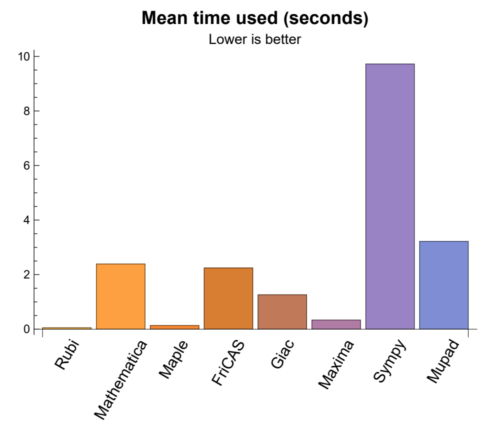
System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.05	102.68	1.02	96.50	1.00
Mathematica	2.39	153.34	1.57	128.00	1.23
Maple	0.13	103.01	1.85	52.00	1.08
Maxima	0.33	688.23	7.43	195.00	3.38
Fricas	2.25	83.90	1.21	73.00	1.02
Sympy	9.72	138.65	1.76	54.00	1.22
Giac	1.26	17104.77	69.35	159.50	2.56
Mupad	3.22	77.91	1.09	59.00	0.92

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.



Normalized mean size of antiderivative
Lower is better
0
10
20
30
40
50
60
70
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad



Mean time used (seconds)
Lower is better
0
2
4
6
8
10
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{260, 302\}$

Mathematica $\{132, 153, 155, 156, 157, 178, 204, 206, 207, 208, 229, 264, 265, 276, 282, 306, 308, 316, 318, 324\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

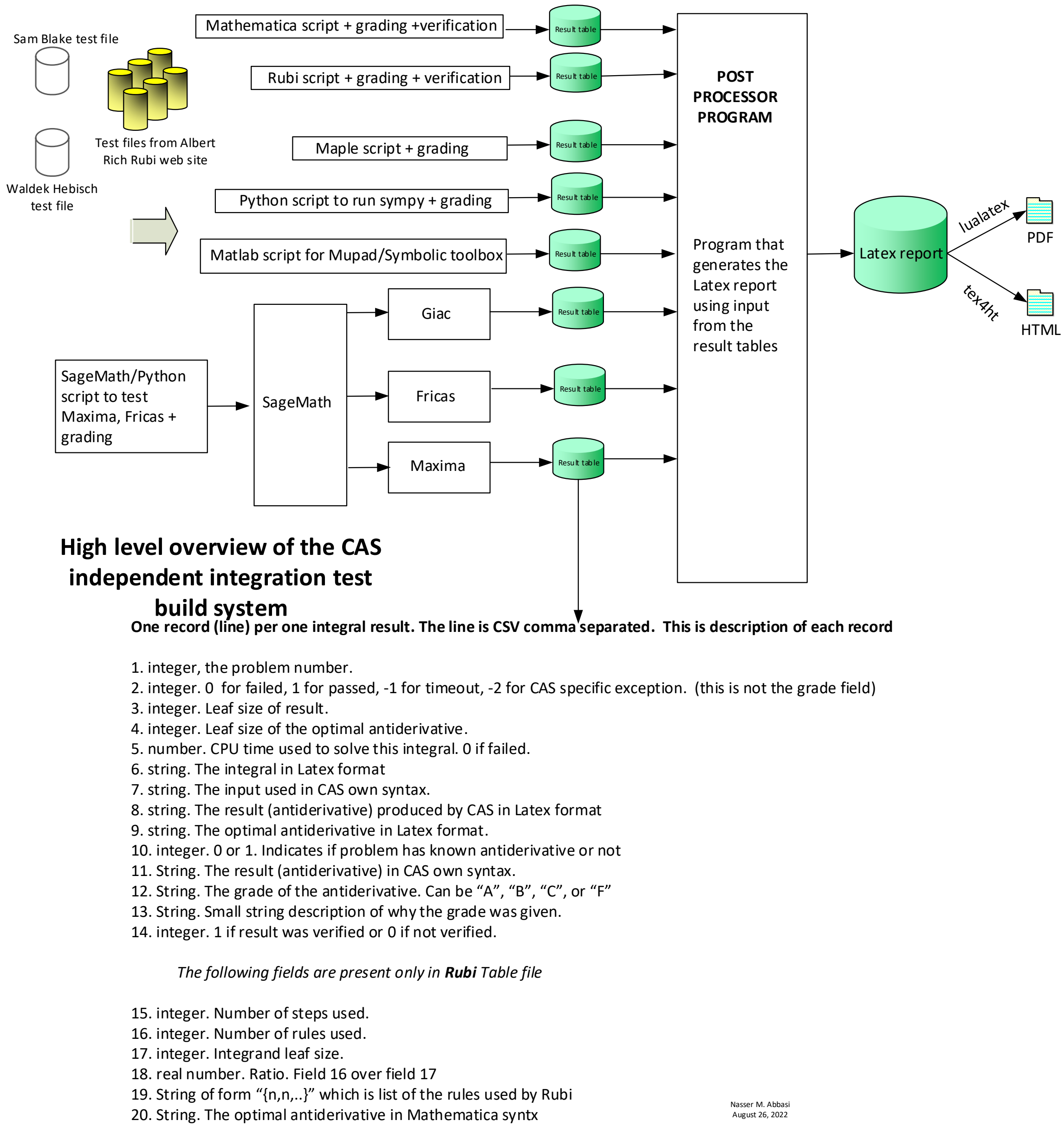
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330 }

B grade: { }

C grade: { 259, 260, 301, 302 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 30, 37, 40, 44, 48, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 108, 111, 113, 115, 116, 117, 119, 120, 121, 122, 123, 124, 126, 127, 129, 132, 133, 134, 136, 138, 139, 140, 142, 144, 146, 147, 148, 150, 151, 152, 154, 155, 156, 157, 162, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 179, 181, 183, 187, 189, 190, 191, 193, 195, 197, 199, 201, 202, 203, 205, 206, 207, 208, 213, 216, 217, 218, 219, 221, 222, 223, 225, 226, 228, 230, 232, 234, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 256, 259, 260, 264, 265, 266, 267, 268, 269, 270, 271, 273, 274, 275, 277, 278, 280, 281, 283, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 296, 297, 298, 300, 301, 302, 306, 307, 308, 309, 310, 311, 312, 313, 315, 316, 317, 319, 322, 323, 325, 327, 328, 329, 330 }

B grade: { 75, 77, 89, 110, 112, 114, 118, 128, 130, 131, 135, 137, 141, 143, 145, 149, 153, 158, 159, 160, 161, 163, 164, 176, 178, 186, 188, 192, 194, 196, 200, 204, 209, 210, 211, 212, 214, 215, 227, 229, 254, 255, 257, 258, 261, 262, 263, 272, 276, 279, 282, 284, 292, 299, 303, 304, 305, 314, 318, 320, 321, 324, 326 }

C grade: { 72, 125, 180, 182, 184, 185, 198, 220, 224, 231, 233, 235, 236 }

F grade: { 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 38, 39, 41, 42, 43, 45, 46, 47, 49, 51, 104, 105, 106, 107, 109 }

2.1.3 Maple

A grade: { 4, 10, 16, 22, 30, 37, 44, 55, 60, 64, 66, 68, 89, 94, 99, 102, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 162, 169, 172, 173, 174, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 213, 220, 223, 224, 225, 231, 232, 233, 234, 235, 236, 240, 246, 251, 256, 292, 296, 298, 300, 309, 311, 313, 315, 317, 319 }

B grade: { 48, 50, 52, 111, 113, 115, 119, 121, 267, 269, 271, 273, 275, 277 }

C grade: { 25, 103, 117, 259, 261, 262, 263, 301, 303, 304, 305 }

F grade: { 1, 2, 3, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 23, 24, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 45, 46, 47, 49, 51, 53, 54, 56, 57, 58, 59, 61, 62, 63, 65, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 95, 96, 97, 98, 100, 101, 104, 105, 106, 107, 108, 109, 110, 112, 114, 116, 118, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 170, 171, 175, 176, 177, 178, 179, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 221, 222, 226, 227, 228, 229, 230, 237, 238, 239, 241, 242, 243, 244, 245, 247, 248, 249, 250, 252, 253, 254, 255, 257, 258, 260, 264, 265, 266, 268, 270, 272, 274, 276, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 297, 299, 302, 306, 307, 308, 310, 312, 314, 316, 318, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330 }

2.1.4 Maxima

A grade: { 4, 10, 22, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 89, 94, 102, 103, 104, 105, 106, 107, 108, 109, 139, 147, 162, 190, 198, 213, 240, 292 }

B grade: { 1, 2, 3, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 69, 70, 71, 72, 73, 86, 87, 88, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 122, 123, 124, 125, 126, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 148, 169, 172, 173, 174, 186, 187, 188, 189, 191, 192, 193, 194, 195, 196, 197, 199, 220, 223, 224, 225, 246, 256, 259, 260, 261, 262, 263, 296, 298, 300, 301, 302, 303, 304, 305 }

C grade: { }

F grade: { 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 127, 128, 129, 130, 131, 132, 133, 134, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 170, 171, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 221, 222, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 245, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 297, 299, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 30, 37, 44, 48, 69, 70, 71, 72, 73, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 122, 123, 124, 125, 126, 135, 136, 137, 138, 139, 141, 142, 143, 144, 145, 148, 149, 162, 172, 186, 188, 190, 191, 192, 193, 194, 196, 197, 199, 200, 213, 223, 246, 251, 256, 259, 261, 262, 264, 296, 300, 301, 303, 304, 306 }

B grade: { 50, 52, 140, 146, 147, 169, 173, 174, 187, 189, 195, 198, 220, 224, 225, 240, 263, 265, 292, 298, 305, 307 }

C grade: { 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 45, 46, 47, 49, 51, 55, 60, 64, 66, 68, 104, 105, 106, 107, 108, 109, 111, 113, 115, 117, 119, 121, 260, 267, 269, 271, 273, 275, 277, 302, 309, 311, 313, 315, 317, 319 }

F grade: { 53, 54, 56, 57, 58, 59, 61, 62, 63, 65, 67, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 110, 112, 114, 116, 118, 120, 127, 128, 129, 130, 131, 132, 133, 134, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 170, 171, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 221, 222, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 245, 247, 248, 249, 250, 252, 253, 254, 255, 257, 258, 266, 268, 270, 272, 274, 276, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 297, 299, 308, 310, 312, 314, 316, 318, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330 }

2.1.6 Sympy

A grade: { 10, 22, 25, 30, 31, 32, 37, 38, 44, 45, 46, 94, 102, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 162, 172, 173, 174, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 224, 240, 292 }

B grade: { 4, 16, 39, 89, 99, 213, 223, 225 }

C grade: { 5, 6, 11, 12, 17, 18, 23, 24, 90, 95, 100 }

F grade: { 1, 2, 3, 7, 8, 9, 13, 14, 15, 19, 20, 21, 26, 27, 28, 29, 33, 34, 35, 36, 40, 41, 42, 43, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 91, 92, 93, 96, 97, 98, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 220, 221, 222, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330 }

2.1.7 Giac

A grade: { 25, 27, 28, 29, 30, 34, 35, 36, 37, 44, 48, 50, 103, 105, 107, 135, 136, 137, 138, 140, 141, 142, 147, 148, 186, 187, 188, 193, 195, 199, 262, 304 }

B grade: { 1, 2, 3, 7, 8, 9, 13, 14, 15, 19, 20, 21, 70, 71, 72, 73, 86, 87, 88, 91, 92, 93, 96, 97, 98, 101, 123, 124, 125, 126, 139, 143, 144, 145, 146, 149, 189, 190, 191, 192, 194, 196, 197, 198, 200, 263, 305 }

C grade: { 26, 33, 40, 47, 49, 51, 104, 106, 108, 260, 302 }

F grade: { 4, 5, 6, 10, 11, 12, 16, 17, 18, 22, 23, 24, 31, 32, 38, 39, 41, 42, 43, 45, 46, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 89, 90, 94, 95, 99, 100, 102, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 127, 128, 129, 130, 131, 132, 133, 134, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 261, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 7, 8, 9, 10, 13, 14, 15, 16, 19, 20, 21, 22, 25, 26, 27, 28, 29, 30, 33, 34, 35, 36, 37, 40, 42, 43, 44, 47, 49, 51, 55, 60, 64, 66, 68, 69, 70, 71, 72, 73, 83, 86, 87, 88, 89, 91, 92, 93, 94, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 113, 115, 117, 119, 121, 122, 123, 124, 125, 126, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 162, 169, 172, 173, 174, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 213, 220, 223, 224, 225, 231, 232, 233, 234, 235, 236, 240, 246, 251, 256, 259, 260, 261, 262, 263, 267, 292, 296, 298, 300, 301, 302, 303, 304, 305, 309 }

C grade: { }

F grade: { 5, 6, 11, 12, 17, 18, 23, 24, 31, 32, 38, 39, 41, 45, 46, 48, 50, 52, 53, 54, 56, 57, 58, 59, 61, 62, 63, 65, 67, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 90, 95, 100, 110, 112, 114, 116, 118, 120, 127, 128, 129, 130, 131, 132, 133, 134, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 170, 171, 175, 176, 177, 178, 179, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 221, 222, 226, 227, 228, 229, 230, 237, 238, 239, 241, 242, 243, 244, 245, 247, 248, 249, 250, 252, 253, 254, 255, 257, 258, 264, 265, 266, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 297, 299, 306, 307, 308, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	44	0	219	49	0	923	44
N.S.	1	1.00	0.77	0.00	3.84	0.86	0.00	16.19	0.77
time (sec)	N/A	0.012	0.088	0.013	0.297	1.621	0.000	0.475	2.494

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	44	0	219	49	0	923	44
N.S.	1	1.00	0.77	0.00	3.84	0.86	0.00	16.19	0.77
time (sec)	N/A	0.009	0.074	0.010	0.297	1.328	0.000	0.455	2.392

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	40	0	206	45	0	882	40
N.S.	1	1.00	0.77	0.00	3.96	0.87	0.00	16.96	0.77
time (sec)	N/A	0.008	0.062	0.013	0.295	1.262	0.000	0.404	2.329

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	38	20	19	20	36	0	19
N.S.	1	1.00	2.00	1.05	1.00	1.05	1.89	0.00	1.00
time (sec)	N/A	0.011	0.035	0.029	0.285	1.658	0.230	0.000	2.256

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	40	0	209	44	192	0	-1
N.S.	1	1.00	0.70	0.00	3.67	0.77	3.37	0.00	-0.02
time (sec)	N/A	0.012	0.073	0.013	0.298	2.455	1.288	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	44	0	216	46	228	0	-1
N.S.	1	1.00	0.77	0.00	3.79	0.81	4.00	0.00	-0.02
time (sec)	N/A	0.010	0.077	0.012	0.289	3.341	3.268	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	61	0	301	80	0	833	67
N.S.	1	1.00	0.63	0.00	3.10	0.82	0.00	8.59	0.69
time (sec)	N/A	0.021	0.169	0.068	0.295	4.611	0.000	0.534	3.279

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	57	0	282	78	0	820	67
N.S.	1	1.00	0.58	0.00	2.88	0.80	0.00	8.37	0.68
time (sec)	N/A	0.018	0.126	0.050	0.291	2.340	0.000	0.548	2.565

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	56	0	280	73	0	786	56
N.S.	1	1.00	0.64	0.00	3.18	0.83	0.00	8.93	0.64
time (sec)	N/A	0.013	0.106	0.049	0.301	2.759	0.000	0.496	2.467

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	36	45	55	40	51	0	32
N.S.	1	1.00	0.92	1.15	1.41	1.03	1.31	0.00	0.82
time (sec)	N/A	0.021	0.087	0.049	0.281	2.975	1.865	0.000	2.399

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	57	0	283	71	301	0	-1
N.S.	1	1.00	0.60	0.00	2.98	0.75	3.17	0.00	-0.01
time (sec)	N/A	0.018	0.119	0.053	0.293	2.043	7.560	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	58	0	280	69	468	0	-1
N.S.	1	1.00	0.59	0.00	2.86	0.70	4.78	0.00	-0.01
time (sec)	N/A	0.019	0.121	0.053	0.302	3.466	3.237	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	122	0	1008	138	0	18085	122
N.S.	1	1.00	0.76	0.00	6.30	0.86	0.00	113.03	0.76
time (sec)	N/A	0.038	0.582	0.059	0.326	2.011	0.000	1.332	3.121

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	125	0	1016	140	0	18117	122
N.S.	1	1.00	0.79	0.00	6.43	0.89	0.00	114.66	0.77
time (sec)	N/A	0.034	0.545	0.052	0.311	3.209	0.000	1.138	3.048

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	121	0	990	130	0	17522	114
N.S.	1	1.00	0.81	0.00	6.64	0.87	0.00	117.60	0.77
time (sec)	N/A	0.027	0.540	0.054	0.324	3.214	0.000	0.791	2.893

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	45	35	233	37	73	0	37
N.S.	1	1.00	1.05	0.81	5.42	0.86	1.70	0.00	0.86
time (sec)	N/A	0.024	0.071	0.046	0.282	2.020	1.807	0.000	2.432

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	125	0	995	127	772	0	-1
N.S.	1	1.00	0.79	0.00	6.30	0.80	4.89	0.00	-0.01
time (sec)	N/A	0.033	0.369	0.055	0.323	2.131	51.272	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	125	0	1007	129	882	0	-1
N.S.	1	1.00	0.79	0.00	6.37	0.82	5.58	0.00	-0.01
time (sec)	N/A	0.033	0.442	0.052	0.327	2.391	55.453	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	171	0	1107	178	0	17035	127
N.S.	1	1.00	0.85	0.00	5.48	0.88	0.00	84.33	0.63
time (sec)	N/A	0.049	0.558	0.083	0.322	1.297	0.000	1.210	3.116

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	169	0	1085	177	0	16984	127
N.S.	1	1.00	0.80	0.00	5.17	0.84	0.00	80.88	0.60
time (sec)	N/A	0.043	0.494	0.064	0.323	1.356	0.000	1.017	3.038

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	168	0	1078	165	0	16422	117
N.S.	1	1.00	0.88	0.00	5.64	0.86	0.00	85.98	0.61
time (sec)	N/A	0.035	0.449	0.066	0.339	1.403	0.000	0.742	2.864

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	51	61	93	59	100	0	51
N.S.	1	1.00	0.70	0.84	1.27	0.81	1.37	0.00	0.70
time (sec)	N/A	0.033	0.111	0.050	0.300	1.216	14.145	0.000	2.582

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	170	0	1085	162	957	0	-1
N.S.	1	1.00	0.84	0.00	5.37	0.80	4.74	0.00	-0.00
time (sec)	N/A	0.045	0.567	0.070	0.322	1.013	147.185	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	169	0	1082	163	1068	0	-1
N.S.	1	1.00	0.80	0.00	5.15	0.78	5.09	0.00	-0.00
time (sec)	N/A	0.043	0.518	0.076	0.331	1.410	134.854	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	44	27	33	56	35	36
N.S.	1	1.00	0.74	1.13	0.69	0.85	1.44	0.90	0.92
time (sec)	N/A	0.010	0.021	0.042	0.268	0.749	0.190	0.399	2.158

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	C	F	C	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	0	0	82	63	0	272	135
N.S.	1	1.00	0.00	0.00	0.62	0.47	0.00	2.05	1.02
time (sec)	N/A	0.198	0.286	0.019	0.320	1.247	0.000	0.994	3.942

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	C	F	A	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	0	0	31	42	0	1	85
N.S.	1	1.00	0.00	0.00	0.35	0.48	0.00	0.01	0.97
time (sec)	N/A	0.071	0.169	0.014	0.291	1.158	0.000	0.522	3.021

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	C	F	A	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	0	0	31	42	0	1	85
N.S.	1	1.00	0.00	0.00	0.35	0.48	0.00	0.01	0.97
time (sec)	N/A	0.034	0.137	0.013	0.287	1.170	0.000	0.513	2.805

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	C	F	A	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	0	0	29	42	0	1	81
N.S.	1	1.00	0.00	0.00	0.35	0.51	0.00	0.01	0.99
time (sec)	N/A	0.033	0.097	0.014	0.298	1.134	0.000	0.462	2.726

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	6	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	1.00	1.20	1.00
time (sec)	N/A	0.002	0.001	0.021	0.261	1.195	0.006	0.393	0.031

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	C	A	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	0	0	33	45	95	0	-1
N.S.	1	1.00	0.00	0.00	0.38	0.52	1.10	0.00	-0.01
time (sec)	N/A	0.044	0.088	0.012	0.298	1.394	1.775	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	C	A	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	0	0	35	45	105	0	-1
N.S.	1	1.00	0.00	0.00	0.40	0.51	1.19	0.00	-0.01
time (sec)	N/A	0.037	0.100	0.012	0.280	1.618	6.073	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	C	F	C	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	0	0	173	107	0	498	145
N.S.	1	1.00	0.00	0.00	1.48	0.91	0.00	4.26	1.24
time (sec)	N/A	0.107	0.403	0.083	0.306	1.672	0.000	2.824	3.846

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	C	F	A	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	0	0	47	59	0	1	92
N.S.	1	1.00	0.00	0.00	0.62	0.78	0.00	0.01	1.21
time (sec)	N/A	0.052	0.292	0.072	0.289	1.479	0.000	0.806	2.967

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	C	F	A	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	0	0	47	60	0	1	92
N.S.	1	1.00	0.00	0.00	0.62	0.79	0.00	0.01	1.21
time (sec)	N/A	0.039	0.176	0.069	0.287	1.634	0.000	0.747	2.888

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	C	F	A	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	0	0	41	57	0	1	86
N.S.	1	1.00	0.00	0.00	0.60	0.84	0.00	0.01	1.26
time (sec)	N/A	0.040	0.113	0.057	0.302	1.591	0.000	0.656	2.661

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	10	7	8	7
N.S.	1	1.00	1.00	1.14	1.00	1.43	1.00	1.14	1.00
time (sec)	N/A	0.004	0.001	0.016	0.274	1.466	0.007	0.419	0.021

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	C	A	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	0	0	48	62	117	0	-1
N.S.	1	1.00	0.00	0.00	0.65	0.84	1.58	0.00	-0.01
time (sec)	N/A	0.044	0.173	0.063	0.296	1.580	14.409	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	C	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	0	0	54	65	221	0	-1
N.S.	1	1.00	0.00	0.00	0.71	0.86	2.91	0.00	-0.01
time (sec)	N/A	0.041	0.150	0.056	0.291	1.406	6.115	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	169	0	195	128	0	1870	297
N.S.	1	1.00	0.75	0.00	0.86	0.57	0.00	8.27	1.31
time (sec)	N/A	0.053	1.394	0.072	0.323	2.354	0.000	5.780	4.706

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	C	F(-1)	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	0	0	90	82	0	0	-1
N.S.	1	1.00	0.00	0.00	0.52	0.48	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.259	0.063	0.299	1.565	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	C	F	F(-2)	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	0	0	112	84	0	0	163
N.S.	1	1.00	0.00	0.00	0.63	0.47	0.00	0.00	0.92
time (sec)	N/A	0.082	0.289	0.063	0.305	1.920	0.000	0.000	3.317

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	C	F	F(-2)	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	0	0	106	84	0	0	155
N.S.	1	1.00	0.00	0.00	0.63	0.50	0.00	0.00	0.92
time (sec)	N/A	0.071	0.158	0.060	0.289	1.725	0.000	0.000	2.985

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	12	7	8	7
N.S.	1	1.00	1.00	1.14	1.00	1.71	1.00	1.14	1.00
time (sec)	N/A	0.004	0.001	0.014	0.268	2.673	0.008	0.393	2.116

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	C	A	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	0	0	122	87	167	0	-1
N.S.	1	1.00	0.00	0.00	0.69	0.49	0.95	0.00	-0.01
time (sec)	N/A	0.089	0.205	0.063	0.297	5.513	52.083	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	C	A	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	0	0	128	87	184	0	-1
N.S.	1	1.00	0.00	0.00	0.72	0.49	1.03	0.00	-0.01
time (sec)	N/A	0.076	0.243	0.056	0.302	2.145	61.242	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	C	F	C	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	0	0	48	51	0	189	139
N.S.	1	1.00	0.00	0.00	0.43	0.46	0.00	1.69	1.24
time (sec)	N/A	0.133	0.264	0.014	0.279	2.719	0.000	0.564	3.129

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	44	106	31	24	0	29	-1
N.S.	1	1.00	0.85	2.04	0.60	0.46	0.00	0.56	-0.02
time (sec)	N/A	0.027	0.066	0.046	0.266	3.030	0.000	0.427	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	C	F	C	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	0	0	134	75	0	350	149
N.S.	1	1.00	0.00	0.00	1.26	0.71	0.00	3.30	1.41
time (sec)	N/A	0.099	0.367	0.065	0.287	3.008	0.000	0.994	3.044

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	60	173	47	145	0	32	-1
N.S.	1	1.00	1.13	3.26	0.89	2.74	0.00	0.60	-0.02
time (sec)	N/A	0.037	0.112	0.124	0.291	3.778	0.000	0.491	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	C	F	C	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	0	0	206	98	0	1297	291
N.S.	1	1.00	0.00	0.00	0.94	0.45	0.00	5.95	1.33
time (sec)	N/A	0.227	0.495	0.056	0.290	3.019	0.000	1.581	4.078

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	103	284	75	204	0	0	-1
N.S.	1	1.00	1.05	2.90	0.77	2.08	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.137	0.133	0.277	6.059	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	145	0	0	0	0	0	-1
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	6.840	0.034	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	148	0	0	0	0	0	-1
N.S.	1	1.00	1.35	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	6.488	0.026	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	32	129	0	90	0	0	26
N.S.	1	1.00	1.10	4.45	0.00	3.10	0.00	0.00	0.90
time (sec)	N/A	0.020	0.114	0.309	0.000	0.503	0.000	0.000	2.323

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	149	0	0	0	0	0	-1
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	6.788	0.026	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	145	0	0	0	0	0	-1
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	6.819	0.026	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	218	0	0	0	0	0	-1
N.S.	1	1.00	1.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	1.213	0.027	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	218	0	0	0	0	0	-1
N.S.	1	1.00	2.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	1.089	0.026	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	58	131	0	111	0	0	65
N.S.	1	1.00	0.85	1.93	0.00	1.63	0.00	0.00	0.96
time (sec)	N/A	0.029	0.168	0.178	0.000	0.505	0.000	0.000	2.532

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	220	0	0	0	0	0	-1
N.S.	1	1.00	1.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	1.159	0.026	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	216	0	0	0	0	0	-1
N.S.	1	1.00	1.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	1.172	0.024	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	132	0	0	0	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.467	0.029	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	32	102	0	82	0	0	26
N.S.	1	1.00	1.10	3.52	0.00	2.83	0.00	0.00	0.90
time (sec)	N/A	0.019	0.112	0.122	0.000	0.396	0.000	0.000	2.553

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	164	0	0	0	0	0	-1
N.S.	1	1.00	1.50	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	6.548	0.027	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	57	190	0	156	0	0	65
N.S.	1	1.00	0.89	2.97	0.00	2.44	0.00	0.00	1.02
time (sec)	N/A	0.028	0.249	0.191	0.000	0.378	0.000	0.000	2.734

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	191	0	0	0	0	0	-1
N.S.	1	1.00	1.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	3.657	0.029	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	61	131	0	177	0	0	65
N.S.	1	1.00	0.90	1.93	0.00	2.60	0.00	0.00	0.96
time (sec)	N/A	0.031	0.245	0.184	0.000	0.458	0.000	0.000	2.962

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	81	0	402	43	0	0	50
N.S.	1	1.00	1.65	0.00	8.20	0.88	0.00	0.00	1.02
time (sec)	N/A	0.027	0.140	0.021	0.619	0.800	0.000	0.000	2.937

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	341	0	16750	461	0	706991	175
N.S.	1	1.00	1.01	0.00	49.70	1.37	0.00	2097.90	0.52
time (sec)	N/A	0.117	2.062	0.135	1.001	1.351	0.000	16.642	4.036

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	326	0	11390	287	0	200416	161
N.S.	1	1.00	1.27	0.00	44.49	1.12	0.00	782.88	0.63
time (sec)	N/A	0.083	1.366	0.095	0.625	1.164	0.000	5.294	3.926

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	102	0	2517	149	0	30585	95
N.S.	1	1.00	0.66	0.00	16.34	0.97	0.00	198.60	0.62
time (sec)	N/A	0.032	0.323	0.096	0.381	1.833	0.000	1.227	3.052

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	63	0	1254	80	0	6580	80
N.S.	1	1.00	0.68	0.00	13.63	0.87	0.00	71.52	0.87
time (sec)	N/A	0.014	0.152	0.023	0.331	1.420	0.000	0.592	2.864

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	145	256	0	0	0	0	0	-1
N.S.	1	0.97	1.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	1.807	0.089	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	145	488	0	0	0	0	0	-1
N.S.	1	0.97	3.28	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	6.711	0.073	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	165	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.709	0.077	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	145	544	0	0	0	0	0	-1
N.S.	1	0.97	3.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	5.620	0.075	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	145	205	0	0	0	0	0	-1
N.S.	1	0.97	1.37	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	2.119	0.076	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	174	0	0	0	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	1.408	0.067	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	148	0	0	0	0	0	-1
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.982	0.025	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	144	0	0	0	0	0	-1
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.796	0.019	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	146	0	0	0	0	0	-1
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.726	0.023	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	0	0	0	0	0	77
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.90
time (sec)	N/A	0.044	0.167	0.034	0.000	0.000	0.000	0.000	2.720

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	146	0	0	0	0	0	-1
N.S.	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.807	0.022	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	142	0	0	0	0	0	-1
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.792	0.021	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	43	0	218	48	0	923	43
N.S.	1	1.00	0.77	0.00	3.89	0.86	0.00	16.48	0.77
time (sec)	N/A	0.012	0.090	0.020	0.303	3.066	0.000	0.450	2.455

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	43	0	218	48	0	915	43
N.S.	1	1.00	0.77	0.00	3.89	0.86	0.00	16.34	0.77
time (sec)	N/A	0.009	0.075	0.011	0.311	2.884	0.000	0.453	2.426

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	0	205	43	0	878	39
N.S.	1	1.00	0.76	0.00	4.02	0.84	0.00	17.22	0.76
time (sec)	N/A	0.008	0.058	0.030	0.297	2.604	0.000	0.458	2.349

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	37	19	18	19	34	0	18
N.S.	1	1.00	2.06	1.06	1.00	1.06	1.89	0.00	1.00
time (sec)	N/A	0.011	0.036	0.033	0.271	2.305	0.259	0.000	2.283

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	41	0	208	45	190	0	-1
N.S.	1	1.00	0.73	0.00	3.71	0.80	3.39	0.00	-0.02
time (sec)	N/A	0.011	0.066	0.013	0.296	4.620	1.302	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	61	0	301	76	0	833	66
N.S.	1	1.00	0.63	0.00	3.10	0.78	0.00	8.59	0.68
time (sec)	N/A	0.021	0.185	0.057	0.303	2.272	0.000	0.578	2.696

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	54	0	282	74	0	820	66
N.S.	1	1.00	0.55	0.00	2.88	0.76	0.00	8.37	0.67
time (sec)	N/A	0.016	0.124	0.045	0.294	2.292	0.000	0.550	2.629

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	54	0	280	68	0	786	56
N.S.	1	1.00	0.61	0.00	3.18	0.77	0.00	8.93	0.64
time (sec)	N/A	0.012	0.096	0.045	0.316	3.072	0.000	0.525	2.531

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	36	45	53	39	51	0	32
N.S.	1	1.00	0.92	1.15	1.36	1.00	1.31	0.00	0.82
time (sec)	N/A	0.022	0.078	0.055	0.297	1.954	1.380	0.000	2.437

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	57	0	285	68	299	0	-1
N.S.	1	1.00	0.60	0.00	3.00	0.72	3.15	0.00	-0.01
time (sec)	N/A	0.018	0.150	0.046	0.308	3.629	5.106	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	120	0	1007	127	0	18053	122
N.S.	1	1.00	0.75	0.00	6.29	0.79	0.00	112.83	0.76
time (sec)	N/A	0.036	0.587	0.075	0.337	4.178	0.000	1.354	3.060

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	123	0	1015	129	0	18069	122
N.S.	1	1.00	0.78	0.00	6.42	0.82	0.00	114.36	0.77
time (sec)	N/A	0.032	0.548	0.059	0.325	2.221	0.000	1.171	2.948

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	117	0	989	119	0	17458	114
N.S.	1	1.00	0.79	0.00	6.64	0.80	0.00	117.17	0.77
time (sec)	N/A	0.027	0.459	0.062	0.323	2.844	0.000	0.831	2.825

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	35	232	36	71	0	37
N.S.	1	1.00	1.00	0.83	5.52	0.86	1.69	0.00	0.88
time (sec)	N/A	0.022	0.064	0.056	0.298	7.340	1.757	0.000	2.353

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	122	0	994	119	774	0	-1
N.S.	1	1.00	0.77	0.00	6.29	0.75	4.90	0.00	-0.01
time (sec)	N/A	0.034	0.505	0.059	0.327	2.954	34.678	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	167	0	1078	144	0	16422	116
N.S.	1	1.00	0.87	0.00	5.64	0.75	0.00	85.98	0.61
time (sec)	N/A	0.035	0.462	0.072	0.317	2.580	0.000	0.802	2.818

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	51	61	93	59	100	0	50
N.S.	1	1.00	0.70	0.84	1.27	0.81	1.37	0.00	0.68
time (sec)	N/A	0.032	0.119	0.058	0.301	2.361	10.517	0.000	2.550

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	22	34	20	25	0	25	21
N.S.	1	1.00	0.76	1.17	0.69	0.86	0.00	0.86	0.72
time (sec)	N/A	0.009	0.017	0.037	0.273	1.956	0.000	0.410	2.168

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	C	F	C	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	82	60	0	267	131
N.S.	1	1.00	0.00	0.00	0.81	0.59	0.00	2.64	1.30
time (sec)	N/A	0.105	0.278	0.029	0.310	1.435	0.000	1.041	3.780

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	C	F	A	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	0	0	29	40	0	1	83
N.S.	1	1.00	0.00	0.00	0.47	0.65	0.00	0.02	1.34
time (sec)	N/A	0.034	0.092	0.125	0.299	1.433	0.000	0.482	2.775

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	C	F	C	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	0	0	172	107	0	498	143
N.S.	1	1.00	0.00	0.00	1.47	0.91	0.00	4.26	1.22
time (sec)	N/A	0.089	0.388	0.073	0.325	1.202	0.000	3.399	3.709

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	C	F	A	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	0	0	41	57	0	1	86
N.S.	1	1.00	0.00	0.00	0.60	0.84	0.00	0.01	1.26
time (sec)	N/A	0.043	0.115	0.067	0.311	3.288	0.000	0.678	2.709

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	158	0	195	128	0	1870	277
N.S.	1	1.00	0.70	0.00	0.86	0.57	0.00	8.27	1.23
time (sec)	N/A	0.057	1.564	0.089	0.361	2.036	0.000	8.017	4.710

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	C	F	F(-2)	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	0	0	106	84	0	0	158
N.S.	1	1.00	0.00	0.00	0.83	0.66	0.00	0.00	1.23
time (sec)	N/A	0.071	0.171	0.078	0.300	1.997	0.000	0.000	3.015

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	361	0	0	0	0	0	-1
N.S.	1	1.00	3.28	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	5.538	0.033	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	181	0	84	0	0	23
N.S.	1	1.00	1.00	7.54	0.00	3.50	0.00	0.00	0.96
time (sec)	N/A	0.019	0.097	0.225	0.000	0.549	0.000	0.000	2.368

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	220	0	0	0	0	0	-1
N.S.	1	1.00	2.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	1.039	0.029	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	54	247	0	107	0	0	56
N.S.	1	1.00	0.86	3.92	0.00	1.70	0.00	0.00	0.89
time (sec)	N/A	0.032	0.121	0.254	0.000	0.405	0.000	0.000	2.300

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	681	0	0	0	0	0	-1
N.S.	1	1.00	6.19	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	7.232	0.029	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	58	280	0	113	0	0	65
N.S.	1	1.00	0.92	4.44	0.00	1.79	0.00	0.00	1.03
time (sec)	N/A	0.030	0.139	0.233	0.000	0.783	0.000	0.000	2.377

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	134	0	0	0	0	0	-1
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.465	0.027	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	26	0	78	0	0	23
N.S.	1	1.00	1.00	1.08	0.00	3.25	0.00	0.00	0.96
time (sec)	N/A	0.019	0.105	0.065	0.000	0.748	0.000	0.000	2.366

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	847	0	0	0	0	0	-1
N.S.	1	1.00	7.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	7.407	0.026	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	250	0	150	0	0	65
N.S.	1	1.00	0.92	4.24	0.00	2.54	0.00	0.00	1.10
time (sec)	N/A	0.028	0.173	0.244	0.000	0.802	0.000	0.000	2.672

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	188	0	0	0	0	0	-1
N.S.	1	1.00	1.72	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	2.563	0.025	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	54	291	0	149	0	0	65
N.S.	1	1.00	0.86	4.62	0.00	2.37	0.00	0.00	1.03
time (sec)	N/A	0.029	0.191	0.260	0.000	0.420	0.000	0.000	2.709

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	82	0	187	39	0	0	48
N.S.	1	1.00	1.71	0.00	3.90	0.81	0.00	0.00	1.00
time (sec)	N/A	0.027	0.122	0.027	0.533	1.916	0.000	0.000	2.792

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	260	312	0	3537	273	0	225232	152
N.S.	1	0.98	1.17	0.00	13.30	1.03	0.00	846.74	0.57
time (sec)	N/A	0.090	4.258	0.078	0.467	2.271	0.000	5.246	3.592

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	292	0	2352	190	0	159584	140
N.S.	1	1.00	1.45	0.00	11.70	0.95	0.00	793.95	0.70
time (sec)	N/A	0.052	2.046	0.071	0.401	1.857	0.000	3.968	3.530

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	91	0	646	105	0	8742	82
N.S.	1	1.00	0.76	0.00	5.38	0.88	0.00	72.85	0.68
time (sec)	N/A	0.021	0.385	0.063	0.321	2.757	0.000	0.710	2.788

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	53	0	313	58	0	5162	70
N.S.	1	1.00	0.76	0.00	4.47	0.83	0.00	73.74	1.00
time (sec)	N/A	0.011	0.152	0.014	0.300	2.134	0.000	0.550	2.672

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	126	238	0	0	0	0	0	-1
N.S.	1	0.97	1.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	2.435	0.029	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	126	436	0	0	0	0	0	-1
N.S.	1	0.98	3.38	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	6.188	0.031	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	153	0	0	0	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.749	0.029	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	126	487	0	0	0	0	0	-1
N.S.	1	0.97	3.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	5.700	0.033	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	126	263	0	0	0	0	0	-1
N.S.	1	0.97	2.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	3.455	0.029	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	170	0	0	0	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	1.858	0.081	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	141	0	0	0	0	0	-1
N.S.	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.965	0.027	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	143	0	0	0	0	0	-1
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.741	0.020	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	132	37	88	30	37	34	36
N.S.	1	1.00	2.81	0.79	1.87	0.64	0.79	0.72	0.77
time (sec)	N/A	0.042	0.040	0.062	0.293	1.728	0.101	0.454	2.225

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	66	33	149	42	61	26	36
N.S.	1	1.00	1.53	0.77	3.47	0.98	1.42	0.60	0.84
time (sec)	N/A	0.029	0.022	0.051	0.510	1.960	0.101	0.458	2.210

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	114	26	70	21	26	25	25
N.S.	1	1.00	3.45	0.79	2.12	0.64	0.79	0.76	0.76
time (sec)	N/A	0.023	0.028	0.044	0.287	3.565	0.090	0.460	2.187

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	42	22	122	33	27	30	25
N.S.	1	1.00	1.56	0.81	4.52	1.22	1.00	1.11	0.93
time (sec)	N/A	0.010	0.011	0.040	0.521	2.475	0.140	0.444	2.166

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	17	10	16	17	73	16
N.S.	1	1.00	1.00	1.21	0.71	1.14	1.21	5.21	1.14
time (sec)	N/A	0.009	0.028	0.040	0.267	3.239	0.139	0.432	3.729

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	44	24	127	39	27	28	27
N.S.	1	1.00	1.52	0.83	4.38	1.34	0.93	0.97	0.93
time (sec)	N/A	0.023	0.029	0.045	0.517	2.844	0.178	0.469	2.268

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	132	36	94	37	39	33	35
N.S.	1	1.00	3.77	1.03	2.69	1.06	1.11	0.94	1.00
time (sec)	N/A	0.026	0.037	0.060	0.280	2.454	0.280	0.465	2.291

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	70	35	156	53	53	28	40
N.S.	1	1.00	1.56	0.78	3.47	1.18	1.18	0.62	0.89
time (sec)	N/A	0.028	0.030	0.053	0.543	2.790	0.232	0.479	2.301

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	155	52	217	64	54	261	51
N.S.	1	1.00	2.46	0.83	3.44	1.02	0.86	4.14	0.81
time (sec)	N/A	0.062	0.207	0.055	0.289	3.250	0.254	0.569	2.252

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	100	48	254	86	66	141	52
N.S.	1	1.00	1.61	0.77	4.10	1.39	1.06	2.27	0.84
time (sec)	N/A	0.055	0.138	0.048	0.520	3.148	0.267	0.569	2.233

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	135	42	185	54	42	221	41
N.S.	1	1.00	2.65	0.82	3.63	1.06	0.82	4.33	0.80
time (sec)	N/A	0.044	0.139	0.044	0.290	3.545	0.152	0.576	2.205

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	70	36	218	77	51	114	42
N.S.	1	1.00	1.52	0.78	4.74	1.67	1.11	2.48	0.91
time (sec)	N/A	0.023	0.096	0.040	0.506	3.673	0.215	0.508	2.205

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	28	24	17	30	22	17	16
N.S.	1	1.00	1.56	1.33	0.94	1.67	1.22	0.94	0.89
time (sec)	N/A	0.016	0.046	0.036	0.468	3.763	0.166	0.431	2.376

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	72	38	223	78	54	73	45
N.S.	1	1.00	1.20	0.63	3.72	1.30	0.90	1.22	0.75
time (sec)	N/A	0.046	0.124	0.046	0.553	3.268	0.303	0.555	2.197

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	150	51	0	74	61	178	56
N.S.	1	1.00	2.73	0.93	0.00	1.35	1.11	3.24	1.02
time (sec)	N/A	0.043	0.207	0.049	0.000	3.470	0.417	0.567	2.211

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	124	0	0	0	0	0	-1
N.S.	1	1.00	1.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.219	0.066	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	86	0	0	0	0	0	-1
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.178	0.013	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	125	0	0	0	0	0	-1
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.175	0.237	0.016	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	330	0	0	0	0	0	-1
N.S.	1	1.00	2.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.740	0.041	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	157	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.701	0.023	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	240	0	0	0	0	0	-1
N.S.	1	1.00	2.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.575	0.039	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	240	0	0	0	0	0	-1
N.S.	1	1.00	2.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.573	0.040	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	240	0	0	0	0	0	-1
N.S.	1	1.00	2.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.559	0.040	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	146	0	0	0	0	0	-1
N.S.	1	1.00	2.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	6.799	0.016	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	155	0	0	0	0	0	-1
N.S.	1	1.00	2.07	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	6.433	0.014	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	146	0	0	0	0	0	-1
N.S.	1	1.00	2.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	6.661	0.014	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	151	0	0	0	0	0	-1
N.S.	1	1.00	2.25	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	11.372	0.026	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	30	24	35	44	0	38
N.S.	1	1.00	0.96	1.15	0.92	1.35	1.69	0.00	1.46
time (sec)	N/A	0.013	0.059	0.064	0.277	2.797	2.368	0.000	3.783

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	153	0	0	0	0	0	-1
N.S.	1	1.00	2.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	4.474	0.016	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	147	0	0	0	0	0	-1
N.S.	1	1.00	2.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	4.189	0.016	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	179	0	0	0	0	0	-1
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	7.079	0.016	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	189	0	0	0	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	6.874	0.016	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	179	0	0	0	0	0	-1
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	6.972	0.016	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	185	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	11.709	0.014	0.000	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	51	41	320	85	0	0	39
N.S.	1	1.00	1.76	1.41	11.03	2.93	0.00	0.00	1.34
time (sec)	N/A	0.020	0.099	0.034	0.295	2.107	0.000	0.000	3.836

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	184	0	0	0	0	0	-1
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	4.887	0.015	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	179	0	0	0	0	0	-1
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	4.177	0.016	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	38	42	1242	69	63	0	105
N.S.	1	1.00	0.88	0.98	28.88	1.60	1.47	0.00	2.44
time (sec)	N/A	0.023	0.186	0.073	0.309	2.819	0.971	0.000	4.718

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	62	49	2171	140	65	0	183
N.S.	1	1.00	1.38	1.09	48.24	3.11	1.44	0.00	4.07
time (sec)	N/A	0.025	0.117	0.053	0.356	2.603	2.133	0.000	8.043

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	55	57	4466	129	82	0	247
N.S.	1	1.00	0.82	0.85	66.66	1.93	1.22	0.00	3.69
time (sec)	N/A	0.031	0.185	0.143	0.383	3.303	4.762	0.000	6.587

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	186	0	0	0	0	0	-1
N.S.	1	1.00	1.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	15.293	0.022	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	550	0	0	0	0	0	-1
N.S.	1	1.00	2.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.146	17.764	0.020	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	642	0	0	0	0	0	-1
N.S.	1	1.00	1.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.349	18.008	0.026	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	458	0	0	0	0	0	-1
N.S.	1	1.00	2.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	1.490	0.037	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	205	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.135	0.831	0.052	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	50	139	0	0	0	0	79
N.S.	1	1.00	0.25	0.69	0.00	0.00	0.00	0.00	0.39
time (sec)	N/A	0.097	0.194	0.190	0.000	0.000	0.000	0.000	3.387

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	175	139	0	0	0	0	78
N.S.	1	1.00	0.88	0.70	0.00	0.00	0.00	0.00	0.39
time (sec)	N/A	0.091	0.190	0.045	0.000	0.000	0.000	0.000	3.312

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	48	122	0	0	0	0	131
N.S.	1	1.00	0.27	0.69	0.00	0.00	0.00	0.00	0.74
time (sec)	N/A	0.082	0.069	0.045	0.000	0.000	0.000	0.000	2.628

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	142	122	0	0	0	0	59
N.S.	1	1.00	0.81	0.69	0.00	0.00	0.00	0.00	0.34
time (sec)	N/A	0.087	0.099	0.046	0.000	0.000	0.000	0.000	2.956

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	46	139	0	0	0	0	79
N.S.	1	1.00	0.23	0.70	0.00	0.00	0.00	0.00	0.40
time (sec)	N/A	0.089	0.094	0.047	0.000	0.000	0.000	0.000	2.921

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	48	139	0	0	0	0	78
N.S.	1	1.00	0.24	0.69	0.00	0.00	0.00	0.00	0.39
time (sec)	N/A	0.091	0.156	0.047	0.000	0.000	0.000	0.000	4.080

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	137	39	131	32	39	50	38
N.S.	1	1.00	2.80	0.80	2.67	0.65	0.80	1.02	0.78
time (sec)	N/A	0.042	0.025	0.063	0.279	3.777	0.110	0.442	2.222

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	66	33	126	78	63	47	40
N.S.	1	1.00	1.53	0.77	2.93	1.81	1.47	1.09	0.93
time (sec)	N/A	0.029	0.014	0.066	0.280	4.300	0.109	0.429	2.198

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	118	28	109	23	27	41	27
N.S.	1	1.00	3.37	0.80	3.11	0.66	0.77	1.17	0.77
time (sec)	N/A	0.024	0.018	0.052	0.272	2.768	0.105	0.416	2.199

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	42	22	94	49	29	38	29
N.S.	1	1.00	1.56	0.81	3.48	1.81	1.07	1.41	1.07
time (sec)	N/A	0.011	0.008	0.050	0.278	3.210	0.097	0.436	2.182

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	25	17	10	18	17	75	21
N.S.	1	1.00	1.79	1.21	0.71	1.29	1.21	5.36	1.50
time (sec)	N/A	0.009	0.022	0.043	0.271	2.735	0.146	0.434	2.252

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	44	24	99	36	29	40	31
N.S.	1	1.00	1.52	0.83	3.41	1.24	1.00	1.38	1.07
time (sec)	N/A	0.024	0.017	0.051	0.277	2.691	0.127	0.422	2.214

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	136	38	135	39	39	49	37
N.S.	1	1.00	3.78	1.06	3.75	1.08	1.08	1.36	1.03
time (sec)	N/A	0.025	0.023	0.059	0.281	2.374	0.207	0.425	2.232

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	70	35	139	55	54	49	44
N.S.	1	1.00	1.56	0.78	3.09	1.22	1.20	1.09	0.98
time (sec)	N/A	0.029	0.017	0.062	0.283	2.253	0.166	0.445	2.212

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	162	54	345	70	54	139	55
N.S.	1	1.00	2.42	0.81	5.15	1.04	0.81	2.07	0.82
time (sec)	N/A	0.065	0.137	0.063	0.292	2.703	0.183	0.460	2.231

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	100	48	335	102	60	83	57
N.S.	1	1.00	1.56	0.75	5.23	1.59	0.94	1.30	0.89
time (sec)	N/A	0.056	0.091	0.058	0.298	2.813	0.198	0.479	2.222

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	142	44	290	61	42	118	45
N.S.	1	1.00	2.58	0.80	5.27	1.11	0.76	2.15	0.82
time (sec)	N/A	0.043	0.091	0.054	0.300	2.481	0.161	0.459	2.193

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	70	36	270	72	42	79	44
N.S.	1	1.00	1.46	0.75	5.62	1.50	0.88	1.65	0.92
time (sec)	N/A	0.025	0.058	0.053	0.295	3.196	0.156	0.435	2.188

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	34	29	19	34	20	32	16
N.S.	1	1.00	1.89	1.61	1.06	1.89	1.11	1.78	0.89
time (sec)	N/A	0.016	0.043	0.040	0.491	3.222	0.169	0.416	2.492

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	72	38	276	74	46	87	47
N.S.	1	1.00	1.12	0.59	4.31	1.16	0.72	1.36	0.73
time (sec)	N/A	0.046	0.089	0.060	0.315	2.104	0.216	0.457	2.209

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	153	53	0	81	60	190	60
N.S.	1	1.00	2.68	0.93	0.00	1.42	1.05	3.33	1.05
time (sec)	N/A	0.044	0.169	0.063	0.000	2.867	0.284	0.451	2.226

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	103	0	0	0	0	0	-1
N.S.	1	1.00	1.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.189	0.037	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	84	0	0	0	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.158	0.024	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	122	0	0	0	0	0	-1
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.227	0.038	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	330	0	0	0	0	0	-1
N.S.	1	1.00	2.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.666	0.072	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	157	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.708	0.034	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	238	0	0	0	0	0	-1
N.S.	1	1.00	1.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.555	0.066	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	238	0	0	0	0	0	-1
N.S.	1	1.00	1.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.535	0.066	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	238	0	0	0	0	0	-1
N.S.	1	1.00	1.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.542	0.067	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	220	0	0	0	0	0	-1
N.S.	1	1.00	3.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	5.681	0.027	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	229	0	0	0	0	0	-1
N.S.	1	1.00	3.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	6.180	0.019	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	219	0	0	0	0	0	-1
N.S.	1	1.00	3.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	6.179	0.017	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	141	0	0	0	0	0	-1
N.S.	1	1.00	2.14	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	10.911	0.019	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	40	30	24	35	46	0	37
N.S.	1	1.00	1.60	1.20	0.96	1.40	1.84	0.00	1.48
time (sec)	N/A	0.013	0.070	0.057	0.282	2.664	2.556	0.000	3.802

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	217	0	0	0	0	0	-1
N.S.	1	1.00	3.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	5.178	0.021	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	211	0	0	0	0	0	-1
N.S.	1	1.00	3.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	4.582	0.022	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	175	0	0	0	0	0	-1
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	5.131	0.025	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	185	0	0	0	0	0	-1
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	5.803	0.022	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	175	0	0	0	0	0	-1
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	5.664	0.019	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	178	0	0	0	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	9.976	0.019	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	51	46	322	78	0	0	39
N.S.	1	1.00	1.70	1.53	10.73	2.60	0.00	0.00	1.30
time (sec)	N/A	0.021	0.134	0.036	0.292	3.179	0.000	0.000	3.863

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	181	0	0	0	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	4.876	0.021	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	175	0	0	0	0	0	-1
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	4.362	0.019	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	52	42	1713	70	97	0	106
N.S.	1	1.00	1.18	0.95	38.93	1.59	2.20	0.00	2.41
time (sec)	N/A	0.025	0.253	0.076	0.336	3.614	4.945	0.000	4.693

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	46	50	2172	132	65	0	182
N.S.	1	1.00	1.05	1.14	49.36	3.00	1.48	0.00	4.14
time (sec)	N/A	0.028	0.132	0.057	0.364	2.427	1.788	0.000	8.104

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	69	57	5998	129	136	0	246
N.S.	1	1.00	1.05	0.86	90.88	1.95	2.06	0.00	3.73
time (sec)	N/A	0.032	0.254	0.137	0.440	3.155	24.074	0.000	6.604

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	182	0	0	0	0	0	-1
N.S.	1	1.00	1.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	14.834	0.025	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	547	0	0	0	0	0	-1
N.S.	1	1.00	2.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	17.380	0.024	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	639	0	0	0	0	0	-1
N.S.	1	1.00	1.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.333	17.849	0.049	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	458	0	0	0	0	0	-1
N.S.	1	1.00	2.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	1.356	0.056	0.000	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	205	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.134	1.249	0.056	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	50	139	0	0	0	0	79
N.S.	1	1.00	0.25	0.69	0.00	0.00	0.00	0.00	0.39
time (sec)	N/A	0.097	0.312	0.233	0.000	0.000	0.000	0.000	3.390

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	175	139	0	0	0	0	80
N.S.	1	1.00	0.88	0.70	0.00	0.00	0.00	0.00	0.40
time (sec)	N/A	0.094	0.344	0.069	0.000	0.000	0.000	0.000	3.325

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	48	122	0	0	0	0	58
N.S.	1	1.00	0.27	0.69	0.00	0.00	0.00	0.00	0.33
time (sec)	N/A	0.081	0.112	0.066	0.000	0.000	0.000	0.000	2.618

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	142	122	0	0	0	0	57
N.S.	1	1.00	0.81	0.69	0.00	0.00	0.00	0.00	0.32
time (sec)	N/A	0.083	0.170	0.069	0.000	0.000	0.000	0.000	2.937

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	46	139	0	0	0	0	79
N.S.	1	1.00	0.23	0.70	0.00	0.00	0.00	0.00	0.40
time (sec)	N/A	0.093	0.150	0.067	0.000	0.000	0.000	0.000	2.952

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	48	139	0	0	0	0	80
N.S.	1	1.00	0.24	0.69	0.00	0.00	0.00	0.00	0.40
time (sec)	N/A	0.088	0.229	0.068	0.000	0.000	0.000	0.000	4.132

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	86	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	1.105	0.020	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	82	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	1.050	0.017	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	84	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.868	0.043	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	32	31	43	51	0	66
N.S.	1	1.00	1.00	1.68	1.63	2.26	2.68	0.00	3.47
time (sec)	N/A	0.011	0.053	0.054	0.274	3.003	1.317	0.000	3.900

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	85	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.734	0.020	0.000	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	81	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.757	0.019	0.000	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	160	0	0	0	0	0	-1
N.S.	1	1.00	1.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	5.974	0.069	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	149	0	0	0	0	0	-1
N.S.	1	1.00	1.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	5.627	0.059	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	147	0	0	0	0	0	-1
N.S.	1	1.00	1.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	6.748	0.061	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	165	33	0	0	29
N.S.	1	1.00	1.00	1.06	9.17	1.83	0.00	0.00	1.61
time (sec)	N/A	0.018	0.097	0.091	0.303	3.001	0.000	0.000	3.840

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	160	0	0	0	0	0	-1
N.S.	1	1.00	1.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	4.096	0.069	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	150	0	0	0	0	0	-1
N.S.	1	1.00	1.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	3.950	0.069	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	118	0	0	0	0	0	-1
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	6.072	0.079	0.000	0.000	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	120	0	0	0	0	0	-1
N.S.	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	5.476	0.071	0.000	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	59	0	100	0	0	178
N.S.	1	1.00	1.00	1.07	0.00	1.82	0.00	0.00	3.24
time (sec)	N/A	0.027	0.083	0.257	0.000	2.381	0.000	0.000	6.305

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	123	0	0	0	0	0	-1
N.S.	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	5.735	0.077	0.000	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	119	0	0	0	0	0	-1
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	5.754	0.075	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	204	0	0	0	0	0	-1
N.S.	1	1.00	2.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	13.522	0.081	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	517	0	0	0	0	0	-1
N.S.	1	1.00	6.08	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	11.133	0.081	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	36	37	1323	52	0	0	49
N.S.	1	1.00	0.86	0.88	31.50	1.24	0.00	0.00	1.17
time (sec)	N/A	0.024	0.130	0.142	0.329	3.180	0.000	0.000	9.056

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	215	0	0	0	0	0	-1
N.S.	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	10.182	0.081	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	203	0	0	0	0	0	-1
N.S.	1	1.00	2.57	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	10.207	0.081	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	C	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	175	29	525	1696	47	0	0	87
N.S.	1	4.27	0.71	12.80	41.37	1.15	0.00	0.00	2.12
time (sec)	N/A	0.098	1.669	0.704	0.668	3.288	0.000	0.000	3.418

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	F	B	C	F(-2)	C	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	146	198	0	976	81	0	834	176
N.S.	1	1.33	1.80	0.00	8.87	0.74	0.00	7.58	1.60
time (sec)	N/A	0.139	2.288	0.149	0.375	2.070	0.000	5.793	6.962

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	127	209	139	55	0	0	46
N.S.	1	1.00	2.82	4.64	3.09	1.22	0.00	0.00	1.02
time (sec)	N/A	0.032	0.171	0.259	0.298	2.888	0.000	0.000	4.432

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	48	137	208	151	55	0	74	56
N.S.	1	0.83	2.36	3.59	2.60	0.95	0.00	1.28	0.97
time (sec)	N/A	0.024	0.128	0.241	0.317	2.066	0.000	1.104	4.481

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	139	238	162	57	0	83	39
N.S.	1	1.00	2.90	4.96	3.38	1.19	0.00	1.73	0.81
time (sec)	N/A	0.026	0.158	0.242	0.332	2.521	0.000	1.105	6.281

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	117	0	0	149	0	0	-1
N.S.	1	1.00	1.23	0.00	0.00	1.57	0.00	0.00	-0.01
time (sec)	N/A	0.062	2.016	0.113	0.000	2.104	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	117	0	0	149	0	0	-1
N.S.	1	1.00	1.67	0.00	0.00	2.13	0.00	0.00	-0.01
time (sec)	N/A	0.049	1.963	0.108	0.000	2.164	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	99	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.358	0.112	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	181	0	78	0	0	51
N.S.	1	1.00	1.00	3.35	0.00	1.44	0.00	0.00	0.94
time (sec)	N/A	0.031	0.134	0.354	0.000	0.652	0.000	0.000	2.568

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	190	0	0	0	0	0	-1
N.S.	1	1.00	1.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	9.749	0.092	0.000	0.000	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	68	250	0	112	0	0	-1
N.S.	1	1.00	0.76	2.81	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.151	0.375	0.000	0.360	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	124	0	0	0	0	0	-1
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	1.339	0.102	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	69	291	0	145	0	0	-1
N.S.	1	1.00	0.74	3.13	0.00	1.56	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.187	0.351	0.000	0.896	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	364	0	0	0	0	0	-1
N.S.	1	1.00	3.31	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	4.876	0.091	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	181	0	84	0	0	-1
N.S.	1	1.00	1.00	3.35	0.00	1.56	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.108	0.295	0.000	0.507	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	168	0	0	0	0	0	-1
N.S.	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	1.521	0.103	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	72	247	0	107	0	0	-1
N.S.	1	1.00	0.77	2.66	0.00	1.15	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.168	0.331	0.000	0.506	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	861	0	0	0	0	0	-1
N.S.	1	1.00	7.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	8.842	0.107	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	83	280	0	113	0	0	-1
N.S.	1	1.00	0.89	3.01	0.00	1.22	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.194	0.362	0.000	0.705	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	139	0	0	0	0	0	-1
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	7.482	0.092	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	482	0	0	0	0	0	-1
N.S.	1	1.00	4.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	17.608	0.081	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	99	99	0	0	0	0	0	-1
N.S.	1	0.96	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	1.816	0.026	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	126	209	0	0	0	0	0	-1
N.S.	1	0.97	1.61	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	7.793	0.082	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	126	470	0	0	0	0	0	-1
N.S.	1	0.97	3.62	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	10.098	0.084	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	165	0	0	0	0	0	-1
N.S.	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	5.682	0.093	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	126	437	0	0	0	0	0	-1
N.S.	1	0.98	3.39	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	7.368	0.092	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	126	230	0	0	0	0	0	-1
N.S.	1	0.97	1.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	7.976	0.086	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	133	169	0	0	0	0	0	-1
N.S.	1	0.96	1.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	1.541	0.105	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	142	0	0	0	0	0	-1
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	1.075	0.082	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	142	0	0	0	0	0	-1
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.890	0.060	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	82	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	1.573	0.075	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	78	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	1.534	0.070	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	80	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	1.282	0.070	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	54	33	32	45	49	0	68
N.S.	1	1.00	2.70	1.65	1.60	2.25	2.45	0.00	3.40
time (sec)	N/A	0.013	0.072	0.115	0.266	2.552	1.316	0.000	3.997

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	82	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	1.164	0.071	0.000	0.000	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	78	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	1.154	0.071	0.000	0.000	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	146	0	0	0	0	0	-1
N.S.	1	1.00	1.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	5.200	0.079	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	168	34	0	0	29
N.S.	1	1.00	1.00	1.05	8.84	1.79	0.00	0.00	1.53
time (sec)	N/A	0.019	0.073	0.086	0.294	1.805	0.000	0.000	3.896

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	117	0	0	0	0	0	-1
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	4.515	0.097	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	107	61	2168	110	0	0	177
N.S.	1	1.00	1.95	1.11	39.42	2.00	0.00	0.00	3.22
time (sec)	N/A	0.027	0.065	0.259	0.346	2.479	0.000	0.000	6.433

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	221	0	0	0	0	0	-1
N.S.	1	1.00	2.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	10.686	0.099	0.000	0.000	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	56	36	1332	71	0	0	49
N.S.	1	1.00	1.30	0.84	30.98	1.65	0.00	0.00	1.14
time (sec)	N/A	0.023	0.098	0.164	0.297	1.831	0.000	0.000	9.231

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	C	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	172	30	523	1701	50	0	0	85
N.S.	1	4.10	0.71	12.45	40.50	1.19	0.00	0.00	2.02
time (sec)	N/A	0.091	0.519	0.750	0.501	6.703	0.000	0.000	3.261

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	F	B	C	F(-2)	C	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	142	79	0	974	83	0	839	171
N.S.	1	1.29	0.72	0.00	8.85	0.75	0.00	7.63	1.55
time (sec)	N/A	0.131	2.252	0.149	0.384	1.081	0.000	5.824	6.962

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	127	211	139	56	0	0	45
N.S.	1	1.00	2.59	4.31	2.84	1.14	0.00	0.00	0.92
time (sec)	N/A	0.030	0.199	0.233	0.314	2.830	0.000	0.000	4.408

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	51	137	209	153	57	0	74	55
N.S.	1	0.88	2.36	3.60	2.64	0.98	0.00	1.28	0.95
time (sec)	N/A	0.024	0.160	0.237	0.314	1.939	0.000	1.109	4.536

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	137	239	162	57	0	83	38
N.S.	1	1.00	2.69	4.69	3.18	1.12	0.00	1.63	0.75
time (sec)	N/A	0.028	0.167	0.231	0.313	1.632	0.000	1.110	6.317

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	155	0	0	150	0	0	-1
N.S.	1	1.00	1.61	0.00	0.00	1.56	0.00	0.00	-0.01
time (sec)	N/A	0.061	1.979	0.165	0.000	4.014	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	128	0	0	150	0	0	-1
N.S.	1	1.00	1.80	0.00	0.00	2.11	0.00	0.00	-0.01
time (sec)	N/A	0.052	2.980	0.167	0.000	1.339	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	115	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.458	0.127	0.000	0.000	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	58	102	0	78	0	0	89
N.S.	1	1.00	0.98	1.73	0.00	1.32	0.00	0.00	1.51
time (sec)	N/A	0.027	0.134	0.277	0.000	0.486	0.000	0.000	2.623

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	127	0	0	0	0	0	-1
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	7.426	0.118	0.000	0.000	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	72	190	0	111	0	0	-1
N.S.	1	1.00	0.77	2.02	0.00	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.166	0.348	0.000	1.225	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	174	0	0	0	0	0	-1
N.S.	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	1.674	0.130	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	73	131	0	145	0	0	-1
N.S.	1	1.00	0.74	1.34	0.00	1.48	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.223	0.303	0.000	0.518	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	367	0	0	0	0	0	-1
N.S.	1	1.00	3.34	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	5.346	0.117	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	58	129	0	82	0	0	-1
N.S.	1	1.00	0.98	2.19	0.00	1.39	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.119	0.283	0.000	0.523	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	186	0	0	0	0	0	-1
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	2.159	0.122	0.000	0.000	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	76	131	0	107	0	0	-1
N.S.	1	1.00	0.78	1.34	0.00	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.198	0.373	0.000	0.925	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	862	0	0	0	0	0	-1
N.S.	1	1.00	7.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	8.869	0.121	0.000	0.000	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	88	205	0	130	0	0	-1
N.S.	1	1.00	0.90	2.09	0.00	1.33	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.232	0.316	0.000	0.263	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	367	0	0	0	0	0	-1
N.S.	1	1.00	3.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	2.477	0.089	0.000	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	534	0	0	0	0	0	-1
N.S.	1	1.00	4.49	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	6.575	0.085	0.000	0.000	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	118	181	0	0	0	0	0	-1
N.S.	1	0.96	1.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.353	0.078	0.000	0.000	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	126	207	0	0	0	0	0	-1
N.S.	1	0.97	1.59	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	6.383	0.123	0.000	0.000	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	126	466	0	0	0	0	0	-1
N.S.	1	0.97	3.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	10.342	0.119	0.000	0.000	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	163	0	0	0	0	0	-1
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	4.407	0.119	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	126	441	0	0	0	0	0	-1
N.S.	1	0.98	3.42	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	7.818	0.125	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	126	228	0	0	0	0	0	-1
N.S.	1	0.97	1.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	6.980	0.115	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	133	169	0	0	0	0	0	-1
N.S.	1	0.96	1.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	1.568	0.146	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	142	0	0	0	0	0	-1
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	1.190	0.115	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	142	0	0	0	0	0	-1
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.932	0.096	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [259] had the largest ratio of [44]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	15	0.067
2	A	1	1	1.00	13	0.077
3	A	1	1	1.00	11	0.091
4	A	2	1	1.00	15	0.067
5	A	1	1	1.00	15	0.067
6	A	1	1	1.00	15	0.067
7	A	2	2	1.00	17	0.118
8	A	2	2	1.00	15	0.133
9	A	2	2	1.00	13	0.154
10	A	3	2	1.00	17	0.118
11	A	2	2	1.00	17	0.118
12	A	2	2	1.00	17	0.118
13	A	2	2	1.00	17	0.118
14	A	2	2	1.00	15	0.133
15	A	2	2	1.00	13	0.154
16	A	3	1	1.00	17	0.059
17	A	2	2	1.00	17	0.118
18	A	2	2	1.00	17	0.118
19	A	3	2	1.00	17	0.118
20	A	3	2	1.00	15	0.133
21	A	3	2	1.00	13	0.154
22	A	4	2	1.00	17	0.118
23	A	3	2	1.00	17	0.118
24	A	3	2	1.00	17	0.118
25	A	2	1	1.00	7	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	3	2	1.00	28	0.071
27	A	3	2	1.00	24	0.083
28	A	3	2	1.00	22	0.091
29	A	3	2	1.00	19	0.105
30	A	2	2	1.00	6	0.333
31	A	3	2	1.00	23	0.087
32	A	3	2	1.00	24	0.083
33	A	3	2	1.00	33	0.061
34	A	3	2	1.00	28	0.071
35	A	3	2	1.00	23	0.087
36	A	3	2	1.00	24	0.083
37	A	2	2	1.00	8	0.250
38	A	3	2	1.00	28	0.071
39	A	3	2	1.00	25	0.080
40	A	2	2	1.00	33	0.061
41	A	3	2	1.00	25	0.080
42	A	3	2	1.00	26	0.077
43	A	3	2	1.00	24	0.083
44	A	2	2	1.00	8	0.250
45	A	3	2	1.00	28	0.071
46	A	3	2	1.00	28	0.071
47	A	3	2	1.00	28	0.071
48	A	3	2	1.00	15	0.133
49	A	3	2	1.00	30	0.067
50	A	3	2	1.00	17	0.118
51	A	3	2	1.00	30	0.067
52	A	3	2	1.00	17	0.118
53	A	3	3	1.00	17	0.176
54	A	3	3	1.00	15	0.200
55	A	2	1	1.00	19	0.053
56	A	3	3	1.00	19	0.158
57	A	3	3	1.00	19	0.158
58	A	3	3	1.00	17	0.176
59	A	3	3	1.00	15	0.200
60	A	3	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	3	3	1.00	19	0.158
62	A	3	3	1.00	19	0.158
63	A	3	3	1.00	15	0.200
64	A	2	1	1.00	19	0.053
65	A	3	3	1.00	15	0.200
66	A	3	2	1.00	19	0.105
67	A	3	3	1.00	15	0.200
68	A	3	2	1.00	19	0.105
69	A	3	3	1.00	15	0.200
70	A	3	2	1.00	21	0.095
71	A	2	2	1.00	21	0.095
72	A	2	2	1.00	21	0.095
73	A	1	1	1.00	19	0.053
74	A	3	3	0.97	23	0.130
75	A	3	3	0.97	23	0.130
76	A	3	3	1.00	23	0.130
77	A	3	3	0.97	23	0.130
78	A	3	3	0.97	23	0.130
79	A	3	3	1.00	21	0.143
80	A	3	3	1.00	17	0.176
81	A	3	3	1.00	15	0.200
82	A	3	3	1.00	13	0.231
83	A	2	1	1.00	17	0.059
84	A	3	3	1.00	17	0.176
85	A	3	3	1.00	17	0.176
86	A	1	1	1.00	15	0.067
87	A	1	1	1.00	13	0.077
88	A	1	1	1.00	11	0.091
89	A	2	1	1.00	15	0.067
90	A	1	1	1.00	15	0.067
91	A	2	2	1.00	17	0.118
92	A	2	2	1.00	15	0.133
93	A	2	2	1.00	13	0.154
94	A	3	2	1.00	17	0.118
95	A	2	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	2	2	1.00	17	0.118
97	A	2	2	1.00	15	0.133
98	A	2	2	1.00	13	0.154
99	A	3	1	1.00	17	0.059
100	A	2	2	1.00	17	0.118
101	A	3	2	1.00	13	0.154
102	A	4	2	1.00	17	0.118
103	A	2	1	1.00	7	0.143
104	A	3	2	1.00	28	0.071
105	A	3	2	1.00	19	0.105
106	A	3	2	1.00	33	0.061
107	A	3	2	1.00	24	0.083
108	A	2	2	1.00	33	0.061
109	A	3	2	1.00	24	0.083
110	A	3	3	1.00	15	0.200
111	A	2	1	1.00	19	0.053
112	A	3	3	1.00	15	0.200
113	A	3	2	1.00	19	0.105
114	A	3	3	1.00	15	0.200
115	A	3	2	1.00	19	0.105
116	A	3	3	1.00	15	0.200
117	A	2	1	1.00	19	0.053
118	A	3	3	1.00	15	0.200
119	A	3	2	1.00	19	0.105
120	A	3	3	1.00	15	0.200
121	A	3	2	1.00	19	0.105
122	A	3	3	1.00	15	0.200
123	A	3	2	0.98	17	0.118
124	A	2	2	1.00	17	0.118
125	A	2	2	1.00	17	0.118
126	A	1	1	1.00	15	0.067
127	A	3	3	0.97	19	0.158
128	A	3	3	0.98	19	0.158
129	A	3	3	1.00	19	0.158
130	A	3	3	0.97	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	3	3	0.97	19	0.158
132	A	3	3	1.00	21	0.143
133	A	3	3	1.00	15	0.200
134	A	3	3	1.00	13	0.231
135	A	5	4	1.00	13	0.308
136	A	5	5	1.00	13	0.385
137	A	5	4	1.00	11	0.364
138	A	4	4	1.00	9	0.444
139	A	2	1	1.00	13	0.077
140	A	4	4	1.00	13	0.308
141	A	4	3	1.00	13	0.231
142	A	5	5	1.00	13	0.385
143	A	5	4	1.00	15	0.267
144	A	6	6	1.00	15	0.400
145	A	5	4	1.00	13	0.308
146	A	6	5	1.00	11	0.454
147	A	3	2	1.00	15	0.133
148	A	5	5	1.00	15	0.333
149	A	4	3	1.00	15	0.200
150	A	4	4	1.00	15	0.267
151	A	5	5	1.00	17	0.294
152	A	6	6	1.00	17	0.353
153	A	4	4	1.00	9	0.444
154	A	4	4	1.00	15	0.267
155	A	4	4	1.00	7	0.571
156	A	4	4	1.00	9	0.444
157	A	4	4	1.00	9	0.444
158	A	4	4	1.00	17	0.235
159	A	4	4	1.00	17	0.235
160	A	4	4	1.00	15	0.267
161	A	4	4	1.00	13	0.308
162	A	2	1	1.00	17	0.059
163	A	4	4	1.00	17	0.235
164	A	4	4	1.00	17	0.235
165	A	5	5	1.00	19	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	5	5	1.00	19	0.263
167	A	5	5	1.00	17	0.294
168	A	5	5	1.00	15	0.333
169	A	3	2	1.00	19	0.105
170	A	5	5	1.00	19	0.263
171	A	5	5	1.00	19	0.263
172	A	3	2	1.00	17	0.118
173	A	4	2	1.00	17	0.118
174	A	4	2	1.00	17	0.118
175	A	4	4	1.00	19	0.210
176	A	5	5	1.00	21	0.238
177	A	6	6	1.00	21	0.286
178	A	5	5	1.00	15	0.333
179	A	5	5	1.00	21	0.238
180	A	13	9	1.00	19	0.474
181	A	13	9	1.00	19	0.474
182	A	12	8	1.00	19	0.421
183	A	12	8	1.00	19	0.421
184	A	13	9	1.00	19	0.474
185	A	13	9	1.00	19	0.474
186	A	5	4	1.00	13	0.308
187	A	5	5	1.00	13	0.385
188	A	5	4	1.00	11	0.364
189	A	4	4	1.00	9	0.444
190	A	2	1	1.00	13	0.077
191	A	4	4	1.00	13	0.308
192	A	4	3	1.00	13	0.231
193	A	5	5	1.00	13	0.385
194	A	5	4	1.00	15	0.267
195	A	6	6	1.00	15	0.400
196	A	5	4	1.00	13	0.308
197	A	6	5	1.00	11	0.454
198	A	3	2	1.00	15	0.133
199	A	5	5	1.00	15	0.333
200	A	4	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	4	4	1.00	15	0.267
202	A	5	5	1.00	17	0.294
203	A	6	6	1.00	17	0.353
204	A	4	4	1.00	9	0.444
205	A	4	4	1.00	15	0.267
206	A	4	4	1.00	7	0.571
207	A	4	4	1.00	9	0.444
208	A	4	4	1.00	9	0.444
209	A	4	4	1.00	17	0.235
210	A	4	4	1.00	17	0.235
211	A	4	4	1.00	15	0.267
212	A	4	4	1.00	13	0.308
213	A	2	1	1.00	17	0.059
214	A	4	4	1.00	17	0.235
215	A	4	4	1.00	17	0.235
216	A	5	5	1.00	19	0.263
217	A	5	5	1.00	19	0.263
218	A	5	5	1.00	17	0.294
219	A	5	5	1.00	15	0.333
220	A	3	2	1.00	19	0.105
221	A	5	5	1.00	19	0.263
222	A	5	5	1.00	19	0.263
223	A	3	2	1.00	17	0.118
224	A	4	2	1.00	17	0.118
225	A	4	2	1.00	17	0.118
226	A	4	4	1.00	19	0.210
227	A	5	5	1.00	21	0.238
228	A	6	6	1.00	21	0.286
229	A	5	5	1.00	15	0.333
230	A	5	5	1.00	21	0.238
231	A	13	9	1.00	19	0.474
232	A	13	9	1.00	19	0.474
233	A	12	8	1.00	19	0.421
234	A	12	8	1.00	19	0.421
235	A	13	9	1.00	19	0.474

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	13	9	1.00	19	0.474
237	A	3	3	1.00	15	0.200
238	A	3	3	1.00	13	0.231
239	A	3	3	1.00	11	0.273
240	A	2	1	1.00	15	0.067
241	A	3	3	1.00	15	0.200
242	A	3	3	1.00	15	0.200
243	A	3	3	1.00	17	0.176
244	A	3	3	1.00	15	0.200
245	A	3	3	1.00	13	0.231
246	A	3	2	1.00	17	0.118
247	A	3	3	1.00	17	0.176
248	A	3	3	1.00	17	0.176
249	A	3	3	1.00	15	0.200
250	A	3	3	1.00	13	0.231
251	A	3	2	1.00	17	0.118
252	A	3	3	1.00	17	0.176
253	A	3	3	1.00	17	0.176
254	A	3	3	1.00	15	0.200
255	A	3	3	1.00	13	0.231
256	A	3	1	1.00	17	0.059
257	A	3	3	1.00	17	0.176
258	A	3	3	1.00	17	0.176
259	C	7	3	4.27	44	0.068
260	C	3	3	1.33	31	0.097
261	A	3	3	1.00	17	0.176
262	A	3	3	0.83	17	0.176
263	A	3	3	1.00	17	0.176
264	A	3	3	1.00	23	0.130
265	A	3	3	1.00	23	0.130
266	A	3	3	1.00	15	0.200
267	A	3	2	1.00	19	0.105
268	A	3	3	1.00	15	0.200
269	A	4	3	1.00	19	0.158
270	A	3	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	4	3	1.00	19	0.158
272	A	3	3	1.00	15	0.200
273	A	3	2	1.00	19	0.105
274	A	3	3	1.00	15	0.200
275	A	4	3	1.00	19	0.158
276	A	3	3	1.00	15	0.200
277	A	4	3	1.00	19	0.158
278	A	3	3	1.00	17	0.176
279	A	3	3	1.00	17	0.176
280	A	3	3	0.96	15	0.200
281	A	3	3	0.97	19	0.158
282	A	3	3	0.97	19	0.158
283	A	3	3	1.00	19	0.158
284	A	3	3	0.98	19	0.158
285	A	3	3	0.97	19	0.158
286	A	3	3	0.96	21	0.143
287	A	3	3	1.00	15	0.200
288	A	3	3	1.00	13	0.231
289	A	3	3	1.00	15	0.200
290	A	3	3	1.00	13	0.231
291	A	3	3	1.00	11	0.273
292	A	2	1	1.00	15	0.067
293	A	3	3	1.00	15	0.200
294	A	3	3	1.00	15	0.200
295	A	3	3	1.00	13	0.231
296	A	3	2	1.00	17	0.118
297	A	3	3	1.00	13	0.231
298	A	3	2	1.00	17	0.118
299	A	3	3	1.00	13	0.231
300	A	3	1	1.00	17	0.059
301	C	7	3	4.10	44	0.068
302	C	3	3	1.29	31	0.097
303	A	3	3	1.00	17	0.176
304	A	3	3	0.88	17	0.176
305	A	3	3	1.00	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	3	3	1.00	23	0.130
307	A	3	3	1.00	23	0.130
308	A	3	3	1.00	15	0.200
309	A	3	2	1.00	19	0.105
310	A	3	3	1.00	15	0.200
311	A	4	3	1.00	19	0.158
312	A	3	3	1.00	15	0.200
313	A	4	3	1.00	19	0.158
314	A	3	3	1.00	15	0.200
315	A	3	2	1.00	19	0.105
316	A	3	3	1.00	15	0.200
317	A	4	3	1.00	19	0.158
318	A	3	3	1.00	15	0.200
319	A	4	3	1.00	19	0.158
320	A	3	3	1.00	21	0.143
321	A	3	3	1.00	21	0.143
322	A	3	3	0.96	19	0.158
323	A	3	3	0.97	19	0.158
324	A	3	3	0.97	19	0.158
325	A	3	3	1.00	19	0.158
326	A	3	3	0.98	19	0.158
327	A	3	3	0.97	19	0.158
328	A	3	3	0.96	21	0.143
329	A	3	3	1.00	15	0.200
330	A	3	3	1.00	13	0.231

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^2 \sin (a+b \log (c x^n)) \, dx$

Optimal. Leaf size=57

$$-\frac{b n x^3 \cos (a+b \log (c x^n))}{9+b^2 n^2}+\frac{3 x^3 \sin (a+b \log (c x^n))}{9+b^2 n^2}$$

```
[Out] -b*n*x^3*cos(a+b*ln(c*x^n))/(b^2*n^2+9)+3*x^3*sin(a+b*ln(c*x^n))/(b^2*n^2+9
)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {4573}

$$\frac{3 x^3 \sin (a+b \log (c x^n))}{b^2 n^2+9}-\frac{b n x^3 \cos (a+b \log (c x^n))}{b^2 n^2+9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[a + b*Log[c*x^n]],x]
```

```
[Out] -((b*n*x^3*Cos[a + b*Log[c*x^n]])/(9 + b^2*n^2)) + (3*x^3*Sin[a + b*Log[c*x
^n]])/(9 + b^2*n^2)
```

Rule 4573

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] - Simp[b*d*n*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int x^2 \sin (a+b \log (c x^n)) \, dx=-\frac{b n x^3 \cos (a+b \log (c x^n))}{9+b^2 n^2}+\frac{3 x^3 \sin (a+b \log (c x^n))}{9+b^2 n^2}$$

Mathematica [A]
time = 0.09, size = 44, normalized size = 0.77

$$-\frac{x^3 (b n \cos (a+b \log (c x^n))-3 \sin (a+b \log (c x^n)))}{9+b^2 n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[a + b*Log[c*x^n]],x]
```

```
[Out] -((x^3*(b*n*Cos[a + b*Log[c*x^n]] - 3*Sin[a + b*Log[c*x^n]]))/(9 + b^2*n^2)
)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^{2} \sin \left(a +b \ln \left(c \,x^{n}\right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a+b*ln(c*x^n)),x)
```

```
[Out] int(x^2*sin(a+b*ln(c*x^n)),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(57) = 114$.
time = 0.30, size = 219, normalized size = 3.84

$$-\frac{{\left({\left(b \cos\left(2 \, b \log\left(c\right)\right) \cos\left(b \log\left(c\right)\right) + b \sin\left(2 \, b \log\left(c\right)\right) \sin\left(b \log\left(c\right)\right) + b \cos\left(b \log\left(c\right)\right)\right)} n - 3 \, \cos\left(b \log\left(c\right)\right) \sin\left(2 \, b \log\left(c\right)\right) + 3 \, \cos\left(2 \, b \log\left(c\right)\right) \sin\left(b \log\left(c\right)\right) - 3 \, \sin\left(b \log\left(c\right)\right)\right)} x^{3} \cos\left(b \log\left(x^{n}\right) + a\right) - {\left({\left(b \cos\left(b \log\left(c\right)\right) \sin\left(2 \, b \log\left(c\right)\right) - b \cos\left(2 \, b \log\left(c\right)\right) \sin\left(b \log\left(c\right)\right) + b \sin\left(b \log\left(c\right)\right)\right)} n + 3 \, \cos\left(2 \, b \log\left(c\right)\right) \cos\left(b \log\left(c\right)\right) + 3 \, \sin\left(2 \, b \log\left(c\right)\right) \sin\left(b \log\left(c\right)\right) + 3 \, \cos\left(b \log\left(c\right)\right)\right)} x^{3} \sin\left(b \log\left(x^{n}\right) + a\right)}{2 \, {\left({\left(b^{2} \cos\left(b \log\left(c\right)\right)^{2} + b^{2} \sin\left(b \log\left(c\right)\right)^{2}\right)} n^{2} + 9 \, \cos\left(b \log\left(c\right)\right)^{2} + 9 \, \sin\left(b \log\left(c\right)\right)^{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/2*(((b*cos(2*b*log(c))*cos(b*log(c)) + b*sin(2*b*log(c))*sin(b*log(c)) +
 b*cos(b*log(c)))*n - 3*cos(b*log(c))*sin(2*b*log(c)) + 3*cos(2*b*log(c))*s
in(b*log(c)) - 3*sin(b*log(c)))*x^3*cos(b*log(x^n) + a) - ((b*cos(b*log(c))
*sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)) + b*sin(b*log(c)))*n + 3
*cos(2*b*log(c))*cos(b*log(c)) + 3*sin(2*b*log(c))*sin(b*log(c)) + 3*cos(b*
log(c)))*x^3*sin(b*log(x^n) + a))/((b^2*cos(b*log(c))^2 + b^2*sin(b*log(c))
^2)*n^2 + 9*cos(b*log(c))^2 + 9*sin(b*log(c))^2)
```

Fricas [A]
time = 1.62, size = 49, normalized size = 0.86

$$-\frac{b n x^{3} \cos\left(b n \log\left(x\right) + b \log\left(c\right) + a\right) - 3 \, x^{3} \sin\left(b n \log\left(x\right) + b \log\left(c\right) + a\right)}{b^{2} n^{2} + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -(b*n*x^3*cos(b*n*log(x) + b*log(c) + a) - 3*x^3*sin(b*n*log(x) + b*log(c)
+ a))/(b^2*n^2 + 9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int x^2 \sin\left(a - \frac{3i\log(cx^n)}{n}\right)dx & \text{for } b = -\frac{3i}{n} \\ \int x^2 \sin\left(a + \frac{3i\log(cx^n)}{n}\right)dx & \text{for } b = \frac{3i}{n} \\ -\frac{bnx^3\cos(a+b\log(cx^n))}{b^2n^2+9} + \frac{3x^3\sin(a+b\log(cx^n))}{b^2n^2+9} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((Integral(x**2*sin(a - 3*I*log(c*x**n)/n), x), Eq(b, -3*I/n)), (I
ntegral(x**2*sin(a + 3*I*log(c*x**n)/n), x), Eq(b, 3*I/n)), (-b*n*x**3*cos(
a + b*log(c*x**n))/(b**2*n**2 + 9) + 3*x**3*sin(a + b*log(c*x**n))/(b**2*n*
*2 + 9), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 923 vs. 2(57) = 114.
time = 0.48, size = 923, normalized size = 16.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] -1/2*(b*n*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*
b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 + b*n*x^3*e^
(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*
log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 - b*n*x^3*e^(1/2*pi*b*n*sgn
(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/
2*b*log(abs(c)))^2 - b*n*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*
sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2 - 4*b*n*x
^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*
b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a) - 4*b*n*x^3*e^(-1/2*pi*b*n*
sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) +
 1/2*b*log(abs(c)))*tan(1/2*a) - b*n*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n
+ 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*a)^2 - b*n*x^3*e^(-1/2*pi*b*n*sgn(x)
+ 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*a)^2 + 6*x^3*e^(1/2*pi*b
*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)
) + 1/2*b*log(abs(c)))^2*tan(1/2*a) + 6*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*
b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c
)))^2*tan(1/2*a) + 6*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c
) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a)^2 + 6
*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1
```

```
/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a)^2 + b*n*x^3*e^(1/2*pi*b*
n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b) + b*n*x^3*e^(-1/2*pi*b*
n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b) - 6*x^3*e^(1/2*pi*b*n*s
gn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) +
1/2*b*log(abs(c))) - 6*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sg
n(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c))) - 6*x^3*e^(1/
2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*a) - 6*x
^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2
*a))/(b^2*n^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 +
 b^2*n^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2 + b^2*n^2*tan(1/2*a
)^2 + b^2*n^2 + 9*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)
^2 + 9*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2 + 9*tan(1/2*a)^2 + 9)
```

Mupad [B]
time = 2.49, size = 44, normalized size = 0.77

$$\frac{x^3\,\left(3\,\sin\left(a+b\,\ln\left(c\,x^n\right)\right)-b\,n\,\cos\left(a+b\,\ln\left(c\,x^n\right)\right)\right)}{b^2\,n^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a + b*log(c*x^n)),x)
[Out] (x^3*(3*sin(a + b*log(c*x^n)) - b*n*cos(a + b*log(c*x^n))))/(b^2*n^2 + 9)
```

3.2 $\int x \sin(a + b \log(cx^n))\, dx$

Optimal. Leaf size=57

$$-\frac{bnx^2 \cos(a + b \log(cx^n))}{4 + b^2n^2} + \frac{2x^2 \sin(a + b \log(cx^n))}{4 + b^2n^2}$$

```
[Out] -b*n*x^2*cos(a+b*ln(c*x^n))/(b^2*n^2+4)+2*x^2*sin(a+b*ln(c*x^n))/(b^2*n^2+4
)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{4573\}$

$$\frac{2x^2 \sin(a + b \log(cx^n))}{b^2n^2 + 4} - \frac{bnx^2 \cos(a + b \log(cx^n))}{b^2n^2 + 4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[a + b*Log[c*x^n]],x]
```

```
[Out] -((b*n*x^2*Cos[a + b*Log[c*x^n]])/(4 + b^2*n^2)) + (2*x^2*Sin[a + b*Log[c*x
^n]])/(4 + b^2*n^2)
```

Rule 4573

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] - Simp[b*d*n*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int x \sin(a + b \log(cx^n))\, dx = -\frac{bnx^2 \cos(a + b \log(cx^n))}{4 + b^2n^2} + \frac{2x^2 \sin(a + b \log(cx^n))}{4 + b^2n^2}$$

Mathematica [A]
time = 0.07, size = 44, normalized size = 0.77

$$-\frac{x^2(bn \cos(a + b \log(cx^n)) - 2 \sin(a + b \log(cx^n)))}{4 + b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[a + b*Log[c*x^n]],x]
```

```
[Out] -((x^2*(b*n*Cos[a + b*Log[c*x^n]] - 2*Sin[a + b*Log[c*x^n]]))/(4 + b^2*n^2)
)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x \sin \left(a + b \ln \left(c\, x^{n}\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a+b*ln(c*x^n)),x)
```

```
[Out] int(x*sin(a+b*ln(c*x^n)),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(57) = 114$.
time = 0.30, size = 219, normalized size = 3.84

$$-\frac{\left(\left(b \cos\left(2\, b \log\left(c\right)\right) \cos\left(b \log\left(c\right)\right) + b \sin\left(2\, b \log\left(c\right)\right) \sin\left(b \log\left(c\right)\right) + b \cos\left(b \log\left(c\right)\right)\right)n - 2\, \cos\left(b \log\left(c\right)\right) \sin\left(2\, b \log\left(c\right)\right) + 2\, \cos\left(2\, b \log\left(c\right)\right) \sin\left(b \log\left(c\right)\right) - 2\, \sin\left(b \log\left(c\right)\right)\right)x^{2} \cos\left(b \log\left(x^{n}\right) + a\right) - \left(\left(b \cos\left(b \log\left(c\right)\right) \sin\left(2\, b \log\left(c\right)\right) - b \cos\left(2\, b \log\left(c\right)\right) \sin\left(b \log\left(c\right)\right) + b \sin\left(b \log\left(c\right)\right)\right)n + 2\, \cos\left(2\, b \log\left(c\right)\right) \cos\left(b \log\left(c\right)\right) + 2\, \sin\left(2\, b \log\left(c\right)\right) \sin\left(b \log\left(c\right)\right) + 2\, \cos\left(b \log\left(c\right)\right)\right)x^{2} \sin\left(b \log\left(x^{n}\right) + a\right)}{2\, \left(\left(b^{2} \cos\left(b \log\left(c\right)\right)^{2} + b^{2} \sin\left(b \log\left(c\right)\right)^{2}\right)n^{2} + 4\, \cos\left(b \log\left(c\right)\right)^{2} + 4\, \sin\left(b \log\left(c\right)\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/2*(((b*cos(2*b*log(c))*cos(b*log(c)) + b*sin(2*b*log(c))*sin(b*log(c)) +
 b*cos(b*log(c)))*n - 2*cos(b*log(c))*sin(2*b*log(c)) + 2*cos(2*b*log(c))*s
in(b*log(c)) - 2*sin(b*log(c)))*x^2*cos(b*log(x^n) + a) - ((b*cos(b*log(c))
*sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)) + b*sin(b*log(c)))*n + 2
*cos(2*b*log(c))*cos(b*log(c)) + 2*sin(2*b*log(c))*sin(b*log(c)) + 2*cos(b*
log(c)))*x^2*sin(b*log(x^n) + a))/((b^2*cos(b*log(c))^2 + b^2*sin(b*log(c))
^2)*n^2 + 4*cos(b*log(c))^2 + 4*sin(b*log(c))^2)
```

Fricas [A]
time = 1.33, size = 49, normalized size = 0.86

$$-\frac{bnx^{2} \cos\left(bn \log\left(x\right) + b \log\left(c\right) + a\right) - 2\, x^{2} \sin\left(bn \log\left(x\right) + b \log\left(c\right) + a\right)}{b^{2} n^{2} + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -(b*n*x^2*cos(b*n*log(x) + b*log(c) + a) - 2*x^2*sin(b*n*log(x) + b*log(c)
+ a))/(b^2*n^2 + 4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int x \sin\left(a - \frac{2i\log(cx^n)}{n}\right) dx & \text{for } b = -\frac{2i}{n} \\ \int x \sin\left(a + \frac{2i\log(cx^n)}{n}\right) dx & \text{for } b = \frac{2i}{n} \\ -\frac{bnx^2\cos(a+b\log(cx^n))}{b^2n^2+4} + \frac{2x^2\sin(a+b\log(cx^n))}{b^2n^2+4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((Integral(x*sin(a - 2*I*log(c*x**n)/n), x), Eq(b, -2*I/n)), (Inte
gral(x*sin(a + 2*I*log(c*x**n)/n), x), Eq(b, 2*I/n)), (-b*n*x**2*cos(a + b*
log(c*x**n))/(b**2*n**2 + 4) + 2*x**2*sin(a + b*log(c*x**n))/(b**2*n**2 + 4
), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 923 vs. 2(57) = 114.
time = 0.45, size = 923, normalized size = 16.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] -1/2*(b*n*x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*
b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 + b*n*x^2*e^
(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*
log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 - b*n*x^2*e^(1/2*pi*b*n*sgn
(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/
2*b*log(abs(c)))^2 - b*n*x^2*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*
sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2 - 4*b*n*x
^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*
b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a) - 4*b*n*x^2*e^(-1/2*pi*b*n*
sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) +
 1/2*b*log(abs(c)))*tan(1/2*a) - b*n*x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n
+ 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*a)^2 - b*n*x^2*e^(-1/2*pi*b*n*sgn(x)
+ 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*a)^2 + 4*x^2*e^(1/2*pi*b
*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)
) + 1/2*b*log(abs(c)))^2*tan(1/2*a) + 4*x^2*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*
b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c
)))^2*tan(1/2*a) + 4*x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c
) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a)^2 + 4
*x^2*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1
```

```
/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a)^2 + b*n*x^2*e^(1/2*pi*b*
n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b) + b*n*x^2*e^(-1/2*pi*b*
n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b) - 4*x^2*e^(1/2*pi*b*n*s
gn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) +
1/2*b*log(abs(c))) - 4*x^2*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sg
n(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c))) - 4*x^2*e^(1/
2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*a) - 4*x
^2*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2
*a))/(b^2*n^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 +
 b^2*n^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2 + b^2*n^2*tan(1/2*a
)^2 + b^2*n^2 + 4*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)
^2 + 4*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2 + 4*tan(1/2*a)^2 + 4)
```

Mupad [B]
time = 2.39, size = 44, normalized size = 0.77

$$\frac{x^2\,\left(2\,\sin\left(a+b\,\ln\left(c\,x^n\right)\right)-b\,n\,\cos\left(a+b\,\ln\left(c\,x^n\right)\right)\right)}{b^2\,n^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b*log(c*x^n)),x)
[Out] (x^2*(2*sin(a + b*log(c*x^n)) - b*n*cos(a + b*log(c*x^n))))/(b^2*n^2 + 4)
```

3.3 $\int \sin(a+b\log(cx^n))\,dx$

Optimal. Leaf size=52

$$-\frac{bnx\cos(a+b\log(cx^n))}{1+b^2n^2}+\frac{x\sin(a+b\log(cx^n))}{1+b^2n^2}$$

```
[Out] -b*n*x*cos(a+b*ln(c*x^n))/(b^2*n^2+1)+x*sin(a+b*ln(c*x^n))/(b^2*n^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{4563\}$

$$\frac{x\sin(a+b\log(cx^n))}{b^2n^2+1}-\frac{bnx\cos(a+b\log(cx^n))}{b^2n^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Log[c*x^n]],x]
```

```
[Out] -((b*n*x*Cos[a + b*Log[c*x^n]])/(1 + b^2*n^2)) + (x*Sin[a + b*Log[c*x^n]])/
(1 + b^2*n^2)
```

Rule 4563

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*(
Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] - Simp[b*d*n*x*(Cos[d*(a +
 b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

Rubi steps

$$\int \sin(a+b\log(cx^n))\,dx=-\frac{bnx\cos(a+b\log(cx^n))}{1+b^2n^2}+\frac{x\sin(a+b\log(cx^n))}{1+b^2n^2}$$

Mathematica [A]
time = 0.06, size = 40, normalized size = 0.77

$$\frac{x(-bn\cos(a+b\log(cx^n))+\sin(a+b\log(cx^n)))}{1+b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]],x]
```

```
[Out] (x*(-(b*n*Cos[a + b*Log[c*x^n]]) + Sin[a + b*Log[c*x^n]]))/(1 + b^2*n^2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \sin \left(a+b\ln \left(c\,x^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n)),x)
```

```
[Out] int(sin(a+b*ln(c*x^n)),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(52) = 104$.
time = 0.29, size = 206, normalized size = 3.96

$$-\frac{\left(\left(b\cos\left(2\,b\log\left(c\right)\right)\cos\left(b\log\left(c\right)\right)+b\sin\left(2\,b\log\left(c\right)\right)\sin\left(b\log\left(c\right)\right)+b\cos\left(b\log\left(c\right)\right)\right)n-\cos\left(b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)+\cos\left(2\,b\log\left(c\right)\right)\sin\left(b\log\left(c\right)\right)-\sin\left(b\log\left(c\right)\right)\right)x\cos\left(b\log\left(x^{n}\right)+a\right)-\left(\left(b\cos\left(b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)-b\cos\left(2\,b\log\left(c\right)\right)\sin\left(b\log\left(c\right)\right)+b\sin\left(b\log\left(c\right)\right)\right)n+\cos\left(2\,b\log\left(c\right)\right)\cos\left(b\log\left(c\right)\right)+\sin\left(2\,b\log\left(c\right)\right)\sin\left(b\log\left(c\right)\right)+\cos\left(b\log\left(c\right)\right)\right)x\sin\left(b\log\left(x^{n}\right)+a\right)}{2\left(\left(b^{2}\cos\left(b\log\left(c\right)\right)^{2}+b^{2}\sin\left(b\log\left(c\right)\right)^{2}\right)n^{2}+\cos\left(b\log\left(c\right)\right)^{2}+\sin\left(b\log\left(c\right)\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/2*(((b*cos(2*b*log(c))*cos(b*log(c)) + b*sin(2*b*log(c))*sin(b*log(c)) +
 b*cos(b*log(c)))*n - cos(b*log(c))*sin(2*b*log(c)) + cos(2*b*log(c))*sin(b
*log(c)) - sin(b*log(c)))*x*cos(b*log(x^n) + a) - ((b*cos(b*log(c))*sin(2*b
*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)) + b*sin(b*log(c)))*n + cos(2*b*l
og(c))*cos(b*log(c)) + sin(2*b*log(c))*sin(b*log(c)) + cos(b*log(c)))*x*sin
(b*log(x^n) + a))/((b^2*cos(b*log(c))^2 + b^2*sin(b*log(c))^2)*n^2 + cos(b*
log(c))^2 + sin(b*log(c))^2)
```

Fricas [A]
time = 1.26, size = 45, normalized size = 0.87

$$-\frac{bnx\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-x\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{b^{2}n^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -(b*n*x*cos(b*n*log(x) + b*log(c) + a) - x*sin(b*n*log(x) + b*log(c) + a))/
(b^2*n^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \sin\left(a-\frac{i\log\left(cx^{n}\right)}{n}\right)dx & \text{for}\: b=-\frac{i}{n} \\ \int \sin\left(a+\frac{i\log\left(cx^{n}\right)}{n}\right)dx & \text{for}\: b=\frac{i}{n} \\ -\frac{bnx\cos\left(a+b\log\left(cx^{n}\right)\right)}{b^{2}n^{2}+1}+\frac{x\sin\left(a+b\log\left(cx^{n}\right)\right)}{b^{2}n^{2}+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((Integral(sin(a - I*log(c*x**n)/n), x), Eq(b, -I/n)), (Integral(s
in(a + I*log(c*x**n)/n), x), Eq(b, I/n)), (-b*n*x*cos(a + b*log(c*x**n))/(b
**2*n**2 + 1) + x*sin(a + b*log(c*x**n))/(b**2*n**2 + 1), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(52) = 104$.
time = 0.40, size = 882, normalized size = 16.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] -1/2*(b*n*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)
*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 + b*n*x*e^(-1/
2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(
abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 - b*n*x*e^(1/2*pi*b*n*sgn(x) -
1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*lo
g(abs(c)))^2 - b*n*x*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) +
 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2 - 4*b*n*x*e^(1/2*
pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(ab
s(x)) + 1/2*b*log(abs(c)))*tan(1/2*a) - 4*b*n*x*e^(-1/2*pi*b*n*sgn(x) + 1/2
*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(a
bs(c)))*tan(1/2*a) - b*n*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn
(c) - 1/2*pi*b)*tan(1/2*a)^2 - b*n*x*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1
/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*a)^2 + 2*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi
*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(
c)))^2*tan(1/2*a) + 2*x*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c
) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a) + 2
*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*
b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a)^2 + 2*x*e^(-1/2*pi*b*n*sgn(
x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2
*b*log(abs(c)))*tan(1/2*a)^2 + b*n*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/
2*pi*b*sgn(c) - 1/2*pi*b) + b*n*x*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*
pi*b*sgn(c) + 1/2*pi*b) - 2*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*
sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c))) - 2*x*e^(-1
/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log
(abs(x)) + 1/2*b*log(abs(c))) - 2*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2
*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*a) - 2*x*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*
n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*a))/(b^2*n^2*tan(1/2*b*n*log(abs(x)
) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 + b^2*n^2*tan(1/2*b*n*log(abs(x)) + 1
/2*b*log(abs(c)))^2 + b^2*n^2*tan(1/2*a)^2 + b^2*n^2 + tan(1/2*b*n*log(abs(
```

```
x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 + tan(1/2*b*n*log(abs(x)) + 1/2*b*l
og(abs(c)))^2 + tan(1/2*a)^2 + 1)
```

Mupad [B]
time = 2.33, size = 40, normalized size = 0.77

$$\frac{x\left(\sin\left(a+b\,\ln\left(c\,x^n\right)\right)-b\,n\,\cos\left(a+b\,\ln\left(c\,x^n\right)\right)\right)}{b^2\,n^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n)),x)
[Out] (x*(sin(a + b*log(c*x^n)) - b*n*cos(a + b*log(c*x^n))))/(b^2*n^2 + 1)
```

3.4 $\int \frac{\sin(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=19

$$-\frac{\cos\left(a+b\log\left(cx^n\right)\right)}{bn}$$

```
[Out] -cos(a+b*ln(c*x^n))/b/n
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{2718\}$

$$-\frac{\cos\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Log[c*x^n]]/x,x]
```

```
[Out] -(Cos[a + b*Log[c*x^n]]/(b*n))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \sin(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= -\frac{\cos\left(a+b\log\left(cx^n\right)\right)}{bn}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 2.00

$$-\frac{\cos(a)\cos\left(b\log\left(cx^n\right)\right)}{bn}+\frac{\sin(a)\sin\left(b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]/x,x]
```

```
[Out] -((Cos[a]*Cos[b*Log[c*x^n]])/(b*n)) + (Sin[a]*Sin[b*Log[c*x^n]])/(b*n)
```

Maple [A]
time = 0.03, size = 20, normalized size = 1.05

method	result	size
derivativedivides	$-\frac{\cos(a+b\ln(c\,x^{n}))}{bn}$	20
default	$-\frac{\cos(a+b\ln(c\,x^{n}))}{bn}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
```

```
[Out] -cos(a+b*ln(c*x^n))/b/n
```

Maxima [A]
time = 0.28, size = 19, normalized size = 1.00

$$-\frac{\cos\left(b\log\left(cx^{n}\right)+a\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

```
[Out] -cos(b*log(c*x^n) + a)/(b*n)
```

Fricas [A]
time = 1.66, size = 20, normalized size = 1.05

$$-\frac{\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

```
[Out] -cos(b*n*log(x) + b*log(c) + a)/(b*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(15) = 30$.
time = 0.23, size = 36, normalized size = 1.89

$$\begin{cases} \log\left(x\right)\sin\left(a\right) & \text{for}\ b=0 \wedge \left(b=0 \vee n=0\right) \\ \log\left(x\right)\sin\left(a+b\log\left(c\right)\right) & \text{for}\ n=0 \\ -\frac{\cos\left(a+b\log\left(cx^{n}\right)\right)}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))/x,x)
```

```
[Out] Piecewise((log(x)*sin(a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*sin(a
+ b*log(c)), Eq(n, 0)), (-cos(a + b*log(c*x**n))/(b*n), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))/x,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)/x, x)
```

Mupad [B]
time = 2.26, size = 19, normalized size = 1.00

$$-\frac{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))/x,x)
```

```
[Out] -cos(a + b*log(c*x^n))/(b*n)
```

3.5 $\int \frac{\sin(a+b\log(cx^n))}{x^2}\,dx$

Optimal. Leaf size=57

$$-\frac{bn\cos\left(a+b\log\left(cx^n\right)\right)}{\left(1+b^2n^2\right)x}-\frac{\sin\left(a+b\log\left(cx^n\right)\right)}{\left(1+b^2n^2\right)x}$$

```
[Out] -b*n*cos(a+b*ln(c*x^n))/(b^2*n^2+1)/x-sin(a+b*ln(c*x^n))/(b^2*n^2+1)/x
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{4573\}$

$$-\frac{\sin\left(a+b\log\left(cx^n\right)\right)}{x\left(b^2n^2+1\right)}-\frac{bn\cos\left(a+b\log\left(cx^n\right)\right)}{x\left(b^2n^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Log[c*x^n]]/x^2,x]
```

```
[Out] -((b*n*Cos[a + b*Log[c*x^n]])/((1 + b^2*n^2)*x)) - Sin[a + b*Log[c*x^n]]/((
1 + b^2*n^2)*x)
```

Rule 4573

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] - Simp[b*d*n*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int \frac{\sin\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx=-\frac{bn\cos\left(a+b\log\left(cx^n\right)\right)}{\left(1+b^2n^2\right)x}-\frac{\sin\left(a+b\log\left(cx^n\right)\right)}{\left(1+b^2n^2\right)x}$$

Mathematica [A]
time = 0.07, size = 40, normalized size = 0.70

$$-\frac{bn\cos\left(a+b\log\left(cx^n\right)\right)+\sin\left(a+b\log\left(cx^n\right)\right)}{x+b^2n^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]/x^2,x]
```

```
[Out] -((b*n*Cos[a + b*Log[c*x^n]] + Sin[a + b*Log[c*x^n]])/(x + b^2*n^2*x))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sin \left(a+b \ln \left(c\,x^{n}\right)\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))/x^2,x)
```

```
[Out] int(sin(a+b*ln(c*x^n))/x^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(57) = 114$.
time = 0.30, size = 209, normalized size = 3.67

$$-\frac{\left(\left(b \cos \left(2\,b \log \left(c\right)\right) \cos \left(b \log \left(c\right)\right)+b \sin \left(2\,b \log \left(c\right)\right) \sin \left(b \log \left(c\right)\right)+b \cos \left(b \log \left(c\right)\right)\right) n+\cos \left(b \log \left(c\right)\right) \sin \left(2\,b \log \left(c\right)\right)-\cos \left(2\,b \log \left(c\right)\right) \sin \left(b \log \left(c\right)\right)+\sin \left(b \log \left(c\right)\right)\right) \cos \left(b \log \left(x^{n}\right)+a\right)-\left(\left(b \cos \left(b \log \left(c\right)\right) \sin \left(2\,b \log \left(c\right)\right)-b \cos \left(2\,b \log \left(c\right)\right) \sin \left(b \log \left(c\right)\right)+b \sin \left(b \log \left(c\right)\right)\right) n-\cos \left(2\,b \log \left(c\right)\right) \cos \left(b \log \left(c\right)\right)-\sin \left(2\,b \log \left(c\right)\right) \sin \left(b \log \left(c\right)\right)-\cos \left(b \log \left(c\right)\right)\right) \sin \left(b \log \left(x^{n}\right)+a\right)}{2\,\left(\left(b^{2} \cos \left(b \log \left(c\right)\right)^{2}+b^{2} \sin \left(b \log \left(c\right)\right)^{2}\right) n^{2}+\cos \left(b \log \left(c\right)\right)^{2}+\sin \left(b \log \left(c\right)\right)^{2}\right) x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(((b*cos(2*b*log(c))*cos(b*log(c)) + b*sin(2*b*log(c))*sin(b*log(c)) +
 b*cos(b*log(c)))*n + cos(b*log(c))*sin(2*b*log(c)) - cos(2*b*log(c))*sin(b
*log(c)) + sin(b*log(c)))*cos(b*log(x^n) + a) - ((b*cos(b*log(c))*sin(2*b*l
og(c)) - b*cos(2*b*log(c))*sin(b*log(c)) + b*sin(b*log(c)))*n - cos(2*b*log
(c))*cos(b*log(c)) - sin(2*b*log(c))*sin(b*log(c)) - cos(b*log(c)))*sin(b*l
og(x^n) + a))/(((b^2*cos(b*log(c))^2 + b^2*sin(b*log(c))^2)*n^2 + cos(b*log
(c))^2 + sin(b*log(c))^2)*x)
```

Fricas [A]
time = 2.45, size = 44, normalized size = 0.77

$$-\frac{bn \cos \left(bn \log \left(x\right)+b \log \left(c\right)+a\right)+\sin \left(bn \log \left(x\right)+b \log \left(c\right)+a\right)}{(b^{2} n^{2}+1) x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))/x^2,x, algorithm="fricas")
```

```
[Out] -(b*n*cos(b*n*log(x) + b*log(c) + a) + sin(b*n*log(x) + b*log(c) + a))/((b^
2*n^2 + 1)*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.29, size = 192, normalized size = 3.37

$$\begin{cases} -\frac{\sin\left(a-\frac{i\log\left(cx^{n}\right)}{n}\right)}{2x}+\frac{\log\left(cx^{n}\right)\sin\left(a-\frac{i\log\left(cx^{n}\right)}{n}\right)}{2nx}-\frac{i\log\left(cx^{n}\right)\cos\left(a-\frac{i\log\left(cx^{n}\right)}{n}\right)}{2nx} & \text{for } b=-\frac{i}{n} \\ \frac{i\cos\left(a+\frac{i\log\left(cx^{n}\right)}{n}\right)}{2x}+\frac{\log\left(cx^{n}\right)\sin\left(a+\frac{i\log\left(cx^{n}\right)}{n}\right)}{2nx}+\frac{i\log\left(cx^{n}\right)\cos\left(a+\frac{i\log\left(cx^{n}\right)}{n}\right)}{2nx} & \text{for } b=\frac{i}{n} \\ -\frac{bn\cos\left(a+b\log\left(cx^{n}\right)\right)}{b^{2}n^{2}x+x}-\frac{\sin\left(a+b\log\left(cx^{n}\right)\right)}{b^{2}n^{2}x+x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))/x**2,x)
```

```
[Out] Piecewise((-sin(a - I*log(c*x**n)/n)/(2*x) + log(c*x**n)*sin(a - I*log(c*x*
*n)/n)/(2*n*x) - I*log(c*x**n)*cos(a - I*log(c*x**n)/n)/(2*n*x), Eq(b, -I/n
)), (I*cos(a + I*log(c*x**n)/n)/(2*x) + log(c*x**n)*sin(a + I*log(c*x**n)/n
)/(2*n*x) + I*log(c*x**n)*cos(a + I*log(c*x**n)/n)/(2*n*x), Eq(b, I/n)), (-
b*n*cos(a + b*log(c*x**n))/(b**2*n**2*x + x) - sin(a + b*log(c*x**n))/(b**2
*n**2*x + x), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))/x^2,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))/x^2,x)
```

```
[Out] int(sin(a + b*log(c*x^n))/x^2, x)
```

3.6 $\int \frac{\sin(a+b\log(cx^n))}{x^3}\,dx$

Optimal. Leaf size=57

$$-\frac{bn\cos\left(a+b\log\left(cx^n\right)\right)}{\left(4+b^2n^2\right)x^2}-\frac{2\sin\left(a+b\log\left(cx^n\right)\right)}{\left(4+b^2n^2\right)x^2}$$

```
[Out] -b*n*cos(a+b*ln(c*x^n))/(b^2*n^2+4)/x^2-2*sin(a+b*ln(c*x^n))/(b^2*n^2+4)/x^
2
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = $\{4573\}$

$$-\frac{2\sin\left(a+b\log\left(cx^n\right)\right)}{x^2\left(b^2n^2+4\right)}-\frac{bn\cos\left(a+b\log\left(cx^n\right)\right)}{x^2\left(b^2n^2+4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Log[c*x^n]]/x^3,x]
```

```
[Out] -((b*n*Cos[a + b*Log[c*x^n]])/((4 + b^2*n^2)*x^2)) - (2*Sin[a + b*Log[c*x^n
]])/((4 + b^2*n^2)*x^2)
```

Rule 4573

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] - Simp[b*d*n*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int \frac{\sin\left(a+b\log\left(cx^n\right)\right)}{x^3}\,dx=-\frac{bn\cos\left(a+b\log\left(cx^n\right)\right)}{\left(4+b^2n^2\right)x^2}-\frac{2\sin\left(a+b\log\left(cx^n\right)\right)}{\left(4+b^2n^2\right)x^2}$$

Mathematica [A]
time = 0.08, size = 44, normalized size = 0.77

$$-\frac{bn\cos\left(a+b\log\left(cx^n\right)\right)+2\sin\left(a+b\log\left(cx^n\right)\right)}{\left(4+b^2n^2\right)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]/x^3,x]
```

```
[Out] -((b*n*Cos[a + b*Log[c*x^n]] + 2*Sin[a + b*Log[c*x^n]])/((4 + b^2*n^2)*x^2)
)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+b\ln\left(c\,x^{n}\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))/x^3,x)
```

```
[Out] int(sin(a+b*ln(c*x^n))/x^3,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(57) = 114$.
time = 0.29, size = 216, normalized size = 3.79

$$-\frac{\left(\left(b\cos\left(2\,b\log\left(c\right)\right)\cos\left(b\log\left(c\right)\right)+b\sin\left(2\,b\log\left(c\right)\right)\sin\left(b\log\left(c\right)\right)+b\cos\left(b\log\left(c\right)\right)\right)n+2\,\cos\left(b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)-2\,\cos\left(2\,b\log\left(c\right)\right)\sin\left(b\log\left(c\right)\right)+2\,\sin\left(b\log\left(c\right)\right)\right)\cos\left(b\log\left(x^{n}\right)+a\right)-\left(\left(b\cos\left(b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)-b\cos\left(2\,b\log\left(c\right)\right)\sin\left(b\log\left(c\right)\right)+b\sin\left(b\log\left(c\right)\right)\right)n-2\,\cos\left(2\,b\log\left(c\right)\right)\cos\left(b\log\left(c\right)\right)-2\,\sin\left(2\,b\log\left(c\right)\right)\sin\left(b\log\left(c\right)\right)-2\,\cos\left(b\log\left(c\right)\right)\right)\sin\left(b\log\left(x^{n}\right)+a\right)}{2\,\left(\left(b^{2}\cos\left(b\log\left(c\right)\right)^{2}+b^{2}\sin\left(b\log\left(c\right)\right)^{2}\right)n^{2}+4\,\cos\left(b\log\left(c\right)\right)^{2}+4\,\sin\left(b\log\left(c\right)\right)^{2}\right)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(((b*cos(2*b*log(c))*cos(b*log(c)) + b*sin(2*b*log(c))*sin(b*log(c)) +
 b*cos(b*log(c)))*n + 2*cos(b*log(c))*sin(2*b*log(c)) - 2*cos(2*b*log(c))*s
in(b*log(c)) + 2*sin(b*log(c)))*cos(b*log(x^n) + a) - ((b*cos(b*log(c))*sin
(2*b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)) + b*sin(b*log(c)))*n - 2*cos
(2*b*log(c))*cos(b*log(c)) - 2*sin(2*b*log(c))*sin(b*log(c)) - 2*cos(b*log(
c)))*sin(b*log(x^n) + a))/(((b^2*cos(b*log(c))^2 + b^2*sin(b*log(c))^2)*n^2
 + 4*cos(b*log(c))^2 + 4*sin(b*log(c))^2)*x^2)
```

Fricas [A]
time = 3.34, size = 46, normalized size = 0.81

$$-\frac{bn\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+2\,\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{(b^{2}n^{2}+4)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))/x^3,x, algorithm="fricas")
```

```
[Out] -(b*n*cos(b*n*log(x) + b*log(c) + a) + 2*sin(b*n*log(x) + b*log(c) + a))/((
b^2*n^2 + 4)*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.27, size = 228, normalized size = 4.00

$$\begin{cases} -\frac{\sin\left(a-\frac{2i\log\left(cx^n\right)}{n}\right)}{4x^2}+\frac{\log\left(cx^n\right)\sin\left(a-\frac{2i\log\left(cx^n\right)}{n}\right)}{2nx^2}-\frac{i\log\left(cx^n\right)\cos\left(a-\frac{2i\log\left(cx^n\right)}{n}\right)}{2nx^2} & \text{for } b=-\frac{2i}{n} \\ \frac{i\cos\left(a+\frac{2i\log\left(cx^n\right)}{n}\right)}{4x^2}+\frac{\log\left(cx^n\right)\sin\left(a+\frac{2i\log\left(cx^n\right)}{n}\right)}{2nx^2}+\frac{i\log\left(cx^n\right)\cos\left(a+\frac{2i\log\left(cx^n\right)}{n}\right)}{2nx^2} & \text{for } b=\frac{2i}{n} \\ -\frac{bn\cos\left(a+b\log\left(cx^n\right)\right)}{b^2n^2x^2+4x^2}-\frac{2\sin\left(a+b\log\left(cx^n\right)\right)}{b^2n^2x^2+4x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))/x**3,x)
```

```
[Out] Piecewise((-sin(a - 2*I*log(c*x**n)/n)/(4*x**2) + log(c*x**n)*sin(a - 2*I*l
og(c*x**n)/n)/(2*n*x**2) - I*log(c*x**n)*cos(a - 2*I*log(c*x**n)/n)/(2*n*x*
*2), Eq(b, -2*I/n)), (I*cos(a + 2*I*log(c*x**n)/n)/(4*x**2) + log(c*x**n)*s
in(a + 2*I*log(c*x**n)/n)/(2*n*x**2) + I*log(c*x**n)*cos(a + 2*I*log(c*x**n
)/n)/(2*n*x**2), Eq(b, 2*I/n)), (-b*n*cos(a + b*log(c*x**n))/(b**2*n**2*x**
2 + 4*x**2) - 2*sin(a + b*log(c*x**n))/(b**2*n**2*x**2 + 4*x**2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))/x^3,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))/x^3,x)
```

```
[Out] int(sin(a + b*log(c*x^n))/x^3, x)
```

3.7 $\int x^2 \sin^2(a+b\log(cx^n))\,dx$

Optimal. Leaf size=97

$$\frac{2b^2n^2x^3}{3(9+4b^2n^2)}-\frac{2bnx^3\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{9+4b^2n^2}+\frac{3x^3\sin^2(a+b\log(cx^n))}{9+4b^2n^2}$$

```
[Out] 2/3*b^2*n^2*x^3/(4*b^2*n^2+9)-2*b*n*x^3*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n
))/(4*b^2*n^2+9)+3*x^3*sin(a+b*ln(c*x^n))^2/(4*b^2*n^2+9)
```

Rubi [A]
time = 0.02, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4575, 30}

$$\frac{3x^3\sin^2(a+b\log(cx^n))}{4b^2n^2+9}-\frac{2bnx^3\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+9}+\frac{2b^2n^2x^3}{3(4b^2n^2+9)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (2*b^2*n^2*x^3)/(3*(9 + 4*b^2*n^2)) - (2*b*n*x^3*Cos[a + b*Log[c*x^n]]*Sin[
a + b*Log[c*x^n]])/(9 + 4*b^2*n^2) + (3*x^3*Sin[a + b*Log[c*x^n]]^2)/(9 + 4
*b^2*n^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4575

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2
*n^2*p^2 + (m + 1)^2)), Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x],
x] - Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a + b*Log[c*x^n])]*(Sin[d*(a + b*Log
[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x]) /; FreeQ[{a, b, c
, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\sin^2(a+b\log(cx^n))\,dx &= -\frac{2bnx^3\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{9+4b^2n^2}+\frac{3x^3\sin^2(a+b\log(cx^n))}{9+4b^2n^2}\\ &= \frac{2b^2n^2x^3}{3(9+4b^2n^2)}-\frac{2bnx^3\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{9+4b^2n^2}+\frac{3x^3\sin^2(a}{9+}\end{aligned}$$

Mathematica [**A**]
time = 0.17, size = 61, normalized size = 0.63

$$\frac{x^3(9+4b^2n^2-9\cos(2(a+b\log(cx^n)))-6bn\sin(2(a+b\log(cx^n))))}{6(9+4b^2n^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x^3*(9 + 4*b^2*n^2 - 9*Cos[2*(a + b*Log[c*x^n])] - 6*b*n*Sin[2*(a + b*Log[
c*x^n])]))/(6*(9 + 4*b^2*n^2))
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int x^2\left(\sin^2\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(x^2*sin(a+b*ln(c*x^n))^2,x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(95) = 190$.
time = 0.30, size = 301, normalized size = 3.10

$$-\frac{3\left(2\left(b\cos\left(2\,b\log\left(c\right)\right)\sin\left(4\,b\log\left(c\right)\right)-b\cos\left(4\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)+b\sin\left(2\,b\log\left(c\right)\right)\right)n+3\,\cos\left(4\,b\log\left(c\right)\right)\cos\left(2\,b\log\left(c\right)\right)+3\,\sin\left(4\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)+3\,\cos\left(2\,b\log\left(c\right)\right)\right)x^3\cos\left(2\,b\log\left(x^n\right)+2\,a\right)+3\left(2\left(b\cos\left(4\,b\log\left(c\right)\right)\cos\left(2\,b\log\left(c\right)\right)+b\sin\left(4\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)+b\cos\left(2\,b\log\left(c\right)\right)\right)n-3\,\cos\left(2\,b\log\left(c\right)\right)\sin\left(4\,b\log\left(c\right)\right)+3\,\cos\left(4\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)-3\,\sin\left(2\,b\log\left(c\right)\right)\right)x^3\sin\left(2\,b\log\left(x^n\right)+2\,a\right)-2\left(4\left(b^2\cos\left(2\,b\log\left(c\right)\right)^2+b^2\sin\left(2\,b\log\left(c\right)\right)^2\right)n^2+9\,\cos\left(2\,b\log\left(c\right)\right)^2+9\,\sin\left(2\,b\log\left(c\right)\right)^2\right)x^3}{12\left(4\left(b^2\cos\left(2\,b\log\left(c\right)\right)^2+b^2\sin\left(2\,b\log\left(c\right)\right)^2\right)n^2+9\,\cos\left(2\,b\log\left(c\right)\right)^2+9\,\sin\left(2\,b\log\left(c\right)\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] -1/12*(3*(2*(b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*
log(c)) + b*sin(2*b*log(c)))*n + 3*cos(4*b*log(c))*cos(2*b*log(c)) + 3*sin(
4*b*log(c))*sin(2*b*log(c)) + 3*cos(2*b*log(c)))*x^3*cos(2*b*log(x^n) + 2*a
) + 3*(2*(b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log
(c)) + b*cos(2*b*log(c)))*n - 3*cos(2*b*log(c))*sin(4*b*log(c)) + 3*cos(4*b
*log(c))*sin(2*b*log(c)) - 3*sin(2*b*log(c)))*x^3*sin(2*b*log(x^n) + 2*a) -
 2*(4*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + 9*cos(2*b*log(c
))^2 + 9*sin(2*b*log(c))^2)*x^3)/(4*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*lo
g(c))^2)*n^2 + 9*cos(2*b*log(c))^2 + 9*sin(2*b*log(c))^2)
```

Fricas [**A**]
time = 4.61, size = 80, normalized size = 0.82

$$-\frac{6\,bnx^3\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+9\,x^3\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2-\left(2\,b^2n^2+9\right)x^3}{3\left(4\,b^2n^2+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*b*n*x^3*cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(x) + b*log(c) +
a) + 9*x^3*cos(b*n*log(x) + b*log(c) + a)^2 - (2*b^2*n^2 + 9)*x^3)/(4*b^2*n
^2 + 9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int x^2 \sin^2\left(a - \frac{3i\log\left(cx^n\right)}{2n}\right)dx & \text{for } b = -\frac{3i}{2n} \\ \int x^2 \sin^2\left(a + \frac{3i\log\left(cx^n\right)}{2n}\right)dx & \text{for } b = \frac{3i}{2n} \\ \frac{2b^2n^2x^3\sin^2\left(a+b\log\left(cx^n\right)\right)}{12b^2n^2+27} + \frac{2b^2n^2x^3\cos^2\left(a+b\log\left(cx^n\right)\right)}{12b^2n^2+27} - \frac{6bnx^3\sin\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{12b^2n^2+27} + \frac{9x^3\sin^2\left(a+b\log\left(cx^n\right)\right)}{12b^2n^2+27} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(a+b*ln(c*x**n))**2,x)
```

```
[Out] Piecewise((Integral(x**2*sin(a - 3*I*log(c*x**n)/(2*n))**2, x), Eq(b, -3*I/
(2*n))), (Integral(x**2*sin(a + 3*I*log(c*x**n)/(2*n))**2, x), Eq(b, 3*I/(2
*n))), (2*b**2*n**2*x**3*sin(a + b*log(c*x**n))**2/(12*b**2*n**2 + 27) + 2*
b**2*n**2*x**3*cos(a + b*log(c*x**n))**2/(12*b**2*n**2 + 27) - 6*b*n*x**3*s
in(a + b*log(c*x**n))*cos(a + b*log(c*x**n))/(12*b**2*n**2 + 27) + 9*x**3*s
in(a + b*log(c*x**n))**2/(12*b**2*n**2 + 27), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 833 vs. 2(95) = 190.
time = 0.53, size = 833, normalized size = 8.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3 + 1/4*(4*b*n*x^3*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*ta
n(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 4*b*n*x^3*e^(-pi*b*n*sgn(x) +
 pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)
 + 4*b*n*x^3*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(ab
s(x)) + b*log(abs(c)))*tan(a)^2 + 4*b*n*x^3*e^(-pi*b*n*sgn(x) + pi*b*n - pi
*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a)^2 - 3*x^3*e^(
pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(ab
s(c)))^2*tan(a)^2 - 3*x^3*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*
tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 - 4*b*n*x^3*e^(pi*b*n*sgn(x
) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c))) - 4*b
*n*x^3*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x))
```

```
 + b*log(abs(c))) - 4*b*n*x^3*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*
b)*tan(a) - 4*b*n*x^3*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(
a) + 3*x^3*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(
x)) + b*log(abs(c)))^2 + 3*x^3*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + p
i*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + 12*x^3*e^(pi*b*n*sgn(x) - pi*
b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a) + 12*
x^3*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) +
b*log(abs(c)))*tan(a) + 3*x^3*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*
b)*tan(a)^2 + 3*x^3*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(a)
^2 - 3*x^3*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b) - 3*x^3*e^(-pi*b
*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b))/(4*b^2*n^2*tan(b*n*log(abs(x)) +
b*log(abs(c)))^2*tan(a)^2 + 4*b^2*n^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^
2 + 4*b^2*n^2*tan(a)^2 + 4*b^2*n^2 + 9*tan(b*n*log(abs(x)) + b*log(abs(c)))
^2*tan(a)^2 + 9*tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + 9*tan(a)^2 + 9)
```

Mupad [B]
time = 3.28, size = 67, normalized size = 0.69

$$\frac{x^3}{6}-\frac{x^3\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,2\mathrm{i}}}\,1\mathrm{i}}{8\,b\,n+12\mathrm{i}}-\frac{x^3\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}}{12+b\,n\,8\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a + b*log(c*x^n))^2,x)
```

```
[Out] x^3/6 - (x^3*exp(-a*2i)/(c*x^n)^(b*2i)*1i)/(8*b*n + 12i) - (x^3*exp(a*2i)*(
c*x^n)^(b*2i))/(b*n*8i + 12)
```

3.8 $\int x \sin^2(a + b\log(cx^n))\, dx$

Optimal. Leaf size=98

$$\frac{b^2n^2x^2}{4(1+b^2n^2)} - \frac{bnx^2\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{2(1+b^2n^2)} + \frac{x^2\sin^2(a+b\log(cx^n))}{2(1+b^2n^2)}$$

```
[Out] 1/4*b^2*n^2*x^2/(b^2*n^2+1)-1/2*b*n*x^2*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n
))/(b^2*n^2+1)+1/2*x^2*sin(a+b*ln(c*x^n))^2/(b^2*n^2+1)
```

Rubi [A]
time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4575, 30}

$$\frac{x^2\sin^2(a+b\log(cx^n))}{2(b^2n^2+1)} - \frac{bnx^2\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{2(b^2n^2+1)} + \frac{b^2n^2x^2}{4(b^2n^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (b^2*n^2*x^2)/(4*(1 + b^2*n^2)) - (b*n*x^2*Cos[a + b*Log[c*x^n]]*Sin[a + b*
Log[c*x^n]])/(2*(1 + b^2*n^2)) + (x^2*Sin[a + b*Log[c*x^n]]^2)/(2*(1 + b^2*
n^2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4575

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2
*n^2*p^2 + (m + 1)^2)), Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x],
x] - Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a + b*Log[c*x^n])]*(Sin[d*(a + b*Log
[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x]) /; FreeQ[{a, b, c
, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$\begin{aligned}\int x\sin^2(a+b\log(cx^n))\,dx &= -\frac{bnx^2\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{2(1+b^2n^2)} + \frac{x^2\sin^2(a+b\log(cx^n))}{2(1+b^2n^2)} + \cdots \\ &= \frac{b^2n^2x^2}{4(1+b^2n^2)} - \frac{bnx^2\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{2(1+b^2n^2)} + \frac{x^2\sin^2(a+b\log \cdots}{2(1+b^2 \cdots}\end{aligned}$$

Mathematica [**A**]
time = 0.13, size = 57, normalized size = 0.58

$$\frac{x^2(1+b^2n^2-\cos(2(a+b\log(cx^n)))-bn\sin(2(a+b\log(cx^n))))}{4+4b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x^2*(1 + b^2*n^2 - Cos[2*(a + b*Log[c*x^n])] - b*n*Sin[2*(a + b*Log[c*x^n]
)]))/(4 + 4*b^2*n^2)
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int x\left(\sin^2(a+b\ln(c\,x^n))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(x*sin(a+b*ln(c*x^n))^2,x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(92) = 184.
time = 0.29, size = 282, normalized size = 2.88

$$-\frac{((b\cos(2\,b\log(c))\sin(4\,b\log(c))-b\cos(4\,b\log(c))\sin(2\,b\log(c))+b\sin(2\,b\log(c)))n+\cos(4\,b\log(c))\cos(2\,b\log(c))+\sin(4\,b\log(c))\sin(2\,b\log(c))+\cos(2\,b\log(c)))x^2\cos(2\,b\log(x^n)+2\,a)+((b\cos(4\,b\log(c))\cos(2\,b\log(c))+b\sin(4\,b\log(c))\sin(2\,b\log(c))+b\cos(2\,b\log(c)))n-\cos(2\,b\log(c))\sin(4\,b\log(c))+\cos(4\,b\log(c))\sin(2\,b\log(c))-\sin(2\,b\log(c)))x^2\sin(2\,b\log(x^n)+2\,a)-2\left(\left(b^2\cos(2\,b\log(c))^2+b^2\sin(2\,b\log(c))^2\right)n^2+\cos(2\,b\log(c))^2+\sin(2\,b\log(c))^2\right)x^2}{8\left(\left(b^2\cos(2\,b\log(c))^2+b^2\sin(2\,b\log(c))^2\right)n^2+\cos(2\,b\log(c))^2+\sin(2\,b\log(c))^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(((b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c
)) + b*sin(2*b*log(c)))*n + cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c
))*sin(2*b*log(c)) + cos(2*b*log(c)))*x^2*cos(2*b*log(x^n) + 2*a) + ((b*cos
(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log(c)) + b*cos(2*
b*log(c)))*n - cos(2*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c))*sin(2*b*lo
g(c)) - sin(2*b*log(c)))*x^2*sin(2*b*log(x^n) + 2*a) - 2*((b^2*cos(2*b*log(
c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + cos(2*b*log(c))^2 + sin(2*b*log(c))^2)
*x^2)/((b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + cos(2*b*log(c)
)^2 + sin(2*b*log(c))^2)
```

Fricas [**A**]
time = 2.34, size = 78, normalized size = 0.80

$$-\frac{2\,bnx^2\cos(bn\log(x)+b\log(c)+a)\sin(bn\log(x)+b\log(c)+a)+2\,x^2\cos(bn\log(x)+b\log(c)+a)^2-(b^2n^2+2)x^2}{4\,(b^2n^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*n*x^2*cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(x) + b*log(c) +
a) + 2*x^2*cos(b*n*log(x) + b*log(c) + a)^2 - (b^2*n^2 + 2)*x^2)/(b^2*n^2 +
 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int x \sin^2\left(a - \frac{i \log(cx^n)}{n}\right) dx & \text{for } b = -\frac{i}{n} \\ \int x \sin^2\left(a + \frac{i \log(cx^n)}{n}\right) dx & \text{for } b = \frac{i}{n} \\ \frac{b^2n^2x^2 \sin^2(a+b\log(cx^n))}{4b^2n^2+4} + \frac{b^2n^2x^2 \cos^2(a+b\log(cx^n))}{4b^2n^2+4} - \frac{2bnx^2 \sin(a+b\log(cx^n)) \cos(a+b\log(cx^n))}{4b^2n^2+4} + \frac{2x^2 \sin^2(a+b\log(cx^n))}{4b^2n^2+4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*ln(c*x**n))**2,x)
```

```
[Out] Piecewise((Integral(x*sin(a - I*log(c*x**n)/n)**2, x), Eq(b, -I/n)), (Integ
ral(x*sin(a + I*log(c*x**n)/n)**2, x), Eq(b, I/n)), (b**2*n**2*x**2*sin(a +
 b*log(c*x**n))**2/(4*b**2*n**2 + 4) + b**2*n**2*x**2*cos(a + b*log(c*x**n)
)**2/(4*b**2*n**2 + 4) - 2*b*n*x**2*sin(a + b*log(c*x**n))*cos(a + b*log(c*
x**n))/(4*b**2*n**2 + 4) + 2*x**2*sin(a + b*log(c*x**n))**2/(4*b**2*n**2 +
4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 820 vs. 2(92) = 184.
time = 0.55, size = 820, normalized size = 8.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/4*x^2 + 1/8*(2*b*n*x^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*ta
n(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 2*b*n*x^2*e^(-pi*b*n*sgn(x) +
 pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)
 + 2*b*n*x^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(ab
s(x)) + b*log(abs(c)))*tan(a)^2 + 2*b*n*x^2*e^(-pi*b*n*sgn(x) + pi*b*n - pi
*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a)^2 - x^2*e^(pi
*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(
c)))^2*tan(a)^2 - x^2*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(
b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 - 2*b*n*x^2*e^(pi*b*n*sgn(x) -
pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c))) - 2*b*n*x
^2*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b
```

```
*log(abs(c))) - 2*b*n*x^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*t
an(a) - 2*b*n*x^2*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(a) +
 x^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) +
b*log(abs(c)))^2 + x^2*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan
(b*n*log(abs(x)) + b*log(abs(c)))^2 + 4*x^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*
b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a) + 4*x^2*e^(-pi
*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(
c)))*tan(a) + x^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(a)^2
+ x^2*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(a)^2 - x^2*e^(pi
*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b) - x^2*e^(-pi*b*n*sgn(x) + pi*b*n
 - pi*b*sgn(c) + pi*b))/(b^2*n^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan
(a)^2 + b^2*n^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + b^2*n^2*tan(a)^2 +
 b^2*n^2 + tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 + tan(b*n*log(ab
s(x)) + b*log(abs(c)))^2 + tan(a)^2 + 1)
```

Mupad [B]
time = 2.57, size = 67, normalized size = 0.68

$$\frac{x^2}{4}-\frac{x^2\,{\mathrm{e}}^{-a\,2{}\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,2{}\mathrm{i}}}\,1{}\mathrm{i}}{8\,b\,n+8{}\mathrm{i}}-\frac{x^2\,{\mathrm{e}}^{a\,2{}\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2{}\mathrm{i}}}{8+b\,n\,8{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b*log(c*x^n))^2,x)
```

```
[Out] x^2/4 - (x^2*exp(-a*2i)/(c*x^n)^(b*2i)*1i)/(8*b*n + 8i) - (x^2*exp(a*2i)*(c
*x^n)^(b*2i))/(b*n*8i + 8)
```

3.9 $\int \sin^2(a+b\log(cx^n))\,dx$

Optimal. Leaf size=88

$$\frac{2b^2n^2x}{1+4b^2n^2}-\frac{2bnx\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{1+4b^2n^2}+\frac{x\sin^2(a+b\log(cx^n))}{1+4b^2n^2}$$

```
[Out] 2*b^2*n^2*x/(4*b^2*n^2+1)-2*b*n*x*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))/(4*
b^2*n^2+1)+x*sin(a+b*ln(c*x^n))^2/(4*b^2*n^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4565, 8}

$$\frac{x\sin^2(a+b\log(cx^n))}{4b^2n^2+1}-\frac{2bnx\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+1}+\frac{2b^2n^2x}{4b^2n^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (2*b^2*n^2*x)/(1 + 4*b^2*n^2) - (2*b*n*x*Cos[a + b*Log[c*x^n]]*Sin[a + b*Lo
g[c*x^n]])/(1 + 4*b^2*n^2) + (x*Sin[a + b*Log[c*x^n]]^2)/(1 + 4*b^2*n^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4565

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Sim
p[x*(Sin[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 + 1)), x] + (Dist[b^2*d^2
*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + 1)), Int[Sin[d*(a + b*Log[c*x^n])]^(p -
2), x], x] - Simp[b*d*n*p*x*Cos[d*(a + b*Log[c*x^n])]*(Sin[d*(a + b*Log[c*x
^n])]^(p - 1)/(b^2*d^2*n^2*p^2 + 1)), x]) /; FreeQ[{a, b, c, d, n}, x] && I
GtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+b\log(cx^n))\,dx &= -\frac{2bnx\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{1+4b^2n^2}+\frac{x\sin^2(a+b\log(cx^n))}{1+4b^2n^2}+\frac{(2b}{1} \\ &= \frac{2b^2n^2x}{1+4b^2n^2}-\frac{2bnx\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{1+4b^2n^2}+\frac{x\sin^2(a+b\log(c}{1+4b^2n^2}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 56, normalized size = 0.64

$$\frac{x(1+4b^2n^2-\cos(2(a+b\log(cx^n)))-2bn\sin(2(a+b\log(cx^n))))}{2+8b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x*(1 + 4*b^2*n^2 - Cos[2*(a + b*Log[c*x^n])] - 2*b*n*Sin[2*(a + b*Log[c*x^
n])]))/(2 + 8*b^2*n^2)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \sin^2(a+b\ln(cx^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(sin(a+b*ln(c*x^n))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(88) = 176.
time = 0.30, size = 280, normalized size = 3.18

$$-\frac{(2(b\cos(2b\log(c))\sin(4b\log(c))-b\cos(4b\log(c))\sin(2b\log(c))+b\sin(2b\log(c)))n+\cos(4b\log(c))\cos(2b\log(c))+\sin(4b\log(c))\sin(2b\log(c))+\cos(2b\log(c)))x\cos(2b\log(x^n)+2a)+(2(b\cos(4b\log(c))\cos(2b\log(c))+b\sin(4b\log(c))\sin(2b\log(c))+b\cos(2b\log(c)))n-\cos(2b\log(c))\sin(4b\log(c))+\cos(4b\log(c))\sin(2b\log(c))-\sin(2b\log(c)))x\sin(2b\log(x^n)+2a)-2(4(b^2\cos(2b\log(c))^2+b^2\sin(2b\log(c))^2)n^2+\cos(2b\log(c))^2+\sin(2b\log(c))^2)x}{4(4(b^2\cos(2b\log(c))^2+b^2\sin(2b\log(c))^2)n^2+\cos(2b\log(c))^2+\sin(2b\log(c))^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] -1/4*((2*(b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log
(c)) + b*sin(2*b*log(c)))*n + cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log
(c))*sin(2*b*log(c)) + cos(2*b*log(c)))*x*cos(2*b*log(x^n) + 2*a) + (2*(b*c
os(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log(c)) + b*cos(
2*b*log(c)))*n - cos(2*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c))*sin(2*b*
log(c)) - sin(2*b*log(c)))*x*sin(2*b*log(x^n) + 2*a) - 2*(4*(b^2*cos(2*b*lo
g(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + cos(2*b*log(c))^2 + sin(2*b*log(c))^
2)*x)/(4*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + cos(2*b*log(
c))^2 + sin(2*b*log(c))^2)
```

Fricas [A]
time = 2.76, size = 73, normalized size = 0.83

$$-\frac{2\,bnx\cos(bn\log(x)+b\log(c)+a)\sin(bn\log(x)+b\log(c)+a)+x\cos(bn\log(x)+b\log(c)+a)^2-(2\,b^2n^2+1)x}{4\,b^2n^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] -(2*b*n*x*cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(x) + b*log(c) + a) + x
*cos(b*n*log(x) + b*log(c) + a)^2 - (2*b^2*n^2 + 1)*x)/(4*b^2*n^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \sin^2\left(a - \frac{i\log\left(cx^n\right)}{2n}\right)dx & \text{for } b = -\frac{i}{2n} \\ \int \sin^2\left(a + \frac{i\log\left(cx^n\right)}{2n}\right)dx & \text{for } b = \frac{i}{2n} \\ \frac{2b^2n^2x\sin^2\left(a+b\log\left(cx^n\right)\right)}{4b^2n^2+1} + \frac{2b^2n^2x\cos^2\left(a+b\log\left(cx^n\right)\right)}{4b^2n^2+1} - \frac{2bnx\sin\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{4b^2n^2+1} + \frac{x\sin^2\left(a+b\log\left(cx^n\right)\right)}{4b^2n^2+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**2,x)
```

```
[Out] Piecewise((Integral(sin(a - I*log(c*x**n)/(2*n))**2, x), Eq(b, -I/(2*n))),
(Integral(sin(a + I*log(c*x**n)/(2*n))**2, x), Eq(b, I/(2*n))), (2*b**2*n**
2*x*sin(a + b*log(c*x**n))**2/(4*b**2*n**2 + 1) + 2*b**2*n**2*x*cos(a + b*l
og(c*x**n))**2/(4*b**2*n**2 + 1) - 2*b*n*x*sin(a + b*log(c*x**n))*cos(a + b
*log(c*x**n))/(4*b**2*n**2 + 1) + x*sin(a + b*log(c*x**n))**2/(4*b**2*n**2
+ 1), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 786 vs. 2(88) = 176.
time = 0.50, size = 786, normalized size = 8.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*(4*b*n*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*
n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 4*b*n*x*e^(-pi*b*n*sgn(x) + pi*b*
n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 4*b
*n*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) +
b*log(abs(c)))*tan(a)^2 + 4*b*n*x*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c)
+ pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a)^2 - x*e^(pi*b*n*sgn(x)
- pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a
)^2 - x*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)
) + b*log(abs(c)))^2*tan(a)^2 - 4*b*n*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sg
n(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c))) - 4*b*n*x*e^(-pi*b*n*sgn(
x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c))) - 4*
b*n*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(a) - 4*b*n*x*e^(-
```

```
pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(a) + x*e^(pi*b*n*sgn(x) -
pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + x*e^(
-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(a
bs(c)))^2 + 4*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log
(abs(x)) + b*log(abs(c)))*tan(a) + 4*x*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sg
n(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a) + x*e^(pi*b*n*sgn(
x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(a)^2 + x*e^(-pi*b*n*sgn(x) + pi*b*n -
 pi*b*sgn(c) + pi*b)*tan(a)^2 - x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) -
 pi*b) - x*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b))/(4*b^2*n^2*tan
(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 + 4*b^2*n^2*tan(b*n*log(abs(x)
) + b*log(abs(c)))^2 + 4*b^2*n^2*tan(a)^2 + 4*b^2*n^2 + tan(b*n*log(abs(x))
 + b*log(abs(c)))^2*tan(a)^2 + tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + tan
(a)^2 + 1)
```

Mupad [B]
time = 2.47, size = 56, normalized size = 0.64

$$\frac{x\left(2\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^2+4\,b^2\,n^2-2\,b\,n\,\sin\left(2\,a+2\,b\,\ln\left(c\,x^n\right)\right)\right)}{8\,b^2\,n^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))^2,x)
```

```
[Out] (x*(2*sin(a + b*log(c*x^n))^2 + 4*b^2*n^2 - 2*b*n*sin(2*a + 2*b*log(c*x^n))
))/(8*b^2*n^2 + 2)
```

3.10 $\int \frac{\sin^2(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=39

$$\frac{\log(x)}{2}-\frac{\cos\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\left(cx^n\right)\right)}{2bn}$$

```
[Out] 1/2*ln(x)-1/2*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2715, 8}

$$\frac{\log(x)}{2}-\frac{\sin\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Log[c*x^n]]^2/x,x]
[Out] Log[x]/2 - (Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/(2*b*n)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \sin^2(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\
&= -\frac{\cos\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\left(cx^n\right)\right)}{2bn}+\frac{\text{Subst}\left(\int 1\,dx,x,\log\left(cx^n\right)\right)}{2n}\\
&= \frac{\log(x)}{2}-\frac{\cos\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\left(cx^n\right)\right)}{2bn}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 36, normalized size = 0.92

$$-\frac{-2(a+b\log\left(cx^n\right))+\sin\left(2(a+b\log\left(cx^n\right))\right)}{4bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] -1/4*(-2*(a + b*Log[c*x^n]) + Sin[2*(a + b*Log[c*x^n])])/(b*n)
```

Maple [A]
time = 0.05, size = 45, normalized size = 1.15

method	result	size
derivativedivides	$\frac{-\frac{\cos(a+b\ln(c\,x^{n}))\sin(a+b\ln(c\,x^{n}))}{2}+\frac{b\ln(c\,x^{n})}{2}+\frac{a}{2}}{nb}$	45
default	$\frac{-\frac{\cos(a+b\ln(c\,x^{n}))\sin(a+b\ln(c\,x^{n}))}{2}+\frac{b\ln(c\,x^{n})}{2}+\frac{a}{2}}{nb}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(-1/2*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))+1/2*b*ln(c*x^n)+1/2*a)
```

Maxima [A]
time = 0.28, size = 55, normalized size = 1.41

$$\frac{2\,bn\log\left(x\right)-\cos\left(2\,b\log\left(x^{n}\right)+2\,a\right)\sin\left(2\,b\log\left(c\right)\right)-\cos\left(2\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(x^{n}\right)+2\,a\right)}{4\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*n*log(x) - cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) - cos(2*b*log(c
))*sin(2*b*log(x^n) + 2*a))/(b*n)
```

Fricas [A]
time = 2.98, size = 40, normalized size = 1.03

$$\frac{bn\log\left(x\right)-\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

```
[Out] 1/2*(b*n*log(x) - cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(x) + b*log(c)
+ a))/(b*n)
```

Sympy [A]
time = 1.86, size = 51, normalized size = 1.31

$$-\frac{\begin{cases}\log\left(x\right)\cos\left(2a\right) & \text{for}\: b=0\wedge\left(b=0\vee n=0\right)\\\log\left(x\right)\cos\left(2a+2b\log\left(c\right)\right) & \text{for}\: n=0\\\frac{\sin\left(2a+2b\log\left(cx^{n}\right)\right)}{2bn} & \text{otherwise}\end{cases}}{2}+\frac{\log\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**2/x,x)
```

```
[Out] -Piecewise((log(x)*cos(2*a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*cos
(2*a + 2*b*log(c)), Eq(n, 0)), (sin(2*a + 2*b*log(c*x**n))/(2*b*n), True))/
2 + log(x)/2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^2/x, x)
```

Mupad [**B**]
time = 2.40, size = 32, normalized size = 0.82

$$\frac{\ln\left(x^n\right)}{2\,n}-\frac{\sin\left(2\,a+2\,b\,\ln\left(c\,x^n\right)\right)}{4\,b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))^2/x,x)
```

```
[Out] log(x^n)/(2*n) - sin(2*a + 2*b*log(c*x^n))/(4*b*n)
```

3.11 $\int \frac{\sin^2(a+b\log(cx^n))}{x^2}\,dx$

Optimal. Leaf size=95

$$-\frac{2b^2n^2}{(1+4b^2n^2)\,x}-\frac{2bn\cos\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\left(cx^n\right)\right)}{(1+4b^2n^2)\,x}-\frac{\sin^2\left(a+b\log\left(cx^n\right)\right)}{(1+4b^2n^2)\,x}$$

```
[Out] -2*b^2*n^2/(4*b^2*n^2+1)/x-2*b*n*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))/(4*b
^2*n^2+1)/x-sin(a+b*ln(c*x^n))^2/(4*b^2*n^2+1)/x
```

Rubi [A]
time = 0.02, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4575, 30}

$$-\frac{\sin^2\left(a+b\log\left(cx^n\right)\right)}{x\left(4b^2n^2+1\right)}-\frac{2bn\sin\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{x\left(4b^2n^2+1\right)}-\frac{2b^2n^2}{x\left(4b^2n^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Log[c*x^n]]^2/x^2,x]
```

```
[Out] (-2*b^2*n^2)/((1 + 4*b^2*n^2)*x) - (2*b*n*Cos[a + b*Log[c*x^n]]*Sin[a + b*L
og[c*x^n]])/((1 + 4*b^2*n^2)*x) - Sin[a + b*Log[c*x^n]]^2/((1 + 4*b^2*n^2)*
x)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4575

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2
*n^2*p^2 + (m + 1)^2)), Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x],
x] - Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a + b*Log[c*x^n])]*(Sin[d*(a + b*Log
[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x]) /; FreeQ[{a, b, c
, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx &= -\frac{2bn\cos\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\left(cx^n\right)\right)}{(1+4b^2n^2)\,x}-\frac{\sin^2\left(a+b\log\left(cx^n\right)\right)}{(1+4b^2n^2)\,x}+\frac{(2b^2}{1}\\ &= -\frac{2b^2n^2}{(1+4b^2n^2)\,x}-\frac{2bn\cos\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\left(cx^n\right)\right)}{(1+4b^2n^2)\,x}-\frac{\sin^2\left(a+b\log\right.}{(1+4b^2}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 57, normalized size = 0.60

$$\frac{-1-4b^2n^2+\cos\left(2(a+b\log\left(cx^n\right))\right)-2bn\sin\left(2(a+b\log\left(cx^n\right))\right)}{2\left(x+4b^2n^2x\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]^2/x^2,x]
```

```
[Out] (-1 - 4*b^2*n^2 + Cos[2*(a + b*Log[c*x^n])] - 2*b*n*Sin[2*(a + b*Log[c*x^n]
)])/(2*(x + 4*b^2*n^2*x))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(a+b\ln\left(c\,x^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))^2/x^2,x)
```

```
[Out] int(sin(a+b*ln(c*x^n))^2/x^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(95) = 190.
time = 0.29, size = 283, normalized size = 2.98

$$-\frac{8\left(b^2\cos\left(2\,b\log\left(c\right)\right)^2+b^2\sin\left(2\,b\log\left(c\right)\right)^2\right)n^2+2\,\cos\left(2\,b\log\left(c\right)\right)^2+\left(2\left(b\cos\left(2\,b\log\left(c\right)\right)\sin\left(4\,b\log\left(c\right)\right)-b\cos\left(4\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)+b\sin\left(2\,b\log\left(c\right)\right)\right)n-\cos\left(4\,b\log\left(c\right)\right)\cos\left(2\,b\log\left(c\right)\right)-\sin\left(4\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)-\cos\left(2\,b\log\left(c\right)\right)\right)\cos\left(2\,b\log\left(x^n\right)+2\,a\right)+2\,\sin\left(2\,b\log\left(c\right)\right)^2+\left(2\left(b\cos\left(4\,b\log\left(c\right)\right)\cos\left(2\,b\log\left(c\right)\right)+b\sin\left(4\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)+b\cos\left(2\,b\log\left(c\right)\right)\right)n+\cos\left(2\,b\log\left(c\right)\right)\sin\left(4\,b\log\left(c\right)\right)-\cos\left(4\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)+\sin\left(2\,b\log\left(c\right)\right)\right)\sin\left(2\,b\log\left(x^n\right)+2\,a\right)}{4\left(4\left(b^2\cos\left(2\,b\log\left(c\right)\right)^2+b^2\sin\left(2\,b\log\left(c\right)\right)^2\right)n^2+\cos\left(2\,b\log\left(c\right)\right)^2+\sin\left(2\,b\log\left(c\right)\right)^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*(8*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + 2*cos(2*b*log
(c))^2 + (2*(b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*
log(c)) + b*sin(2*b*log(c)))*n - cos(4*b*log(c))*cos(2*b*log(c)) - sin(4*b*
log(c))*sin(2*b*log(c)) - cos(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 2*sin(
2*b*log(c))^2 + (2*(b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*s
in(2*b*log(c)) + b*cos(2*b*log(c)))*n + cos(2*b*log(c))*sin(4*b*log(c)) - c
os(4*b*log(c))*sin(2*b*log(c)) + sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a))/
((4*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + cos(2*b*log(c))^2
 + sin(2*b*log(c))^2)*x)
```

Fricas [A]
time = 2.04, size = 71, normalized size = 0.75

$$-\frac{2\,b^2n^2+2\,bn\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+1}{(4\,b^2n^2+1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] -(2*b^2*n^2 + 2*b*n*cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(x) + b*log(c
) + a) - cos(b*n*log(x) + b*log(c) + a)^2 + 1)/((4*b^2*n^2 + 1)*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 7.56, size = 301, normalized size = 3.17

$$\begin{cases} \frac{\cos\left(2a - \frac{i\log\left(cx^n\right)}{n}\right)}{4x} - \frac{1}{2x} - \frac{i\log\left(cx^n\right)\sin\left(2a - \frac{i\log\left(cx^n\right)}{n}\right)}{4nx} - \frac{\log\left(cx^n\right)\cos\left(2a - \frac{i\log\left(cx^n\right)}{n}\right)}{4nx} & \text{for}\ b = -\frac{i}{2n} \\ \frac{\cos\left(2a + \frac{i\log\left(cx^n\right)}{n}\right)}{4x} - \frac{1}{2x} + \frac{i\log\left(cx^n\right)\sin\left(2a + \frac{i\log\left(cx^n\right)}{n}\right)}{4nx} - \frac{\log\left(cx^n\right)\cos\left(2a + \frac{i\log\left(cx^n\right)}{n}\right)}{4nx} & \text{for}\ b = \frac{i}{2n} \\ -\frac{2b^2n^2\sin^2\left(a + b\log\left(cx^n\right)\right)}{4b^2n^2x + x} - \frac{2b^2n^2\cos^2\left(a + b\log\left(cx^n\right)\right)}{4b^2n^2x + x} - \frac{2bn\sin\left(a + b\log\left(cx^n\right)\right)\cos\left(a + b\log\left(cx^n\right)\right)}{4b^2n^2x + x} - \frac{\sin^2\left(a + b\log\left(cx^n\right)\right)}{4b^2n^2x + x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**2/x**2,x)
```

```
[Out] Piecewise((cos(2*a - I*log(c*x**n)/n)/(4*x) - 1/(2*x) - I*log(c*x**n)*sin(2
*a - I*log(c*x**n)/n)/(4*n*x) - log(c*x**n)*cos(2*a - I*log(c*x**n)/n)/(4*n
*x), Eq(b, -I/(2*n))), (cos(2*a + I*log(c*x**n)/n)/(4*x) - 1/(2*x) + I*log(
c*x**n)*sin(2*a + I*log(c*x**n)/n)/(4*n*x) - log(c*x**n)*cos(2*a + I*log(c*
x**n)/n)/(4*n*x), Eq(b, I/(2*n))), (-2*b**2*n**2*sin(a + b*log(c*x**n))**2/
(4*b**2*n**2*x + x) - 2*b**2*n**2*cos(a + b*log(c*x**n))**2/(4*b**2*n**2*x
+ x) - 2*b*n*sin(a + b*log(c*x**n))*cos(a + b*log(c*x**n))/(4*b**2*n**2*x +
 x) - sin(a + b*log(c*x**n))**2/(4*b**2*n**2*x + x), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a + b\,\ln\left(c\,x^n\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))^2/x^2,x)
```

```
[Out] int(sin(a + b*log(c*x^n))^2/x^2, x)
```

3.12 $\int \frac{\sin^2(a+b\log(cx^n))}{x^3}\,dx$

Optimal. Leaf size=98

$$-\frac{b^2n^2}{4\left(1+b^2n^2\right)x^2}-\frac{bn\cos\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\left(cx^n\right)\right)}{2\left(1+b^2n^2\right)x^2}-\frac{\sin^2\left(a+b\log\left(cx^n\right)\right)}{2\left(1+b^2n^2\right)x^2}$$

```
[Out] -1/4*b^2*n^2/(b^2*n^2+1)/x^2-1/2*b*n*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))/
(b^2*n^2+1)/x^2-1/2*sin(a+b*ln(c*x^n))^2/(b^2*n^2+1)/x^2
```

Rubi [A]
time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4575, 30}

$$-\frac{\sin^2\left(a+b\log\left(cx^n\right)\right)}{2x^2\left(b^2n^2+1\right)}-\frac{bn\sin\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{2x^2\left(b^2n^2+1\right)}-\frac{b^2n^2}{4x^2\left(b^2n^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Log[c*x^n]]^2/x^3,x]
```

```
[Out] -1/4*(b^2*n^2)/((1 + b^2*n^2)*x^2) - (b*n*Cos[a + b*Log[c*x^n]]*Sin[a + b*L
og[c*x^n]])/(2*(1 + b^2*n^2)*x^2) - Sin[a + b*Log[c*x^n]]^2/(2*(1 + b^2*n^2
)*x^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4575

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2
*n^2*p^2 + (m + 1)^2)), Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x],
x] - Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a + b*Log[c*x^n])]*(Sin[d*(a + b*Log
[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x]) /; FreeQ[{a, b, c
, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2\left(a+b\log\left(cx^n\right)\right)}{x^3}\,dx &= -\frac{bn\cos\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\left(cx^n\right)\right)}{2\left(1+b^2n^2\right)x^2}-\frac{\sin^2\left(a+b\log\left(cx^n\right)\right)}{2\left(1+b^2n^2\right)x^2}+\frac{\left(b^2n^2\right)\int}{2\left(1+\right.}\\ &= -\frac{b^2n^2}{4\left(1+b^2n^2\right)x^2}-\frac{bn\cos\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\left(cx^n\right)\right)}{2\left(1+b^2n^2\right)x^2}-\frac{\sin^2\left(a+b\log\right.}{2\left(1+b^2n\right.}\end{aligned}$$

Mathematica [**A**]
time = 0.12, size = 58, normalized size = 0.59

$$-\frac{1+b^2n^2-\cos\left(2(a+b\log\left(cx^n\right))\right)+bn\sin\left(2(a+b\log\left(cx^n\right))\right)}{4\left(1+b^2n^2\right)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]^2/x^3,x]
```

```
[Out] -1/4*(1 + b^2*n^2 - Cos[2*(a + b*Log[c*x^n])] + b*n*Sin[2*(a + b*Log[c*x^n]
)])/((1 + b^2*n^2)*x^2)
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(a+b\ln\left(c\,x^n\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))^2/x^3,x)
```

```
[Out] int(sin(a+b*ln(c*x^n))^2/x^3,x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(92) = 184.
time = 0.30, size = 280, normalized size = 2.86

$$-\frac{2\left(b^2\cos\left(2\,b\log\left(c\right)\right)^2+b^2\sin\left(2\,b\log\left(c\right)\right)^2\right)n^2+2\,\cos\left(2\,b\log\left(c\right)\right)^2+\left(\left(b\cos\left(2\,b\log\left(c\right)\right)\sin\left(4\,b\log\left(c\right)\right)-b\cos\left(4\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)+b\sin\left(2\,b\log\left(c\right)\right)\right)n-\cos\left(4\,b\log\left(c\right)\right)\cos\left(2\,b\log\left(c\right)\right)-\sin\left(4\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)-\cos\left(2\,b\log\left(c\right)\right)\right)\cos\left(2\,b\log\left(x^n\right)+2\,a\right)+2\,\sin\left(2\,b\log\left(c\right)\right)^2+\left(\left(b\cos\left(4\,b\log\left(c\right)\right)\cos\left(2\,b\log\left(c\right)\right)+b\sin\left(4\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)+b\cos\left(2\,b\log\left(c\right)\right)\right)n+\cos\left(2\,b\log\left(c\right)\right)\sin\left(4\,b\log\left(c\right)\right)-\cos\left(4\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)+\sin\left(2\,b\log\left(c\right)\right)\right)\sin\left(2\,b\log\left(x^n\right)+2\,a\right)}{8\left(\left(b^2\cos\left(2\,b\log\left(c\right)\right)^2+b^2\sin\left(2\,b\log\left(c\right)\right)^2\right)n^2+\cos\left(2\,b\log\left(c\right)\right)^2+\sin\left(2\,b\log\left(c\right)\right)^2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*(2*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + 2*cos(2*b*log
(c))^2 + ((b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*lo
g(c)) + b*sin(2*b*log(c)))*n - cos(4*b*log(c))*cos(2*b*log(c)) - sin(4*b*lo
g(c))*sin(2*b*log(c)) - cos(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 2*sin(2*
b*log(c))^2 + ((b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(2
*b*log(c)) + b*cos(2*b*log(c)))*n + cos(2*b*log(c))*sin(4*b*log(c)) - cos(4
*b*log(c))*sin(2*b*log(c)) + sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a))/(((b
^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + cos(2*b*log(c))^2 + sin
(2*b*log(c))^2)*x^2)
```

Fricas [**A**]
time = 3.47, size = 69, normalized size = 0.70

$$-\frac{b^2n^2+2\,bn\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-2\,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+2}{4\,(b^2n^2+1)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*n^2 + 2*b*n*cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(x) + b*log
(c) + a) - 2*cos(b*n*log(x) + b*log(c) + a)^2 + 2)/((b^2*n^2 + 1)*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.24, size = 468, normalized size = 4.78

$$\begin{cases} -\frac{\sin^2\left(a-\frac{i\log(cx^n)}{n}\right)}{2x^2}+\frac{i\sin\left(a-\frac{i\log(cx^n)}{n}\right)\cos\left(a-\frac{i\log(cx^n)}{n}\right)}{4x^2}+\frac{\log(cx^n)\sin^2\left(a-\frac{i\log(cx^n)}{n}\right)}{4nx^2}-\frac{i\log(cx^n)\sin\left(a-\frac{i\log(cx^n)}{n}\right)\cos\left(a-\frac{i\log(cx^n)}{n}\right)}{2nx^2}-\frac{\log(cx^n)\cos^2\left(a-\frac{i\log(cx^n)}{n}\right)}{4nx^2} & \text{for } b=-\frac{i}{n} \\ \frac{3i\sin\left(a+\frac{i\log(cx^n)}{n}\right)\cos\left(a+\frac{i\log(cx^n)}{n}\right)}{4x^2}-\frac{\cos^2\left(a+\frac{i\log(cx^n)}{n}\right)}{2x^2}+\frac{\log(cx^n)\sin^2\left(a+\frac{i\log(cx^n)}{n}\right)}{4nx^2}+\frac{i\log(cx^n)\sin\left(a+\frac{i\log(cx^n)}{n}\right)\cos\left(a+\frac{i\log(cx^n)}{n}\right)}{2nx^2}-\frac{\log(cx^n)\cos^2\left(a+\frac{i\log(cx^n)}{n}\right)}{4nx^2} & \text{for } b=\frac{i}{n} \\ -\frac{b^2n^2\sin^2(a+b\log(cx^n))}{4b^2n^2x^2+4x^2}-\frac{b^2n^2\cos^2(a+b\log(cx^n))}{4b^2n^2x^2+4x^2}-\frac{2bn\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2x^2+4x^2}-\frac{2\sin^2(a+b\log(cx^n))}{4b^2n^2x^2+4x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**2/x**3,x)
```

```
[Out] Piecewise((-sin(a - I*log(c*x**n)/n)**2/(2*x**2) + I*sin(a - I*log(c*x**n)/
n)*cos(a - I*log(c*x**n)/n)/(4*x**2) + log(c*x**n)*sin(a - I*log(c*x**n)/n)
**2/(4*n*x**2) - I*log(c*x**n)*sin(a - I*log(c*x**n)/n)*cos(a - I*log(c*x**
n)/n)/(2*n*x**2) - log(c*x**n)*cos(a - I*log(c*x**n)/n)**2/(4*n*x**2), Eq(b
, -I/n)), (3*I*sin(a + I*log(c*x**n)/n)*cos(a + I*log(c*x**n)/n)/(4*x**2) -
 cos(a + I*log(c*x**n)/n)**2/(2*x**2) + log(c*x**n)*sin(a + I*log(c*x**n)/n
)**2/(4*n*x**2) + I*log(c*x**n)*sin(a + I*log(c*x**n)/n)*cos(a + I*log(c*x*
*n)/n)/(2*n*x**2) - log(c*x**n)*cos(a + I*log(c*x**n)/n)**2/(4*n*x**2), Eq(
b, I/n)), (-b**2*n**2*sin(a + b*log(c*x**n))**2/(4*b**2*n**2*x**2 + 4*x**2)
 - b**2*n**2*cos(a + b*log(c*x**n))**2/(4*b**2*n**2*x**2 + 4*x**2) - 2*b*n*
sin(a + b*log(c*x**n))*cos(a + b*log(c*x**n))/(4*b**2*n**2*x**2 + 4*x**2) -
 2*sin(a + b*log(c*x**n))**2/(4*b**2*n**2*x**2 + 4*x**2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^2/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))^2/x^3,x)
```

```
[Out] int(sin(a + b*log(c*x^n))^2/x^3, x)
```

3.13 $\int x^2 \sin^3(a + b\log(cx^n))\, dx$

Optimal. Leaf size=160

$$-\frac{2b^3n^3x^3\cos(a+b\log(cx^n))}{3(9+10b^2n^2+b^4n^4)}+\frac{2b^2n^2x^3\sin(a+b\log(cx^n))}{9+10b^2n^2+b^4n^4}-\frac{bnx^3\cos(a+b\log(cx^n))\sin^2(a+b\log(cx^n)}{3(1+b^2n^2)}$$

```
[Out] -2/3*b^3*n^3*x^3*cos(a+b*ln(c*x^n))/(b^4*n^4+10*b^2*n^2+9)+2*b^2*n^2*x^3*si
n(a+b*ln(c*x^n))/(b^4*n^4+10*b^2*n^2+9)-1/3*b*n*x^3*cos(a+b*ln(c*x^n))*sin(
a+b*ln(c*x^n))^2/(b^2*n^2+1)+1/3*x^3*sin(a+b*ln(c*x^n))^3/(b^2*n^2+1)
```

Rubi [A]
time = 0.04, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4575, 4573}

$$\frac{x^3\sin^3(a+b\log(cx^n))}{3(b^2n^2+1)}-\frac{bnx^3\sin^2(a+b\log(cx^n))\cos(a+b\log(cx^n))}{3(b^2n^2+1)}+\frac{2b^2n^2x^3\sin(a+b\log(cx^n))}{b^4n^4+10b^2n^2+9}-\frac{2b^3n^3x^3\cos(a+b\log(cx^n))}{3(b^4n^4+10b^2n^2+9)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (-2*b^3*n^3*x^3*Cos[a + b*Log[c*x^n]])/(3*(9 + 10*b^2*n^2 + b^4*n^4)) + (2*
b^2*n^2*x^3*Sin[a + b*Log[c*x^n]])/(9 + 10*b^2*n^2 + b^4*n^4) - (b*n*x^3*Co
s[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^2)/(3*(1 + b^2*n^2)) + (x^3*Sin[a
 + b*Log[c*x^n]]^3)/(3*(1 + b^2*n^2))
```

Rule 4573

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] - Simp[b*d*n*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Rule 4575

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2
*n^2*p^2 + (m + 1)^2)), Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x],
x] - Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a + b*Log[c*x^n])]*(Sin[d*(a + b*Log
[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x]) /; FreeQ[{a, b, c
, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int x^2 \sin^3(a+b\log(cx^n))\,dx = -\frac{bnx^3\cos(a+b\log(cx^n))\sin^2(a+b\log(cx^n))}{3(1+b^2n^2)} + \frac{x^3\sin^3(a+b\log(cx^n))}{3(1+b^2n^2)} +$$
$$= -\frac{2b^3n^3x^3\cos(a+b\log(cx^n))}{3(9+10b^2n^2+b^4n^4)} + \frac{2b^2n^2x^3\sin(a+b\log(cx^n))}{9+10b^2n^2+b^4n^4} - \frac{bnx^3\cos(a}{}$$

Mathematica [A]
time = 0.58, size = 122, normalized size = 0.76

$$\frac{x^3(-9bn(1+b^2n^2)\cos(a+b\log(cx^n))+bn(9+b^2n^2)\cos(3(a+b\log(cx^n)))-2(-9-13b^2n^2+(9+b^2n^2)\cos(2(a+b\log(cx^n))))\sin(a+b\log(cx^n)))}{12(9+10b^2n^2+b^4n^4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[a + b*Log[c*x^n]]^3,x]
[Out] (x^3*(-9*b*n*(1 + b^2*n^2)*Cos[a + b*Log[c*x^n]] + b*n*(9 + b^2*n^2)*Cos[3*
(a + b*Log[c*x^n])] - 2*(-9 - 13*b^2*n^2 + (9 + b^2*n^2)*Cos[2*(a + b*Log[c
*x^n])])*Sin[a + b*Log[c*x^n]]))/(12*(9 + 10*b^2*n^2 + b^4*n^4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int x^2\left(\sin^3(a+b\ln(c\,x^n))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a+b*ln(c*x^n))^3,x)
[Out] int(x^2*sin(a+b*ln(c*x^n))^3,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1008 vs. 2(154) = 308.
time = 0.33, size = 1008, normalized size = 6.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b*log(c*x^n))^3,x, algorithm="maxima")
[Out] 1/24*(((b^3*cos(6*b*log(c))*cos(3*b*log(c)) + b^3*sin(6*b*log(c))*sin(3*b*l
og(c)) + b^3*cos(3*b*log(c)))*n^3 - (b^2*cos(3*b*log(c))*sin(6*b*log(c)) -
b^2*cos(6*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c)))*n^2 + 9*(b*cos(6
*b*log(c))*cos(3*b*log(c)) + b*sin(6*b*log(c))*sin(3*b*log(c)) + b*cos(3*b*
log(c)))*n - 9*cos(3*b*log(c))*sin(6*b*log(c)) + 9*cos(6*b*log(c))*sin(3*b*
```

```
log(c)) - 9*sin(3*b*log(c)))*x^3*cos(3*b*log(x^n) + 3*a) - 9*((b^3*cos(4*b*
log(c))*cos(3*b*log(c)) + b^3*cos(3*b*log(c))*cos(2*b*log(c)) + b^3*sin(4*b
*log(c))*sin(3*b*log(c)) + b^3*sin(3*b*log(c))*sin(2*b*log(c)))*n^3 - 3*(b^
2*cos(3*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(3*b*log(c)) + b
^2*cos(2*b*log(c))*sin(3*b*log(c)) - b^2*cos(3*b*log(c))*sin(2*b*log(c)))*n
^2 + (b*cos(4*b*log(c))*cos(3*b*log(c)) + b*cos(3*b*log(c))*cos(2*b*log(c))
 + b*sin(4*b*log(c))*sin(3*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*log(c)))*n
 - 3*cos(3*b*log(c))*sin(4*b*log(c)) + 3*cos(4*b*log(c))*sin(3*b*log(c)) -
3*cos(2*b*log(c))*sin(3*b*log(c)) + 3*cos(3*b*log(c))*sin(2*b*log(c)))*x^3*
cos(b*log(x^n) + a) - ((b^3*cos(3*b*log(c))*sin(6*b*log(c)) - b^3*cos(6*b*l
og(c))*sin(3*b*log(c)) + b^3*sin(3*b*log(c)))*n^3 + (b^2*cos(6*b*log(c))*co
s(3*b*log(c)) + b^2*sin(6*b*log(c))*sin(3*b*log(c)) + b^2*cos(3*b*log(c)))*
n^2 + 9*(b*cos(3*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(3*b*log(
c)) + b*sin(3*b*log(c)))*n + 9*cos(6*b*log(c))*cos(3*b*log(c)) + 9*sin(6*b*
log(c))*sin(3*b*log(c)) + 9*cos(3*b*log(c)))*x^3*sin(3*b*log(x^n) + 3*a) +
9*((b^3*cos(3*b*log(c))*sin(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(3*b*log(c
)) + b^3*cos(2*b*log(c))*sin(3*b*log(c)) - b^3*cos(3*b*log(c))*sin(2*b*log(
c)))*n^3 + 3*(b^2*cos(4*b*log(c))*cos(3*b*log(c)) + b^2*cos(3*b*log(c))*cos
(2*b*log(c)) + b^2*sin(4*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c))*si
n(2*b*log(c)))*n^2 + (b*cos(3*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))
*sin(3*b*log(c)) + b*cos(2*b*log(c))*sin(3*b*log(c)) - b*cos(3*b*log(c))*si
n(2*b*log(c)))*n + 3*cos(4*b*log(c))*cos(3*b*log(c)) + 3*cos(3*b*log(c))*co
s(2*b*log(c)) + 3*sin(4*b*log(c))*sin(3*b*log(c)) + 3*sin(3*b*log(c))*sin(2
*b*log(c)))*x^3*sin(b*log(x^n) + a))/((b^4*cos(3*b*log(c))^2 + b^4*sin(3*b*
log(c))^2)*n^4 + 10*(b^2*cos(3*b*log(c))^2 + b^2*sin(3*b*log(c))^2)*n^2 + 9
*cos(3*b*log(c))^2 + 9*sin(3*b*log(c))^2)
```

Fricas [**A**]
time = 2.01, size = 138, normalized size = 0.86

$$\frac{(b^3n^3+9\,bn)x^3\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^3-3\,(b^3n^3+3\,bn)x^3\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-\left((b^2n^2+9)x^3\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2-(7\,b^2n^2+9)x^3\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{3\,(b^4n^4+10\,b^2n^2+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] 1/3*((b^3*n^3 + 9*b*n)*x^3*cos(b*n*log(x) + b*log(c) + a)^3 - 3*(b^3*n^3 +
3*b*n)*x^3*cos(b*n*log(x) + b*log(c) + a) - ((b^2*n^2 + 9)*x^3*cos(b*n*log(
x) + b*log(c) + a)^2 - (7*b^2*n^2 + 9)*x^3)*sin(b*n*log(x) + b*log(c) + a))
/(b^4*n^4 + 10*b^2*n^2 + 9)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(a+b*ln(c*x**n))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 18085 vs. $2(154) = 308$.
time = 1.33, size = 18085, normalized size = 113.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] 1/24*(b^3*n^3*x^3*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2
*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x
)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 - 9*b^3*n^3*x^3*e^(1/2*
pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(ab
s(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2
*tan(3/2*a)^2*tan(1/2*a)^2 - 9*b^3*n^3*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b
*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)
))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)
^2 + b^3*n^3*x^3*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/2
*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x
)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 + b^3*n^3*x^3*e^(3/2*pi
*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(
x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*t
an(3/2*a)^2 + 9*b^3*n^3*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sg
n(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n
*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2 + 9*b^3*n^3*x^3*e^(-1/2*pi
*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(
x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*t
an(3/2*a)^2 + b^3*n^3*x^3*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn
(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*
log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2 + 36*b^3*n^3*x^3*e^(1/2*pi*
b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x
)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(
3/2*a)^2*tan(1/2*a) + 36*b^3*n^3*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1
/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*t
an(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(3/2*a)^2*tan(1/2*a) - b^3*n
^3*x^3*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(
3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*
log(abs(c)))^2*tan(1/2*a)^2 - 9*b^3*n^3*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b
*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)
))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 - 9*b^3*n^
3*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(
3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*
```

```
log(abs(c)))^2*tan(1/2*a)^2 - b^3*n^3*x^3*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*
n - 3/2*pi*b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c))
)^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 - 4*b^3*n^3
*x^3*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/
2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(
abs(c)))^2*tan(3/2*a)*tan(1/2*a)^2 - 4*b^3*n^3*x^3*e^(-3/2*pi*b*n*sgn(x) +
3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*lo
g(abs(c)))*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)*tan(1/
2*a)^2 + b^3*n^3*x^3*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) -
3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1
/2*a)^2 + 9*b^3*n^3*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c)
 - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(3/2*a)^2*ta
n(1/2*a)^2 + 9*b^3*n^3*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sg
n(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(3/2*a)^
2*tan(1/2*a)^2 + b^3*n^3*x^3*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*
sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(3/2*a
)^2*tan(1/2*a)^2 - b^3*n^3*x^3*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b
*sgn(c) - 3/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*
a)^2*tan(1/2*a)^2 - 9*b^3*n^3*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*p
i*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3
/2*a)^2*tan(1/2*a)^2 - 9*b^3*n^3*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1
/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*t
an(3/2*a)^2*tan(1/2*a)^2 - b^3*n^3*x^3*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n -
 3/2*pi*b*sgn(c) + 3/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2
*tan(3/2*a)^2*tan(1/2*a)^2 - 54*b^2*n^2*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b
*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)
))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)
 - 54*b^2*n^2*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/
2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(
x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a) + 2*b^2*n^2*x^3*e^(3/2*p
i*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs
(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*
tan(3/2*a)*tan(1/2*a)^2 + 2*b^2*n^2*x^3*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n
- 3/2*pi*b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log(a...
```

Mupad [B]
time = 3.12, size = 122, normalized size = 0.76

$$-\frac{x^3\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,1\mathrm{i}}}\,3\mathrm{i}}{-24+b\,n\,8\mathrm{i}}-\frac{3\,x^3\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,1\mathrm{i}}}{8\,b\,n-24\mathrm{i}}+\frac{x^3\,\mathrm{e}^{-a\,3\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,3\mathrm{i}}}\,1\mathrm{i}}{-24+b\,n\,24\mathrm{i}}+\frac{x^3\,\mathrm{e}^{a\,3\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,3\mathrm{i}}}{24\,b\,n-24\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a + b*log(c*x^n))^3,x)
[Out] (x^3*exp(-a*3i)/(c*x^n)^(b*3i)*1i)/(b*n*24i - 24) - (3*x^3*exp(a*1i)*(c*x^n
```

```
)^(b*1i))/(8*b*n - 24i) - (x^3*exp(-a*1i)/(c*x^n)^(b*1i)*3i)/(b*n*8i - 24)
+ (x^3*exp(a*3i)*(c*x^n)^(b*3i))/(24*b*n - 24i)
```

3.14 $\int x \sin^3(a+b\log(cx^n))\,dx$

Optimal. Leaf size=158

$$-\frac{6b^3n^3x^2\cos(a+b\log(cx^n))}{16+40b^2n^2+9b^4n^4}+\frac{12b^2n^2x^2\sin(a+b\log(cx^n))}{16+40b^2n^2+9b^4n^4}-\frac{3bnx^2\cos(a+b\log(cx^n))\sin^2(a+b\log(cx$$

```
[Out] -6*b^3*n^3*x^2*cos(a+b*ln(c*x^n))/(9*b^4*n^4+40*b^2*n^2+16)+12*b^2*n^2*x^2*
sin(a+b*ln(c*x^n))/(9*b^4*n^4+40*b^2*n^2+16)-3*b*n*x^2*cos(a+b*ln(c*x^n))*s
in(a+b*ln(c*x^n))^2/(9*b^2*n^2+4)+2*x^2*sin(a+b*ln(c*x^n))^3/(9*b^2*n^2+4)
```

Rubi [A]
time = 0.03, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4575, 4573}

$$\frac{2x^2\sin^3(a+b\log(cx^n))}{9b^2n^2+4}-\frac{3bnx^2\sin^2(a+b\log(cx^n))\cos(a+b\log(cx^n))}{9b^2n^2+4}+\frac{12b^2n^2x^2\sin(a+b\log(cx^n))}{9b^4n^4+40b^2n^2+16}-\frac{6b^3n^3x^2\cos(a+b\log(cx^n))}{9b^4n^4+40b^2n^2+16}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (-6*b^3*n^3*x^2*Cos[a + b*Log[c*x^n]])/(16 + 40*b^2*n^2 + 9*b^4*n^4) + (12*
b^2*n^2*x^2*Sin[a + b*Log[c*x^n]])/(16 + 40*b^2*n^2 + 9*b^4*n^4) - (3*b*n*x
^2*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^2)/(4 + 9*b^2*n^2) + (2*x^2*
Sin[a + b*Log[c*x^n]]^3)/(4 + 9*b^2*n^2)
```

Rule 4573

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] - Simp[b*d*n*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Rule 4575

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2
*n^2*p^2 + (m + 1)^2)), Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x],
x] - Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a + b*Log[c*x^n])]*(Sin[d*(a + b*Log
[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x]) /; FreeQ[{a, b, c
, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Rule steps

$$\int x\sin^3(a+b\log(cx^n))\,dx = -\frac{3bnx^2\cos(a+b\log(cx^n))\sin^2(a+b\log(cx^n))}{4+9b^2n^2}+\frac{2x^2\sin^3(a+b\log(cx^n))}{4+9b^2n^2}$$
$$= -\frac{6b^3n^3x^2\cos(a+b\log(cx^n))}{16+40b^2n^2+9b^4n^4}+\frac{12b^2n^2x^2\sin(a+b\log(cx^n))}{16+40b^2n^2+9b^4n^4}-\frac{3bnx^2\cos(a}{}$$

Mathematica [A]
time = 0.55, size = 125, normalized size = 0.79

$$\frac{x^2(-3bn(4+9b^2n^2)\cos(a+b\log(cx^n))+3bn(4+b^2n^2)\cos(3(a+b\log(cx^n)))-4(-4-13b^2n^2+(4+b^2n^2)\cos(2(a+b\log(cx^n))))\sin(a+b\log(cx^n)))}{4(16+40b^2n^2+9b^4n^4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (x^2*(-3*b*n*(4 + 9*b^2*n^2)*Cos[a + b*Log[c*x^n]] + 3*b*n*(4 + b^2*n^2)*Co
s[3*(a + b*Log[c*x^n])] - 4*(-4 - 13*b^2*n^2 + (4 + b^2*n^2)*Cos[2*(a + b*L
og[c*x^n])])*Sin[a + b*Log[c*x^n]]))/(4*(16 + 40*b^2*n^2 + 9*b^4*n^4))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int x\left(\sin^3(a+b\ln(c\,x^n))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(x*sin(a+b*ln(c*x^n))^3,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1016 vs. 2(158) = 316.
time = 0.31, size = 1016, normalized size = 6.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*(b^3*cos(6*b*log(c))*cos(3*b*log(c)) + b^3*sin(6*b*log(c))*sin(3*b*
log(c)) + b^3*cos(3*b*log(c)))*n^3 - 2*(b^2*cos(3*b*log(c))*sin(6*b*log(c))
 - b^2*cos(6*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c)))*n^2 + 12*(b*c
os(6*b*log(c))*cos(3*b*log(c)) + b*sin(6*b*log(c))*sin(3*b*log(c)) + b*cos(
3*b*log(c)))*n - 8*cos(3*b*log(c))*sin(6*b*log(c)) + 8*cos(6*b*log(c))*sin(
3*b*log(c)) - 8*sin(3*b*log(c)))*x^2*cos(3*b*log(x^n) + 3*a) - 3*(9*(b^3*co
```

```
s(4*b*log(c))*cos(3*b*log(c)) + b^3*cos(3*b*log(c))*cos(2*b*log(c)) + b^3*s
in(4*b*log(c))*sin(3*b*log(c)) + b^3*sin(3*b*log(c))*sin(2*b*log(c)))*n^3 -
 18*(b^2*cos(3*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(3*b*log(
c)) + b^2*cos(2*b*log(c))*sin(3*b*log(c)) - b^2*cos(3*b*log(c))*sin(2*b*log
(c)))*n^2 + 4*(b*cos(4*b*log(c))*cos(3*b*log(c)) + b*cos(3*b*log(c))*cos(2*
b*log(c)) + b*sin(4*b*log(c))*sin(3*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*l
og(c)))*n - 8*cos(3*b*log(c))*sin(4*b*log(c)) + 8*cos(4*b*log(c))*sin(3*b*l
og(c)) - 8*cos(2*b*log(c))*sin(3*b*log(c)) + 8*cos(3*b*log(c))*sin(2*b*log(
c)))*x^2*cos(b*log(x^n) + a) - (3*(b^3*cos(3*b*log(c))*sin(6*b*log(c)) - b^
3*cos(6*b*log(c))*sin(3*b*log(c)) + b^3*sin(3*b*log(c)))*n^3 + 2*(b^2*cos(6
*b*log(c))*cos(3*b*log(c)) + b^2*sin(6*b*log(c))*sin(3*b*log(c)) + b^2*cos(
3*b*log(c)))*n^2 + 12*(b*cos(3*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c)
)*sin(3*b*log(c)) + b*sin(3*b*log(c)))*n + 8*cos(6*b*log(c))*cos(3*b*log(c)
) + 8*sin(6*b*log(c))*sin(3*b*log(c)) + 8*cos(3*b*log(c)))*x^2*sin(3*b*log(
x^n) + 3*a) + 3*(9*(b^3*cos(3*b*log(c))*sin(4*b*log(c)) - b^3*cos(4*b*log(c
))*sin(3*b*log(c)) + b^3*cos(2*b*log(c))*sin(3*b*log(c)) - b^3*cos(3*b*log(
c))*sin(2*b*log(c)))*n^3 + 18*(b^2*cos(4*b*log(c))*cos(3*b*log(c)) + b^2*co
s(3*b*log(c))*cos(2*b*log(c)) + b^2*sin(4*b*log(c))*sin(3*b*log(c)) + b^2*s
in(3*b*log(c))*sin(2*b*log(c)))*n^2 + 4*(b*cos(3*b*log(c))*sin(4*b*log(c))
- b*cos(4*b*log(c))*sin(3*b*log(c)) + b*cos(2*b*log(c))*sin(3*b*log(c)) - b
*cos(3*b*log(c))*sin(2*b*log(c)))*n + 8*cos(4*b*log(c))*cos(3*b*log(c)) + 8
*cos(3*b*log(c))*cos(2*b*log(c)) + 8*sin(4*b*log(c))*sin(3*b*log(c)) + 8*si
n(3*b*log(c))*sin(2*b*log(c)))*x^2*sin(b*log(x^n) + a))/(9*(b^4*cos(3*b*log
(c))^2 + b^4*sin(3*b*log(c))^2)*n^4 + 40*(b^2*cos(3*b*log(c))^2 + b^2*sin(3
*b*log(c))^2)*n^2 + 16*cos(3*b*log(c))^2 + 16*sin(3*b*log(c))^2)
```

Fricas [A]
time = 3.21, size = 140, normalized size = 0.89

$$\frac{3\,(b^3n^3+4\,bn)x^2\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^3-3\,(3\,b^3n^3+4\,bn)x^2\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-2\,\left((b^2n^2+4)x^2\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2-(7\,b^2n^2+4)x^2\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{9\,b^4n^4+40\,b^2n^2+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] (3*(b^3*n^3 + 4*b*n)*x^2*cos(b*n*log(x) + b*log(c) + a)^3 - 3*(3*b^3*n^3 +
4*b*n)*x^2*cos(b*n*log(x) + b*log(c) + a) - 2*((b^2*n^2 + 4)*x^2*cos(b*n*lo
g(x) + b*log(c) + a)^2 - (7*b^2*n^2 + 4)*x^2)*sin(b*n*log(x) + b*log(c) + a
))/(9*b^4*n^4 + 40*b^2*n^2 + 16)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int x\sin^3\left(a-\frac{2i\log\left(cx^n\right)}{n}\right)dx & \text{for}\ b=-\frac{2i}{n} \\ \int x\sin^3\left(a-\frac{2i\log\left(cx^n\right)}{3n}\right)dx & \text{for}\ b=-\frac{2i}{3n} \\ \int x\sin^3\left(a+\frac{2i\log\left(cx^n\right)}{3n}\right)dx & \text{for}\ b=\frac{2i}{3n} \\ \int x\sin^3\left(a+\frac{2i\log\left(cx^n\right)}{n}\right)dx & \text{for}\ b=\frac{2i}{n} \\ -\frac{9b^3n^3x^2\sin^2\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{9b^4n^4+40b^2n^2+16}-\frac{6b^3n^3x^2\cos^3\left(a+b\log\left(cx^n\right)\right)}{9b^4n^4+40b^2n^2+16}+\frac{14b^2n^2x^2\sin^3\left(a+b\log\left(cx^n\right)\right)}{9b^4n^4+40b^2n^2+16}+\frac{12b^2n^2x^2\sin\left(a+b\log\left(cx^n\right)\right)\cos^2\left(a+b\log\left(cx^n\right)\right)}{9b^4n^4+40b^2n^2+16}-\frac{12bnx^2\sin^2\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{9b^4n^4+40b^2n^2+16}+\frac{8x^2\sin^3\left(a+b\log\left(cx^n\right)\right)}{9b^4n^4+40b^2n^2+16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*ln(c*x**n))**3,x)
```

```
[Out] Piecewise((Integral(x*sin(a - 2*I*log(c*x**n)/n)**3, x), Eq(b, -2*I/n)), (I
ntegral(x*sin(a - 2*I*log(c*x**n)/(3*n))**3, x), Eq(b, -2*I/(3*n))), (Integ
ral(x*sin(a + 2*I*log(c*x**n)/(3*n))**3, x), Eq(b, 2*I/(3*n))), (Integral(x
*sin(a + 2*I*log(c*x**n)/n)**3, x), Eq(b, 2*I/n)), (-9*b**3*n**3*x**2*sin(a
 + b*log(c*x**n))**2*cos(a + b*log(c*x**n))/(9*b**4*n**4 + 40*b**2*n**2 + 1
6) - 6*b**3*n**3*x**2*cos(a + b*log(c*x**n))**3/(9*b**4*n**4 + 40*b**2*n**2
 + 16) + 14*b**2*n**2*x**2*sin(a + b*log(c*x**n))**3/(9*b**4*n**4 + 40*b**2
*n**2 + 16) + 12*b**2*n**2*x**2*sin(a + b*log(c*x**n))*cos(a + b*log(c*x**n
))**2/(9*b**4*n**4 + 40*b**2*n**2 + 16) - 12*b*n*x**2*sin(a + b*log(c*x**n)
)**2*cos(a + b*log(c*x**n))/(9*b**4*n**4 + 40*b**2*n**2 + 16) + 8*x**2*sin(
a + b*log(c*x**n))**3/(9*b**4*n**4 + 40*b**2*n**2 + 16), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 18117 vs. $2(158) = 316$.
time = 1.14, size = 18117, normalized size = 114.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^3*n^3*x^2*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/
2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(
x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 - 27*b^3*n^3*x^2*e^(1/
2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(
abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))
^2*tan(3/2*a)^2*tan(1/2*a)^2 - 27*b^3*n^3*x^2*e^(-1/2*pi*b*n*sgn(x) + 1/2*p
i*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs
(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2
*a)^2 + 3*b^3*n^3*x^2*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c)
+ 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(
abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 + 3*b^3*n^3*x^2*e^
(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*l
og(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c
)))^2*tan(3/2*a)^2 + 27*b^3*n^3*x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2
*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan
(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2 + 27*b^3*n^3*x^2*e
^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n
*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs
(c)))^2*tan(3/2*a)^2 + 3*b^3*n^3*x^2*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3
/2*pi*b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*t
an(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2 + 108*b^3*n^3*x^
```

2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(3/2*a)^2*tan(1/2*a) + 108*b^3*n^3*x^2*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(3/2*a)^2*tan(1/2*a) - 3*b^3*n^3*x^2*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 - 27*b^3*n^3*x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 - 27*b^3*n^3*x^2*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 - 3*b^3*n^3*x^2*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 - 12*b^3*n^3*x^2*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)*tan(1/2*a)^2 - 12*b^3*n^3*x^2*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)*tan(1/2*a)^2 + 3*b^3*n^3*x^2*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 + 27*b^3*n^3*x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 + 27*b^3*n^3*x^2*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 + 3*b^3*n^3*x^2*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 - 3*b^3*n^3*x^2*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 - 27*b^3*n^3*x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 - 27*b^3*n^3*x^2*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 - 3*b^3*n^3*x^2*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 - 108*b^2*n^2*x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a) - 108*b^2*n^2*x^2*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a) + 4*b^2*n^2*x^2*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)*tan(1/2*a)^2 + 4*b^2*n^2*x^2*

```
e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*s...
```

Mupad [B]
time = 3.05, size = 122, normalized size = 0.77

$$-\frac{x^2\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,1\mathrm{i}}}\,3\mathrm{i}}{-16+b\,n\,8\mathrm{i}}-\frac{3\,x^2\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,1\mathrm{i}}}{8\,b\,n-16\mathrm{i}}+\frac{x^2\,\mathrm{e}^{-a\,3\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,3\mathrm{i}}}\,1\mathrm{i}}{-16+b\,n\,24\mathrm{i}}+\frac{x^2\,\mathrm{e}^{a\,3\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,3\mathrm{i}}}{24\,b\,n-16\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b*log(c*x^n))^3,x)
```

```
[Out] (x^2*exp(-a*3i)/(c*x^n)^(b*3i)*1i)/(b*n*24i - 16) - (3*x^2*exp(a*1i)*(c*x^n
)^(b*1i))/(8*b*n - 16i) - (x^2*exp(-a*1i)/(c*x^n)^(b*1i)*3i)/(b*n*8i - 16)
+ (x^2*exp(a*3i)*(c*x^n)^(b*3i))/(24*b*n - 16i)
```

3.15 $\int \sin^3(a + b\log(cx^n))\,dx$

Optimal. Leaf size=149

$$-\frac{6b^3n^3x\cos(a+b\log(cx^n))}{1+10b^2n^2+9b^4n^4}+\frac{6b^2n^2x\sin(a+b\log(cx^n))}{1+10b^2n^2+9b^4n^4}-\frac{3bnx\cos(a+b\log(cx^n))\sin^2(a+b\log(cx^n))}{1+9b^2n^2}-$$

```
[Out] -6*b^3*n^3*x*cos(a+b*ln(c*x^n))/(9*b^4*n^4+10*b^2*n^2+1)+6*b^2*n^2*x*sin(a+
b*ln(c*x^n))/(9*b^4*n^4+10*b^2*n^2+1)-3*b*n*x*cos(a+b*ln(c*x^n))*sin(a+b*ln
(c*x^n))^2/(9*b^2*n^2+1)+x*sin(a+b*ln(c*x^n))^3/(9*b^2*n^2+1)
```

Rubi [A]
time = 0.03, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4565, 4563}

$$\frac{x\sin^3(a+b\log(cx^n))}{9b^2n^2+1}-\frac{3bnx\sin^2(a+b\log(cx^n))\cos(a+b\log(cx^n))}{9b^2n^2+1}+\frac{6b^2n^2x\sin(a+b\log(cx^n))}{9b^4n^4+10b^2n^2+1}-\frac{6b^3n^3x\cos(a+b\log(cx^n))}{9b^4n^4+10b^2n^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (-6*b^3*n^3*x*Cos[a + b*Log[c*x^n]])/(1 + 10*b^2*n^2 + 9*b^4*n^4) + (6*b^2*
n^2*x*Sin[a + b*Log[c*x^n]])/(1 + 10*b^2*n^2 + 9*b^4*n^4) - (3*b*n*x*Cos[a
+ b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^2)/(1 + 9*b^2*n^2) + (x*Sin[a + b*Log
[c*x^n]]^3)/(1 + 9*b^2*n^2)
```

Rule 4563

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*(
Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] - Simp[b*d*n*x*(Cos[d*(a +
 b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

Rule 4565

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Sim
p[x*(Sin[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 + 1)), x] + (Dist[b^2*d^2
*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + 1)), Int[Sin[d*(a + b*Log[c*x^n])]^(p -
2), x], x] - Simp[b*d*n*p*x*Cos[d*(a + b*Log[c*x^n])]*(Sin[d*(a + b*Log[c*x
^n])]^(p - 1)/(b^2*d^2*n^2*p^2 + 1)), x]) /; FreeQ[{a, b, c, d, n}, x] && I
GtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + 1, 0]
```

Rubi steps

$$\int \sin^3(a+b\log(cx^n))\,dx = -\frac{3bnx\cos(a+b\log(cx^n))\sin^2(a+b\log(cx^n))}{1+9b^2n^2} + \frac{x\sin^3(a+b\log(cx^n))}{1+9b^2n^2} + \frac{(6$$
$$= -\frac{6b^3n^3x\cos(a+b\log(cx^n))}{1+10b^2n^2+9b^4n^4} + \frac{6b^2n^2x\sin(a+b\log(cx^n))}{1+10b^2n^2+9b^4n^4} - \frac{3bnx\cos(a+b\log$$

Mathematica [A]
time = 0.54, size = 121, normalized size = 0.81

$$-\frac{x(3bn(1+9b^2n^2)\cos(a+b\log(cx^n)) - 3(bn+b^3n^3)\cos(3(a+b\log(cx^n))) + 2(-1-13b^2n^2+(1+b^2n^2)\cos(2(a+b\log(cx^n))))\sin(a+b\log(cx^n)))}{4+40b^2n^2+36b^4n^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]^3,x]
```

```
[Out] -((x*(3*b*n*(1 + 9*b^2*n^2)*Cos[a + b*Log[c*x^n]] - 3*(b*n + b^3*n^3)*Cos[3
*(a + b*Log[c*x^n])] + 2*(-1 - 13*b^2*n^2 + (1 + b^2*n^2)*Cos[2*(a + b*Log[
c*x^n])])*Sin[a + b*Log[c*x^n]]))/(4 + 40*b^2*n^2 + 36*b^4*n^4))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \sin^3(a+b\ln(cx^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(sin(a+b*ln(c*x^n))^3,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 990 vs. $2(149) = 298$.
time = 0.32, size = 990, normalized size = 6.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*(b^3*cos(6*b*log(c))*cos(3*b*log(c)) + b^3*sin(6*b*log(c))*sin(3*b*
log(c)) + b^3*cos(3*b*log(c)))*n^3 - (b^2*cos(3*b*log(c))*sin(6*b*log(c)) -
 b^2*cos(6*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c)))*n^2 + 3*(b*cos(
6*b*log(c))*cos(3*b*log(c)) + b*sin(6*b*log(c))*sin(3*b*log(c)) + b*cos(3*b
*log(c)))*n - cos(3*b*log(c))*sin(6*b*log(c)) + cos(6*b*log(c))*sin(3*b*log
(c)) - sin(3*b*log(c)))*x*cos(3*b*log(x^n) + 3*a) - 3*(9*(b^3*cos(4*b*log(c
```

```
))*cos(3*b*log(c)) + b^3*cos(3*b*log(c))*cos(2*b*log(c)) + b^3*sin(4*b*log(
c))*sin(3*b*log(c)) + b^3*sin(3*b*log(c))*sin(2*b*log(c)))*n^3 - 9*(b^2*cos
(3*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(3*b*log(c)) + b^2*co
s(2*b*log(c))*sin(3*b*log(c)) - b^2*cos(3*b*log(c))*sin(2*b*log(c)))*n^2 +
(b*cos(4*b*log(c))*cos(3*b*log(c)) + b*cos(3*b*log(c))*cos(2*b*log(c)) + b*
sin(4*b*log(c))*sin(3*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*log(c)))*n - co
s(3*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c))*sin(3*b*log(c)) - cos(2*b*l
og(c))*sin(3*b*log(c)) + cos(3*b*log(c))*sin(2*b*log(c)))*x*cos(b*log(x^n)
+ a) - (3*(b^3*cos(3*b*log(c))*sin(6*b*log(c)) - b^3*cos(6*b*log(c))*sin(3*
b*log(c)) + b^3*sin(3*b*log(c)))*n^3 + (b^2*cos(6*b*log(c))*cos(3*b*log(c))
 + b^2*sin(6*b*log(c))*sin(3*b*log(c)) + b^2*cos(3*b*log(c)))*n^2 + 3*(b*co
s(3*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(3*b*log(c)) + b*sin(3
*b*log(c)))*n + cos(6*b*log(c))*cos(3*b*log(c)) + sin(6*b*log(c))*sin(3*b*l
og(c)) + cos(3*b*log(c)))*x*sin(3*b*log(x^n) + 3*a) + 3*(9*(b^3*cos(3*b*log
(c))*sin(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(3*b*log(c)) + b^3*cos(2*b*lo
g(c))*sin(3*b*log(c)) - b^3*cos(3*b*log(c))*sin(2*b*log(c)))*n^3 + 9*(b^2*c
os(4*b*log(c))*cos(3*b*log(c)) + b^2*cos(3*b*log(c))*cos(2*b*log(c)) + b^2*
sin(4*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c))*sin(2*b*log(c)))*n^2
+ (b*cos(3*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(3*b*log(c)) +
b*cos(2*b*log(c))*sin(3*b*log(c)) - b*cos(3*b*log(c))*sin(2*b*log(c)))*n +
cos(4*b*log(c))*cos(3*b*log(c)) + cos(3*b*log(c))*cos(2*b*log(c)) + sin(4*b
*log(c))*sin(3*b*log(c)) + sin(3*b*log(c))*sin(2*b*log(c)))*x*sin(b*log(x^n
) + a))/(9*(b^4*cos(3*b*log(c))^2 + b^4*sin(3*b*log(c))^2)*n^4 + 10*(b^2*co
s(3*b*log(c))^2 + b^2*sin(3*b*log(c))^2)*n^2 + cos(3*b*log(c))^2 + sin(3*b*
log(c))^2)
```

Fricas [A]
time = 3.21, size = 130, normalized size = 0.87

$$\frac{3\,(b^3n^3+bn)x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^3-3\,(3\,b^3n^3+bn)x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-\left((b^2n^2+1)x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2-(7\,b^2n^2+1)x\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{9\,b^4n^4+10\,b^2n^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] (3*(b^3*n^3 + b*n)*x*cos(b*n*log(x) + b*log(c) + a)^3 - 3*(3*b^3*n^3 + b*n)
*x*cos(b*n*log(x) + b*log(c) + a) - ((b^2*n^2 + 1)*x*cos(b*n*log(x) + b*log
(c) + a)^2 - (7*b^2*n^2 + 1)*x)*sin(b*n*log(x) + b*log(c) + a))/(9*b^4*n^4
+ 10*b^2*n^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \sin^{3}{\left(a - \frac{i \log{\left(c x^{n} \right)}}{n} \right)}\, dx & \text{for}\: b = - \frac{i}{n} \\ \int \sin^{3}{\left(a - \frac{i \log{\left(c x^{n} \right)}}{3 n} \right)}\, dx & \text{for}\: b = - \frac{i}{3 n} \\ \int \sin^{3}{\left(a + \frac{i \log{\left(c x^{n} \right)}}{3 n} \right)}\, dx & \text{for}\: b = \frac{i}{3 n} \\ \int \sin^{3}{\left(a + \frac{i \log{\left(c x^{n} \right)}}{n} \right)}\, dx & \text{for}\: b = \frac{i}{n} \\ - \frac{9 b^{3} n^{3} x \sin^{2}{\left(a + b \log{\left(c x^{n} \right)} \right)} \cos{\left(a + b \log{\left(c x^{n} \right)} \right)}}{9 b^{4} n^{4} + 10 b^{2} n^{2} + 1} - \frac{6 b^{3} n^{3} x \cos^{3}{\left(a + b \log{\left(c x^{n} \right)} \right)}}{9 b^{4} n^{4} + 10 b^{2} n^{2} + 1} + \frac{7 b^{2} n^{2} x \sin^{3}{\left(a + b \log{\left(c x^{n} \right)} \right)}}{9 b^{4} n^{4} + 10 b^{2} n^{2} + 1} + \frac{6 b^{2} n^{2} x \sin{\left(a + b \log{\left(c x^{n} \right)} \right)} \cos^{2}{\left(a + b \log{\left(c x^{n} \right)} \right)}}{9 b^{4} n^{4} + 10 b^{2} n^{2} + 1} - \frac{3 b n x \sin^{2}{\left(a + b \log{\left(c x^{n} \right)} \right)} \cos{\left(a + b \log{\left(c x^{n} \right)} \right)}}{9 b^{4} n^{4} + 10 b^{2} n^{2} + 1} + \frac{x \sin^{3}{\left(a + b \log{\left(c x^{n} \right)} \right)}}{9 b^{4} n^{4} + 10 b^{2} n^{2} + 1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**3,x)
```

```
[Out] Piecewise((Integral(sin(a - I*log(c*x**n)/n)**3, x), Eq(b, -I/n)), (Integra
l(sin(a - I*log(c*x**n)/(3*n))**3, x), Eq(b, -I/(3*n))), (Integral(sin(a +
I*log(c*x**n)/(3*n))**3, x), Eq(b, I/(3*n))), (Integral(sin(a + I*log(c*x**
n)/n)**3, x), Eq(b, I/n)), (-9*b**3*n**3*x*sin(a + b*log(c*x**n))**2*cos(a
+ b*log(c*x**n))/(9*b**4*n**4 + 10*b**2*n**2 + 1) - 6*b**3*n**3*x*cos(a + b
*log(c*x**n))**3/(9*b**4*n**4 + 10*b**2*n**2 + 1) + 7*b**2*n**2*x*sin(a + b
*log(c*x**n))**3/(9*b**4*n**4 + 10*b**2*n**2 + 1) + 6*b**2*n**2*x*sin(a + b
*log(c*x**n))*cos(a + b*log(c*x**n))**2/(9*b**4*n**4 + 10*b**2*n**2 + 1) -
3*b*n*x*sin(a + b*log(c*x**n))**2*cos(a + b*log(c*x**n))/(9*b**4*n**4 + 10*
b**2*n**2 + 1) + x*sin(a + b*log(c*x**n))**3/(9*b**4*n**4 + 10*b**2*n**2 +
1), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 17522 vs. $2(149) = 298$.
time = 0.79, size = 17522, normalized size = 117.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^3*n^3*x*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*
pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)
) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 - 27*b^3*n^3*x*e^(1/2*pi
*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(
x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*t
an(3/2*a)^2*tan(1/2*a)^2 - 27*b^3*n^3*x*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n
- 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^
2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2
+ 3*b^3*n^3*x*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/2*pi
*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x))
+ 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 + 3*b^3*n^3*x*e^(3/2*pi*b*
n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x))
 + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(
3/2*a)^2 + 27*b^3*n^3*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c)
 - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log
(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2 + 27*b^3*n^3*x*e^(-1/2*pi*b*n*
sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) +
 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/
2*a)^2 + 3*b^3*n^3*x*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) +
 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(a
bs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2 + 108*b^3*n^3*x*e^(1/2*pi*b*n*sg
```

n(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3
/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(3/2*a)
^2*tan(1/2*a) + 108*b^3*n^3*x*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b
*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*
b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(3/2*a)^2*tan(1/2*a) - 3*b^3*n^3*x*
e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n
*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs
(c)))^2*tan(1/2*a)^2 - 27*b^3*n^3*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2
*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan
(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 - 27*b^3*n^3*x*e^(
-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*l
og(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c
)))^2*tan(1/2*a)^2 - 3*b^3*n^3*x*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*p
i*b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1
/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 - 12*b^3*n^3*x*e^(3/
2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(
abs(x)) + 3/2*b*log(abs(c)))*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2
*tan(3/2*a)*tan(1/2*a)^2 - 12*b^3*n^3*x*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n
- 3/2*pi*b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))*
tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)*tan(1/2*a)^2 + 3*
b^3*n^3*x*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*t
an(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 + 2
7*b^3*n^3*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)
*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 +
 27*b^3*n^3*x*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi
*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^
2 + 3*b^3*n^3*x*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/2*
pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a
)^2 - 3*b^3*n^3*x*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2
*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*
a)^2 - 27*b^3*n^3*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1
/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/
2*a)^2 - 27*b^3*n^3*x*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c)
+ 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan
(1/2*a)^2 - 3*b^3*n^3*x*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c
) + 3/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*t
an(1/2*a)^2 - 54*b^2*n^2*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn
(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*
log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a) - 54*b^2*n^2*x*e
^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n
*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs
(c)))^2*tan(3/2*a)^2*tan(1/2*a) + 2*b^2*n^2*x*e^(3/2*pi*b*n*sgn(x) - 3/2*pi
*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(
c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)*tan(1/2*a)
^2 + 2*b^2*n^2*x*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/2

```
*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(...
```

Mupad [B]
time = 2.89, size = 114, normalized size = 0.77

$$-\frac{x\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,1\mathrm{i}}}\,3\mathrm{i}}{-8+b\,n\,8\mathrm{i}}-\frac{3\,x\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,1\mathrm{i}}}{8\,b\,n-8\mathrm{i}}+\frac{x\,\mathrm{e}^{-a\,3\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,3\mathrm{i}}}\,1\mathrm{i}}{-8+b\,n\,24\mathrm{i}}+\frac{x\,\mathrm{e}^{a\,3\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,3\mathrm{i}}}{24\,b\,n-8\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))^3,x)
```

```
[Out] (x*exp(-a*3i)/(c*x^n)^(b*3i)*1i)/(b*n*24i - 8) - (3*x*exp(a*1i)*(c*x^n)^(b*
1i))/(8*b*n - 8i) - (x*exp(-a*1i)/(c*x^n)^(b*1i)*3i)/(b*n*8i - 8) + (x*exp(
a*3i)*(c*x^n)^(b*3i))/(24*b*n - 8i)
```

3.16 $\int \frac{\sin^3(a+b\log(cx^n))}{x} dx$

Optimal. Leaf size=43

$$-\frac{\cos(a+b\log(cx^n))}{bn}+\frac{\cos^3(a+b\log(cx^n))}{3bn}$$

```
[Out] -cos(a+b*ln(c*x^n))/b/n+1/3*cos(a+b*ln(c*x^n))^3/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2713}

$$\frac{\cos^3(a+b\log(cx^n))}{3bn}-\frac{\cos(a+b\log(cx^n))}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] -(Cos[a + b*Log[c*x^n]]/(b*n)) + Cos[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \sin^3(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\
&= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(a+b\log(cx^n))\right)}{bn}\\
&= -\frac{\cos(a+b\log(cx^n))}{bn}+\frac{\cos^3(a+b\log(cx^n))}{3bn}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 45, normalized size = 1.05

$$-\frac{3\cos(a+b\log(cx^n))}{4bn}+\frac{\cos(3(a+b\log(cx^n)))}{12bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] (-3*Cos[a + b*Log[c*x^n]])/(4*b*n) + Cos[3*(a + b*Log[c*x^n])]/(12*b*n)
```

Maple [A]
time = 0.05, size = 35, normalized size = 0.81

method	result	size
derivativedivides	$-\frac{\left(2+\sin^{2}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)\cos\left(a+b\ln\left(c\,x^{n}\right)\right)}{3nb}$	35
default	$-\frac{\left(2+\sin^{2}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)\cos\left(a+b\ln\left(c\,x^{n}\right)\right)}{3nb}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/n/b*(2+sin(a+b*ln(c*x^n))^2)*cos(a+b*ln(c*x^n))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(41) = 82$.
time = 0.28, size = 233, normalized size = 5.42

$$\frac{\left(\cos\left(6\,b\log\left(c\right)\right)\cos\left(3\,b\log\left(c\right)\right)+\sin\left(6\,b\log\left(c\right)\right)\sin\left(3\,b\log\left(c\right)\right)+\cos\left(3\,b\log\left(c\right)\right)\right)\cos\left(3\,b\log\left(x^{n}\right)+3\,a\right)-9\left(\cos\left(4\,b\log\left(c\right)\right)\cos\left(3\,b\log\left(c\right)\right)+\cos\left(3\,b\log\left(c\right)\right)\cos\left(2\,b\log\left(c\right)\right)+\sin\left(4\,b\log\left(c\right)\right)\sin\left(3\,b\log\left(c\right)\right)+\sin\left(3\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)\right)\cos\left(b\log\left(x^{n}\right)+a\right)-\left(\cos\left(3\,b\log\left(c\right)\right)\sin\left(6\,b\log\left(c\right)\right)-\cos\left(6\,b\log\left(c\right)\right)\sin\left(3\,b\log\left(c\right)\right)+\sin\left(3\,b\log\left(c\right)\right)\right)\sin\left(3\,b\log\left(x^{n}\right)+3\,a\right)+9\left(\cos\left(3\,b\log\left(c\right)\right)\sin\left(4\,b\log\left(c\right)\right)-\cos\left(4\,b\log\left(c\right)\right)\sin\left(3\,b\log\left(c\right)\right)+\cos\left(2\,b\log\left(c\right)\right)\sin\left(3\,b\log\left(c\right)\right)-\cos\left(3\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)\right)\sin\left(b\log\left(x^{n}\right)+a\right)}{24\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] 1/24*((cos(6*b*log(c))*cos(3*b*log(c)) + sin(6*b*log(c))*sin(3*b*log(c)) +
cos(3*b*log(c)))*cos(3*b*log(x^n) + 3*a) - 9*(cos(4*b*log(c))*cos(3*b*log(c
)) + cos(3*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(3*b*log(c)) + si
n(3*b*log(c))*sin(2*b*log(c)))*cos(b*log(x^n) + a) - (cos(3*b*log(c))*sin(6
*b*log(c)) - cos(6*b*log(c))*sin(3*b*log(c)) + sin(3*b*log(c)))*sin(3*b*log
(x^n) + 3*a) + 9*(cos(3*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(3*b
*log(c)) + cos(2*b*log(c))*sin(3*b*log(c)) - cos(3*b*log(c))*sin(2*b*log(c)
))*sin(b*log(x^n) + a))/(b*n)
```

Fricas [A]
time = 2.02, size = 37, normalized size = 0.86

$$\frac{\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}-3\,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{3\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(b*n*log(x) + b*log(c) + a)^3 - 3*cos(b*n*log(x) + b*log(c) + a))/(
b*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(32) = 64$.
time = 1.81, size = 73, normalized size = 1.70

$$\begin{cases} \log(x)\sin^3(a) & \text{for } b = 0 \wedge (b = 0 \vee n = 0) \\ \log(x)\sin^3(a + b\log(c)) & \text{for } n = 0 \\ -\frac{\sin^2(a+b\log(cx^n))\cos(a+b\log(cx^n))}{bn} - \frac{2\cos^3(a+b\log(cx^n))}{3bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**3/x,x)
```

```
[Out] Piecewise((log(x)*sin(a)**3, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*sin
(a + b*log(c))**3, Eq(n, 0)), (-sin(a + b*log(c*x**n))**2*cos(a + b*log(c*x
**n))/(b*n) - 2*cos(a + b*log(c*x**n))**3/(3*b*n), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^3/x, x)
```

Mupad [B]
time = 2.43, size = 37, normalized size = 0.86

$$-\frac{3\,\cos\left(a+b\,\ln\left(c\,x^n\right)\right)-{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}^3}{3\,b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))^3/x,x)
```

```
[Out] -(3*cos(a + b*log(c*x^n)) - cos(a + b*log(c*x^n))^3)/(3*b*n)
```

3.17 $\int \frac{\sin^3(a+b\log(cx^n))}{x^2}\,dx$

Optimal. Leaf size=158

$$-\frac{6b^3n^3\cos\left(a+b\log\left(cx^n\right)\right)}{\left(1+10b^2n^2+9b^4n^4\right)x}-\frac{6b^2n^2\sin\left(a+b\log\left(cx^n\right)\right)}{\left(1+10b^2n^2+9b^4n^4\right)x}-\frac{3bn\cos\left(a+b\log\left(cx^n\right)\right)\sin^2\left(a+b\log\left(cx^n\right)\right)}{\left(1+9b^2n^2\right)x}-\frac{\sin^3}{(}$$

```
[Out] -6*b^3*n^3*cos(a+b*ln(c*x^n))/(9*b^4*n^4+10*b^2*n^2+1)/x-6*b^2*n^2*sin(a+b*
ln(c*x^n))/(9*b^4*n^4+10*b^2*n^2+1)/x-3*b*n*cos(a+b*ln(c*x^n))*sin(a+b*ln(c
*x^n))^2/(9*b^2*n^2+1)/x-sin(a+b*ln(c*x^n))^3/(9*b^2*n^2+1)/x
```

Rubi [A]
time = 0.03, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4575, 4573}

$$-\frac{\sin^3\left(a+b\log\left(cx^n\right)\right)}{x\left(9b^2n^2+1\right)}-\frac{3bn\sin^2\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{x\left(9b^2n^2+1\right)}-\frac{6b^2n^2\sin\left(a+b\log\left(cx^n\right)\right)}{x\left(9b^4n^4+10b^2n^2+1\right)}-\frac{6b^3n^3\cos\left(a+b\log\left(cx^n\right)\right)}{x\left(9b^4n^4+10b^2n^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Log[c*x^n]]^3/x^2,x]
```

```
[Out] (-6*b^3*n^3*Cos[a + b*Log[c*x^n]])/((1 + 10*b^2*n^2 + 9*b^4*n^4)*x) - (6*b^
2*n^2*Sin[a + b*Log[c*x^n]])/((1 + 10*b^2*n^2 + 9*b^4*n^4)*x) - (3*b*n*Cos[
a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^2)/((1 + 9*b^2*n^2)*x) - Sin[a + b*
Log[c*x^n]]^3/((1 + 9*b^2*n^2)*x)
```

Rule 4573

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] - Simp[b*d*n*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Rule 4575

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2
*n^2*p^2 + (m + 1)^2)), Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x],
x] - Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a + b*Log[c*x^n])]*(Sin[d*(a + b*Log
[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x]) /; FreeQ[{a, b, c
, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int \frac{\sin^3(a+b\log(cx^n))}{x^2}\,dx = -\frac{3bn\cos(a+b\log(cx^n))\sin^2(a+b\log(cx^n))}{(1+9b^2n^2)\,x} - \frac{\sin^3(a+b\log(cx^n))}{(1+9b^2n^2)\,x} + \frac{(6b^2}{}$$
$$= -\frac{6b^3n^3\cos(a+b\log(cx^n))}{(1+10b^2n^2+9b^4n^4)\,x} - \frac{6b^2n^2\sin(a+b\log(cx^n))}{(1+10b^2n^2+9b^4n^4)\,x} - \frac{3bn\cos(a+b\log}{(1}$$

Mathematica [A]
time = 0.37, size = 125, normalized size = 0.79

$$\frac{-3bn(1+9b^2n^2)\cos(a+b\log(cx^n))+3(bn+b^3n^3)\cos(3(a+b\log(cx^n)))+2(-1-13b^2n^2+(1+b^2n^2)\cos(2(a+b\log(cx^n))))\sin(a+b\log(cx^n))}{4(1+10b^2n^2+9b^4n^4)\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]^3/x^2,x]
```

```
[Out] (-3*b*n*(1 + 9*b^2*n^2)*Cos[a + b*Log[c*x^n]] + 3*(b*n + b^3*n^3)*Cos[3*(a
+ b*Log[c*x^n])] + 2*(-1 - 13*b^2*n^2 + (1 + b^2*n^2)*Cos[2*(a + b*Log[c*x^
n])])*Sin[a + b*Log[c*x^n]])/(4*(1 + 10*b^2*n^2 + 9*b^4*n^4)*x)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+b\ln(c\,x^n))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))^3/x^2,x)
```

```
[Out] int(sin(a+b*ln(c*x^n))^3/x^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 995 vs. 2(158) = 316.
time = 0.32, size = 995, normalized size = 6.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/8*((3*(b^3*cos(6*b*log(c))*cos(3*b*log(c)) + b^3*sin(6*b*log(c))*sin(3*b*
log(c)) + b^3*cos(3*b*log(c)))*n^3 + (b^2*cos(3*b*log(c))*sin(6*b*log(c)) -
 b^2*cos(6*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c)))*n^2 + 3*(b*cos(
6*b*log(c))*cos(3*b*log(c)) + b*sin(6*b*log(c))*sin(3*b*log(c)) + b*cos(3*b
*log(c)))*n + cos(3*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c))*sin(3*b*log
```

```
(c)) + sin(3*b*log(c)))*cos(3*b*log(x^n) + 3*a) - 3*(9*(b^3*cos(4*b*log(c))
*cos(3*b*log(c)) + b^3*cos(3*b*log(c))*cos(2*b*log(c)) + b^3*sin(4*b*log(c)
)*sin(3*b*log(c)) + b^3*sin(3*b*log(c))*sin(2*b*log(c)))*n^3 + 9*(b^2*cos(3
*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(3*b*log(c)) + b^2*cos(
2*b*log(c))*sin(3*b*log(c)) - b^2*cos(3*b*log(c))*sin(2*b*log(c)))*n^2 + (b
*cos(4*b*log(c))*cos(3*b*log(c)) + b*cos(3*b*log(c))*cos(2*b*log(c)) + b*si
n(4*b*log(c))*sin(3*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*log(c)))*n + cos(
3*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(3*b*log(c)) + cos(2*b*log
(c))*sin(3*b*log(c)) - cos(3*b*log(c))*sin(2*b*log(c)))*cos(b*log(x^n) + a)
 - (3*(b^3*cos(3*b*log(c))*sin(6*b*log(c)) - b^3*cos(6*b*log(c))*sin(3*b*lo
g(c)) + b^3*sin(3*b*log(c)))*n^3 - (b^2*cos(6*b*log(c))*cos(3*b*log(c)) + b
^2*sin(6*b*log(c))*sin(3*b*log(c)) + b^2*cos(3*b*log(c)))*n^2 + 3*(b*cos(3*
b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(3*b*log(c)) + b*sin(3*b*l
og(c)))*n - cos(6*b*log(c))*cos(3*b*log(c)) - sin(6*b*log(c))*sin(3*b*log(c
)) - cos(3*b*log(c)))*sin(3*b*log(x^n) + 3*a) + 3*(9*(b^3*cos(3*b*log(c))*s
in(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(3*b*log(c)) + b^3*cos(2*b*log(c))*
sin(3*b*log(c)) - b^3*cos(3*b*log(c))*sin(2*b*log(c)))*n^3 - 9*(b^2*cos(4*b
*log(c))*cos(3*b*log(c)) + b^2*cos(3*b*log(c))*cos(2*b*log(c)) + b^2*sin(4*
b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c))*sin(2*b*log(c)))*n^2 + (b*c
os(3*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(3*b*log(c)) + b*cos(
2*b*log(c))*sin(3*b*log(c)) - b*cos(3*b*log(c))*sin(2*b*log(c)))*n - cos(4*
b*log(c))*cos(3*b*log(c)) - cos(3*b*log(c))*cos(2*b*log(c)) - sin(4*b*log(c
))*sin(3*b*log(c)) - sin(3*b*log(c))*sin(2*b*log(c)))*sin(b*log(x^n) + a))/
((9*(b^4*cos(3*b*log(c))^2 + b^4*sin(3*b*log(c))^2)*n^4 + 10*(b^2*cos(3*b*l
og(c))^2 + b^2*sin(3*b*log(c))^2)*n^2 + cos(3*b*log(c))^2 + sin(3*b*log(c))
^2)*x)
```

Fricas [A]
time = 2.13, size = 127, normalized size = 0.80

$$\frac{3\left(b^{3} n^{3}+b n\right) \cos \left(b n \log \left(x\right)+b \log \left(c\right)+a\right)^{3}-3\left(3 b^{3} n^{3}+b n\right) \cos \left(b n \log \left(x\right)+b \log \left(c\right)+a\right)-\left(7 b^{2} n^{2}-\left(b^{2} n^{2}+1\right) \cos \left(b n \log \left(x\right)+b \log \left(c\right)+a\right)^{2}+1\right) \sin \left(b n \log \left(x\right)+b \log \left(c\right)+a\right)}{\left(9 b^{4} n^{4}+10 b^{2} n^{2}+1\right) x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^3/x^2,x, algorithm="fricas")
```

```
[Out] (3*(b^3*n^3 + b*n)*cos(b*n*log(x) + b*log(c) + a)^3 - 3*(3*b^3*n^3 + b*n)*c
os(b*n*log(x) + b*log(c) + a) - (7*b^2*n^2 - (b^2*n^2 + 1)*cos(b*n*log(x) +
 b*log(c) + a)^2 + 1)*sin(b*n*log(x) + b*log(c) + a))/((9*b^4*n^4 + 10*b^2*
n^2 + 1)*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 51.27, size = 772, normalized size = 4.89

$$\begin{cases} -\frac{3 \sin \left(a-\frac{i \log \left(c x^{n}\right)}{n}\right)}{8 x}-\frac{\sin \left(3 a-\frac{3 i \log \left(c x^{n}\right)}{n}\right)}{32 x}+\frac{3 i \cos \left(3 a-\frac{3 i \log \left(c x^{n}\right)}{n}\right)}{32 x}+\frac{3 \log \left(c x^{n}\right) \sin \left(a-\frac{i \log \left(c x^{n}\right)}{n}\right)}{8 n x}-\frac{3 i \log \left(c x^{n}\right) \cos \left(a-\frac{i \log \left(c x^{n}\right)}{n}\right)}{8 n x} & \text{for}\: b=-\frac{i}{n} \\ -\frac{27 \sin \left(a-\frac{i \log \left(c x^{n}\right)}{3 n}\right)}{32 x}+\frac{\sin \left(3 a-\frac{i \log \left(c x^{n}\right)}{n}\right)}{8 x}+\frac{9 i \cos \left(a-\frac{i \log \left(c x^{n}\right)}{3 n}\right)}{32 x}-\frac{\log \left(c x^{n}\right) \sin \left(3 a-\frac{i \log \left(c x^{n}\right)}{n}\right)}{8 n x}+\frac{i \log \left(c x^{n}\right) \cos \left(3 a-\frac{i \log \left(c x^{n}\right)}{n}\right)}{8 n x} & \text{for}\: b=-\frac{i}{3 n} \\ -\frac{27 \sin \left(a+\frac{i \log \left(c x^{n}\right)}{3 n}\right)}{32 x}+\frac{\sin \left(3 a+\frac{i \log \left(c x^{n}\right)}{n}\right)}{8 x}-\frac{9 i \cos \left(a+\frac{i \log \left(c x^{n}\right)}{3 n}\right)}{32 x}-\frac{\log \left(c x^{n}\right) \sin \left(3 a+\frac{i \log \left(c x^{n}\right)}{n}\right)}{8 n x}-\frac{i \log \left(c x^{n}\right) \cos \left(3 a+\frac{i \log \left(c x^{n}\right)}{n}\right)}{8 n x} & \text{for}\: b=\frac{i}{3 n} \\ -\frac{\sin \left(3 a+\frac{3 i \log \left(c x^{n}\right)}{n}\right)}{32 x}+\frac{3 i \cos \left(a+\frac{i \log \left(c x^{n}\right)}{n}\right)}{8 x}-\frac{3 i \cos \left(3 a+\frac{3 i \log \left(c x^{n}\right)}{n}\right)}{32 x}+\frac{3 \log \left(c x^{n}\right) \sin \left(a+\frac{i \log \left(c x^{n}\right)}{n}\right)}{8 n x}+\frac{3 i \log \left(c x^{n}\right) \cos \left(a+\frac{i \log \left(c x^{n}\right)}{n}\right)}{8 n x} & \text{for}\: b=\frac{i}{n} \\ -\frac{9 b^{3} n^{3} \sin^{2}\left(a+b \log \left(c x^{n}\right)\right) \cos \left(a+b \log \left(c x^{n}\right)\right)}{9 b^{4} n^{4} x+10 b^{2} n^{2} x+x}-\frac{6 b^{3} n^{3} \cos^{3}\left(a+b \log \left(c x^{n}\right)\right)}{9 b^{4} n^{4} x+10 b^{2} n^{2} x+x}-\frac{7 b^{2} n^{2} \sin^{3}\left(a+b \log \left(c x^{n}\right)\right)}{9 b^{4} n^{4} x+10 b^{2} n^{2} x+x}-\frac{6 b^{2} n^{2} \sin \left(a+b \log \left(c x^{n}\right)\right) \cos^{2}\left(a+b \log \left(c x^{n}\right)\right)}{9 b^{4} n^{4} x+10 b^{2} n^{2} x+x}-\frac{3 b n \sin^{2}\left(a+b \log \left(c x^{n}\right)\right) \cos \left(a+b \log \left(c x^{n}\right)\right)}{9 b^{4} n^{4} x+10 b^{2} n^{2} x+x}-\frac{\sin^{3}\left(a+b \log \left(c x^{n}\right)\right)}{9 b^{4} n^{4} x+10 b^{2} n^{2} x+x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**3/x**2,x)
```

```
[Out] Piecewise((-3*sin(a - I*log(c*x**n)/n)/(8*x) - sin(3*a - 3*I*log(c*x**n)/n)
/(32*x) + 3*I*cos(3*a - 3*I*log(c*x**n)/n)/(32*x) + 3*log(c*x**n)*sin(a - I
*log(c*x**n)/n)/(8*n*x) - 3*I*log(c*x**n)*cos(a - I*log(c*x**n)/n)/(8*n*x),
 Eq(b, -I/n)), (-27*sin(a - I*log(c*x**n)/(3*n))/(32*x) + sin(3*a - I*log(c
*x**n)/n)/(8*x) + 9*I*cos(a - I*log(c*x**n)/(3*n))/(32*x) - log(c*x**n)*sin
(3*a - I*log(c*x**n)/n)/(8*n*x) + I*log(c*x**n)*cos(3*a - I*log(c*x**n)/n)/
(8*n*x), Eq(b, -I/(3*n))), (-27*sin(a + I*log(c*x**n)/(3*n))/(32*x) + sin(3
*a + I*log(c*x**n)/n)/(8*x) - 9*I*cos(a + I*log(c*x**n)/(3*n))/(32*x) - log
(c*x**n)*sin(3*a + I*log(c*x**n)/n)/(8*n*x) - I*log(c*x**n)*cos(3*a + I*log
(c*x**n)/n)/(8*n*x), Eq(b, I/(3*n))), (-sin(3*a + 3*I*log(c*x**n)/n)/(32*x)
 + 3*I*cos(a + I*log(c*x**n)/n)/(8*x) - 3*I*cos(3*a + 3*I*log(c*x**n)/n)/(3
2*x) + 3*log(c*x**n)*sin(a + I*log(c*x**n)/n)/(8*n*x) + 3*I*log(c*x**n)*cos
(a + I*log(c*x**n)/n)/(8*n*x), Eq(b, I/n)), (-9*b**3*n**3*sin(a + b*log(c*x
**n))**2*cos(a + b*log(c*x**n))/(9*b**4*n**4*x + 10*b**2*n**2*x + x) - 6*b*
*3*n**3*cos(a + b*log(c*x**n))**3/(9*b**4*n**4*x + 10*b**2*n**2*x + x) - 7*
b**2*n**2*sin(a + b*log(c*x**n))**3/(9*b**4*n**4*x + 10*b**2*n**2*x + x) -
6*b**2*n**2*sin(a + b*log(c*x**n))*cos(a + b*log(c*x**n))**2/(9*b**4*n**4*x
 + 10*b**2*n**2*x + x) - 3*b*n*sin(a + b*log(c*x**n))**2*cos(a + b*log(c*x*
*n))/(9*b**4*n**4*x + 10*b**2*n**2*x + x) - sin(a + b*log(c*x**n))**3/(9*b*
*4*n**4*x + 10*b**2*n**2*x + x), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^3/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))^3/x^2,x)
```

```
[Out] int(sin(a + b*log(c*x^n))^3/x^2, x)
```

3.18 $\int \frac{\sin^3(a+b\log(cx^n))}{x^3}\,dx$

Optimal. Leaf size=158

$$-\frac{6b^3n^3\cos\left(a+b\log\left(cx^n\right)\right)}{\left(16+40b^2n^2+9b^4n^4\right)x^2}-\frac{12b^2n^2\sin\left(a+b\log\left(cx^n\right)\right)}{\left(16+40b^2n^2+9b^4n^4\right)x^2}-\frac{3bn\cos\left(a+b\log\left(cx^n\right)\right)\sin^2\left(a+b\log\left(cx^n\right)\right)}{\left(4+9b^2n^2\right)x^2}-2\,\mathrm{si}$$

```
[Out] -6*b^3*n^3*cos(a+b*ln(c*x^n))/(9*b^4*n^4+40*b^2*n^2+16)/x^2-12*b^2*n^2*sin(
a+b*ln(c*x^n))/(9*b^4*n^4+40*b^2*n^2+16)/x^2-3*b*n*cos(a+b*ln(c*x^n))*sin(a
+b*ln(c*x^n))^2/(9*b^2*n^2+4)/x^2-2*sin(a+b*ln(c*x^n))^3/(9*b^2*n^2+4)/x^2
```

Rubi [A]
time = 0.03, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4575, 4573}

$$-\frac{2\sin^3\left(a+b\log\left(cx^n\right)\right)}{x^2\left(9b^2n^2+4\right)}-\frac{3bn\sin^2\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{x^2\left(9b^2n^2+4\right)}-\frac{12b^2n^2\sin\left(a+b\log\left(cx^n\right)\right)}{x^2\left(9b^4n^4+40b^2n^2+16\right)}-\frac{6b^3n^3\cos\left(a+b\log\left(cx^n\right)\right)}{x^2\left(9b^4n^4+40b^2n^2+16\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Log[c*x^n]]^3/x^3,x]
```

```
[Out] (-6*b^3*n^3*Cos[a + b*Log[c*x^n]])/((16 + 40*b^2*n^2 + 9*b^4*n^4)*x^2) - (1
2*b^2*n^2*Sin[a + b*Log[c*x^n]])/((16 + 40*b^2*n^2 + 9*b^4*n^4)*x^2) - (3*b
*n*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^2)/((4 + 9*b^2*n^2)*x^2) - (
2*Sin[a + b*Log[c*x^n]]^3)/((4 + 9*b^2*n^2)*x^2)
```

Rule 4573

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] - Simp[b*d*n*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Rule 4575

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2
*n^2*p^2 + (m + 1)^2)), Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x],
x] - Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a + b*Log[c*x^n])]*(Sin[d*(a + b*Log
[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x]) /; FreeQ[{a, b, c
, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int \frac{\sin^3(a+b\log(cx^n))}{x^3}\,dx = -\frac{3bn\cos(a+b\log(cx^n))\sin^2(a+b\log(cx^n))}{(4+9b^2n^2)\,x^2} - \frac{2\sin^3(a+b\log(cx^n))}{(4+9b^2n^2)\,x^2} + \frac{(6}{}$$
$$= -\frac{6b^3n^3\cos(a+b\log(cx^n))}{(16+40b^2n^2+9b^4n^4)\,x^2} - \frac{12b^2n^2\sin(a+b\log(cx^n))}{(16+40b^2n^2+9b^4n^4)\,x^2} - \frac{3bn\cos(a+b\log}{(4}$$

Mathematica [A]
time = 0.44, size = 125, normalized size = 0.79

$$\frac{-3bn(4+9b^2n^2)\cos(a+b\log(cx^n))+3bn(4+b^2n^2)\cos(3(a+b\log(cx^n)))+4(-4-13b^2n^2+(4+b^2n^2)\cos(2(a+b\log(cx^n))))\sin(a+b\log(cx^n))}{4(16+40b^2n^2+9b^4n^4)\,x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]^3/x^3,x]
```

```
[Out] (-3*b*n*(4 + 9*b^2*n^2)*Cos[a + b*Log[c*x^n]] + 3*b*n*(4 + b^2*n^2)*Cos[3*(
a + b*Log[c*x^n])] + 4*(-4 - 13*b^2*n^2 + (4 + b^2*n^2)*Cos[2*(a + b*Log[c*
x^n])])*Sin[a + b*Log[c*x^n]])/(4*(16 + 40*b^2*n^2 + 9*b^4*n^4)*x^2)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+b\ln(c\,x^n))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))^3/x^3,x)
```

```
[Out] int(sin(a+b*ln(c*x^n))^3/x^3,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1007 vs. 2(158) = 316.
time = 0.33, size = 1007, normalized size = 6.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*(b^3*cos(6*b*log(c))*cos(3*b*log(c)) + b^3*sin(6*b*log(c))*sin(3*b*
log(c)) + b^3*cos(3*b*log(c)))*n^3 + 2*(b^2*cos(3*b*log(c))*sin(6*b*log(c))
 - b^2*cos(6*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c)))*n^2 + 12*(b*c
os(6*b*log(c))*cos(3*b*log(c)) + b*sin(6*b*log(c))*sin(3*b*log(c)) + b*cos(
3*b*log(c)))*n + 8*cos(3*b*log(c))*sin(6*b*log(c)) - 8*cos(6*b*log(c))*sin(
```

```
3*b*log(c)) + 8*sin(3*b*log(c)))*cos(3*b*log(x^n) + 3*a) - 3*(9*(b^3*cos(4*
b*log(c))*cos(3*b*log(c)) + b^3*cos(3*b*log(c))*cos(2*b*log(c)) + b^3*sin(4
*b*log(c))*sin(3*b*log(c)) + b^3*sin(3*b*log(c))*sin(2*b*log(c)))*n^3 + 18*
(b^2*cos(3*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(3*b*log(c))
+ b^2*cos(2*b*log(c))*sin(3*b*log(c)) - b^2*cos(3*b*log(c))*sin(2*b*log(c))
)*n^2 + 4*(b*cos(4*b*log(c))*cos(3*b*log(c)) + b*cos(3*b*log(c))*cos(2*b*lo
g(c)) + b*sin(4*b*log(c))*sin(3*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*log(c
)))*n + 8*cos(3*b*log(c))*sin(4*b*log(c)) - 8*cos(4*b*log(c))*sin(3*b*log(c
)) + 8*cos(2*b*log(c))*sin(3*b*log(c)) - 8*cos(3*b*log(c))*sin(2*b*log(c)))
*cos(b*log(x^n) + a) - (3*(b^3*cos(3*b*log(c))*sin(6*b*log(c)) - b^3*cos(6*
b*log(c))*sin(3*b*log(c)) + b^3*sin(3*b*log(c)))*n^3 - 2*(b^2*cos(6*b*log(c
))*cos(3*b*log(c)) + b^2*sin(6*b*log(c))*sin(3*b*log(c)) + b^2*cos(3*b*log(
c)))*n^2 + 12*(b*cos(3*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(3*
b*log(c)) + b*sin(3*b*log(c)))*n - 8*cos(6*b*log(c))*cos(3*b*log(c)) - 8*si
n(6*b*log(c))*sin(3*b*log(c)) - 8*cos(3*b*log(c)))*sin(3*b*log(x^n) + 3*a)
+ 3*(9*(b^3*cos(3*b*log(c))*sin(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(3*b*l
og(c)) + b^3*cos(2*b*log(c))*sin(3*b*log(c)) - b^3*cos(3*b*log(c))*sin(2*b*
log(c)))*n^3 - 18*(b^2*cos(4*b*log(c))*cos(3*b*log(c)) + b^2*cos(3*b*log(c)
)*cos(2*b*log(c)) + b^2*sin(4*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c
))*sin(2*b*log(c)))*n^2 + 4*(b*cos(3*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*
log(c))*sin(3*b*log(c)) + b*cos(2*b*log(c))*sin(3*b*log(c)) - b*cos(3*b*log
(c))*sin(2*b*log(c)))*n - 8*cos(4*b*log(c))*cos(3*b*log(c)) - 8*cos(3*b*log
(c))*cos(2*b*log(c)) - 8*sin(4*b*log(c))*sin(3*b*log(c)) - 8*sin(3*b*log(c)
)*sin(2*b*log(c)))*sin(b*log(x^n) + a))/((9*(b^4*cos(3*b*log(c))^2 + b^4*si
n(3*b*log(c))^2)*n^4 + 40*(b^2*cos(3*b*log(c))^2 + b^2*sin(3*b*log(c))^2)*n
^2 + 16*cos(3*b*log(c))^2 + 16*sin(3*b*log(c))^2)*x^2)
```

Fricas [A]
time = 2.39, size = 129, normalized size = 0.82

$$\frac{3\left(b^{3}n^{3}+4\,bn\right)\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}-3\left(3\,b^{3}n^{3}+4\,bn\right)\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-2\left(7\,b^{2}n^{2}-\left(b^{2}n^{2}+4\right)\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+4\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{\left(9\,b^{4}n^{4}+40\,b^{2}n^{2}+16\right)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^3/x^3,x, algorithm="fricas")
```

```
[Out] (3*(b^3*n^3 + 4*b*n)*cos(b*n*log(x) + b*log(c) + a)^3 - 3*(3*b^3*n^3 + 4*b*
n)*cos(b*n*log(x) + b*log(c) + a) - 2*(7*b^2*n^2 - (b^2*n^2 + 4)*cos(b*n*lo
g(x) + b*log(c) + a)^2 + 4)*sin(b*n*log(x) + b*log(c) + a))/((9*b^4*n^4 + 4
0*b^2*n^2 + 16)*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 55.45, size = 882, normalized size = 5.58

$$\begin{cases} -\frac{3\sin\left(a-\frac{2i\log\left(cx^{n}\right)}{n}\right)}{16x^{2}}-\frac{\sin\left(3a-\frac{6i\log\left(cx^{n}\right)}{n}\right)}{64x^{2}}+\frac{3i\cos\left(3a-\frac{6i\log\left(cx^{n}\right)}{n}\right)}{64x^{2}}+\frac{3\log\left(cx^{n}\right)\sin\left(a-\frac{2i\log\left(cx^{n}\right)}{n}\right)}{8nx^{2}}-\frac{3i\log\left(cx^{n}\right)\cos\left(a-\frac{2i\log\left(cx^{n}\right)}{n}\right)}{8nx^{2}} & \text{for}\: b=-\frac{2i}{n} \\ -\frac{27\sin\left(a-\frac{2i\log\left(cx^{n}\right)}{3n}\right)}{64x^{2}}+\frac{\sin\left(3a-\frac{2i\log\left(cx^{n}\right)}{n}\right)}{16x^{2}}+\frac{9i\cos\left(a-\frac{2i\log\left(cx^{n}\right)}{3n}\right)}{64x^{2}}-\frac{\log\left(cx^{n}\right)\sin\left(3a-\frac{2i\log\left(cx^{n}\right)}{n}\right)}{8nx^{2}}+\frac{i\log\left(cx^{n}\right)\cos\left(3a-\frac{2i\log\left(cx^{n}\right)}{n}\right)}{8nx^{2}} & \text{for}\: b=-\frac{2i}{3n} \\ -\frac{27\sin\left(a+\frac{2i\log\left(cx^{n}\right)}{3n}\right)}{64x^{2}}+\frac{\sin\left(3a+\frac{2i\log\left(cx^{n}\right)}{n}\right)}{16x^{2}}-\frac{9i\cos\left(a+\frac{2i\log\left(cx^{n}\right)}{3n}\right)}{64x^{2}}-\frac{\log\left(cx^{n}\right)\sin\left(3a+\frac{2i\log\left(cx^{n}\right)}{n}\right)}{8nx^{2}}-\frac{i\log\left(cx^{n}\right)\cos\left(3a+\frac{2i\log\left(cx^{n}\right)}{n}\right)}{8nx^{2}} & \text{for}\: b=\frac{2i}{3n} \\ -\frac{\sin\left(3a+\frac{6i\log\left(cx^{n}\right)}{n}\right)}{64x^{2}}+\frac{3i\cos\left(a+\frac{2i\log\left(cx^{n}\right)}{n}\right)}{16x^{2}}-\frac{3i\cos\left(3a+\frac{6i\log\left(cx^{n}\right)}{n}\right)}{64x^{2}}+\frac{3\log\left(cx^{n}\right)\sin\left(a+\frac{2i\log\left(cx^{n}\right)}{n}\right)}{8nx^{2}}+\frac{3i\log\left(cx^{n}\right)\cos\left(a+\frac{2i\log\left(cx^{n}\right)}{n}\right)}{8nx^{2}} & \text{for}\: b=\frac{2i}{n} \\ -\frac{9b^{3}n^{3}\sin^{2}\left(a+b\log\left(cx^{n}\right)\right)\cos\left(a+b\log\left(cx^{n}\right)\right)}{9b^{4}n^{4}x^{2}+40b^{2}n^{2}x^{2}+16x^{2}}-\frac{6b^{3}n^{3}\cos^{3}\left(a+b\log\left(cx^{n}\right)\right)}{9b^{4}n^{4}x^{2}+40b^{2}n^{2}x^{2}+16x^{2}}-\frac{14b^{2}n^{2}\sin^{3}\left(a+b\log\left(cx^{n}\right)\right)}{9b^{4}n^{4}x^{2}+40b^{2}n^{2}x^{2}+16x^{2}}-\frac{12b^{2}n^{2}\sin\left(a+b\log\left(cx^{n}\right)\right)\cos^{2}\left(a+b\log\left(cx^{n}\right)\right)}{9b^{4}n^{4}x^{2}+40b^{2}n^{2}x^{2}+16x^{2}}-\frac{12bn\sin^{2}\left(a+b\log\left(cx^{n}\right)\right)\cos\left(a+b\log\left(cx^{n}\right)\right)}{9b^{4}n^{4}x^{2}+40b^{2}n^{2}x^{2}+16x^{2}}-\frac{8\sin^{3}\left(a+b\log\left(cx^{n}\right)\right)}{9b^{4}n^{4}x^{2}+40b^{2}n^{2}x^{2}+16x^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**3/x**3,x)
```

```
[Out] Piecewise((-3*sin(a - 2*I*log(c*x**n)/n)/(16*x**2) - sin(3*a - 6*I*log(c*x*
*n)/n)/(64*x**2) + 3*I*cos(3*a - 6*I*log(c*x**n)/n)/(64*x**2) + 3*log(c*x**
n)*sin(a - 2*I*log(c*x**n)/n)/(8*n*x**2) - 3*I*log(c*x**n)*cos(a - 2*I*log(
c*x**n)/n)/(8*n*x**2), Eq(b, -2*I/n)), (-27*sin(a - 2*I*log(c*x**n)/(3*n))/
(64*x**2) + sin(3*a - 2*I*log(c*x**n)/n)/(16*x**2) + 9*I*cos(a - 2*I*log(c*
x**n)/(3*n))/(64*x**2) - log(c*x**n)*sin(3*a - 2*I*log(c*x**n)/n)/(8*n*x**2
) + I*log(c*x**n)*cos(3*a - 2*I*log(c*x**n)/n)/(8*n*x**2), Eq(b, -2*I/(3*n)
)), (-27*sin(a + 2*I*log(c*x**n)/(3*n))/(64*x**2) + sin(3*a + 2*I*log(c*x**
n)/n)/(16*x**2) - 9*I*cos(a + 2*I*log(c*x**n)/(3*n))/(64*x**2) - log(c*x**n
)*sin(3*a + 2*I*log(c*x**n)/n)/(8*n*x**2) - I*log(c*x**n)*cos(3*a + 2*I*log
(c*x**n)/n)/(8*n*x**2), Eq(b, 2*I/(3*n))), (-sin(3*a + 6*I*log(c*x**n)/n)/(
64*x**2) + 3*I*cos(a + 2*I*log(c*x**n)/n)/(16*x**2) - 3*I*cos(3*a + 6*I*log
(c*x**n)/n)/(64*x**2) + 3*log(c*x**n)*sin(a + 2*I*log(c*x**n)/n)/(8*n*x**2)
 + 3*I*log(c*x**n)*cos(a + 2*I*log(c*x**n)/n)/(8*n*x**2), Eq(b, 2*I/n)), (-
9*b**3*n**3*sin(a + b*log(c*x**n))**2*cos(a + b*log(c*x**n))/(9*b**4*n**4*x
**2 + 40*b**2*n**2*x**2 + 16*x**2) - 6*b**3*n**3*cos(a + b*log(c*x**n))**3/
(9*b**4*n**4*x**2 + 40*b**2*n**2*x**2 + 16*x**2) - 14*b**2*n**2*sin(a + b*l
og(c*x**n))**3/(9*b**4*n**4*x**2 + 40*b**2*n**2*x**2 + 16*x**2) - 12*b**2*n
**2*sin(a + b*log(c*x**n))*cos(a + b*log(c*x**n))**2/(9*b**4*n**4*x**2 + 40
*b**2*n**2*x**2 + 16*x**2) - 12*b*n*sin(a + b*log(c*x**n))**2*cos(a + b*log
(c*x**n))/(9*b**4*n**4*x**2 + 40*b**2*n**2*x**2 + 16*x**2) - 8*sin(a + b*lo
g(c*x**n))**3/(9*b**4*n**4*x**2 + 40*b**2*n**2*x**2 + 16*x**2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^3/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))^3/x^3,x)
```

```
[Out] int(sin(a + b*log(c*x^n))^3/x^3, x)
```

3.19 $\int x^2 \sin^4(a + b\log(cx^n))\, dx$

Optimal. Leaf size=202

$$\frac{8b^4n^4x^3}{81 + 180b^2n^2 + 64b^4n^4} - \frac{24b^3n^3x^3\cos(a + b\log(cx^n))\sin(a + b\log(cx^n))}{81 + 180b^2n^2 + 64b^4n^4} + \frac{36b^2n^2x^3\sin^2(a + b\log(cx^n))}{81 + 180b^2n^2 + 64b^4n^4} -$$

```
[Out] 8*b^4*n^4*x^3/(64*b^4*n^4+180*b^2*n^2+81)-24*b^3*n^3*x^3*cos(a+b*ln(c*x^n))
*sin(a+b*ln(c*x^n))/(64*b^4*n^4+180*b^2*n^2+81)+36*b^2*n^2*x^3*sin(a+b*ln(c
*x^n))^2/(64*b^4*n^4+180*b^2*n^2+81)-4*b*n*x^3*cos(a+b*ln(c*x^n))*sin(a+b*l
n(c*x^n))^3/(16*b^2*n^2+9)+3*x^3*sin(a+b*ln(c*x^n))^4/(16*b^2*n^2+9)
```

Rubi [A]
time = 0.05, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4575, 30}

$$\frac{3x^3\sin^4(a + b\log(cx^n))}{16b^2n^2 + 9} - \frac{4bnx^3\sin^3(a + b\log(cx^n))\cos(a + b\log(cx^n))}{16b^2n^2 + 9} + \frac{36b^2n^2x^3\sin^2(a + b\log(cx^n))}{64b^4n^4 + 180b^2n^2 + 81} - \frac{24b^3n^3x^3\sin(a + b\log(cx^n))\cos(a + b\log(cx^n))}{64b^4n^4 + 180b^2n^2 + 81} + \frac{8b^4n^4x^3}{64b^4n^4 + 180b^2n^2 + 81}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (8*b^4*n^4*x^3)/(81 + 180*b^2*n^2 + 64*b^4*n^4) - (24*b^3*n^3*x^3*Cos[a + b
*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/(81 + 180*b^2*n^2 + 64*b^4*n^4) + (36*b
^2*n^2*x^3*Sin[a + b*Log[c*x^n]]^2)/(81 + 180*b^2*n^2 + 64*b^4*n^4) - (4*b*
n*x^3*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^3)/(9 + 16*b^2*n^2) + (3*
x^3*Sin[a + b*Log[c*x^n]]^4)/(9 + 16*b^2*n^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4575

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2
*n^2*p^2 + (m + 1)^2)), Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x],
x] - Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a + b*Log[c*x^n])]*(Sin[d*(a + b*Log
[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x]) /; FreeQ[{a, b, c
, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int x^2 \sin^4(a+b\log(cx^n))\,dx = -\frac{4bnx^3\cos(a+b\log(cx^n))\sin^3(a+b\log(cx^n))}{9+16b^2n^2} + \frac{3x^3\sin^4(a+b\log(cx^n}{9+16b^2n^2}$$
$$= -\frac{24b^3n^3x^3\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{81+180b^2n^2+64b^4n^4} + \frac{36b^2n^2x^3\sin^2(a+b}{81+180b^2n^2+6}$$
$$= \frac{8b^4n^4x^3}{81+180b^2n^2+64b^4n^4} - \frac{24b^3n^3x^3\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{81+180b^2n^2+64b^4n^4}$$

Mathematica [A]
time = 0.56, size = 171, normalized size = 0.85

$$\frac{x^3(81+180b^2n^2+64b^4n^4-12(9+16b^2n^2)\cos(2(a+b\log(cx^n)))+3(9+4b^2n^2)\cos(4(a+b\log(cx^n)))-72bn\sin(2(a+b\log(cx^n)))-128b^3n^3\sin(2(a+b\log(cx^n)))+36bn\sin(4(a+b\log(cx^n)))+16b^3n^3\sin(4(a+b\log(cx^n))))}{8(81+180b^2n^2+64b^4n^4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x^3*(81 + 180*b^2*n^2 + 64*b^4*n^4 - 12*(9 + 16*b^2*n^2)*Cos[2*(a + b*Log[
c*x^n])] + 3*(9 + 4*b^2*n^2)*Cos[4*(a + b*Log[c*x^n])] - 72*b*n*Sin[2*(a +
b*Log[c*x^n])] - 128*b^3*n^3*Sin[2*(a + b*Log[c*x^n])] + 36*b*n*Sin[4*(a +
b*Log[c*x^n])] + 16*b^3*n^3*Sin[4*(a + b*Log[c*x^n])]))/(8*(81 + 180*b^2*n^
2 + 64*b^4*n^4))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int x^2\left(\sin^4(a+b\ln(c\,x^n))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a+b*ln(c*x^n))^4,x)
```

```
[Out] int(x^2*sin(a+b*ln(c*x^n))^4,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1107 vs. $2(202) = 404$.
time = 0.32, size = 1107, normalized size = 5.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

```
[Out] 1/16*((16*(b^3*cos(4*b*log(c))*sin(8*b*log(c)) - b^3*cos(8*b*log(c))*sin(4*
b*log(c)) + b^3*sin(4*b*log(c)))*n^3 + 12*(b^2*cos(8*b*log(c))*cos(4*b*log(
```

```
c)) + b^2*sin(8*b*log(c))*sin(4*b*log(c)) + b^2*cos(4*b*log(c)))*n^2 + 36*(
b*cos(4*b*log(c))*sin(8*b*log(c)) - b*cos(8*b*log(c))*sin(4*b*log(c)) + b*s
in(4*b*log(c)))*n + 27*cos(8*b*log(c))*cos(4*b*log(c)) + 27*sin(8*b*log(c))
*sin(4*b*log(c)) + 27*cos(4*b*log(c)))*x^3*cos(4*b*log(x^n) + 4*a) - 4*(32*
(b^3*cos(4*b*log(c))*sin(6*b*log(c)) - b^3*cos(6*b*log(c))*sin(4*b*log(c))
+ b^3*cos(2*b*log(c))*sin(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(2*b*log(c))
)*n^3 + 48*(b^2*cos(6*b*log(c))*cos(4*b*log(c)) + b^2*cos(4*b*log(c))*cos(2
*b*log(c)) + b^2*sin(6*b*log(c))*sin(4*b*log(c)) + b^2*sin(4*b*log(c))*sin(
2*b*log(c)))*n^2 + 18*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c)
)*sin(4*b*log(c)) + b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*s
in(2*b*log(c)))*n + 27*cos(6*b*log(c))*cos(4*b*log(c)) + 27*cos(4*b*log(c))
*cos(2*b*log(c)) + 27*sin(6*b*log(c))*sin(4*b*log(c)) + 27*sin(4*b*log(c))*
sin(2*b*log(c)))*x^3*cos(2*b*log(x^n) + 2*a) + (16*(b^3*cos(8*b*log(c))*cos
(4*b*log(c)) + b^3*sin(8*b*log(c))*sin(4*b*log(c)) + b^3*cos(4*b*log(c)))*n
^3 - 12*(b^2*cos(4*b*log(c))*sin(8*b*log(c)) - b^2*cos(8*b*log(c))*sin(4*b*
log(c)) + b^2*sin(4*b*log(c)))*n^2 + 36*(b*cos(8*b*log(c))*cos(4*b*log(c))
+ b*sin(8*b*log(c))*sin(4*b*log(c)) + b*cos(4*b*log(c)))*n - 27*cos(4*b*log
(c))*sin(8*b*log(c)) + 27*cos(8*b*log(c))*sin(4*b*log(c)) - 27*sin(4*b*log(
c)))*x^3*sin(4*b*log(x^n) + 4*a) - 4*(32*(b^3*cos(6*b*log(c))*cos(4*b*log(c
)) + b^3*cos(4*b*log(c))*cos(2*b*log(c)) + b^3*sin(6*b*log(c))*sin(4*b*log(
c)) + b^3*sin(4*b*log(c))*sin(2*b*log(c)))*n^3 - 48*(b^2*cos(4*b*log(c))*si
n(6*b*log(c)) - b^2*cos(6*b*log(c))*sin(4*b*log(c)) + b^2*cos(2*b*log(c))*s
in(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(2*b*log(c)))*n^2 + 18*(b*cos(6*b*l
og(c))*cos(4*b*log(c)) + b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(6*b*log(
c))*sin(4*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log(c)))*n - 27*cos(4*b*log
(c))*sin(6*b*log(c)) + 27*cos(6*b*log(c))*sin(4*b*log(c)) - 27*cos(2*b*log(
c))*sin(4*b*log(c)) + 27*cos(4*b*log(c))*sin(2*b*log(c)))*x^3*sin(2*b*log(x
^n) + 2*a) + 2*(64*(b^4*cos(4*b*log(c))^2 + b^4*sin(4*b*log(c))^2)*n^4 + 18
0*(b^2*cos(4*b*log(c))^2 + b^2*sin(4*b*log(c))^2)*n^2 + 81*cos(4*b*log(c))^
2 + 81*sin(4*b*log(c))^2)*x^3)/(64*(b^4*cos(4*b*log(c))^2 + b^4*sin(4*b*log
(c))^2)*n^4 + 180*(b^2*cos(4*b*log(c))^2 + b^2*sin(4*b*log(c))^2)*n^2 + 81*
cos(4*b*log(c))^2 + 81*sin(4*b*log(c))^2)
```

Fricas [A]
time = 1.30, size = 178, normalized size = 0.88

$$\frac{3\,(4\,b^2n^2+9)x^3\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^4-6\,(10\,b^2n^2+9)x^3\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+(8\,b^4n^4+48\,b^2n^2+27)x^3+4\,\left((4\,b^3n^3+9\,bn)x^3\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^3-(10\,b^3n^3+9\,bn)x^3\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{64\,b^4n^4+180\,b^2n^2+81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] (3*(4*b^2*n^2 + 9)*x^3*cos(b*n*log(x) + b*log(c) + a)^4 - 6*(10*b^2*n^2 + 9
)*x^3*cos(b*n*log(x) + b*log(c) + a)^2 + (8*b^4*n^4 + 48*b^2*n^2 + 27)*x^3
+ 4*((4*b^3*n^3 + 9*b*n)*x^3*cos(b*n*log(x) + b*log(c) + a)^3 - (10*b^3*n^3
 + 9*b*n)*x^3*cos(b*n*log(x) + b*log(c) + a))*sin(b*n*log(x) + b*log(c) + a
))/(64*b^4*n^4 + 180*b^2*n^2 + 81)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(a+b*ln(c*x**n))**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 17035 vs. $2(202) = 404$.
time = 1.21, size = 17035, normalized size = 84.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] 1/8*x^3 + 1/16*(256*b^3*n^3*x^3*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - p
i*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log
(abs(c)))^2*tan(2*a)^2*tan(a) + 256*b^3*n^3*x^3*e^(-pi*b*n*sgn(x) + pi*b*n
- pi*b*sgn(c) + pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*lo
g(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a) - 32*b^3*n^3*x^3*e^(2*pi*b*n
*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*lo
g(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)*tan(a)^2 - 32*
b^3*n^3*x^3*e^(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c) + 2*pi*b)*tan(2*
b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2
*tan(2*a)*tan(a)^2 + 256*b^3*n^3*x^3*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c
) - pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) +
b*log(abs(c)))*tan(2*a)^2*tan(a)^2 + 256*b^3*n^3*x^3*e^(-pi*b*n*sgn(x) + pi
*b*n - pi*b*sgn(c) + pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b
*n*log(abs(x)) + b*log(abs(c)))*tan(2*a)^2*tan(a)^2 - 32*b^3*n^3*x^3*e^(2*p
i*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b*n*log(abs(x)) + 2
*b*log(abs(c)))*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a)^2
- 32*b^3*n^3*x^3*e^(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c) + 2*pi*b)*t
an(2*b*n*log(abs(x)) + 2*b*log(abs(c)))*tan(b*n*log(abs(x)) + b*log(abs(c))
)^2*tan(2*a)^2*tan(a)^2 + 12*b^2*n^2*x^3*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*
pi*b*sgn(c) - 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*lo
g(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a)^2 - 192*b^2*n^2*x^3*e^(pi*b*
n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs
(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a)^2 - 192*b^
2*n^2*x^3*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(2*b*n*log(ab
s(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^
2*tan(a)^2 + 12*b^2*n^2*x^3*e^(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c)
```

```
+ 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) +
b*log(abs(c)))^2*tan(2*a)^2*tan(a)^2 - 32*b^3*n^3*x^3*e^(2*pi*b*n*sgn(x) -
2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))
^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a) - 32*b^3*n^3*x^3*e^(-2*p
i*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c) + 2*pi*b)*tan(2*b*n*log(abs(x)) + 2
*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a) - 256*b^3
*n^3*x^3*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(2*b*n*log(abs(
x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(2*a)^2 -
256*b^3*n^3*x^3*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(2*b*n*
log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(2
*a)^2 - 32*b^3*n^3*x^3*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi
*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))*tan(b*n*log(abs(x)) + b*log(ab
s(c)))^2*tan(2*a)^2 - 32*b^3*n^3*x^3*e^(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*
b*sgn(c) + 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))*tan(b*n*log(abs
(x)) + b*log(abs(c)))^2*tan(2*a)^2 + 256*b^3*n^3*x^3*e^(pi*b*n*sgn(x) - pi*
b*n + pi*b*sgn(c) - pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*
n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 256*b^3*n^3*x^3*e^(-pi*b*n*sgn(x)
 + pi*b*n - pi*b*sgn(c) + pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*
tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) - 256*b^3*n^3*x^3*e^(pi*b*n*s
gn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)
))^2*tan(2*a)^2*tan(a) - 256*b^3*n^3*x^3*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*
sgn(c) + pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(2*a)^2*tan(a)
 + 256*b^3*n^3*x^3*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*
log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a) + 256*b^3*n^3*x^3*e^(-pi*b
*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)
))^2*tan(2*a)^2*tan(a) + 256*b^3*n^3*x^3*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*s
gn(c) - pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)
) + b*log(abs(c)))*tan(a)^2 + 256*b^3*n^3*x^3*e^(-pi*b*n*sgn(x) + pi*b*n -
pi*b*sgn(c) + pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(
abs(x)) + b*log(abs(c)))*tan(a)^2 + 32*b^3*n^3*x^3*e^(2*pi*b*n*sgn(x) - 2*p
i*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))*ta
n(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 + 32*b^3*n^3*x^3*e^(-2*pi*b*n
*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c) + 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*lo
g(abs(c)))*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 - 32*b^3*n^3*x^3
*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b*n*log(abs(
x)) + 2*b*log(abs(c)))^2*tan(2*a)*tan(a)^2 - 32*b^3*n^3*x^3*e^(-2*pi*b*n*sg
n(x) + 2*pi*b*n - 2*pi*b*sgn(c) + 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(a
bs(c)))^2*tan(2*a)*tan(a)^2 + 32*b^3*n^3*x^3*e^(2*pi*b*n*sgn(x) - 2*pi*b*n
+ 2*pi*b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)*t
an(a)^2 + 32*b^3*n^3*x^3*e^(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c) + 2
*pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*t...
```

Mupad [B]

time = 3.12, size = 127, normalized size = 0.63

$$\frac{x^3}{8}-\frac{x^3\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{(c\,x^n)^{b\,2\mathrm{i}}}\,1\mathrm{i}}{8\,b\,n+12\mathrm{i}}-\frac{x^3\,\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}}}{12+b\,n\,8\mathrm{i}}+\frac{x^3\,\mathrm{e}^{-a\,4\mathrm{i}}\,\frac{1}{(c\,x^n)^{b\,4\mathrm{i}}}\,1\mathrm{i}}{64\,b\,n+48\mathrm{i}}+\frac{x^3\,\mathrm{e}^{a\,4\mathrm{i}}\,(c\,x^n)^{b\,4\mathrm{i}}}{48+b\,n\,64\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a + b*log(c*x^n))^4,x)
```

```
[Out] x^3/8 - (x^3*exp(-a*2i)/(c*x^n)^(b*2i)*1i)/(8*b*n + 12i) - (x^3*exp(a*2i)*(
c*x^n)^(b*2i))/(b*n*8i + 12) + (x^3*exp(-a*4i)/(c*x^n)^(b*4i)*1i)/(64*b*n +
 48i) + (x^3*exp(a*4i)*(c*x^n)^(b*4i))/(b*n*64i + 48)
```

3.20 $\int x \sin^4(a + b \log(cx^n)) \, dx$

Optimal. Leaf size=210

$$\frac{3b^4n^4x^2}{4(1+5b^2n^2+4b^4n^4)} - \frac{3b^3n^3x^2\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{2(1+5b^2n^2+4b^4n^4)} + \frac{3b^2n^2x^2\sin^2(a+b\log(cx^n))}{2(1+5b^2n^2+4b^4n^4)} - bnx$$

```
[Out] 3/4*b^4*n^4*x^2/(4*b^4*n^4+5*b^2*n^2+1)-3/2*b^3*n^3*x^2*cos(a+b*ln(c*x^n))*
sin(a+b*ln(c*x^n))/(4*b^4*n^4+5*b^2*n^2+1)+3/2*b^2*n^2*x^2*sin(a+b*ln(c*x^n
))^2/(4*b^4*n^4+5*b^2*n^2+1)-b*n*x^2*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))^
3/(4*b^2*n^2+1)+1/2*x^2*sin(a+b*ln(c*x^n))^4/(4*b^2*n^2+1)
```

Rubi [A]
time = 0.04, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4575, 30}

$$\frac{x^2\sin^4(a+b\log(cx^n))}{2(4b^2n^2+1)} - \frac{bnx^2\sin^3(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+1} + \frac{3b^2n^2x^2\sin^2(a+b\log(cx^n))}{2(4b^4n^4+5b^2n^2+1)} - \frac{3b^3n^3x^2\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{2(4b^4n^4+5b^2n^2+1)} + \frac{3b^4n^4x^2}{4(4b^4n^4+5b^2n^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (3*b^4*n^4*x^2)/(4*(1 + 5*b^2*n^2 + 4*b^4*n^4)) - (3*b^3*n^3*x^2*Cos[a + b*
Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/(2*(1 + 5*b^2*n^2 + 4*b^4*n^4)) + (3*b^2
*n^2*x^2*Sin[a + b*Log[c*x^n]]^2)/(2*(1 + 5*b^2*n^2 + 4*b^4*n^4)) - (b*n*x^
2*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^3)/(1 + 4*b^2*n^2) + (x^2*Sin
[a + b*Log[c*x^n]]^4)/(2*(1 + 4*b^2*n^2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4575

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2
*n^2*p^2 + (m + 1)^2)), Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x],
x] - Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a + b*Log[c*x^n])]*(Sin[d*(a + b*Log
[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x]) /; FreeQ[{a, b, c
, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int x \sin^4\left(a+b\log\left(cx^n\right)\right)\,dx = -\frac{bnx^2\cos\left(a+b\log\left(cx^n\right)\right)\sin^3\left(a+b\log\left(cx^n\right)\right)}{1+4b^2n^2}+\frac{x^2\sin^4\left(a+b\log\left(cx^n\right)\right)}{2\left(1+4b^2n^2\right)}-$$
$$= -\frac{3b^3n^3x^2\cos\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\left(cx^n\right)\right)}{2\left(1+5b^2n^2+4b^4n^4\right)}+\frac{3b^2n^2x^2\sin^2\left(a+b\log\right.}{2\left(1+5b^2n^2+4b^4n\right.}$$
$$= \frac{3b^4n^4x^2}{4\left(1+5b^2n^2+4b^4n^4\right)}-\frac{3b^3n^3x^2\cos\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\left(cx^n\right)\right)}{2\left(1+5b^2n^2+4b^4n^4\right)}+$$

Mathematica [A]
time = 0.49, size = 169, normalized size = 0.80

$$\frac{x^2\left(3+15b^2n^2+12b^4n^4-4\left(1+4b^2n^2\right)\cos\left(2\left(a+b\log\left(cx^n\right)\right)\right)+\left(1+b^2n^2\right)\cos\left(4\left(a+b\log\left(cx^n\right)\right)\right)-4bn\sin\left(2\left(a+b\log\left(cx^n\right)\right)\right)-16b^3n^3\sin\left(2\left(a+b\log\left(cx^n\right)\right)\right)+2bn\sin\left(4\left(a+b\log\left(cx^n\right)\right)\right)+2b^3n^3\sin\left(4\left(a+b\log\left(cx^n\right)\right)\right)\right)}{16\left(1+5b^2n^2+4b^4n^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x^2*(3 + 15*b^2*n^2 + 12*b^4*n^4 - 4*(1 + 4*b^2*n^2)*Cos[2*(a + b*Log[c*x^
n])] + (1 + b^2*n^2)*Cos[4*(a + b*Log[c*x^n])] - 4*b*n*Sin[2*(a + b*Log[c*x
^n])] - 16*b^3*n^3*Sin[2*(a + b*Log[c*x^n])] + 2*b*n*Sin[4*(a + b*Log[c*x^n
])] + 2*b^3*n^3*Sin[4*(a + b*Log[c*x^n])]))/(16*(1 + 5*b^2*n^2 + 4*b^4*n^4)
)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int x\left(\sin^4\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a+b*ln(c*x^n))^4,x)
```

```
[Out] int(x*sin(a+b*ln(c*x^n))^4,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1085 vs. $2(202) = 404$.
time = 0.32, size = 1085, normalized size = 5.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

```
[Out] 1/32*((2*(b^3*cos(4*b*log(c))*sin(8*b*log(c)) - b^3*cos(8*b*log(c))*sin(4*b
*log(c)) + b^3*sin(4*b*log(c)))*n^3 + (b^2*cos(8*b*log(c))*cos(4*b*log(c))
```

```
+ b^2*sin(8*b*log(c))*sin(4*b*log(c)) + b^2*cos(4*b*log(c)))*n^2 + 2*(b*cos
(4*b*log(c))*sin(8*b*log(c)) - b*cos(8*b*log(c))*sin(4*b*log(c)) + b*sin(4*
b*log(c)))*n + cos(8*b*log(c))*cos(4*b*log(c)) + sin(8*b*log(c))*sin(4*b*lo
g(c)) + cos(4*b*log(c)))*x^2*cos(4*b*log(x^n) + 4*a) - 4*(4*(b^3*cos(4*b*lo
g(c))*sin(6*b*log(c)) - b^3*cos(6*b*log(c))*sin(4*b*log(c)) + b^3*cos(2*b*l
og(c))*sin(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(2*b*log(c)))*n^3 + 4*(b^2*
cos(6*b*log(c))*cos(4*b*log(c)) + b^2*cos(4*b*log(c))*cos(2*b*log(c)) + b^2
*sin(6*b*log(c))*sin(4*b*log(c)) + b^2*sin(4*b*log(c))*sin(2*b*log(c)))*n^2
 + (b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b*log(c)) +
 b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*n +
 cos(6*b*log(c))*cos(4*b*log(c)) + cos(4*b*log(c))*cos(2*b*log(c)) + sin(6*
b*log(c))*sin(4*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*x^2*cos(2*b*lo
g(x^n) + 2*a) + (2*(b^3*cos(8*b*log(c))*cos(4*b*log(c)) + b^3*sin(8*b*log(c
))*sin(4*b*log(c)) + b^3*cos(4*b*log(c)))*n^3 - (b^2*cos(4*b*log(c))*sin(8*
b*log(c)) - b^2*cos(8*b*log(c))*sin(4*b*log(c)) + b^2*sin(4*b*log(c)))*n^2
+ 2*(b*cos(8*b*log(c))*cos(4*b*log(c)) + b*sin(8*b*log(c))*sin(4*b*log(c))
+ b*cos(4*b*log(c)))*n - cos(4*b*log(c))*sin(8*b*log(c)) + cos(8*b*log(c))*
sin(4*b*log(c)) - sin(4*b*log(c)))*x^2*sin(4*b*log(x^n) + 4*a) - 4*(4*(b^3*
cos(6*b*log(c))*cos(4*b*log(c)) + b^3*cos(4*b*log(c))*cos(2*b*log(c)) + b^3
*sin(6*b*log(c))*sin(4*b*log(c)) + b^3*sin(4*b*log(c))*sin(2*b*log(c)))*n^3
 - 4*(b^2*cos(4*b*log(c))*sin(6*b*log(c)) - b^2*cos(6*b*log(c))*sin(4*b*log
(c)) + b^2*cos(2*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(2*b*lo
g(c)))*n^2 + (b*cos(6*b*log(c))*cos(4*b*log(c)) + b*cos(4*b*log(c))*cos(2*b
*log(c)) + b*sin(6*b*log(c))*sin(4*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*lo
g(c)))*n - cos(4*b*log(c))*sin(6*b*log(c)) + cos(6*b*log(c))*sin(4*b*log(c)
) - cos(2*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c))*sin(2*b*log(c)))*x^2*
sin(2*b*log(x^n) + 2*a) + 6*(4*(b^4*cos(4*b*log(c))^2 + b^4*sin(4*b*log(c))
^2)*n^4 + 5*(b^2*cos(4*b*log(c))^2 + b^2*sin(4*b*log(c))^2)*n^2 + cos(4*b*l
og(c))^2 + sin(4*b*log(c))^2)*x^2)/(4*(b^4*cos(4*b*log(c))^2 + b^4*sin(4*b*
log(c))^2)*n^4 + 5*(b^2*cos(4*b*log(c))^2 + b^2*sin(4*b*log(c))^2)*n^2 + co
s(4*b*log(c))^2 + sin(4*b*log(c))^2)
```

Fricas [A]
time = 1.36, size = 177, normalized size = 0.84

$$\frac{2\,(b^2n^2+1)x^2\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^4-2\,(5\,b^2n^2+2)x^2\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+(3\,b^4n^4+8\,b^2n^2+2)x^2+2\,\left(2\,(b^3n^3+bn)x^2\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^3-(5\,b^3n^3+2\,bn)x^2\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{4\,(4\,b^4n^4+5\,b^2n^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^2*n^2 + 1)*x^2*cos(b*n*log(x) + b*log(c) + a)^4 - 2*(5*b^2*n^2 +
2)*x^2*cos(b*n*log(x) + b*log(c) + a)^2 + (3*b^4*n^4 + 8*b^2*n^2 + 2)*x^2 +
 2*(2*(b^3*n^3 + b*n)*x^2*cos(b*n*log(x) + b*log(c) + a)^3 - (5*b^3*n^3 + 2
*b*n)*x^2*cos(b*n*log(x) + b*log(c) + a))*sin(b*n*log(x) + b*log(c) + a))/(
4*b^4*n^4 + 5*b^2*n^2 + 1)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*ln(c*x**n))**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 16984 vs. 2(202) = 404.
time = 1.02, size = 16984, normalized size = 80.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] 3/16*x^2 + 1/32*(32*b^3*n^3*x^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - p
i*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log
(abs(c)))^2*tan(2*a)^2*tan(a) + 32*b^3*n^3*x^2*e^(-pi*b*n*sgn(x) + pi*b*n -
 pi*b*sgn(c) + pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log
(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a) - 4*b^3*n^3*x^2*e^(2*pi*b*n*s
gn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(
abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)*tan(a)^2 - 4*b^3
*n^3*x^2*e^(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c) + 2*pi*b)*tan(2*b*n
*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*ta
n(2*a)*tan(a)^2 + 32*b^3*n^3*x^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) -
pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*lo
g(abs(c)))*tan(2*a)^2*tan(a)^2 + 32*b^3*n^3*x^2*e^(-pi*b*n*sgn(x) + pi*b*n
- pi*b*sgn(c) + pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*lo
g(abs(x)) + b*log(abs(c)))*tan(2*a)^2*tan(a)^2 - 4*b^3*n^3*x^2*e^(2*pi*b*n*
sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log
(abs(c)))*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a)^2 - 4*b^
3*n^3*x^2*e^(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c) + 2*pi*b)*tan(2*b*
n*log(abs(x)) + 2*b*log(abs(c)))*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan
(2*a)^2*tan(a)^2 + b^2*n^2*x^2*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c
) - 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x))
+ b*log(abs(c)))^2*tan(2*a)^2*tan(a)^2 - 16*b^2*n^2*x^2*e^(pi*b*n*sgn(x) -
pi*b*n + pi*b*sgn(c) - pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan
(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a)^2 - 16*b^2*n^2*x^2*e^
(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(2*b*n*log(abs(x)) + 2*b*
log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a)^2 +
 b^2*n^2*x^2*e^(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c) + 2*pi*b)*tan(2
```

```
*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^
2*tan(2*a)^2*tan(a)^2 - 4*b^3*n^3*x^2*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*
b*sgn(c) - 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(a
bs(x)) + b*log(abs(c)))^2*tan(2*a) - 4*b^3*n^3*x^2*e^(-2*pi*b*n*sgn(x) + 2*
pi*b*n - 2*pi*b*sgn(c) + 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2
*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a) - 32*b^3*n^3*x^2*e^(pi*b*n
*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(
c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(2*a)^2 - 32*b^3*n^3*x^2*e^(
-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(2*b*n*log(abs(x)) + 2*b*l
og(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(2*a)^2 - 4*b^3*n^3*x
^2*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b*n*log(ab
s(x)) + 2*b*log(abs(c)))*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2
- 4*b^3*n^3*x^2*e^(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c) + 2*pi*b)*ta
n(2*b*n*log(abs(x)) + 2*b*log(abs(c)))*tan(b*n*log(abs(x)) + b*log(abs(c)))
^2*tan(2*a)^2 + 32*b^3*n^3*x^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi
*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(
abs(c)))^2*tan(a) + 32*b^3*n^3*x^2*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c)
 + pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b
*log(abs(c)))^2*tan(a) - 32*b^3*n^3*x^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sg
n(c) - pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(2*a)^2*tan(a) -
 32*b^3*n^3*x^2*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(2*b*n*
log(abs(x)) + 2*b*log(abs(c)))^2*tan(2*a)^2*tan(a) + 32*b^3*n^3*x^2*e^(pi*b
*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)
))^2*tan(2*a)^2*tan(a) + 32*b^3*n^3*x^2*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*s
gn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a) + 32
*b^3*n^3*x^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(2*b*n*log(
abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a)^2
+ 32*b^3*n^3*x^2*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(2*b*n
*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(
a)^2 + 4*b^3*n^3*x^2*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b
)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))*tan(b*n*log(abs(x)) + b*log(abs(
c)))^2*tan(a)^2 + 4*b^3*n^3*x^2*e^(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn
(c) + 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))*tan(b*n*log(abs(x))
+ b*log(abs(c)))^2*tan(a)^2 - 4*b^3*n^3*x^2*e^(2*pi*b*n*sgn(x) - 2*pi*b*n +
 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(2*a
)*tan(a)^2 - 4*b^3*n^3*x^2*e^(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c) +
 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(2*a)*tan(a)^2 + 4*b
^3*n^3*x^2*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(b*n*
log(abs(x)) + b*log(abs(c)))^2*tan(2*a)*tan(a)^2 + 4*b^3*n^3*x^2*e^(-2*pi*b
*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c) + 2*pi*b)*tan(b*n*log(abs(x)) + b*log(
abs(c)))^2*tan(2*a)*tan(a)^2 - 4*b^3*n^3*x^2*e^...
```

Mupad [B]

time = 3.04, size = 127, normalized size = 0.60

$$\frac{3\,x^2}{16}-\frac{x^2\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{(c\,x^n)^{b\,2\mathrm{i}}}\,1\mathrm{i}}{8\,b\,n+8\mathrm{i}}-\frac{x^2\,\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}}}{8+b\,n\,8\mathrm{i}}+\frac{x^2\,\mathrm{e}^{-a\,4\mathrm{i}}\,\frac{1}{(c\,x^n)^{b\,4\mathrm{i}}}\,1\mathrm{i}}{64\,b\,n+32\mathrm{i}}+\frac{x^2\,\mathrm{e}^{a\,4\mathrm{i}}\,(c\,x^n)^{b\,4\mathrm{i}}}{32+b\,n\,64\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b*log(c*x^n))^4,x)
```

```
[Out] (3*x^2)/16 - (x^2*exp(-a*2i)/(c*x^n)^(b*2i)*1i)/(8*b*n + 8i) - (x^2*exp(a*2
i)*(c*x^n)^(b*2i))/(b*n*8i + 8) + (x^2*exp(-a*4i)/(c*x^n)^(b*4i)*1i)/(64*b*
n + 32i) + (x^2*exp(a*4i)*(c*x^n)^(b*4i))/(b*n*64i + 32)
```

3.21 $\int \sin^4(a + b\log(cx^n))\,dx$

Optimal. Leaf size=191

$$\frac{24b^4n^4x}{1+20b^2n^2+64b^4n^4} - \frac{24b^3n^3x\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{1+20b^2n^2+64b^4n^4} + \frac{12b^2n^2x\sin^2(a+b\log(cx^n))}{1+20b^2n^2+64b^4n^4} - 4bnx$$

```
[Out] 24*b^4*n^4*x/(64*b^4*n^4+20*b^2*n^2+1)-24*b^3*n^3*x*cos(a+b*ln(c*x^n))*sin(
a+b*ln(c*x^n))/(64*b^4*n^4+20*b^2*n^2+1)+12*b^2*n^2*x*sin(a+b*ln(c*x^n))^2/
(64*b^4*n^4+20*b^2*n^2+1)-4*b*n*x*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))^3/(
16*b^2*n^2+1)+x*sin(a+b*ln(c*x^n))^4/(16*b^2*n^2+1)
```

Rubi [A]
time = 0.03, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4565, 8}

$$\frac{x\sin^4(a+b\log(cx^n))}{16b^2n^2+1} - \frac{4bnx\sin^3(a+b\log(cx^n))\cos(a+b\log(cx^n))}{16b^2n^2+1} + \frac{12b^2n^2x\sin^2(a+b\log(cx^n))}{64b^4n^4+20b^2n^2+1} - \frac{24b^3n^3x\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{64b^4n^4+20b^2n^2+1} + \frac{24b^4n^4x}{64b^4n^4+20b^2n^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (24*b^4*n^4*x)/(1 + 20*b^2*n^2 + 64*b^4*n^4) - (24*b^3*n^3*x*Cos[a + b*Log[
c*x^n]]*Sin[a + b*Log[c*x^n]])/(1 + 20*b^2*n^2 + 64*b^4*n^4) + (12*b^2*n^2*
x*Sin[a + b*Log[c*x^n]]^2)/(1 + 20*b^2*n^2 + 64*b^4*n^4) - (4*b*n*x*Cos[a +
 b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^3)/(1 + 16*b^2*n^2) + (x*Sin[a + b*Log
[c*x^n]]^4)/(1 + 16*b^2*n^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4565

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Sim
p[x*(Sin[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 + 1)), x] + (Dist[b^2*d^2
*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + 1)), Int[Sin[d*(a + b*Log[c*x^n])]^(p -
2), x], x] - Simp[b*d*n*p*x*Cos[d*(a + b*Log[c*x^n])]*(Sin[d*(a + b*Log[c*x
^n])]^(p - 1)/(b^2*d^2*n^2*p^2 + 1)), x]) /; FreeQ[{a, b, c, d, n}, x] && I
GtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^4\left(a+b\log\left(cx^n\right)\right)\,dx &= -\frac{4bnx\cos\left(a+b\log\left(cx^n\right)\right)\sin^3\left(a+b\log\left(cx^n\right)\right)}{1+16b^2n^2}+\frac{x\sin^4\left(a+b\log\left(cx^n\right)\right)}{1+16b^2n^2}+\\ &= -\frac{24b^3n^3x\cos\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\left(cx^n\right)\right)}{1+20b^2n^2+64b^4n^4}+\frac{12b^2n^2x\sin^2\left(a+b\log\left(c\right.\right.}{1+20b^2n^2+64b^4n}\\ &= \frac{24b^4n^4x}{1+20b^2n^2+64b^4n^4}-\frac{24b^3n^3x\cos\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\left(cx^n\right)\right)}{1+20b^2n^2+64b^4n^4}+\frac{12}{}\end{aligned}$$

Mathematica [A]
time = 0.45, size = 168, normalized size = 0.88

$$\frac{x(3+60b^2n^2+192b^4n^4-4(1+16b^2n^2)\cos\left(2(a+b\log\left(cx^n\right))\right)+(1+4b^2n^2)\cos\left(4(a+b\log\left(cx^n\right))\right)-8bn\sin\left(2(a+b\log\left(cx^n\right))\right)-128b^3n^3\sin\left(2(a+b\log\left(cx^n\right))\right)+4bn\sin\left(4(a+b\log\left(cx^n\right))\right)+16b^3n^3\sin\left(4(a+b\log\left(cx^n\right))\right))}{8\left(1+20b^2n^2+64b^4n^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x*(3 + 60*b^2*n^2 + 192*b^4*n^4 - 4*(1 + 16*b^2*n^2)*Cos[2*(a + b*Log[c*x^
n])] + (1 + 4*b^2*n^2)*Cos[4*(a + b*Log[c*x^n])] - 8*b*n*Sin[2*(a + b*Log[c
*x^n])] - 128*b^3*n^3*Sin[2*(a + b*Log[c*x^n])] + 4*b*n*Sin[4*(a + b*Log[c*
x^n])] + 16*b^3*n^3*Sin[4*(a + b*Log[c*x^n])]))/(8*(1 + 20*b^2*n^2 + 64*b^4
*n^4))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \sin^4\left(a+b\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))^4,x)
```

```
[Out] int(sin(a+b*ln(c*x^n))^4,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1078 vs. $2(191) = 382$.
time = 0.34, size = 1078, normalized size = 5.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

```
[Out] 1/16*((16*(b^3*cos(4*b*log(c))*sin(8*b*log(c)) - b^3*cos(8*b*log(c))*sin(4*
b*log(c)) + b^3*sin(4*b*log(c)))*n^3 + 4*(b^2*cos(8*b*log(c))*cos(4*b*log(c
```

```
)) + b^2*sin(8*b*log(c))*sin(4*b*log(c)) + b^2*cos(4*b*log(c)))*n^2 + 4*(b*
cos(4*b*log(c))*sin(8*b*log(c)) - b*cos(8*b*log(c))*sin(4*b*log(c)) + b*sin
(4*b*log(c)))*n + cos(8*b*log(c))*cos(4*b*log(c)) + sin(8*b*log(c))*sin(4*b
*log(c)) + cos(4*b*log(c)))*x*cos(4*b*log(x^n) + 4*a) - 4*(32*(b^3*cos(4*b*
log(c))*sin(6*b*log(c)) - b^3*cos(6*b*log(c))*sin(4*b*log(c)) + b^3*cos(2*b
*log(c))*sin(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(2*b*log(c)))*n^3 + 16*(b
^2*cos(6*b*log(c))*cos(4*b*log(c)) + b^2*cos(4*b*log(c))*cos(2*b*log(c)) +
b^2*sin(6*b*log(c))*sin(4*b*log(c)) + b^2*sin(4*b*log(c))*sin(2*b*log(c)))*
n^2 + 2*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b*log(
c)) + b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c))
)*n + cos(6*b*log(c))*cos(4*b*log(c)) + cos(4*b*log(c))*cos(2*b*log(c)) + s
in(6*b*log(c))*sin(4*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*x*cos(2*b
*log(x^n) + 2*a) + (16*(b^3*cos(8*b*log(c))*cos(4*b*log(c)) + b^3*sin(8*b*l
og(c))*sin(4*b*log(c)) + b^3*cos(4*b*log(c)))*n^3 - 4*(b^2*cos(4*b*log(c))*
sin(8*b*log(c)) - b^2*cos(8*b*log(c))*sin(4*b*log(c)) + b^2*sin(4*b*log(c))
)*n^2 + 4*(b*cos(8*b*log(c))*cos(4*b*log(c)) + b*sin(8*b*log(c))*sin(4*b*lo
g(c)) + b*cos(4*b*log(c)))*n - cos(4*b*log(c))*sin(8*b*log(c)) + cos(8*b*lo
g(c))*sin(4*b*log(c)) - sin(4*b*log(c)))*x*sin(4*b*log(x^n) + 4*a) - 4*(32*
(b^3*cos(6*b*log(c))*cos(4*b*log(c)) + b^3*cos(4*b*log(c))*cos(2*b*log(c))
+ b^3*sin(6*b*log(c))*sin(4*b*log(c)) + b^3*sin(4*b*log(c))*sin(2*b*log(c))
)*n^3 - 16*(b^2*cos(4*b*log(c))*sin(6*b*log(c)) - b^2*cos(6*b*log(c))*sin(4
*b*log(c)) + b^2*cos(2*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(
2*b*log(c)))*n^2 + 2*(b*cos(6*b*log(c))*cos(4*b*log(c)) + b*cos(4*b*log(c))
*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(4*b*log(c)) + b*sin(4*b*log(c))*si
n(2*b*log(c)))*n - cos(4*b*log(c))*sin(6*b*log(c)) + cos(6*b*log(c))*sin(4*
b*log(c)) - cos(2*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c))*sin(2*b*log(c
)))*x*sin(2*b*log(x^n) + 2*a) + 6*(64*(b^4*cos(4*b*log(c))^2 + b^4*sin(4*b*
log(c))^2)*n^4 + 20*(b^2*cos(4*b*log(c))^2 + b^2*sin(4*b*log(c))^2)*n^2 + c
os(4*b*log(c))^2 + sin(4*b*log(c))^2)*x)/(64*(b^4*cos(4*b*log(c))^2 + b^4*s
in(4*b*log(c))^2)*n^4 + 20*(b^2*cos(4*b*log(c))^2 + b^2*sin(4*b*log(c))^2)*
n^2 + cos(4*b*log(c))^2 + sin(4*b*log(c))^2)
```

Fricas [A]
time = 1.40, size = 165, normalized size = 0.86

$$\frac{(4\,b^2n^2+1)x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^4-2\,(10\,b^2n^2+1)x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+(24\,b^4n^4+16\,b^2n^2+1)x+4\,\left((4\,b^3n^3+bn)x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^3-(10\,b^3n^3+bn)x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{64\,b^4n^4+20\,b^2n^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] ((4*b^2*n^2 + 1)*x*cos(b*n*log(x) + b*log(c) + a)^4 - 2*(10*b^2*n^2 + 1)*x*
cos(b*n*log(x) + b*log(c) + a)^2 + (24*b^4*n^4 + 16*b^2*n^2 + 1)*x + 4*((4*
b^3*n^3 + b*n)*x*cos(b*n*log(x) + b*log(c) + a)^3 - (10*b^3*n^3 + b*n)*x*co
s(b*n*log(x) + b*log(c) + a))*sin(b*n*log(x) + b*log(c) + a))/(64*b^4*n^4 +
 20*b^2*n^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \sin^4\left(a - \frac{i\log(cx^n)}{2n}\right) dx & \text{for } b = -\frac{i}{2n} \\ \int \sin^4\left(a - \frac{i\log(cx^n)}{4n}\right) dx & \text{for } b = -\frac{i}{4n} \\ \int \sin^4\left(a + \frac{i\log(cx^n)}{4n}\right) dx & \text{for } b = \frac{i}{4n} \\ \int \sin^4\left(a + \frac{i\log(cx^n)}{2n}\right) dx & \text{for } b = \frac{i}{2n} \\ \frac{24b^4n^4x\sin^4(a+b\log(cx^n))}{64b^4n^4+20b^2n^2+1} + \frac{48b^4n^4x\sin^2(a+b\log(cx^n))\cos^2(a+b\log(cx^n))}{64b^4n^4+20b^2n^2+1} + \frac{24b^4n^4x\cos^4(a+b\log(cx^n))}{64b^4n^4+20b^2n^2+1} - \frac{40b^3n^3x\sin^3(a+b\log(cx^n))\cos(a+b\log(cx^n))}{64b^4n^4+20b^2n^2+1} - \frac{24b^3n^3x\sin(a+b\log(cx^n))\cos^3(a+b\log(cx^n))}{64b^4n^4+20b^2n^2+1} + \frac{16b^2n^2x\sin^4(a+b\log(cx^n))}{64b^4n^4+20b^2n^2+1} + \frac{12b^2n^2x\sin^2(a+b\log(cx^n))\cos^2(a+b\log(cx^n))}{64b^4n^4+20b^2n^2+1} - \frac{4bnx\sin^3(a+b\log(cx^n))\cos(a+b\log(cx^n))}{64b^4n^4+20b^2n^2+1} + \frac{x\sin^4(a+b\log(cx^n))}{64b^4n^4+20b^2n^2+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**4,x)
```

```
[Out] Piecewise((Integral(sin(a - I*log(c*x**n)/(2*n))**4, x), Eq(b, -I/(2*n))),
(Integral(sin(a - I*log(c*x**n)/(4*n))**4, x), Eq(b, -I/(4*n))), (Integral(
sin(a + I*log(c*x**n)/(4*n))**4, x), Eq(b, I/(4*n))), (Integral(sin(a + I*l
og(c*x**n)/(2*n))**4, x), Eq(b, I/(2*n))), (24*b**4*n**4*x*sin(a + b*log(c*
x**n))**4/(64*b**4*n**4 + 20*b**2*n**2 + 1) + 48*b**4*n**4*x*sin(a + b*log(
c*x**n))**2*cos(a + b*log(c*x**n))**2/(64*b**4*n**4 + 20*b**2*n**2 + 1) + 2
4*b**4*n**4*x*cos(a + b*log(c*x**n))**4/(64*b**4*n**4 + 20*b**2*n**2 + 1) -
 40*b**3*n**3*x*sin(a + b*log(c*x**n))**3*cos(a + b*log(c*x**n))/(64*b**4*n
**4 + 20*b**2*n**2 + 1) - 24*b**3*n**3*x*sin(a + b*log(c*x**n))*cos(a + b*l
og(c*x**n))**3/(64*b**4*n**4 + 20*b**2*n**2 + 1) + 16*b**2*n**2*x*sin(a + b
*log(c*x**n))**4/(64*b**4*n**4 + 20*b**2*n**2 + 1) + 12*b**2*n**2*x*sin(a +
 b*log(c*x**n))**2*cos(a + b*log(c*x**n))**2/(64*b**4*n**4 + 20*b**2*n**2 +
 1) - 4*b*n*x*sin(a + b*log(c*x**n))**3*cos(a + b*log(c*x**n))/(64*b**4*n**
4 + 20*b**2*n**2 + 1) + x*sin(a + b*log(c*x**n))**4/(64*b**4*n**4 + 20*b**2
*n**2 + 1), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 16422 vs. $2(191) = 382$.
time = 0.74, size = 16422, normalized size = 85.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/16*(256*b^3*n^3*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)
*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs
(c)))^2*tan(2*a)^2*tan(a) + 256*b^3*n^3*x*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b
*sgn(c) + pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(
x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a) - 32*b^3*n^3*x*e^(2*pi*b*n*sgn(x)
- 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)
))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)*tan(a)^2 - 32*b^3*n^3*
x*e^(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c) + 2*pi*b)*tan(2*b*n*log(ab
s(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)*
tan(a)^2 + 256*b^3*n^3*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*ta
```

```
n(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)
))*tan(2*a)^2*tan(a)^2 + 256*b^3*n^3*x*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sg
n(c) + pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x))
 + b*log(abs(c)))*tan(2*a)^2*tan(a)^2 - 32*b^3*n^3*x*e^(2*pi*b*n*sgn(x) - 2
*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))*
tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a)^2 - 32*b^3*n^3*x*e
^(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c) + 2*pi*b)*tan(2*b*n*log(abs(x
)) + 2*b*log(abs(c)))*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan
(a)^2 + 4*b^2*n^2*x*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)
*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs
(c)))^2*tan(2*a)^2*tan(a)^2 - 64*b^2*n^2*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b
*sgn(c) - pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(
x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a)^2 - 64*b^2*n^2*x*e^(-pi*b*n*sgn(x)
 + pi*b*n - pi*b*sgn(c) + pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*
tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a)^2 + 4*b^2*n^2*x*e^
(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c) + 2*pi*b)*tan(2*b*n*log(abs(x)
) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*ta
n(a)^2 - 32*b^3*n^3*x*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*
b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(a
bs(c)))^2*tan(2*a) - 32*b^3*n^3*x*e^(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*s
gn(c) + 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(
x)) + b*log(abs(c)))^2*tan(2*a) - 256*b^3*n^3*x*e^(pi*b*n*sgn(x) - pi*b*n +
 pi*b*sgn(c) - pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log
(abs(x)) + b*log(abs(c)))*tan(2*a)^2 - 256*b^3*n^3*x*e^(-pi*b*n*sgn(x) + pi
*b*n - pi*b*sgn(c) + pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b
*n*log(abs(x)) + b*log(abs(c)))*tan(2*a)^2 - 32*b^3*n^3*x*e^(2*pi*b*n*sgn(x
) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(
c)))*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2 - 32*b^3*n^3*x*e^(-2
*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c) + 2*pi*b)*tan(2*b*n*log(abs(x)) +
 2*b*log(abs(c)))*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2 + 256*b
^3*n^3*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(2*b*n*log(abs(
x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 25
6*b^3*n^3*x*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(2*b*n*log(
abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)
 - 256*b^3*n^3*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(2*b*n*l
og(abs(x)) + 2*b*log(abs(c)))^2*tan(2*a)^2*tan(a) - 256*b^3*n^3*x*e^(-pi*b*
n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs
(c)))^2*tan(2*a)^2*tan(a) + 256*b^3*n^3*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*
sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a) + 2
56*b^3*n^3*x*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(a
bs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a) + 256*b^3*n^3*x*e^(pi*b*n*sgn(x
) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2
*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a)^2 + 256*b^3*n^3*x*e^(-pi*b*n*s
gn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)
))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a)^2 + 32*b^3*n^3*x*e^(2*pi*b
```

```
*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*
log(abs(c)))*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 + 32*b^3*n^3*x
*e^(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c) + 2*pi*b)*tan(2*b*n*log(abs
(x)) + 2*b*log(abs(c)))*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 - 3
2*b^3*n^3*x*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b
*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(2*a)*tan(a)^2 - 32*b^3*n^3*x*e^(-2*
pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c) + 2*pi*b)*tan(2*b*n*log(abs(x)) +
2*b*log(abs(c)))^2*tan(2*a)*tan(a)^2 + 32*b^3*n^3*x*e^(2*pi*b*n*sgn(x) - 2*
pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan
(2*a)*tan(a)^2 + 32*b^3*n^3*x*e^(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c
) + 2*pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)*tan(a)^2 - 32*b
^3*n^3*x*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b...
```

Mupad [B]
time = 2.86, size = 117, normalized size = 0.61

$$\frac{3\,x}{8}-\frac{x\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,2\mathrm{i}}}\,1\mathrm{i}}{8\,b\,n+4\mathrm{i}}-\frac{x\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}}{4+b\,n\,8\mathrm{i}}+\frac{x\,\mathrm{e}^{-a\,4\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,4\mathrm{i}}}\,1\mathrm{i}}{64\,b\,n+16\mathrm{i}}+\frac{x\,\mathrm{e}^{a\,4\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,4\mathrm{i}}}{16+b\,n\,64\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))^4,x)
```

```
[Out] (3*x)/8 - (x*exp(-a*2i)/(c*x^n)^(b*2i)*1i)/(8*b*n + 4i) - (x*exp(a*2i)*(c*x
^n)^(b*2i))/(b*n*8i + 4) + (x*exp(-a*4i)/(c*x^n)^(b*4i)*1i)/(64*b*n + 16i)
+ (x*exp(a*4i)*(c*x^n)^(b*4i))/(b*n*64i + 16)
```

3.22 $\int \frac{\sin^4(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=73

$$\frac{3\log(x)}{8}-\frac{3\cos\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\left(cx^n\right)\right)}{8bn}-\frac{\cos\left(a+b\log\left(cx^n\right)\right)\sin^3\left(a+b\log\left(cx^n\right)\right)}{4bn}$$

```
[Out] 3/8*ln(x)-3/8*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))/b/n-1/4*cos(a+b*ln(c*x^
n))*sin(a+b*ln(c*x^n))^3/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2715, 8}

$$-\frac{\sin^3\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{4bn}-\frac{3\sin\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{8bn}+\frac{3\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] (3*Log[x])/8 - (3*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/(8*b*n) - (C
os[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^3)/(4*b*n)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^4\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \sin^4(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\ &= -\frac{\cos\left(a+b\log\left(cx^n\right)\right)\sin^3\left(a+b\log\left(cx^n\right)\right)}{4bn}+\frac{3\text{Subst}\left(\int \sin^2(a+bx)\,dx,x,\log\left(\right.\right.}{4n}\\ &= -\frac{3\cos\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\left(cx^n\right)\right)}{8bn}-\frac{\cos\left(a+b\log\left(cx^n\right)\right)\sin^3\left(a+b\text{lo}\right.}{4bn}\\ &= \frac{3\log(x)}{8}-\frac{3\cos\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\left(cx^n\right)\right)}{8bn}-\frac{\cos\left(a+b\log\left(cx^n\right)\right)\text{sin}}{4bn}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 51, normalized size = 0.70

$$\frac{12(a+b\log(cx^n))-8\sin(2(a+b\log(cx^n)))+\sin(4(a+b\log(cx^n)))}{32bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] (12*(a + b*Log[c*x^n]) - 8*Sin[2*(a + b*Log[c*x^n])] + Sin[4*(a + b*Log[c*x
^n])])/(32*b*n)
```

Maple [A]
time = 0.05, size = 61, normalized size = 0.84

method	result	size
derivativedivides	$\frac{-\frac{\left(\sin^3(a+b\ln(cx^n))+\frac{3\sin(a+b\ln(cx^n))}{2}\right)\cos(a+b\ln(cx^n))}{4}+\frac{3b\ln(cx^n)}{8}+\frac{3a}{8}}{nb}$	61
default	$\frac{-\frac{\left(\sin^3(a+b\ln(cx^n))+\frac{3\sin(a+b\ln(cx^n))}{2}\right)\cos(a+b\ln(cx^n))}{4}+\frac{3b\ln(cx^n)}{8}+\frac{3a}{8}}{nb}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))^4/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(-1/4*(sin(a+b*ln(c*x^n))^3+3/2*sin(a+b*ln(c*x^n)))*cos(a+b*ln(c*x^n)
)+3/8*b*ln(c*x^n)+3/8*a)
```

Maxima [A]
time = 0.30, size = 93, normalized size = 1.27

$$\frac{12\,bn\log(x)+\cos(4\,b\log(x^n)+4\,a)\sin(4\,b\log(c))-8\,\cos(2\,b\log(x^n)+2\,a)\sin(2\,b\log(c))+\cos(4\,b\log(c))\sin(4\,b\log(x^n)+4\,a)-8\,\cos(2\,b\log(c))\sin(2\,b\log(x^n)+2\,a)}{32\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

```
[Out] 1/32*(12*b*n*log(x) + cos(4*b*log(x^n) + 4*a)*sin(4*b*log(c)) - 8*cos(2*b*l
og(x^n) + 2*a)*sin(2*b*log(c)) + cos(4*b*log(c))*sin(4*b*log(x^n) + 4*a) -
8*cos(2*b*log(c))*sin(2*b*log(x^n) + 2*a))/(b*n)
```

Fricas [A]
time = 1.22, size = 59, normalized size = 0.81

$$\frac{3\,bn\log(x)+\left(2\,\cos(bn\log(x)+b\log(c)+a)^3-5\,\cos(bn\log(x)+b\log(c)+a)\right)\sin(bn\log(x)+b\log(c)+a)}{8\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

```
[Out] 1/8*(3*b*n*log(x) + (2*cos(b*n*log(x) + b*log(c) + a)^3 - 5*cos(b*n*log(x)
+ b*log(c) + a))*sin(b*n*log(x) + b*log(c) + a))/(b*n)
```

Sympy [A]
time = 14.14, size = 100, normalized size = 1.37

$$-\frac{\begin{cases} \log{(x)}\cos{(2a)} & \text{for } b=0 \wedge (b=0 \vee n=0) \\ \log{(x)}\cos{(2a+2b\log{(c)})} & \text{for } n=0 \\ \frac{\sin{(2a+2b\log{(cx^n)})}}{2bn} & \text{otherwise} \end{cases}}{2}+\frac{\begin{cases} \log{(x)}\cos{(4a)} & \text{for } b=0 \wedge (b=0 \vee n=0) \\ \log{(x)}\cos{(4a+4b\log{(c)})} & \text{for } n=0 \\ \frac{\sin{(4a+4b\log{(cx^n)})}}{4bn} & \text{otherwise} \end{cases}}{8}+\frac{3\log{(x)}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] -Piecewise((log(x)*cos(2*a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*cos
(2*a + 2*b*log(c)), Eq(n, 0)), (sin(2*a + 2*b*log(c*x**n))/(2*b*n), True))/
2 + Piecewise((log(x)*cos(4*a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*
cos(4*a + 4*b*log(c)), Eq(n, 0)), (sin(4*a + 4*b*log(c*x**n))/(4*b*n), True
))/8 + 3*log(x)/8
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^4/x, x)
```

Mupad [B]
time = 2.58, size = 51, normalized size = 0.70

$$\frac{3\,\ln\left(x^n\right)}{8\,n}-\frac{\frac{\sin\left(2\,a+2\,b\,\ln\left(c\,x^n\right)\right)}{4}-\frac{\sin\left(4\,a+4\,b\,\ln\left(c\,x^n\right)\right)}{32}}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))^4/x,x)
```

```
[Out] (3*log(x^n))/(8*n) - (sin(2*a + 2*b*log(c*x^n))/4 - sin(4*a + 4*b*log(c*x^n
))/32)/(b*n)
```

3.23 $\int \frac{\sin^4(a+b\log(cx^n))}{x^2}\,dx$

Optimal. Leaf size=202

$$-\frac{24b^4n^4}{(1+20b^2n^2+64b^4n^4)\,x}-\frac{24b^3n^3\cos\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\left(cx^n\right)\right)}{(1+20b^2n^2+64b^4n^4)\,x}-\frac{12b^2n^2\sin^2\left(a+b\log\left(cx^n\right)\right)}{(1+20b^2n^2+64b^4n^4)\,x}-$$

```
[Out] -24*b^4*n^4/(64*b^4*n^4+20*b^2*n^2+1)/x-24*b^3*n^3*cos(a+b*ln(c*x^n))*sin(a
+b*ln(c*x^n))/(64*b^4*n^4+20*b^2*n^2+1)/x-12*b^2*n^2*sin(a+b*ln(c*x^n))^2/(
64*b^4*n^4+20*b^2*n^2+1)/x-4*b*n*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))^3/(1
6*b^2*n^2+1)/x-sin(a+b*ln(c*x^n))^4/(16*b^2*n^2+1)/x
```

Rubi [A]

time = 0.04, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4575, 30}

$$-\frac{\sin^4\left(a+b\log\left(cx^n\right)\right)}{x\left(16b^2n^2+1\right)}-\frac{4bn\sin^3\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{x\left(16b^2n^2+1\right)}-\frac{12b^2n^2\sin^2\left(a+b\log\left(cx^n\right)\right)}{x\left(64b^4n^4+20b^2n^2+1\right)}-\frac{24b^3n^3\sin\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{x\left(64b^4n^4+20b^2n^2+1\right)}-\frac{24b^4n^4}{x\left(64b^4n^4+20b^2n^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Log[c*x^n]]^4/x^2,x]
```

```
[Out] (-24*b^4*n^4)/((1 + 20*b^2*n^2 + 64*b^4*n^4)*x) - (24*b^3*n^3*Cos[a + b*Log
[c*x^n]]*Sin[a + b*Log[c*x^n]])/((1 + 20*b^2*n^2 + 64*b^4*n^4)*x) - (12*b^2
*n^2*Sin[a + b*Log[c*x^n]]^2)/((1 + 20*b^2*n^2 + 64*b^4*n^4)*x) - (4*b*n*Co
s[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^3)/((1 + 16*b^2*n^2)*x) - Sin[a +
 b*Log[c*x^n]]^4/((1 + 16*b^2*n^2)*x)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4575

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2
*n^2*p^2 + (m + 1)^2)), Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x],
x] - Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a + b*Log[c*x^n])]*(Sin[d*(a + b*Log
[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x]) /; FreeQ[{a, b, c
, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int \frac{\sin^4(a+b\log(cx^n))}{x^2}\,dx = -\frac{4bn\cos(a+b\log(cx^n))\sin^3(a+b\log(cx^n))}{(1+16b^2n^2)\,x} - \frac{\sin^4(a+b\log(cx^n))}{(1+16b^2n^2)\,x} + \frac{(12b^2}{}$$

$$= -\frac{24b^3n^3\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{(1+20b^2n^2+64b^4n^4)\,x} - \frac{12b^2n^2\sin^2(a+b\log(cx^n)}{(1+20b^2n^2+64b^4n^4)\,x}$$

$$= -\frac{24b^4n^4}{(1+20b^2n^2+64b^4n^4)\,x} - \frac{24b^3n^3\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{(1+20b^2n^2+64b^4n^4)\,x} -$$

Mathematica [A]
time = 0.57, size = 170, normalized size = 0.84

$$-\frac{3+60b^2n^2+192b^4n^4-4(1+16b^2n^2)\cos(2(a+b\log(cx^n)))+(1+4b^2n^2)\cos(4(a+b\log(cx^n)))+8bn\sin(2(a+b\log(cx^n)))+128b^3n^3\sin(2(a+b\log(cx^n)))-4bn\sin(4(a+b\log(cx^n)))-16b^3n^3\sin(4(a+b\log(cx^n)))}{8(1+20b^2n^2+64b^4n^4)\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]^4/x^2,x]
```

```
[Out] -1/8*(3 + 60*b^2*n^2 + 192*b^4*n^4 - 4*(1 + 16*b^2*n^2)*Cos[2*(a + b*Log[c*
x^n])] + (1 + 4*b^2*n^2)*Cos[4*(a + b*Log[c*x^n])] + 8*b*n*Sin[2*(a + b*Log
[c*x^n])] + 128*b^3*n^3*Sin[2*(a + b*Log[c*x^n])] - 4*b*n*Sin[4*(a + b*Log[
c*x^n])] - 16*b^3*n^3*Sin[4*(a + b*Log[c*x^n])])/((1 + 20*b^2*n^2 + 64*b^4*
n^4)*x)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(a+b\ln(c\,x^n))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))^4/x^2,x)
```

```
[Out] int(sin(a+b*ln(c*x^n))^4/x^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1085 vs. 2(202) = 404.
time = 0.32, size = 1085, normalized size = 5.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^4/x^2,x, algorithm="maxima")
```

```
[Out] -1/16*(384*(b^4*cos(4*b*log(c))^2 + b^4*sin(4*b*log(c))^2)*n^4 + 120*(b^2*c
os(4*b*log(c))^2 + b^2*sin(4*b*log(c))^2)*n^2 + 6*cos(4*b*log(c))^2 - (16*(
b^3*cos(4*b*log(c))*sin(8*b*log(c)) - b^3*cos(8*b*log(c))*sin(4*b*log(c)) +
 b^3*sin(4*b*log(c)))*n^3 - 4*(b^2*cos(8*b*log(c))*cos(4*b*log(c)) + b^2*si
n(8*b*log(c))*sin(4*b*log(c)) + b^2*cos(4*b*log(c)))*n^2 + 4*(b*cos(4*b*log
(c))*sin(8*b*log(c)) - b*cos(8*b*log(c))*sin(4*b*log(c)) + b*sin(4*b*log(c)
))*n - cos(8*b*log(c))*cos(4*b*log(c)) - sin(8*b*log(c))*sin(4*b*log(c)) -
cos(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) + 4*(32*(b^3*cos(4*b*log(c))*sin(6
*b*log(c)) - b^3*cos(6*b*log(c))*sin(4*b*log(c)) + b^3*cos(2*b*log(c))*sin(
4*b*log(c)) - b^3*cos(4*b*log(c))*sin(2*b*log(c)))*n^3 - 16*(b^2*cos(6*b*lo
g(c))*cos(4*b*log(c)) + b^2*cos(4*b*log(c))*cos(2*b*log(c)) + b^2*sin(6*b*l
og(c))*sin(4*b*log(c)) + b^2*sin(4*b*log(c))*sin(2*b*log(c)))*n^2 + 2*(b*co
s(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b*log(c)) + b*cos(2
*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*n - cos(6*b
*log(c))*cos(4*b*log(c)) - cos(4*b*log(c))*cos(2*b*log(c)) - sin(6*b*log(c)
)*sin(4*b*log(c)) - sin(4*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a
) + 6*sin(4*b*log(c))^2 - (16*(b^3*cos(8*b*log(c))*cos(4*b*log(c)) + b^3*si
n(8*b*log(c))*sin(4*b*log(c)) + b^3*cos(4*b*log(c)))*n^3 + 4*(b^2*cos(4*b*l
og(c))*sin(8*b*log(c)) - b^2*cos(8*b*log(c))*sin(4*b*log(c)) + b^2*sin(4*b*
log(c)))*n^2 + 4*(b*cos(8*b*log(c))*cos(4*b*log(c)) + b*sin(8*b*log(c))*sin
(4*b*log(c)) + b*cos(4*b*log(c)))*n + cos(4*b*log(c))*sin(8*b*log(c)) - cos
(8*b*log(c))*sin(4*b*log(c)) + sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) + 4
*(32*(b^3*cos(6*b*log(c))*cos(4*b*log(c)) + b^3*cos(4*b*log(c))*cos(2*b*log
(c)) + b^3*sin(6*b*log(c))*sin(4*b*log(c)) + b^3*sin(4*b*log(c))*sin(2*b*lo
g(c)))*n^3 + 16*(b^2*cos(4*b*log(c))*sin(6*b*log(c)) - b^2*cos(6*b*log(c))*
sin(4*b*log(c)) + b^2*cos(2*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))
*sin(2*b*log(c)))*n^2 + 2*(b*cos(6*b*log(c))*cos(4*b*log(c)) + b*cos(4*b*lo
g(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(4*b*log(c)) + b*sin(4*b*log(c
))*sin(2*b*log(c)))*n + cos(4*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c))*s
in(4*b*log(c)) + cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*
log(c)))*sin(2*b*log(x^n) + 2*a))/((64*(b^4*cos(4*b*log(c))^2 + b^4*sin(4*b
*log(c))^2)*n^4 + 20*(b^2*cos(4*b*log(c))^2 + b^2*sin(4*b*log(c))^2)*n^2 +
cos(4*b*log(c))^2 + sin(4*b*log(c))^2)*x)
```

Fricas [A]
time = 1.01, size = 162, normalized size = 0.80

$$-\frac{24\,b^4n^4+(4\,b^2n^2+1)\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^4+16\,b^2n^2-2\,(10\,b^2n^2+1)\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2-4\,\left((4\,b^3n^3+bn)\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^3-(10\,b^3n^3+bn)\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+1}{(64\,b^4n^4+20\,b^2n^2+1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^4/x^2,x, algorithm="fricas")
```

```
[Out] -(24*b^4*n^4 + (4*b^2*n^2 + 1)*cos(b*n*log(x) + b*log(c) + a)^4 + 16*b^2*n^
2 - 2*(10*b^2*n^2 + 1)*cos(b*n*log(x) + b*log(c) + a)^2 - 4*((4*b^3*n^3 + b
*n)*cos(b*n*log(x) + b*log(c) + a)^3 - (10*b^3*n^3 + b*n)*cos(b*n*log(x) +
```

```
b*log(c) + a))*sin(b*n*log(x) + b*log(c) + a) + 1)/((64*b^4*n^4 + 20*b^2*n^
2 + 1)*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 147.19, size = 957, normalized size = 4.74

$$\begin{cases} \frac{i\sin\left(4a-\frac{2i\log\left(cx^{n}\right)}{n}\right)}{12x}+\frac{\cos\left(2a-\frac{i\log\left(cx^{n}\right)}{n}\right)}{4x}+\frac{\cos\left(4a-\frac{2i\log\left(cx^{n}\right)}{n}\right)}{24x}-\frac{3}{8x}-\frac{i\log\left(cx^{n}\right)\sin\left(2a-\frac{i\log\left(cx^{n}\right)}{n}\right)}{4nx}-\frac{\log\left(cx^{n}\right)\cos\left(2a-\frac{i\log\left(cx^{n}\right)}{n}\right)}{4nx} & \text{for}\ b=-\frac{i}{2n} \\ \frac{i\sin\left(2a-\frac{i\log\left(cx^{n}\right)}{2n}\right)}{3x}+\frac{i\sin\left(4a-\frac{i\log\left(cx^{n}\right)}{n}\right)}{16x}+\frac{2\cos\left(2a-\frac{i\log\left(cx^{n}\right)}{2n}\right)}{3x}-\frac{3}{8x}+\frac{i\log\left(cx^{n}\right)\sin\left(4a-\frac{i\log\left(cx^{n}\right)}{n}\right)}{16nx}+\frac{\log\left(cx^{n}\right)\cos\left(4a-\frac{i\log\left(cx^{n}\right)}{n}\right)}{16nx} & \text{for}\ b=-\frac{i}{4n} \\ -\frac{i\sin\left(2a+\frac{i\log\left(cx^{n}\right)}{2n}\right)}{3x}-\frac{i\sin\left(4a+\frac{i\log\left(cx^{n}\right)}{n}\right)}{16x}+\frac{2\cos\left(2a+\frac{i\log\left(cx^{n}\right)}{2n}\right)}{3x}-\frac{3}{8x}-\frac{i\log\left(cx^{n}\right)\sin\left(4a+\frac{i\log\left(cx^{n}\right)}{n}\right)}{16nx}+\frac{\log\left(cx^{n}\right)\cos\left(4a+\frac{i\log\left(cx^{n}\right)}{n}\right)}{16nx} & \text{for}\ b=\frac{i}{4n} \\ -\frac{i\sin\left(4a+\frac{2i\log\left(cx^{n}\right)}{n}\right)}{12x}+\frac{\cos\left(2a+\frac{i\log\left(cx^{n}\right)}{n}\right)}{4x}+\frac{\cos\left(4a+\frac{2i\log\left(cx^{n}\right)}{n}\right)}{24x}-\frac{3}{8x}+\frac{i\log\left(cx^{n}\right)\sin\left(2a+\frac{i\log\left(cx^{n}\right)}{n}\right)}{4nx}-\frac{\log\left(cx^{n}\right)\cos\left(2a+\frac{i\log\left(cx^{n}\right)}{n}\right)}{4nx} & \text{for}\ b=\frac{i}{2n} \\ -\frac{24b^{4}n^{4}\sin^{4}\left(a+b\log\left(cx^{n}\right)\right)}{64b^{4}n^{4}x+20b^{2}n^{2}x+x}-\frac{48b^{4}n^{4}\sin^{2}\left(a+b\log\left(cx^{n}\right)\right)\cos^{2}\left(a+b\log\left(cx^{n}\right)\right)}{64b^{4}n^{4}x+20b^{2}n^{2}x+x}-\frac{24b^{4}n^{4}\cos^{4}\left(a+b\log\left(cx^{n}\right)\right)}{64b^{4}n^{4}x+20b^{2}n^{2}x+x}-\frac{40b^{3}n^{3}\sin^{3}\left(a+b\log\left(cx^{n}\right)\right)\cos\left(a+b\log\left(cx^{n}\right)\right)}{64b^{4}n^{4}x+20b^{2}n^{2}x+x}-\frac{24b^{3}n^{3}\sin\left(a+b\log\left(cx^{n}\right)\right)\cos^{3}\left(a+b\log\left(cx^{n}\right)\right)}{64b^{4}n^{4}x+20b^{2}n^{2}x+x}-\frac{16b^{2}n^{2}\sin^{4}\left(a+b\log\left(cx^{n}\right)\right)}{64b^{4}n^{4}x+20b^{2}n^{2}x+x}-\frac{12b^{2}n^{2}\sin^{2}\left(a+b\log\left(cx^{n}\right)\right)\cos^{2}\left(a+b\log\left(cx^{n}\right)\right)}{64b^{4}n^{4}x+20b^{2}n^{2}x+x}-\frac{4bn\sin^{3}\left(a+b\log\left(cx^{n}\right)\right)\cos\left(a+b\log\left(cx^{n}\right)\right)}{64b^{4}n^{4}x+20b^{2}n^{2}x+x}-\frac{\sin^{4}\left(a+b\log\left(cx^{n}\right)\right)}{64b^{4}n^{4}x+20b^{2}n^{2}x+x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**4/x**2,x)
```

```
[Out] Piecewise((I*sin(4*a - 2*I*log(c*x**n)/n)/(12*x) + cos(2*a - I*log(c*x**n)/
n)/(4*x) + cos(4*a - 2*I*log(c*x**n)/n)/(24*x) - 3/(8*x) - I*log(c*x**n)*si
n(2*a - I*log(c*x**n)/n)/(4*n*x) - log(c*x**n)*cos(2*a - I*log(c*x**n)/n)/(
4*n*x), Eq(b, -I/(2*n))), (I*sin(2*a - I*log(c*x**n)/(2*n))/(3*x) + I*sin(4
*a - I*log(c*x**n)/n)/(16*x) + 2*cos(2*a - I*log(c*x**n)/(2*n))/(3*x) - 3/(
8*x) + I*log(c*x**n)*sin(4*a - I*log(c*x**n)/n)/(16*n*x) + log(c*x**n)*cos(
4*a - I*log(c*x**n)/n)/(16*n*x), Eq(b, -I/(4*n))), (-I*sin(2*a + I*log(c*x*
*n)/(2*n))/(3*x) - I*sin(4*a + I*log(c*x**n)/n)/(16*x) + 2*cos(2*a + I*log(
c*x**n)/(2*n))/(3*x) - 3/(8*x) - I*log(c*x**n)*sin(4*a + I*log(c*x**n)/n)/(
16*n*x) + log(c*x**n)*cos(4*a + I*log(c*x**n)/n)/(16*n*x), Eq(b, I/(4*n))),
 (-I*sin(4*a + 2*I*log(c*x**n)/n)/(12*x) + cos(2*a + I*log(c*x**n)/n)/(4*x)
 + cos(4*a + 2*I*log(c*x**n)/n)/(24*x) - 3/(8*x) + I*log(c*x**n)*sin(2*a +
I*log(c*x**n)/n)/(4*n*x) - log(c*x**n)*cos(2*a + I*log(c*x**n)/n)/(4*n*x),
Eq(b, I/(2*n))), (-24*b**4*n**4*sin(a + b*log(c*x**n))**4/(64*b**4*n**4*x +
 20*b**2*n**2*x + x) - 48*b**4*n**4*sin(a + b*log(c*x**n))**2*cos(a + b*log
(c*x**n))**2/(64*b**4*n**4*x + 20*b**2*n**2*x + x) - 24*b**4*n**4*cos(a + b
*log(c*x**n))**4/(64*b**4*n**4*x + 20*b**2*n**2*x + x) - 40*b**3*n**3*sin(a
 + b*log(c*x**n))**3*cos(a + b*log(c*x**n))/(64*b**4*n**4*x + 20*b**2*n**2*
x + x) - 24*b**3*n**3*sin(a + b*log(c*x**n))*cos(a + b*log(c*x**n))**3/(64*
b**4*n**4*x + 20*b**2*n**2*x + x) - 16*b**2*n**2*sin(a + b*log(c*x**n))**4/
(64*b**4*n**4*x + 20*b**2*n**2*x + x) - 12*b**2*n**2*sin(a + b*log(c*x**n))
**2*cos(a + b*log(c*x**n))**2/(64*b**4*n**4*x + 20*b**2*n**2*x + x) - 4*b*n
*sin(a + b*log(c*x**n))**3*cos(a + b*log(c*x**n))/(64*b**4*n**4*x + 20*b**2
*n**2*x + x) - sin(a + b*log(c*x**n))**4/(64*b**4*n**4*x + 20*b**2*n**2*x +
 x), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^4/x^2,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^4/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^4}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))^4/x^2,x)
[Out] int(sin(a + b*log(c*x^n))^4/x^2, x)
```

3.24 $\int \frac{\sin^4(a+b\log(cx^n))}{x^3}\,dx$

Optimal. Leaf size=210

$$-\frac{3b^4n^4}{4\left(1+5b^2n^2+4b^4n^4\right)x^2}-\frac{3b^3n^3\cos\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\left(cx^n\right)\right)}{2\left(1+5b^2n^2+4b^4n^4\right)x^2}-\frac{3b^2n^2\sin^2\left(a+b\log\left(cx^n\right)\right)}{2\left(1+5b^2n^2+4b^4n^4\right)x^2}-\frac{bn\,c}{}$$

```
[Out] -3/4*b^4*n^4/(4*b^4*n^4+5*b^2*n^2+1)/x^2-3/2*b^3*n^3*cos(a+b*ln(c*x^n))*sin
(a+b*ln(c*x^n))/(4*b^4*n^4+5*b^2*n^2+1)/x^2-3/2*b^2*n^2*sin(a+b*ln(c*x^n))^
2/(4*b^4*n^4+5*b^2*n^2+1)/x^2-b*n*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))^3/(
4*b^2*n^2+1)/x^2-1/2*sin(a+b*ln(c*x^n))^4/(4*b^2*n^2+1)/x^2
```

Rubi [A]
time = 0.04, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4575, 30}

$$-\frac{\sin^4\left(a+b\log\left(cx^n\right)\right)}{2x^2\left(4b^2n^2+1\right)}-\frac{bn\sin^3\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{x^2\left(4b^2n^2+1\right)}-\frac{3b^2n^2\sin^2\left(a+b\log\left(cx^n\right)\right)}{2x^2\left(4b^4n^4+5b^2n^2+1\right)}-\frac{3b^3n^3\sin\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{2x^2\left(4b^4n^4+5b^2n^2+1\right)}-\frac{3b^4n^4}{4x^2\left(4b^4n^4+5b^2n^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Log[c*x^n]]^4/x^3,x]
```

```
[Out] (-3*b^4*n^4)/(4*(1 + 5*b^2*n^2 + 4*b^4*n^4)*x^2) - (3*b^3*n^3*Cos[a + b*Log
[c*x^n]]*Sin[a + b*Log[c*x^n]])/(2*(1 + 5*b^2*n^2 + 4*b^4*n^4)*x^2) - (3*b^
2*n^2*Sin[a + b*Log[c*x^n]]^2)/(2*(1 + 5*b^2*n^2 + 4*b^4*n^4)*x^2) - (b*n*C
os[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^3)/((1 + 4*b^2*n^2)*x^2) - Sin[a
 + b*Log[c*x^n]]^4/(2*(1 + 4*b^2*n^2)*x^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4575

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2
*n^2*p^2 + (m + 1)^2)), Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x],
x] - Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a + b*Log[c*x^n])]*(Sin[d*(a + b*Log
[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x]) /; FreeQ[{a, b, c
, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int \frac{\sin^4\left(a+b\log\left(cx^n\right)\right)}{x^3}\,dx = -\frac{bn\cos\left(a+b\log\left(cx^n\right)\right)\sin^3\left(a+b\log\left(cx^n\right)\right)}{\left(1+4b^2n^2\right)x^2} - \frac{\sin^4\left(a+b\log\left(cx^n\right)\right)}{2\left(1+4b^2n^2\right)x^2} + \frac{(3b^2 n}{}$$

$$= -\frac{3b^3n^3\cos\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\left(cx^n\right)\right)}{2\left(1+5b^2n^2+4b^4n^4\right)x^2} - \frac{3b^2n^2\sin^2\left(a+b\log\left(cx^n\right)\right)}{2\left(1+5b^2n^2+4b^4n^4\right)x^2}$$

$$= -\frac{3b^4n^4}{4\left(1+5b^2n^2+4b^4n^4\right)x^2} - \frac{3b^3n^3\cos\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\left(cx^n\right)\right)}{2\left(1+5b^2n^2+4b^4n^4\right)x^2} -$$

Mathematica [A]
time = 0.52, size = 169, normalized size = 0.80

$$-\frac{3+15b^2n^2+12b^4n^4-4\left(1+4b^2n^2\right)\cos\left(2\left(a+b\log\left(cx^n\right)\right)\right)+\left(1+b^2n^2\right)\cos\left(4\left(a+b\log\left(cx^n\right)\right)\right)+4bn\sin\left(2\left(a+b\log\left(cx^n\right)\right)\right)+16b^3n^3\sin\left(2\left(a+b\log\left(cx^n\right)\right)\right)-2bn\sin\left(4\left(a+b\log\left(cx^n\right)\right)\right)-2b^3n^3\sin\left(4\left(a+b\log\left(cx^n\right)\right)\right)}{16\left(1+5b^2n^2+4b^4n^4\right)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]^4/x^3,x]
```

```
[Out] -1/16*(3 + 15*b^2*n^2 + 12*b^4*n^4 - 4*(1 + 4*b^2*n^2)*Cos[2*(a + b*Log[c*x
^n])] + (1 + b^2*n^2)*Cos[4*(a + b*Log[c*x^n])] + 4*b*n*Sin[2*(a + b*Log[c*
x^n])] + 16*b^3*n^3*Sin[2*(a + b*Log[c*x^n])] - 2*b*n*Sin[4*(a + b*Log[c*x^
n])] - 2*b^3*n^3*Sin[4*(a + b*Log[c*x^n])])/((1 + 5*b^2*n^2 + 4*b^4*n^4)*x^
2)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sin^4\left(a+b\ln\left(c\,x^n\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))^4/x^3,x)
```

```
[Out] int(sin(a+b*ln(c*x^n))^4/x^3,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1082 vs. $2(202) = 404$.
time = 0.33, size = 1082, normalized size = 5.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^4/x^3,x, algorithm="maxima")
```

```
[Out] -1/32*(24*(b^4*cos(4*b*log(c))^2 + b^4*sin(4*b*log(c))^2)*n^4 + 30*(b^2*cos
(4*b*log(c))^2 + b^2*sin(4*b*log(c))^2)*n^2 + 6*cos(4*b*log(c))^2 - (2*(b^3
*cos(4*b*log(c))*sin(8*b*log(c)) - b^3*cos(8*b*log(c))*sin(4*b*log(c)) + b^
3*sin(4*b*log(c)))*n^3 - (b^2*cos(8*b*log(c))*cos(4*b*log(c)) + b^2*sin(8*b
*log(c))*sin(4*b*log(c)) + b^2*cos(4*b*log(c)))*n^2 + 2*(b*cos(4*b*log(c))*
sin(8*b*log(c)) - b*cos(8*b*log(c))*sin(4*b*log(c)) + b*sin(4*b*log(c)))*n
- cos(8*b*log(c))*cos(4*b*log(c)) - sin(8*b*log(c))*sin(4*b*log(c)) - cos(4
*b*log(c)))*cos(4*b*log(x^n) + 4*a) + 4*(4*(b^3*cos(4*b*log(c))*sin(6*b*log
(c)) - b^3*cos(6*b*log(c))*sin(4*b*log(c)) + b^3*cos(2*b*log(c))*sin(4*b*lo
g(c)) - b^3*cos(4*b*log(c))*sin(2*b*log(c)))*n^3 - 4*(b^2*cos(6*b*log(c))*c
os(4*b*log(c)) + b^2*cos(4*b*log(c))*cos(2*b*log(c)) + b^2*sin(6*b*log(c))*
sin(4*b*log(c)) + b^2*sin(4*b*log(c))*sin(2*b*log(c)))*n^2 + (b*cos(4*b*log
(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b*log(c)) + b*cos(2*b*log(c)
)*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*n - cos(6*b*log(c))*
cos(4*b*log(c)) - cos(4*b*log(c))*cos(2*b*log(c)) - sin(6*b*log(c))*sin(4*b
*log(c)) - sin(4*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 6*sin
(4*b*log(c))^2 - (2*(b^3*cos(8*b*log(c))*cos(4*b*log(c)) + b^3*sin(8*b*log(
c))*sin(4*b*log(c)) + b^3*cos(4*b*log(c)))*n^3 + (b^2*cos(4*b*log(c))*sin(8
*b*log(c)) - b^2*cos(8*b*log(c))*sin(4*b*log(c)) + b^2*sin(4*b*log(c)))*n^2
 + 2*(b*cos(8*b*log(c))*cos(4*b*log(c)) + b*sin(8*b*log(c))*sin(4*b*log(c))
 + b*cos(4*b*log(c)))*n + cos(4*b*log(c))*sin(8*b*log(c)) - cos(8*b*log(c))
*sin(4*b*log(c)) + sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) + 4*(4*(b^3*cos
(6*b*log(c))*cos(4*b*log(c)) + b^3*cos(4*b*log(c))*cos(2*b*log(c)) + b^3*si
n(6*b*log(c))*sin(4*b*log(c)) + b^3*sin(4*b*log(c))*sin(2*b*log(c)))*n^3 +
4*(b^2*cos(4*b*log(c))*sin(6*b*log(c)) - b^2*cos(6*b*log(c))*sin(4*b*log(c)
) + b^2*cos(2*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(2*b*log(c
)))*n^2 + (b*cos(6*b*log(c))*cos(4*b*log(c)) + b*cos(4*b*log(c))*cos(2*b*lo
g(c)) + b*sin(6*b*log(c))*sin(4*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log(c
)))*n + cos(4*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c))*sin(4*b*log(c)) +
 cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)))*sin(2*b
*log(x^n) + 2*a))/((4*(b^4*cos(4*b*log(c))^2 + b^4*sin(4*b*log(c))^2)*n^4 +
 5*(b^2*cos(4*b*log(c))^2 + b^2*sin(4*b*log(c))^2)*n^2 + cos(4*b*log(c))^2
+ sin(4*b*log(c))^2)*x^2)
```

Fricas [A]
time = 1.41, size = 163, normalized size = 0.78

$$-\frac{3\,b^4n^4+2\,(b^2n^2+1)\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^4+8\,b^2n^2-2\,(5\,b^2n^2+2)\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2-2\,\left(2\,(b^3n^3+bn)\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^3-(5\,b^3n^3+2\,bn)\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+2}{4\,(4\,b^4n^4+5\,b^2n^2+1)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^4/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(3*b^4*n^4 + 2*(b^2*n^2 + 1)*cos(b*n*log(x) + b*log(c) + a)^4 + 8*b^2*
n^2 - 2*(5*b^2*n^2 + 2)*cos(b*n*log(x) + b*log(c) + a)^2 - 2*(2*(b^3*n^3 +
b*n)*cos(b*n*log(x) + b*log(c) + a)^3 - (5*b^3*n^3 + 2*b*n)*cos(b*n*log(x)
```

```
+ b*log(c) + a))*sin(b*n*log(x) + b*log(c) + a) + 2)/((4*b^4*n^4 + 5*b^2*n^
2 + 1)*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 134.85, size = 1068, normalized size = 5.09

$$\begin{cases} \frac{i \sin\left(4a - \frac{4i \log\left(cx^n\right)}{n}\right)}{24x^2} + \frac{\cos\left(2a - \frac{2i \log\left(cx^n\right)}{n}\right)}{8x^2} + \frac{\cos\left(4a - \frac{4i \log\left(cx^n\right)}{n}\right)}{48x^2} - \frac{3}{16x^2} - \frac{i \log\left(cx^n\right) \sin\left(2a - \frac{2i \log\left(cx^n\right)}{n}\right)}{4nx^2} - \frac{\log\left(cx^n\right) \cos\left(2a - \frac{2i \log\left(cx^n\right)}{n}\right)}{4nx^2} & \text{for}\ b = -\frac{i}{n} \\ \frac{i \sin\left(2a - \frac{i \log\left(cx^n\right)}{n}\right)}{6x^2} + \frac{i \sin\left(4a - \frac{2i \log\left(cx^n\right)}{n}\right)}{32x^2} + \frac{\cos\left(2a - \frac{i \log\left(cx^n\right)}{n}\right)}{3x^2} - \frac{3}{16x^2} + \frac{i \log\left(cx^n\right) \sin\left(4a - \frac{2i \log\left(cx^n\right)}{n}\right)}{16nx^2} + \frac{\log\left(cx^n\right) \cos\left(4a - \frac{2i \log\left(cx^n\right)}{n}\right)}{16nx^2} & \text{for}\ b = -\frac{i}{2n} \\ -\frac{i \sin\left(2a + \frac{i \log\left(cx^n\right)}{n}\right)}{6x^2} + \frac{\cos\left(2a + \frac{i \log\left(cx^n\right)}{n}\right)}{3x^2} - \frac{\cos\left(4a + \frac{2i \log\left(cx^n\right)}{n}\right)}{32x^2} - \frac{3}{16x^2} - \frac{i \log\left(cx^n\right) \sin\left(4a + \frac{2i \log\left(cx^n\right)}{n}\right)}{16nx^2} + \frac{\log\left(cx^n\right) \cos\left(4a + \frac{2i \log\left(cx^n\right)}{n}\right)}{16nx^2} & \text{for}\ b = \frac{i}{2n} \\ \frac{i \sin\left(2a + \frac{2i \log\left(cx^n\right)}{n}\right)}{8x^2} - \frac{i \sin\left(4a + \frac{4i \log\left(cx^n\right)}{n}\right)}{24x^2} + \frac{\cos\left(4a + \frac{4i \log\left(cx^n\right)}{n}\right)}{48x^2} - \frac{3}{16x^2} + \frac{i \log\left(cx^n\right) \sin\left(2a + \frac{2i \log\left(cx^n\right)}{n}\right)}{4nx^2} - \frac{\log\left(cx^n\right) \cos\left(2a + \frac{2i \log\left(cx^n\right)}{n}\right)}{4nx^2} & \text{for}\ b = \frac{i}{n} \\ -\frac{3b^4n^4 \sin^4\left(a + b\log\left(cx^n\right)\right)}{16b^4n^4x^2 + 20b^2n^2x^2 + 4x^2} - \frac{6b^4n^4 \sin^2\left(a + b\log\left(cx^n\right)\right) \cos^2\left(a + b\log\left(cx^n\right)\right)}{16b^4n^4x^2 + 20b^2n^2x^2 + 4x^2} - \frac{3b^4n^4 \cos^4\left(a + b\log\left(cx^n\right)\right)}{16b^4n^4x^2 + 20b^2n^2x^2 + 4x^2} - \frac{10b^3n^3 \sin^3\left(a + b\log\left(cx^n\right)\right) \cos\left(a + b\log\left(cx^n\right)\right)}{16b^4n^4x^2 + 20b^2n^2x^2 + 4x^2} - \frac{6b^3n^3 \sin\left(a + b\log\left(cx^n\right)\right) \cos^3\left(a + b\log\left(cx^n\right)\right)}{16b^4n^4x^2 + 20b^2n^2x^2 + 4x^2} - \frac{8b^2n^2 \sin^4\left(a + b\log\left(cx^n\right)\right)}{16b^4n^4x^2 + 20b^2n^2x^2 + 4x^2} - \frac{6b^2n^2 \sin^2\left(a + b\log\left(cx^n\right)\right) \cos^2\left(a + b\log\left(cx^n\right)\right)}{16b^4n^4x^2 + 20b^2n^2x^2 + 4x^2} - \frac{4bn \sin^3\left(a + b\log\left(cx^n\right)\right) \cos\left(a + b\log\left(cx^n\right)\right)}{16b^4n^4x^2 + 20b^2n^2x^2 + 4x^2} - \frac{2 \sin^4\left(a + b\log\left(cx^n\right)\right)}{16b^4n^4x^2 + 20b^2n^2x^2 + 4x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**4/x**3,x)
```

```
[Out] Piecewise((I*sin(4*a - 4*I*log(c*x**n)/n)/(24*x**2) + cos(2*a - 2*I*log(c*x
**n)/n)/(8*x**2) + cos(4*a - 4*I*log(c*x**n)/n)/(48*x**2) - 3/(16*x**2) - I
*log(c*x**n)*sin(2*a - 2*I*log(c*x**n)/n)/(4*n*x**2) - log(c*x**n)*cos(2*a
- 2*I*log(c*x**n)/n)/(4*n*x**2), Eq(b, -I/n)), (I*sin(2*a - I*log(c*x**n)/n
)/(6*x**2) + I*sin(4*a - 2*I*log(c*x**n)/n)/(32*x**2) + cos(2*a - I*log(c*x
**n)/n)/(3*x**2) - 3/(16*x**2) + I*log(c*x**n)*sin(4*a - 2*I*log(c*x**n)/n)
/(16*n*x**2) + log(c*x**n)*cos(4*a - 2*I*log(c*x**n)/n)/(16*n*x**2), Eq(b,
-I/(2*n))), (-I*sin(2*a + I*log(c*x**n)/n)/(6*x**2) + cos(2*a + I*log(c*x**
n)/n)/(3*x**2) - cos(4*a + 2*I*log(c*x**n)/n)/(32*x**2) - 3/(16*x**2) - I*l
og(c*x**n)*sin(4*a + 2*I*log(c*x**n)/n)/(16*n*x**2) + log(c*x**n)*cos(4*a +
 2*I*log(c*x**n)/n)/(16*n*x**2), Eq(b, I/(2*n))), (I*sin(2*a + 2*I*log(c*x*
*n)/n)/(8*x**2) - I*sin(4*a + 4*I*log(c*x**n)/n)/(24*x**2) + cos(4*a + 4*I*
log(c*x**n)/n)/(48*x**2) - 3/(16*x**2) + I*log(c*x**n)*sin(2*a + 2*I*log(c*
x**n)/n)/(4*n*x**2) - log(c*x**n)*cos(2*a + 2*I*log(c*x**n)/n)/(4*n*x**2),
Eq(b, I/n)), (-3*b**4*n**4*sin(a + b*log(c*x**n))**4/(16*b**4*n**4*x**2 + 2
0*b**2*n**2*x**2 + 4*x**2) - 6*b**4*n**4*sin(a + b*log(c*x**n))**2*cos(a +
b*log(c*x**n))**2/(16*b**4*n**4*x**2 + 20*b**2*n**2*x**2 + 4*x**2) - 3*b**4
*n**4*cos(a + b*log(c*x**n))**4/(16*b**4*n**4*x**2 + 20*b**2*n**2*x**2 + 4*
x**2) - 10*b**3*n**3*sin(a + b*log(c*x**n))**3*cos(a + b*log(c*x**n))/(16*b
**4*n**4*x**2 + 20*b**2*n**2*x**2 + 4*x**2) - 6*b**3*n**3*sin(a + b*log(c*x
**n))*cos(a + b*log(c*x**n))**3/(16*b**4*n**4*x**2 + 20*b**2*n**2*x**2 + 4*
x**2) - 8*b**2*n**2*sin(a + b*log(c*x**n))**4/(16*b**4*n**4*x**2 + 20*b**2*
n**2*x**2 + 4*x**2) - 6*b**2*n**2*sin(a + b*log(c*x**n))**2*cos(a + b*log(c
*x**n))**2/(16*b**4*n**4*x**2 + 20*b**2*n**2*x**2 + 4*x**2) - 4*b*n*sin(a +
 b*log(c*x**n))**3*cos(a + b*log(c*x**n))/(16*b**4*n**4*x**2 + 20*b**2*n**2
*x**2 + 4*x**2) - 2*sin(a + b*log(c*x**n))**4/(16*b**4*n**4*x**2 + 20*b**2*
n**2*x**2 + 4*x**2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^4/x^3,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^4/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^4}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))^4/x^3,x)
```

```
[Out] int(sin(a + b*log(c*x^n))^4/x^3, x)
```

3.25 $\int \sin(\log(a+bx))\,dx$

Optimal. Leaf size=39

$$-\frac{(a+bx)\cos(\log(a+bx))}{2b}+\frac{(a+bx)\sin(\log(a+bx))}{2b}$$

```
[Out] -1/2*(b*x+a)*cos(ln(b*x+a))/b+1/2*(b*x+a)*sin(ln(b*x+a))/b
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4563}

$$\frac{(a+bx)\sin(\log(a+bx))}{2b}-\frac{(a+bx)\cos(\log(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Log[a + b*x]],x]
[Out] -1/2*((a + b*x)*Cos[Log[a + b*x]])/b + ((a + b*x)*Sin[Log[a + b*x]])/(2*b)
```

Rule 4563

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*(
Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] - Simp[b*d*n*x*(Cos[d*(a +
 b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(\log(a+bx))\,dx &= \frac{\text{Subst}\left(\int \sin(\log(x))\,dx, x, a+bx\right)}{b}\\ &= -\frac{(a+bx)\cos(\log(a+bx))}{2b}+\frac{(a+bx)\sin(\log(a+bx))}{2b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 0.74

$$-\frac{(a+bx)(\cos(\log(a+bx))-\sin(\log(a+bx)))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Log[a + b*x]],x]
```

```
[Out] -1/2*((a + b*x)*(Cos[Log[a + b*x]] - Sin[Log[a + b*x]]))/b
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 44, normalized size = 1.13

method	result	size
risch	$\frac{\left(-\frac{1}{4}-\frac{i}{4}\right)(bx+a)(bx+a)^{i}}{b}+\frac{\left(-\frac{1}{4}+\frac{i}{4}\right)(bx+a)(bx+a)^{-i}}{b}$	44
norman	$\frac{x\tan\left(\frac{\ln(bx+a)}{2}\right)+\frac{a\tan\left(\frac{\ln(bx+a)}{2}\right)}{b}+\frac{a\left(\tan^{2}\left(\frac{\ln(bx+a)}{2}\right)\right)}{b}-\frac{x}{2}+\frac{x\left(\tan^{2}\left(\frac{\ln(bx+a)}{2}\right)\right)}{2}}{1+\tan^{2}\left(\frac{\ln(bx+a)}{2}\right)}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(ln(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/4-1/4*I)*(b*x+a)/b*(b*x+a)^I+(-1/4+1/4*I)*(b*x+a)/b/((b*x+a)^I)
```

Maxima [A]
time = 0.27, size = 27, normalized size = 0.69

$$-\frac{(bx+a)(\cos(\log(bx+a))-\sin(\log(bx+a)))}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/2*(b*x + a)*(cos(log(b*x + a)) - sin(log(b*x + a)))/b
```

Fricas [A]
time = 0.75, size = 33, normalized size = 0.85

$$-\frac{(bx+a)\cos(\log(bx+a))-(bx+a)\sin(\log(bx+a))}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/2*((b*x + a)*cos(log(b*x + a)) - (b*x + a)*sin(log(b*x + a)))/b
```

Sympy [A]
time = 0.19, size = 56, normalized size = 1.44

$$\begin{cases}\frac{a\sin(\log(a+bx))}{2b}-\frac{a\cos(\log(a+bx))}{2b}+\frac{x\sin(\log(a+bx))}{2}-\frac{x\cos(\log(a+bx))}{2} & \text{for } b\neq 0\\ x\sin(\log(a)) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(ln(b*x+a)),x)
```

```
[Out] Piecewise((a*sin(log(a + b*x))/(2*b) - a*cos(log(a + b*x))/(2*b) + x*sin(lo
g(a + b*x))/2 - x*cos(log(a + b*x))/2, Ne(b, 0)), (x*sin(log(a)), True))
```

Giac [A]
time = 0.40, size = 35, normalized size = 0.90

$$-\frac{(bx+a)\cos(\log(bx+a))}{2b}+\frac{(bx+a)\sin(\log(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(b*x+a)),x, algorithm="giac")
```

```
[Out] -1/2*(b*x + a)*cos(log(b*x + a))/b + 1/2*(b*x + a)*sin(log(b*x + a))/b
```

Mupad [B]
time = 2.16, size = 36, normalized size = 0.92

$$\begin{cases} x\sin(\ln(a)) & \text{if } b=0 \\ -\frac{\sqrt{2}\cos\left(\frac{\pi}{4}+\ln(a+b\,x)\right)(a+b\,x)}{2\,b} & \text{if } b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(log(a + b*x)),x)
```

```
[Out] piecewise(b == 0, x*sin(log(a)), b ~= 0, -(2^(1/2)*cos(pi/4 + log(a + b*x))
*(a + b*x))/(2*b))
```

3.26 $\int x^m \sin\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=133

$$-\frac{e^{\frac{a(1+m)}{\sqrt{-\frac{(1+m)^2}{n^2}}\,n}}x^{1+m}\left(cx^n\right)^{\frac{1+m}{n}}}{4\sqrt{-\frac{(1+m)^2}{n^2}}\,n}+\frac{e^{\frac{a\sqrt{-\frac{(1+m)^2}{n^2}}\,n}{1+m}}(1+m)x^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\log(x)}{2\sqrt{-\frac{(1+m)^2}{n^2}}\,n}$$

```
[Out] -1/4*exp(a*(1+m)/n/(-(1+m)^2/n^2)^(1/2))*x^(1+m)*(c*x^n)^((1+m)/n)/n/(-(1+m
)^2/n^2)^(1/2)+1/2*exp(a*n*(-(1+m)^2/n^2)^(1/2)/(1+m))*(1+m)*x^(1+m)*ln(x)/
n/((c*x^n)^((1+m)/n))/(-(1+m)^2/n^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {4581, 4577}

$$\frac{(m+1)x^{m+1}\log(x)e^{\frac{an\sqrt{-\frac{(m+1)^2}{n^2}}}{m+1}}\left(cx^n\right)^{-\frac{m+1}{n}}}{2n\sqrt{-\frac{(m+1)^2}{n^2}}}-\frac{x^{m+1}e^{\frac{a(m+1)}{n\sqrt{-\frac{(m+1)^2}{n^2}}}}\left(cx^n\right)^{\frac{m+1}{n}}}{4n\sqrt{-\frac{(m+1)^2}{n^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sin[a + Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n]],x]
```

```
[Out] -1/4*(E^((a*(1 + m))/(Sqrt[-((1 + m)^2/n^2)]*n))*x^(1 + m)*(c*x^n)^((1 + m)
/n))/(Sqrt[-((1 + m)^2/n^2)]*n) + (E^((a*Sqrt[-((1 + m)^2/n^2)]*n)/(1 + m))
*(1 + m)*x^(1 + m)*Log[x])/(2*Sqrt[-((1 + m)^2/n^2)]*n*(c*x^n)^((1 + m)/n))
```

Rule 4577

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[(m + 1)^p/(2^p*b^p*d^p*p^p), Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^
2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x]
, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m
 + 1)^2, 0]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
```

```
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^m \sin\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)dx = \frac{\left(x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{n}}\sin\left(a+\sqrt{-\frac{(1+m)}{n^2}}\right.\right.}{n}$$

$$= \frac{\left((1+m)x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int\left(\frac{e^{\frac{a\sqrt{-\frac{(1+m)^2}{n^2}}\,n}{1+m}}}{x}-e^{\sqrt{-}}\right.\right.}{2\sqrt{-\frac{(1+m)^2}{n^2}}\,n^2}$$

$$= -\frac{e^{\frac{a(1+m)}{\sqrt{-\frac{(1+m)^2}{n^2}}\,n}}x^{1+m}(cx^n)^{\frac{1+m}{n}}}{4\sqrt{-\frac{(1+m)^2}{n^2}}\,n}+\frac{e^{\frac{a\sqrt{-\frac{(1+m)^2}{n^2}}\,n}{1+m}}(1+m)x^{1+m}(c}{2\sqrt{-\frac{(1+m)^2}{n^2}}}$$

Mathematica [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int x^m \sin\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*Sin[a + Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n]],x]
[Out] Integrate[x^m*Sin[a + Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n]], x]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^m \sin\left(a+\ln\left(c\,x^n\right)\sqrt{-\frac{(1+m)^2}{n^2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(a+ln(c*x^n)*(-(1+m)^2/n^2)^(1/2)),x)
[Out] int(x^m*sin(a+ln(c*x^n)*(-(1+m)^2/n^2)^(1/2)),x)
```

Maxima [A]
time = 0.32, size = 82, normalized size = 0.62

$$\frac{c^{\frac{2m}{n}+\frac{2}{n}}xe^{\left(m\log(x)+\frac{m\log(x^n)}{n}+\frac{\log(x^n)}{n}\right)}\sin(a)+2\,(m\sin(a)+\sin(a))\log(x)}{4\left(c^{\frac{m}{n}+\frac{1}{n}}m+c^{\frac{m}{n}+\frac{1}{n}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+log(c*x^n)*(-(1+m)^2/n^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*(c^(2*m/n + 2/n)*x*e^(m*log(x) + m*log(x^n)/n + log(x^n)/n)*sin(a) + 2*
(m*sin(a) + sin(a))*log(x))/(c^(m/n + 1/n)*m + c^(m/n + 1/n))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.25, size = 63, normalized size = 0.47

$$\frac{\left(i\,x^2x^{2\,m}-2\,(i\,m+i)e^{\left(\frac{2\,(i\,an-(m+1)\log(c))}{n}\right)}\log(x)\right)e^{\left(-\frac{i\,an-(m+1)\log(c)}{n}\right)}}{4\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+log(c*x^n)*(-(1+m)^2/n^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(I*x^2*x^(2*m) - 2*(I*m + I)*e^(2*(I*a*n - (m + 1)*log(c))/n)*log(x))*e
^(-(I*a*n - (m + 1)*log(c))/n)/(m + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\sin\left(a+\sqrt{-\frac{m^2}{n^2}-\frac{2m}{n^2}-\frac{1}{n^2}}\,\log(cx^n)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sin(a+ln(c*x**n)*(-(1+m)**2/n**2)**(1/2)),x)
```

```
[Out] Integral(x**m*sin(a + sqrt(-m**2/n**2 - 2*m/n**2 - 1/n**2)*log(c*x**n)), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.99, size = 272, normalized size = 2.05

$$\frac{-i\,mn^2xx^me^{\left(i\,a-\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}+i\,mn^2xx^me^{\left(-i\,a+\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}-i\,n^2xx^me^{\left(i\,a-\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}-i\,nxx^m|mn+n|e^{\left(i\,a-\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}+i\,n^2xx^me^{\left(-i\,a+\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}-i\,nxx^m|mn+n|e^{\left(-i\,a+\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}}{2\left(m^2n^2+2\,mn^2-(mn+n)^2+n^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+log(c*x^n)*(-(1+m)^2/n^2)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*(-I*m*n^2*x*x^m*e^(I*a - (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/
n^2) + I*m*n^2*x*x^m*e^(-I*a + (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)
)/n^2) - I*n^2*x*x^m*e^(I*a - (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))
/n^2) - I*n*x*x^m*abs(m*n + n)*e^(I*a - (n*abs(m*n + n)*log(x) + abs(m*n +
n)*log(c))/n^2) + I*n^2*x*x^m*e^(-I*a + (n*abs(m*n + n)*log(x) + abs(m*n +
n)*log(c))/n^2) - I*n*x*x^m*abs(m*n + n)*e^(-I*a + (n*abs(m*n + n)*log(x) +
 abs(m*n + n)*log(c))/n^2))/(m^2*n^2 + 2*m*n^2 - (m*n + n)^2 + n^2)
```

Mupad [B]
time = 3.94, size = 135, normalized size = 1.02

$$\frac{x\,x^m\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{(c\,x^n)^{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,1\mathrm{i}}}\,1\mathrm{i}}{2\,m+2-n\,\sqrt{-\frac{(m+1)^2}{n^2}}\,2\mathrm{i}}-\frac{x\,x^m\,\mathrm{e}^{a\,1\mathrm{i}}\,(c\,x^n)^{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,1\mathrm{i}}\,1\mathrm{i}}{2\,m+2+n\,\sqrt{-\frac{(m+1)^2}{n^2}}\,2\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(a + log(c*x^n)*(-(m + 1)^2/n^2)^(1/2)),x)
```

```
[Out] (x*x^m*exp(-a*1i)/(c*x^n)^((- (2*m)/n^2 - 1/n^2 - m^2/n^2)^(1/2)*1i)*1i)/(2
*m - n*(-(m + 1)^2/n^2)^(1/2)*2i + 2) - (x*x^m*exp(a*1i)*(c*x^n)^((- (2*m)/
n^2 - 1/n^2 - m^2/n^2)^(1/2)*1i)*1i)/(2*m + n*(-(m + 1)^2/n^2)^(1/2)*2i + 2
)
```

3.27 $\int x^2 \sin\left(a+3\sqrt{-\frac{1}{n^2}}\,\log(cx^n)\right)\,dx$

Optimal. Leaf size=88

$$\frac{1}{12}e^{-a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,nx^3(cx^n)^{3/n}-\frac{1}{2}e^{a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,nx^3(cx^n)^{-3/n}\log(x)$$

```
[Out] 1/12*n*x^3*(c*x^n)^(3/n)*(-1/n^2)^(1/2)/exp(a*n*(-1/n^2)^(1/2))-1/2*exp(a*n
*(-1/n^2)^(1/2))*n*x^3*ln(x)*(-1/n^2)^(1/2)/((c*x^n)^(3/n))
```

Rubi [A]
time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {4581, 4577}

$$\frac{1}{12}\sqrt{-\frac{1}{n^2}}\,nx^3e^{-a\sqrt{-\frac{1}{n^2}}\,n}(cx^n)^{3/n}-\frac{1}{2}\sqrt{-\frac{1}{n^2}}\,nx^3e^{a\sqrt{-\frac{1}{n^2}}\,n}\log(x)\,(cx^n)^{-3/n}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[a + 3*Sqrt[-n^(-2)]*Log[c*x^n]],x]
```

```
[Out] (Sqrt[-n^(-2)]*n*x^3*(c*x^n)^(3/n))/(12*E^(a*Sqrt[-n^(-2)]*n)) - (E^(a*Sqrt
[-n^(-2)]*n)*Sqrt[-n^(-2)]*n*x^3*Log[x])/(2*(c*x^n)^(3/n))
```

Rule 4577

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[(m + 1)^p/(2^p*b^p*d^p*p^p), Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^
2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x]
, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m
 + 1)^2, 0]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^2 \sin\left(a+3\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx = \frac{\left(x^3(cx^n)^{-3/n}\right)\text{Subst}\left(\int x^{-1+\frac{3}{n}}\sin\left(a+3\sqrt{-\frac{1}{n^2}}\log(x)\right)dx, x,\right.}{n}$$

$$= -\left(\frac{1}{2}\left(\sqrt{-\frac{1}{n^2}}x^3(cx^n)^{-3/n}\right)\text{Subst}\left(\int\left(\frac{e^{a\sqrt{-\frac{1}{n^2}}n}}{x} - e^{-a\sqrt{-\frac{1}{n^2}}n}\right.\right.\right.$$

$$= \frac{1}{12}e^{-a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}nx^3(cx^n)^{3/n} - \frac{1}{2}e^{a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}nx^3(cx^n)^{-}$$

Mathematica [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int x^2 \sin\left(a+3\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*Sin[a + 3*Sqrt[-n^(-2)]*Log[c*x^n]],x]
[Out] Integrate[x^2*Sin[a + 3*Sqrt[-n^(-2)]*Log[c*x^n]], x]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^2 \sin\left(a+3\ln\left(c\,x^n\right)\sqrt{-\frac{1}{n^2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a+3*ln(c*x^n)*(-1/n^2)^(1/2)),x)
[Out] int(x^2*sin(a+3*ln(c*x^n)*(-1/n^2)^(1/2)),x)
```

Maxima [A]
time = 0.29, size = 31, normalized size = 0.35

$$\frac{c^{\frac{6}{n}}x^6\sin\left(a\right)+6\,\log\left(x\right)\sin\left(a\right)}{12\,c^{\frac{3}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+3*log(c*x^n)*(-1/n^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/12*(c^(6/n)*x^6*sin(a) + 6*log(x)*sin(a))/c^(3/n)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.16, size = 42, normalized size = 0.48

$$\frac{1}{12}\left(i\,x^{6}-6i\,e^{\left(\frac{2\,(i\,an-3\,\log(c))}{n}\right)}\log\left(x\right)\right)e^{\left(-\frac{i\,an-3\,\log(c)}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+3*log(c*x^n)*(-1/n^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/12*(I*x^6 - 6*I*e^(2*(I*a*n - 3*log(c))/n)*log(x))*e^(-(I*a*n - 3*log(c))
/n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\sin\left(a+3\sqrt{-\frac{1}{n^{2}}}\,\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(a+3*ln(c*x**n)*(-1/n**2)**(1/2)),x)
```

```
[Out] Integral(x**2*sin(a + 3*sqrt(-1/n**2)*log(c*x**n)), x)
```

Giac [A]
time = 0.52, size = 1, normalized size = 0.01

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+3*log(c*x^n)*(-1/n^2)^(1/2)),x, algorithm="giac")
```

```
[Out] +Infinity
```

Mupad [B]
time = 3.02, size = 85, normalized size = 0.97

$$-\frac{x^{3}\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{{\left(c\,x^{n}\right)}^{\sqrt{-\frac{1}{n^{2}}}\,3\mathrm{i}}}}{6\,n\,\sqrt{-\frac{1}{n^{2}}}+6\mathrm{i}}-\frac{x^{3}\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^{n}\right)}^{\sqrt{-\frac{1}{n^{2}}}\,3\mathrm{i}}}{6\,n\,\sqrt{-\frac{1}{n^{2}}}-6\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a + 3*log(c*x^n)*(-1/n^2)^(1/2)),x)
```

```
[Out] - (x^3*exp(-a*1i)/(c*x^n)^((-1/n^2)^(1/2)*3i))/(6*n*(-1/n^2)^(1/2) + 6i) -
(x^3*exp(a*1i)*(c*x^n)^((-1/n^2)^(1/2)*3i))/(6*n*(-1/n^2)^(1/2) - 6i)
```

3.28 $\int x \sin \left(a+2 \sqrt{-\frac{1}{n^2}} \log \left(c x^n\right)\right) dx$

Optimal. Leaf size=88

$$\frac{1}{8} e^{-a \sqrt{-\frac{1}{n^2}} n} \sqrt{-\frac{1}{n^2}} n x^2 \left(c x^n\right)^{2/n}-\frac{1}{2} e^{a \sqrt{-\frac{1}{n^2}} n} \sqrt{-\frac{1}{n^2}} n x^2 \left(c x^n\right)^{-2/n} \log(x)$$

```
[Out] 1/8*n*x^2*(c*x^n)^(2/n)*(-1/n^2)^(1/2)/exp(a*n*(-1/n^2)^(1/2))-1/2*exp(a*n*
(-1/n^2)^(1/2))*n*x^2*ln(x)*(-1/n^2)^(1/2)/((c*x^n)^(2/n))
```

Rubi [A]
time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4581, 4577}

$$\frac{1}{8} \sqrt{-\frac{1}{n^2}} n x^2 e^{-a \sqrt{-\frac{1}{n^2}} n} \left(c x^n\right)^{2/n}-\frac{1}{2} \sqrt{-\frac{1}{n^2}} n x^2 e^{a \sqrt{-\frac{1}{n^2}} n} \log(x) \left(c x^n\right)^{-2/n}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[a + 2*Sqrt[-n^(-2)]*Log[c*x^n]],x]
```

```
[Out] (Sqrt[-n^(-2)]*n*x^2*(c*x^n)^(2/n))/(8*E^(a*Sqrt[-n^(-2)]*n)) - (E^(a*Sqrt[
-n^(-2)]*n)*Sqrt[-n^(-2)]*n*x^2*Log[x])/(2*(c*x^n)^(2/n))
```

Rule 4577

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[(m + 1)^p/(2^p*b^p*d^p*p^p), Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^
2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x]
, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m
 + 1)^2, 0]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx = \frac{\left(x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int x^{-1+\frac{2}{n}}\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log(x)\right)dx,x,cx\right.}{n}$$

$$= -\left(\frac{1}{2}\left(\sqrt{-\frac{1}{n^2}}x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int\left(\frac{e^{a\sqrt{-\frac{1}{n^2}}n}}{x}-e^{-a\sqrt{-\frac{1}{n^2}}n}x^{-}\right.\right.\right.$$

$$= \frac{1}{8}e^{-a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}nx^2(cx^n)^{2/n}-\frac{1}{2}e^{a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}nx^2(cx^n)^{-2/n}$$

Mathematica [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int x\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*Sin[a + 2*Sqrt[-n^(-2)]*Log[c*x^n]],x]
[Out] Integrate[x*Sin[a + 2*Sqrt[-n^(-2)]*Log[c*x^n]], x]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x\sin\left(a+2\ln\left(c\,x^n\right)\sqrt{-\frac{1}{n^2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a+2*ln(c*x^n)*(-1/n^2)^(1/2)),x)
[Out] int(x*sin(a+2*ln(c*x^n)*(-1/n^2)^(1/2)),x)
```

Maxima [A]
time = 0.29, size = 31, normalized size = 0.35

$$\frac{c^{\frac{4}{n}}x^4\sin\left(a\right)+4\log\left(x\right)\sin\left(a\right)}{8\,c^{\frac{2}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+2*log(c*x^n)*(-1/n^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/8*(c^(4/n)*x^4*sin(a) + 4*log(x)*sin(a))/c^(2/n)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.17, size = 42, normalized size = 0.48

$$\frac{1}{8}\left(i\,x^4-4i\,e^{\left(\frac{2\,(i\,an-2\,\log(c))}{n}\right)}\log\left(x\right)\right)e^{\left(-\frac{i\,an-2\,\log(c)}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+2*log(c*x^n)*(-1/n^2)^(1/2)),x, algorithm="fricas")
[Out] 1/8*(I*x^4 - 4*I*e^(2*(I*a*n - 2*log(c))/n)*log(x))*e^(-(I*a*n - 2*log(c))/
n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sin\left(a+2\sqrt{-\frac{1}{n^2}}\,\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+2*ln(c*x**n)*(-1/n**2)**(1/2)),x)
[Out] Integral(x*sin(a + 2*sqrt(-1/n**2)*log(c*x**n)), x)
```

Giac [A]
time = 0.51, size = 1, normalized size = 0.01

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+2*log(c*x^n)*(-1/n^2)^(1/2)),x, algorithm="giac")
[Out] +Infinity
```

Mupad [B]
time = 2.80, size = 85, normalized size = 0.97

$$-\frac{x^2\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,2\mathrm{i}}}}{4\,n\,\sqrt{-\frac{1}{n^2}}+4\mathrm{i}}-\frac{x^2\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,2\mathrm{i}}}{4\,n\,\sqrt{-\frac{1}{n^2}}-4\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + 2*log(c*x^n)*(-1/n^2)^(1/2)),x)
[Out] - (x^2*exp(-a*1i)/(c*x^n)^((-1/n^2)^(1/2)*2i))/(4*n*(-1/n^2)^(1/2) + 4i) -
(x^2*exp(a*1i)*(c*x^n)^((-1/n^2)^(1/2)*2i))/(4*n*(-1/n^2)^(1/2) - 4i)
```

3.29 $\int \sin\left(a+\sqrt{-\frac{1}{n^2}}\,\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=82

$$\frac{1}{4}e^{-a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,nx(cx^n)^{\frac{1}{n}}-\frac{1}{2}e^{a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,nx(cx^n)^{-1/n}\log(x)$$

```
[Out] 1/4*n*x*(c*x^n)^(1/n)*(-1/n^2)^(1/2)/exp(a*n*(-1/n^2)^(1/2))-1/2*exp(a*n*(-
1/n^2)^(1/2))*n*x*ln(x)*(-1/n^2)^(1/2)/((c*x^n)^(1/n))
```

Rubi [A]
time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {4571, 4577}

$$\frac{1}{4}\sqrt{-\frac{1}{n^2}}\,nxe^{-a\sqrt{-\frac{1}{n^2}}\,n}(cx^n)^{\frac{1}{n}}-\frac{1}{2}\sqrt{-\frac{1}{n^2}}\,nxe^{a\sqrt{-\frac{1}{n^2}}\,n}\log(x)\left(cx^n\right)^{-1/n}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]],x]
```

```
[Out] (Sqrt[-n^(-2)]*n*x*(c*x^n)^n^(-1))/(4*E^(a*Sqrt[-n^(-2)]*n)) - (E^(a*Sqrt[-
n^(-2)]*n)*Sqrt[-n^(-2)]*n*x*Log[x])/(2*(c*x^n)^n^(-1))
```

Rule 4571

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4577

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[(m + 1)^p/(2^p*b^p*d^p*p^p), Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^
2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x]
, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m
 + 1)^2, 0]
```

Rubi steps

$$\int \sin\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\sin\left(a+\sqrt{-\frac{1}{n^2}}\log(x)\right)dx, x, cx^n\right)}{n}$$

$$= -\left(\frac{1}{2}\left(\sqrt{-\frac{1}{n^2}}\,x(cx^n)^{-1/n}\right)\text{Subst}\left(\int\left(\frac{e^{a\sqrt{-\frac{1}{n^2}}\,n}}{x} - e^{-a\sqrt{-\frac{1}{n^2}}\,n}x^{-1+\right.\right.\right.$$

$$= \frac{1}{4}e^{-a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,nx(cx^n)^{\frac{1}{n}} - \frac{1}{2}e^{a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,nx(cx^n)^{-1/n}\log(x$$

__Mathematica__ [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \sin\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]],x]
[Out] Integrate[Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]], x]
```

__Maple__ [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \sin\left(a+\ln\left(c\,x^n\right)\sqrt{-\frac{1}{n^2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+ln(c*x^n)*(-1/n^2)^(1/2)),x)
[Out] int(sin(a+ln(c*x^n)*(-1/n^2)^(1/2)),x)
```

__Maxima__ [A]
time = 0.30, size = 29, normalized size = 0.35

$$\frac{c^{\frac{2}{n}}x^2\sin\left(a\right)+2\,\log\left(x\right)\sin\left(a\right)}{4\,c^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+log(c*x^n)*(-1/n^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*(c^(2/n)*x^2*sin(a) + 2*log(x)*sin(a))/c^(1/n)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.13, size = 42, normalized size = 0.51

$$\frac{1}{4}\left(i\,x^{2}-2i\,e^{\left(\frac{2\,(i\,an-\log(c))}{n}\right)}\log\left(x\right)\right)e^{\left(-\frac{i\,an-\log(c)}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+log(c*x^n)*(-1/n^2)^(1/2)),x, algorithm="fricas")
[Out] 1/4*(I*x^2 - 2*I*e^(2*(I*a*n - log(c))/n)*log(x))*e^(-(I*a*n - log(c))/n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sin\left(a+\sqrt{-\frac{1}{n^{2}}}\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+ln(c*x**n)*(-1/n**2)**(1/2)),x)
[Out] Integral(sin(a + sqrt(-1/n**2)*log(c*x**n)), x)
```

Giac [A]
time = 0.46, size = 1, normalized size = 0.01

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+log(c*x^n)*(-1/n^2)^(1/2)),x, algorithm="giac")
[Out] +Infinity
```

Mupad [B]
time = 2.73, size = 81, normalized size = 0.99

$$-\frac{x\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}}}{2\,n\,\sqrt{-\frac{1}{n^2}}+2\mathrm{i}}-\frac{x\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}}{2\,n\,\sqrt{-\frac{1}{n^2}}-2\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + log(c*x^n)*(-1/n^2)^(1/2)),x)
[Out] - (x*exp(-a*1i)/(c*x^n)^((-1/n^2)^(1/2)*1i))/(2*n*(-1/n^2)^(1/2) + 2i) - (x
*exp(a*1i)*(c*x^n)^((-1/n^2)^(1/2)*1i))/(2*n*(-1/n^2)^(1/2) - 2i)
```

3.30 $\int \frac{\sin(a)}{x} \, dx$

Optimal. Leaf size=5

$$\log(x)\sin(a)$$

```
[Out] ln(x)*sin(a)
```

Rubi [A]
time = 0.00, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {12, 29}

$$\sin(a)\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a]/x,x]
```

```
[Out] Log[x]*Sin[a]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned} \int \frac{\sin(a)}{x} \, dx &= \sin(a) \int \frac{1}{x} \, dx \\ &= \log(x)\sin(a) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 5, normalized size = 1.00

$$\log(x)\sin(a)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a]/x,x]
```

```
[Out] Log[x]*Sin[a]
```

Maple [A]
time = 0.02, size = 6, normalized size = 1.20

method	result	size
default	$\ln(x)\sin(a)$	6
norman	$\ln(x)\sin(a)$	6
risch	$\ln(x)\sin(a)$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a)/x,x,method=_RETURNVERBOSE)
[Out] ln(x)*sin(a)
```

Maxima [A]
time = 0.26, size = 5, normalized size = 1.00

$$\log(x)\sin(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a)/x,x, algorithm="maxima")
[Out] log(x)*sin(a)
```

Fricas [A]
time = 1.20, size = 5, normalized size = 1.00

$$\log(x)\sin(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a)/x,x, algorithm="fricas")
[Out] log(x)*sin(a)
```

Sympy [A]
time = 0.01, size = 5, normalized size = 1.00

$$\log(x)\sin(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a)/x,x)
[Out] log(x)*sin(a)
```

Giac [A]
time = 0.39, size = 6, normalized size = 1.20

$$\log(|x|)\sin(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a)/x,x, algorithm="giac")
```

```
[Out] log(abs(x))*sin(a)
```

Mupad [B]
time = 0.03, size = 5, normalized size = 1.00

$$\sin(a)\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a)/x,x)
```

```
[Out] sin(a)*log(x)
```

3.31 $\int \frac{\sin\left(a+\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{x^2}\,dx$

Optimal. Leaf size=86

$$\frac{e^{a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}\,n(cx^n)^{-1/n}}{4x}+\frac{e^{-a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}\,n(cx^n)^{\frac{1}{n}}\log(x)}{2x}$$

```
[Out] 1/4*exp(a*n*(-1/n^2)^(1/2))*n*(-1/n^2)^(1/2)/x/((c*x^n)^(1/n))+1/2*n*(c*x^n
)^(1/n)*ln(x)*(-1/n^2)^(1/2)/exp(a*n*(-1/n^2)^(1/2))/x
```

Rubi [A]
time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4581, 4577}

$$\frac{\sqrt{-\frac{1}{n^2}}\,ne^{a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{-1/n}}{4x}+\frac{\sqrt{-\frac{1}{n^2}}\,ne^{-a\sqrt{-\frac{1}{n^2}}n}\log(x)\,(cx^n)^{\frac{1}{n}}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]]/x^2,x]
```

```
[Out] (E^(a*Sqrt[-n^(-2)]*n)*Sqrt[-n^(-2)]*n)/(4*x*(c*x^n)^n^(-1)) + (Sqrt[-n^(-2
)]*n*(c*x^n)^n^(-1)*Log[x])/(2*E^(a*Sqrt[-n^(-2)]*n)*x)
```

Rule 4577

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[(m + 1)^p/(2^p*b^p*d^p*p^p), Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^
2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x]
, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m
 + 1)^2, 0]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\sin\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx = \frac{(cx^n)^{\frac{1}{n}}\,\mathrm{Subst}\left(\int x^{-1-\frac{1}{n}}\sin\left(a+\sqrt{-\frac{1}{n^2}}\log(x)\right)dx, x, cx^n\right)}{nx}$$
$$= \frac{\left(\sqrt{-\frac{1}{n^2}}\,(cx^n)^{\frac{1}{n}}\right)\mathrm{Subst}\left(\int\left(\frac{e^{-a\sqrt{-\frac{1}{n^2}}\,n}}{x} - e^{a\sqrt{-\frac{1}{n^2}}\,n}x^{-\frac{2+n}{n}}\right)dx, x, c$$
$$= \frac{e^{a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,n(cx^n)^{-1/n}}{4x} + \frac{e^{-a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,n(cx^n)^{\frac{1}{n}}\log(x)}{2x}$$

Mathematica [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]]/x^2,x]
[Out] Integrate[Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]]/x^2, x]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+\ln\left(c\,x^n\right)\sqrt{-\frac{1}{n^2}}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+ln(c*x^n)*(-1/n^2)^(1/2))/x^2,x)
[Out] int(sin(a+ln(c*x^n)*(-1/n^2)^(1/2))/x^2,x)
```

Maxima [A]
time = 0.30, size = 33, normalized size = 0.38

$$\frac{2\,c^{\frac{2}{n}}x^2\log\left(x\right)\sin\left(a\right)-\sin\left(a\right)}{4\,c^{\left(\frac{1}{n}\right)}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+log(c*x^n)*(-1/n^2)^(1/2))/x^2,x, algorithm="maxima")
[Out] 1/4*(2*c^(2/n)*x^2*log(x)*sin(a) - sin(a))/(c^(1/n)*x^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.39, size = 45, normalized size = 0.52

$$\frac{\left(2i\,x^2\log\left(x\right)+i\,e^{\left(\frac{2\,(i\,an-\log(c))}{n}\right)}\right)e^{\left(-\frac{i\,an-\log(c)}{n}\right)}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+log(c*x^n)*(-1/n^2)^(1/2))/x^2,x, algorithm="fricas")
[Out] 1/4*(2*I*x^2*log(x) + I*e^(2*(I*a*n - log(c))/n))*e^(-(I*a*n - log(c))/n)/x
^2
```

Sympy [A]
time = 1.77, size = 95, normalized size = 1.10

$$\frac{\sqrt{-\frac{1}{n^2}}\,\log\left(cx^n\right)\cos\left(a+\sqrt{-\frac{1}{n^2}}\,\log\left(cx^n\right)\right)}{2x}-\frac{\sin\left(a+\sqrt{-\frac{1}{n^2}}\,\log\left(cx^n\right)\right)}{2x}+\frac{\log\left(cx^n\right)\sin\left(a+\sqrt{-\frac{1}{n^2}}\,\log\left(cx^n\right)\right)}{2nx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+ln(c*x**n)*(-1/n**2)**(1/2))/x**2,x)
[Out] sqrt(-1/n**2)*log(c*x**n)*cos(a + sqrt(-1/n**2)*log(c*x**n))/(2*x) - sin(a
+ sqrt(-1/n**2)*log(c*x**n))/(2*x) + log(c*x**n)*sin(a + sqrt(-1/n**2)*log(
c*x**n))/(2*n*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+log(c*x^n)*(-1/n^2)^(1/2))/x^2,x, algorithm="giac")
[Out] integrate(sin(sqrt(-1/n^2)*log(c*x^n) + a)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+\ln\left(c\,x^n\right)\,\sqrt{-\frac{1}{n^2}}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + log(c*x^n)*(-1/n^2)^(1/2))/x^2,x)
[Out] int(sin(a + log(c*x^n)*(-1/n^2)^(1/2))/x^2, x)
```

3.32 $\int \frac{\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{x^3}\,dx$

Optimal. Leaf size=88

$$\frac{e^{a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}\,n(cx^n)^{-2/n}}{8x^2}+\frac{e^{-a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}\,n(cx^n)^{2/n}\log(x)}{2x^2}$$

```
[Out] 1/8*exp(a*n*(-1/n^2)^(1/2))*n*(-1/n^2)^(1/2)/x^2/((c*x^n)^(2/n))+1/2*n*(c*x
^n)^(2/n)*ln(x)*(-1/n^2)^(1/2)/exp(a*n*(-1/n^2)^(1/2))/x^2
```

Rubi [A]
time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4581, 4577}

$$\frac{\sqrt{-\frac{1}{n^2}}\,ne^{a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{-2/n}}{8x^2}+\frac{\sqrt{-\frac{1}{n^2}}\,ne^{-a\sqrt{-\frac{1}{n^2}}n}\log(x)\,(cx^n)^{2/n}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + 2*Sqrt[-n^(-2)]*Log[c*x^n]]/x^3,x]
```

```
[Out] (E^(a*Sqrt[-n^(-2)]*n)*Sqrt[-n^(-2)]*n)/(8*x^2*(c*x^n)^(2/n)) + (Sqrt[-n^(-
2)]*n*(c*x^n)^(2/n)*Log[x])/(2*E^(a*Sqrt[-n^(-2)]*n)*x^2)
```

Rule 4577

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[(m + 1)^p/(2^p*b^p*d^p*p^p), Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^
2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x]
, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m
 + 1)^2, 0]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^3}\,dx = \frac{(cx^n)^{2/n}\operatorname{Subst}\left(\int x^{-1-\frac{2}{n}}\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log(x)\right)\,dx,x,cx^n\right)}{nx^2}$$
$$= \frac{\left(\sqrt{-\frac{1}{n^2}}\,(cx^n)^{2/n}\right)\operatorname{Subst}\left(\int\left(\frac{e^{-a\sqrt{-\frac{1}{n^2}}\,n}}{x}-e^{a\sqrt{-\frac{1}{n^2}}\,n}x^{-\frac{4+n}{n}}\right)\,dx,x,\right.}{2x^2}$$
$$= \frac{e^{a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,n(cx^n)^{-2/n}}{8x^2}+\frac{e^{-a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,n(cx^n)^{2/n}\log(x)}{2x^2}$$

Mathematica [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + 2*Sqrt[-n^(-2)]*Log[c*x^n]]/x^3,x]
[Out] Integrate[Sin[a + 2*Sqrt[-n^(-2)]*Log[c*x^n]]/x^3, x]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+2\ln\left(c\,x^n\right)\sqrt{-\frac{1}{n^2}}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+2*ln(c*x^n)*(-1/n^2)^(1/2))/x^3,x)
[Out] int(sin(a+2*ln(c*x^n)*(-1/n^2)^(1/2))/x^3,x)
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.40

$$\frac{4\,c^{\frac{4}{n}}x^4\log\left(x\right)\sin\left(a\right)-\sin\left(a\right)}{8\,c^{\frac{2}{n}}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+2*log(c*x^n)*(-1/n^2)^(1/2))/x^3,x, algorithm="maxima")

[Out] 1/8*(4*c^(4/n)*x^4*log(x)*sin(a) - sin(a))/(c^(2/n)*x^4)

Fricas [C] Result contains complex when optimal does not.
time = 1.62, size = 45, normalized size = 0.51

$$\frac{\left(4i\,x^4\log\left(x\right)+i\,e^{\left(\frac{2\,(i\,an-2\,\log(c))}{n}\right)}\right)e^{\left(-\frac{i\,an-2\,\log(c)}{n}\right)}}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+2*log(c*x^n)*(-1/n^2)^(1/2))/x^3,x, algorithm="fricas")

[Out] 1/8*(4*I*x^4*log(x) + I*e^(2*(I*a*n - 2*log(c))/n))*e^(-(I*a*n - 2*log(c))/n)/x^4

Sympy [A]
time = 6.07, size = 105, normalized size = 1.19

$$\frac{\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\cos\left(a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{2x^2}-\frac{\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{4x^2}+\frac{\log\left(cx^n\right)\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{2nx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+2*ln(c*x**n)*(-1/n**2)**(1/2))/x**3,x)

[Out] sqrt(-1/n**2)*log(c*x**n)*cos(a + 2*sqrt(-1/n**2)*log(c*x**n))/(2*x**2) - sin(a + 2*sqrt(-1/n**2)*log(c*x**n))/(4*x**2) + log(c*x**n)*sin(a + 2*sqrt(-1/n**2)*log(c*x**n))/(2*n*x**2)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+2*log(c*x^n)*(-1/n^2)^(1/2))/x^3,x, algorithm="giac")

[Out] integrate(sin(2*sqrt(-1/n^2)*log(c*x^n) + a)/x^3, x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+2\,\ln\left(c\,x^n\right)\,\sqrt{-\frac{1}{n^2}}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a + 2*log(c*x^n)*(-1/n^2)^(1/2))/x^3,x)

[Out] int(sin(a + 2*log(c*x^n)*(-1/n^2)^(1/2))/x^3, x)

3.33 $\int x^m \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)dx$

<u>**Optimal**</u>. Leaf size=117

$$\frac{x^{1+m}}{2(1+m)}-\frac{e^{-\frac{2a\sqrt{-\frac{(1+m)^2}{n^2}}\,n}{1+m}}x^{1+m}(cx^n)^{\frac{1+m}{n}}}{8(1+m)}-\frac{1}{4}e^{\frac{2a\sqrt{-\frac{(1+m)^2}{n^2}}\,n}{1+m}}x^{1+m}(cx^n)^{-\frac{1+m}{n}}\log(x)$$

```
[Out] 1/2*x^(1+m)/(1+m)-1/8*x^(1+m)*(c*x^n)^((1+m)/n)/exp(2*a*n*(-(1+m)^2/n^2)^(1
/2)/(1+m))/(1+m)-1/4*exp(2*a*n*(-(1+m)^2/n^2)^(1/2)/(1+m))*x^(1+m)*ln(x)/((
c*x^n)^((1+m)/n))
```

<u>**Rubi**</u> [A]
time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}=0.061$, Rules used = {4581, 4577}

$$-\frac{x^{m+1}e^{-\frac{2an\sqrt{-\frac{(m+1)^2}{n^2}}}{m+1}}(cx^n)^{\frac{m+1}{n}}}{8(m+1)}-\frac{1}{4}x^{m+1}\log(x)e^{\frac{2an\sqrt{-\frac{(m+1)^2}{n^2}}}{m+1}}(cx^n)^{-\frac{m+1}{n}}+\frac{x^{m+1}}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sin[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2]^2,x]
```

```
[Out] x^(1 + m)/(2*(1 + m)) - (x^(1 + m)*(c*x^n)^((1 + m)/n))/(8*E^((2*a*Sqrt[-((
1 + m)^2/n^2)]*n)/(1 + m))*(1 + m)) - (E^((2*a*Sqrt[-((1 + m)^2/n^2)]*n)/(1
 + m))*x^(1 + m)*Log[x])/(4*(c*x^n)^((1 + m)/n))
```

<u>Rule 4577</u>

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[(m + 1)^p/(2^p*b^p*d^p*p^p), Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^
2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x]
, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m
 + 1)^2, 0]
```

<u>Rule 4581</u>

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

<u>Rubi steps</u>

$$\int x^m \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)dx = \frac{\left(x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\Big(\int x^{-1+\frac{1+m}{n}}\sin^2\Big(a+\frac{1}{2}\sqrt{-\frac{(1}{}}}{n}$$

$$= -\frac{\left(x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\Bigg(\int\Bigg(\frac{e^{\frac{2a\sqrt{-\frac{(1+m)^2}{n^2}}n}{1+m}}}{x}-2x^{-1+\frac{1}{}}}{4n}$$

$$= \frac{x^{1+m}}{2(1+m)} - \frac{e^{-\frac{2a\sqrt{-\frac{(1+m)^2}{n^2}}n}{1+m}}x^{1+m}(cx^n)^{\frac{1+m}{n}}}{8(1+m)} - \frac{1}{4}e^{\frac{2a\sqrt{-\frac{(1+m}{n^2}}}{1+m}}$$

Mathematica [F]
time = 0.40, size = 0, normalized size = 0.00

$$\int x^m \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*Sin[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2]^2,x]
[Out] Integrate[x^m*Sin[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2]^2, x]
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int x^m\left(\sin^2\left(a+\frac{\ln(c\,x^n)\sqrt{-\frac{(1+m)^2}{n^2}}}{2}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(a+1/2*ln(c*x^n)*(-(1+m)^2/n^2)^(1/2))^2,x)
[Out] int(x^m*sin(a+1/2*ln(c*x^n)*(-(1+m)^2/n^2)^(1/2))^2,x)
```

Maxima [A]
time = 0.31, size = 173, normalized size = 1.48

$$\frac{4\left(\cos(2a)^2+\sin(2a)^2\right)c^{\frac{m}{n}+\frac{1}{n}}xx^m - c^{\frac{2m}{n}+\frac{2}{n}}x\cos(2a)\,e^{\left(m\log(x)+\frac{m\log(x^n)}{n}+\frac{\log(x^n)}{n}\right)} - 2\left(\cos(2a)^3+\cos(2a)\sin(2a)^2+\left(\cos(2a)^3+\cos(2a)\sin(2a)^2\right)m\right)\log(x)}{8\left(\left(\cos(2a)^2+\sin(2a)^2\right)c^{\frac{m}{n}+\frac{1}{n}}m+\left(\cos(2a)^2+\sin(2a)^2\right)c^{\frac{m}{n}+\frac{1}{n}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+1/2*log(c*x^n)*(-(1+m)^2/n^2)^(1/2))^2,x, algorithm="ma
xima")
```

```
[Out] 1/8*(4*(cos(2*a)^2 + sin(2*a)^2)*c^(m/n + 1/n)*x*x^m - c^(2*m/n + 2/n)*x*co
s(2*a)*e^(m*log(x) + m*log(x^n)/n + log(x^n)/n) - 2*(cos(2*a)^3 + cos(2*a)*
sin(2*a)^2 + (cos(2*a)^3 + cos(2*a)*sin(2*a)^2)*m)*log(x))/((cos(2*a)^2 + s
in(2*a)^2)*c^(m/n + 1/n)*m + (cos(2*a)^2 + sin(2*a)^2)*c^(m/n + 1/n))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.67, size = 107, normalized size = 0.91

$$-\frac{\left(2\,(m+1)e^{\left(-\frac{2\,((m+1)n\log(x)-2i\,an+(m+1)\log(c))}{n}\right)}\log(x)-4\,e^{\left(-\frac{(m+1)n\log(x)-2i\,an+(m+1)\log(c)}{n}\right)}+1\right)e^{\left(\frac{2\,((m+1)n\log(x)-2i\,an+(m+1)\log(c))}{n}+\frac{2i\,an-(m+1)\log(c)}{n}\right)}}{8\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+1/2*log(c*x^n)*(-(1+m)^2/n^2)^(1/2))^2,x, algorithm="fr
icas")
```

```
[Out] -1/8*(2*(m + 1)*e^(-2*((m + 1)*n*log(x) - 2*I*a*n + (m + 1)*log(c))/n)*log(
x) - 4*e^(-((m + 1)*n*log(x) - 2*I*a*n + (m + 1)*log(c))/n) + 1)*e^(2*((m +
 1)*n*log(x) - 2*I*a*n + (m + 1)*log(c))/n + (2*I*a*n - (m + 1)*log(c))/n)/
(m + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\sin^2\left(a+\frac{\sqrt{-\frac{m^2}{n^2}-\frac{2m}{n^2}-\frac{1}{n^2}}\,\log(cx^n)}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sin(a+1/2*ln(c*x**n)*(-(1+m)**2/n**2)**(1/2))**2,x)
```

```
[Out] Integral(x**m*sin(a + sqrt(-m**2/n**2 - 2*m/n**2 - 1/n**2)*log(c*x**n)/2)**
2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 2.82, size = 498, normalized size = 4.26

$$-\frac{m^2n^2xx^me^{\left(2i\,a-\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}+m^2n^2xx^me^{\left(-2i\,a+\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}-2\,m^2n^2xx^m+2\,mn^2xx^me^{\left(2i\,a-\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}+mnxx^m|mn+n|e^{\left(2i\,a-\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}+2\,mn^2xx^me^{\left(-2i\,a+\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}-mnxx^m|mn+n|e^{\left(-2i\,a+\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}-4\,mn^2xx^m+n^2xx^me^{\left(2i\,a-\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}+nxx^m|mn+n|e^{\left(2i\,a-\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}+n^2xx^me^{\left(-2i\,a+\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}-nxx^m|mn+n|e^{\left(-2i\,a+\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}+2\,(mn+n)^2xx^m-2\,n^2xx^m}{4\,\left(m^3n^2+3\,m^2n^2-(mn+n)^2m+3\,mn^2-(mn+n)^2+n^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+1/2*log(c*x^n)*(-(1+m)^2/n^2)^(1/2))^2,x, algorithm="gi
ac")
```

```
[Out] -1/4*(m^2*n^2*x*x^m*e^(2*I*a - (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)
)/n^2) + m^2*n^2*x*x^m*e^(-2*I*a + (n*abs(m*n + n)*log(x) + abs(m*n + n)*lo
g(c))/n^2) - 2*m^2*n^2*x*x^m + 2*m*n^2*x*x^m*e^(2*I*a - (n*abs(m*n + n)*log
(x) + abs(m*n + n)*log(c))/n^2) + m*n*x*x^m*abs(m*n + n)*e^(2*I*a - (n*abs(
m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 2*m*n^2*x*x^m*e^(-2*I*a + (n*
abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - m*n*x*x^m*abs(m*n + n)*e^
(-2*I*a + (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - 4*m*n^2*x*x^
m + n^2*x*x^m*e^(2*I*a - (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)
 + n*x*x^m*abs(m*n + n)*e^(2*I*a - (n*abs(m*n + n)*log(x) + abs(m*n + n)*lo
g(c))/n^2) + n^2*x*x^m*e^(-2*I*a + (n*abs(m*n + n)*log(x) + abs(m*n + n)*lo
g(c))/n^2) - n*x*x^m*abs(m*n + n)*e^(-2*I*a + (n*abs(m*n + n)*log(x) + abs(
m*n + n)*log(c))/n^2) + 2*(m*n + n)^2*x*x^m - 2*n^2*x*x^m)/(m^3*n^2 + 3*m^2
*n^2 - (m*n + n)^2*m + 3*m*n^2 - (m*n + n)^2 + n^2)
```

Mupad [B]

time = 3.85, size = 145, normalized size = 1.24

$$\frac{x\,x^m}{2\,m+2}-\frac{x\,x^m\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,1\mathrm{i}}}}{4\,m+4-n\,\sqrt{-\frac{{\left(m+1\right)}^2}{n^2}}\,4\mathrm{i}}-\frac{x\,x^m\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,1\mathrm{i}}}{4\,m+4+n\,\sqrt{-\frac{{\left(m+1\right)}^2}{n^2}}\,4\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(a + (log(c*x^n)*(-(m + 1)^2/n^2)^(1/2))/2)^2,x)
```

```
[Out] (x*x^m)/(2*m + 2) - (x*x^m*exp(-a*2i)/(c*x^n)^((- (2*m)/n^2 - 1/n^2 - m^2/n
^2)^(1/2)*1i))/(4*m - n*(-(m + 1)^2/n^2)^(1/2)*4i + 4) - (x*x^m*exp(a*2i)*(
c*x^n)^((- (2*m)/n^2 - 1/n^2 - m^2/n^2)^(1/2)*1i))/(4*m + n*(-(m + 1)^2/n^2
)^(1/2)*4i + 4)
```

3.34 $\int x^2 \sin^2\left(a+\frac{3}{2}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right) dx$

Optimal. Leaf size=76

$$\frac{x^3}{6}-\frac{1}{24}e^{-2a\sqrt{-\frac{1}{n^2}}n}x^3(cx^n)^{3/n}-\frac{1}{4}e^{2a\sqrt{-\frac{1}{n^2}}n}x^3(cx^n)^{-3/n}\log(x)$$

```
[Out] 1/6*x^3-1/24*x^3*(c*x^n)^(3/n)/exp(2*a*n*(-1/n^2)^(1/2))-1/4*exp(2*a*n*(-1/
n^2)^(1/2))*x^3*ln(x)/((c*x^n)^(3/n))
```

Rubi [A]
time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {4581, 4577}

$$-\frac{1}{24}x^3e^{-2a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{3/n}-\frac{1}{4}x^3e^{2a\sqrt{-\frac{1}{n^2}}n}\log(x)(cx^n)^{-3/n}+\frac{x^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[a + (3*Sqrt[-n^(-2)]*Log[c*x^n])/2]^2,x]
```

```
[Out] x^3/6 - (x^3*(c*x^n)^(3/n))/(24*E^(2*a*Sqrt[-n^(-2)]*n)) - (E^(2*a*Sqrt[-n^
(-2)]*n)*x^3*Log[x])/(4*(c*x^n)^(3/n))
```

Rule 4577

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[(m + 1)^p/(2^p*b^p*d^p*p^p), Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^
2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x]
, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m
 + 1)^2, 0]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^2 \sin^2\left(a+\frac{3}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx = \frac{\left(x^3(cx^n)^{-3/n}\right)\text{Subst}\left(\int x^{-1+\frac{3}{n}}\sin^2\left(a+\frac{3}{2}\sqrt{-\frac{1}{n^2}}\log(x)\right)dx,\right.}{n}$$

$$= -\frac{\left(x^3(cx^n)^{-3/n}\right)\text{Subst}\left(\int\left(\frac{e^{2a\sqrt{-\frac{1}{n^2}}n}}{x} - 2x^{-1+\frac{3}{n}} + e^{-2a\sqrt{-\frac{1}{n^2}}n}x\right.\right.}{4n}$$

$$= \frac{x^3}{6} - \frac{1}{24}e^{-2a\sqrt{-\frac{1}{n^2}}n}x^3(cx^n)^{3/n} - \frac{1}{4}e^{2a\sqrt{-\frac{1}{n^2}}n}x^3(cx^n)^{-3/n}\log(x)$$

Mathematica [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int x^2 \sin^2\left(a+\frac{3}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*Sin[a + (3*Sqrt[-n^(-2)]*Log[c*x^n])/2]^2,x]
[Out] Integrate[x^2*Sin[a + (3*Sqrt[-n^(-2)]*Log[c*x^n])/2]^2, x]
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int x^2\left(\sin^2\left(a+\frac{3\ln\left(c\,x^n\right)\sqrt{-\frac{1}{n^2}}}{2}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a+3/2*ln(c*x^n)*(-1/n^2)^(1/2))^2,x)
[Out] int(x^2*sin(a+3/2*ln(c*x^n)*(-1/n^2)^(1/2))^2,x)
```

Maxima [A]
time = 0.29, size = 47, normalized size = 0.62

$$-\frac{c^{\frac{6}{n}}x^6\cos\left(2\,a\right) - 4\,c^{\frac{3}{n}}x^3 + 6\,\cos\left(2\,a\right)\log\left(x\right)}{24\,c^{\frac{3}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+3/2*log(c*x^n)*(-1/n^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] -1/24*(c^(6/n)*x^6*cos(2*a) - 4*c^(3/n)*x^3 + 6*cos(2*a)*log(x))/c^(3/n)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.48, size = 59, normalized size = 0.78

$$-\frac{1}{24}\left(x^{6}-4\,x^{3}e^{\left(\frac{2i\,an-3\,\log(c)}{n}\right)}+6\,e^{\left(\frac{2\,(2i\,an-3\,\log(c))}{n}\right)}\log\left(x\right)\right)e^{\left(-\frac{2i\,an-3\,\log(c)}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+3/2*log(c*x^n)*(-1/n^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/24*(x^6 - 4*x^3*e^((2*I*a*n - 3*log(c))/n) + 6*e^(2*(2*I*a*n - 3*log(c))
/n)*log(x))*e^(-(2*I*a*n - 3*log(c))/n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\sin^{2}\left(a+\frac{3\sqrt{-\frac{1}{n^{2}}}\,\log\left(cx^{n}\right)}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(a+3/2*ln(c*x**n)*(-1/n**2)**(1/2))**2,x)
```

```
[Out] Integral(x**2*sin(a + 3*sqrt(-1/n**2)*log(c*x**n)/2)**2, x)
```

Giac [A]
time = 0.81, size = 1, normalized size = 0.01

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+3/2*log(c*x^n)*(-1/n^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] +Infinity
```

Mupad [B]
time = 2.97, size = 92, normalized size = 1.21

$$\frac{x^{3}}{6}-\frac{x^{3}\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{{\left(c\,x^{n}\right)}^{\sqrt{-\frac{1}{n^{2}}}\,3\mathrm{i}}}\,1\mathrm{i}}{12\,n\,\sqrt{-\frac{1}{n^{2}}}+12\mathrm{i}}+\frac{x^{3}\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^{n}\right)}^{\sqrt{-\frac{1}{n^{2}}}\,3\mathrm{i}}\,1\mathrm{i}}{12\,n\,\sqrt{-\frac{1}{n^{2}}}-12\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a + (3*log(c*x^n)*(-1/n^2)^(1/2))/2)^2,x)
```

```
[Out] x^3/6 - (x^3*exp(-a*2i)/(c*x^n)^((-1/n^2)^(1/2)*3i)*1i)/(12*n*(-1/n^2)^(1/2
) + 12i) + (x^3*exp(a*2i)*(c*x^n)^((-1/n^2)^(1/2)*3i)*1i)/(12*n*(-1/n^2)^(1
/2) - 12i)
```

3.35 $\int x \sin^2\left(a+\sqrt{-\frac{1}{n^2}} \log\left(cx^n\right)\right) dx$

Optimal. Leaf size=76

$$\frac{x^2}{4}-\frac{1}{16}e^{-2a\sqrt{-\frac{1}{n^2}}\,n}x^2(cx^n)^{2/n}-\frac{1}{4}e^{2a\sqrt{-\frac{1}{n^2}}\,n}x^2(cx^n)^{-2/n}\log(x)$$

```
[Out] 1/4*x^2-1/16*x^2*(c*x^n)^(2/n)/exp(2*a*n*(-1/n^2)^(1/2))-1/4*exp(2*a*n*(-1/
n^2)^(1/2))*x^2*ln(x)/((c*x^n)^(2/n))
```

Rubi [A]
time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4581, 4577}

$$-\frac{1}{16}x^2e^{-2a\sqrt{-\frac{1}{n^2}}\,n}(cx^n)^{2/n}-\frac{1}{4}x^2e^{2a\sqrt{-\frac{1}{n^2}}\,n}\log(x)\,(cx^n)^{-2/n}+\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]]^2,x]
```

```
[Out] x^2/4 - (x^2*(c*x^n)^(2/n))/(16*E^(2*a*Sqrt[-n^(-2)]*n)) - (E^(2*a*Sqrt[-n^
(-2)]*n)*x^2*Log[x])/(4*(c*x^n)^(2/n))
```

Rule 4577

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[(m + 1)^p/(2^p*b^p*d^p*p^p), Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^
2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x]
, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m
 + 1)^2, 0]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx = \frac{\left(x^2(cx^n)^{-2/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{2}{n}}\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log(x)\right)dx, x, c\right.}{n}$$

$$= -\frac{\left(x^2(cx^n)^{-2/n}\right)\operatorname{Subst}\left(\int\left(\frac{e^{2a\sqrt{-\frac{1}{n^2}}n}}{x} - 2x^{-1+\frac{2}{n}} + e^{-2a\sqrt{-\frac{1}{n^2}}n}x^{-1}\right.\right.}{4n}$$

$$= \frac{x^2}{4} - \frac{1}{16}e^{-2a\sqrt{-\frac{1}{n^2}}n}x^2(cx^n)^{2/n} - \frac{1}{4}e^{2a\sqrt{-\frac{1}{n^2}}n}x^2(cx^n)^{-2/n}\log(x)$$

Mathematica [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int x\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]]^2,x]
[Out] Integrate[x*Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]]^2, x]
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int x\left(\sin^2\left(a+\ln\left(c\,x^n\right)\sqrt{-\frac{1}{n^2}}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a+ln(c*x^n)*(-1/n^2)^(1/2))^2,x)
[Out] int(x*sin(a+ln(c*x^n)*(-1/n^2)^(1/2))^2,x)
```

Maxima [A]
time = 0.29, size = 47, normalized size = 0.62

$$-\frac{c^{\frac{4}{n}}x^4\cos\left(2\,a\right) - 4\,c^{\frac{2}{n}}x^2 + 4\,\cos\left(2\,a\right)\log\left(x\right)}{16\,c^{\frac{2}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+log(c*x^n)*(-1/n^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] -1/16*(c^(4/n)*x^4*cos(2*a) - 4*c^(2/n)*x^2 + 4*cos(2*a)*log(x))/c^(2/n)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.63, size = 60, normalized size = 0.79

$$-\frac{1}{16}\left(x^4-4\,x^2e^{\left(\frac{2\,(i\,an-\log(c))}{n}\right)}+4\,e^{\left(\frac{4\,(i\,an-\log(c))}{n}\right)}\log\left(x\right)\right)e^{\left(-\frac{2\,(i\,an-\log(c))}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+log(c*x^n)*(-1/n^2)^(1/2))^2,x, algorithm="fricas")
[Out] -1/16*(x^4 - 4*x^2*e^(2*(I*a*n - log(c))/n) + 4*e^(4*(I*a*n - log(c))/n)*lo
g(x))*e^(-2*(I*a*n - log(c))/n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\,\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+ln(c*x**n)*(-1/n**2)**(1/2))**2,x)
[Out] Integral(x*sin(a + sqrt(-1/n**2)*log(c*x**n))**2, x)
```

Giac [A]
time = 0.75, size = 1, normalized size = 0.01

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+log(c*x^n)*(-1/n^2)^(1/2))^2,x, algorithm="giac")
[Out] +Infinity
```

Mupad [B]
time = 2.89, size = 92, normalized size = 1.21

$$\frac{x^2}{4}-\frac{x^2\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,2\mathrm{i}}}\,1\mathrm{i}}{8\,n\,\sqrt{-\frac{1}{n^2}}+8\mathrm{i}}+\frac{x^2\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,2\mathrm{i}}\,1\mathrm{i}}{8\,n\,\sqrt{-\frac{1}{n^2}}-8\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + log(c*x^n)*(-1/n^2)^(1/2))^2,x)
[Out] x^2/4 - (x^2*exp(-a*2i)/(c*x^n)^((-1/n^2)^(1/2)*2i)*1i)/(8*n*(-1/n^2)^(1/2)
 + 8i) + (x^2*exp(a*2i)*(c*x^n)^((-1/n^2)^(1/2)*2i)*1i)/(8*n*(-1/n^2)^(1/2)
 - 8i)
```

3.36 $\int \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=68

$$\frac{x}{2}-\frac{1}{8}e^{-2a\sqrt{-\frac{1}{n^2}}\,n}x(cx^n)^{\frac{1}{n}}-\frac{1}{4}e^{2a\sqrt{-\frac{1}{n^2}}\,n}x(cx^n)^{-1/n}\log(x)$$

```
[Out] 1/2*x-1/8*x*(c*x^n)^(1/n)/exp(2*a*n*(-1/n^2)^(1/2))-1/4*exp(2*a*n*(-1/n^2)^
(1/2))*x*ln(x)/((c*x^n)^(1/n))
```

Rubi [A]
time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4571, 4577}

$$-\frac{1}{8}xe^{-2a\sqrt{-\frac{1}{n^2}}\,n}(cx^n)^{\frac{1}{n}}-\frac{1}{4}xe^{2a\sqrt{-\frac{1}{n^2}}\,n}\log(x)\left(cx^n\right)^{-1/n}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + (Sqrt[-n^(-2)]*Log[c*x^n])/2]^2,x]
```

```
[Out] x/2 - (x*(c*x^n)^n^(-1))/(8*E^(2*a*Sqrt[-n^(-2)]*n)) - (E^(2*a*Sqrt[-n^(-2)
]*n)*x*Log[x])/(4*(c*x^n)^n^(-1))
```

Rule 4571

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4577

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[(m + 1)^p/(2^p*b^p*d^p*p^p), Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^
2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x]
, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m
 + 1)^2, 0]
```

Rubi steps

$$\int \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log(x)\right)dx,x,cx\right.}{n}$$

$$= -\frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int\left(\frac{e^{2a\sqrt{-\frac{1}{n^2}}n}}{x}-2x^{-1+\frac{1}{n}}+e^{-2a\sqrt{-\frac{1}{n^2}}n}x^{-1+\frac{2}{n}}\right.\right.}{4n}$$

$$= \frac{x}{2}-\frac{1}{8}e^{-2a\sqrt{-\frac{1}{n^2}}n}x(cx^n)^{\frac{1}{n}}-\frac{1}{4}e^{2a\sqrt{-\frac{1}{n^2}}n}x(cx^n)^{-1/n}\log(x)$$

Mathematica [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + (Sqrt[-n^(-2)]*Log[c*x^n])/2]^2,x]
[Out] Integrate[Sin[a + (Sqrt[-n^(-2)]*Log[c*x^n])/2]^2, x]
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \sin^2\left(a+\frac{\ln\left(c\,x^n\right)\sqrt{-\frac{1}{n^2}}}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+1/2*ln(c*x^n)*(-1/n^2)^(1/2))^2,x)
[Out] int(sin(a+1/2*ln(c*x^n)*(-1/n^2)^(1/2))^2,x)
```

Maxima [A]
time = 0.30, size = 41, normalized size = 0.60

$$-\frac{c^{\frac{2}{n}}x^2\cos\left(2\,a\right)-4\,c^{\left(\frac{1}{n}\right)}x+2\,\cos\left(2\,a\right)\log\left(x\right)}{8\,c^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/2*log(c*x^n)*(-1/n^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(c^(2/n)*x^2*cos(2*a) - 4*c^(1/n)*x + 2*cos(2*a)*log(x))/c^(1/n)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.59, size = 57, normalized size = 0.84

$$-\frac{1}{8}\left(x^{2}-4\,xe^{\left(\frac{2i\,an-\log\left(c\right)}{n}\right)}+2\,e^{\left(\frac{2\,\left(2i\,an-\log\left(c\right)\right)}{n}\right)}\log\left(x\right)\right)e^{\left(-\frac{2i\,an-\log\left(c\right)}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/2*log(c*x^n)*(-1/n^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(x^2 - 4*x*e^((2*I*a*n - log(c))/n) + 2*e^(2*(2*I*a*n - log(c))/n)*log
(x))*e^(-(2*I*a*n - log(c))/n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sin^{2}\left(a+\frac{\sqrt{-\frac{1}{n^{2}}}\log\left(cx^{n}\right)}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/2*ln(c*x**n)*(-1/n**2)**(1/2))**2,x)
```

```
[Out] Integral(sin(a + sqrt(-1/n**2)*log(c*x**n)/2)**2, x)
```

Giac [A]
time = 0.66, size = 1, normalized size = 0.01

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/2*log(c*x^n)*(-1/n^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] +Infinity
```

Mupad [B]
time = 2.66, size = 86, normalized size = 1.26

$$\frac{x}{2}-\frac{x\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}}\,1\mathrm{i}}{4\,n\,\sqrt{-\frac{1}{n^2}}+4\mathrm{i}}+\frac{x\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}\,1\mathrm{i}}{4\,n\,\sqrt{-\frac{1}{n^2}}-4\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + (log(c*x^n)*(-1/n^2)^(1/2))/2)^2,x)
```

```
[Out] x/2 - (x*exp(-a*2i)/(c*x^n)^((-1/n^2)^(1/2)*1i)*1i)/(4*n*(-1/n^2)^(1/2) + 4
i) + (x*exp(a*2i)*(c*x^n)^((-1/n^2)^(1/2)*1i)*1i)/(4*n*(-1/n^2)^(1/2) - 4i)
```

3.37 $\int \frac{\sin^2(a)}{x} \, dx$

Optimal. Leaf size=7

$$\log(x)\sin^2(a)$$

```
[Out] ln(x)*sin(a)^2
```

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {12, 29}

$$\sin^2(a)\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a]^2/x,x]
```

```
[Out] Log[x]*Sin[a]^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(a)}{x} \, dx &= \sin^2(a) \int \frac{1}{x} \, dx\\ &= \log(x)\sin^2(a)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$\log(x)\sin^2(a)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a]^2/x,x]
```

```
[Out] Log[x]*Sin[a]^2
```

Maple [A]
time = 0.02, size = 8, normalized size = 1.14

method	result	size
default	$\ln(x)(\sin^2(a))$	8
norman	$\ln(x)(\sin^2(a))$	8
risch	$\ln(x)(\sin^2(a))$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a)^2/x,x,method=_RETURNVERBOSE)
[Out] ln(x)*sin(a)^2
```

Maxima [A]
time = 0.27, size = 7, normalized size = 1.00

$$\log(x)\sin(a)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a)^2/x,x, algorithm="maxima")
[Out] log(x)*sin(a)^2
```

Fricas [A]
time = 1.47, size = 10, normalized size = 1.43

$$-\left(\cos(a)^2 - 1\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a)^2/x,x, algorithm="fricas")
[Out] -(cos(a)^2 - 1)*log(x)
```

Sympy [A]
time = 0.01, size = 7, normalized size = 1.00

$$\log(x)\sin^2(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a)**2/x,x)
[Out] log(x)*sin(a)**2
```

Giac [A]
time = 0.42, size = 8, normalized size = 1.14

$$\log(|x|)\sin(a)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a)^2/x,x, algorithm="giac")
```

```
[Out] log(abs(x))*sin(a)^2
```

Mupad [B]
time = 0.02, size = 7, normalized size = 1.00

$$\sin(a)^2 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a)^2/x,x)
```

```
[Out] sin(a)^2*log(x)
```

3.38 $\int \frac{\sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{x^2}\,dx$

Optimal. Leaf size=74

$$-\frac{1}{2x}+\frac{e^{2a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{-1/n}}{8x}-\frac{e^{-2a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{1}{n}}\log(x)}{4x}$$

```
[Out] -1/2/x+1/8*exp(2*a*n*(-1/n^2)^(1/2))/x/((c*x^n)^(1/n))-1/4*(c*x^n)^(1/n)*ln
(x)/exp(2*a*n*(-1/n^2)^(1/2))/x
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {4581, 4577}

$$\frac{e^{2a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{-1/n}}{8x}-\frac{e^{-2a\sqrt{-\frac{1}{n^2}}n}\log(x)\,(cx^n)^{\frac{1}{n}}}{4x}-\frac{1}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + (Sqrt[-n^(-2)]*Log[c*x^n])/2]^2/x^2,x]
```

```
[Out] -1/2*1/x + E^(2*a*Sqrt[-n^(-2)]*n)/(8*x*(c*x^n)^n^(-1)) - ((c*x^n)^n^(-1)*L
og[x])/(4*E^(2*a*Sqrt[-n^(-2)]*n)*x)
```

Rule 4577

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[(m + 1)^p/(2^p*b^p*d^p*p^p), Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^
2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x]
, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m
 + 1)^2, 0]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx = \frac{(cx^n)^{\frac{1}{n}}\operatorname{Subst}\left(\int x^{-1-\frac{1}{n}}\sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log(x)\right)dx,x,cx^n\right)}{nx}$$
$$= -\frac{(cx^n)^{\frac{1}{n}}\operatorname{Subst}\left(\int\left(\frac{e^{-2a\sqrt{-\frac{1}{n^2}}n}}{x}-2x^{-\frac{1+n}{n}}+e^{2a\sqrt{-\frac{1}{n^2}}n}x^{-\frac{2+n}{n}}\right)dx,x}{4nx}$$
$$= -\frac{1}{2x}+\frac{e^{2a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{-1/n}}{8x}-\frac{e^{-2a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{1}{n}}\log(x)}{4x}$$

Mathematica [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + (Sqrt[-n^(-2)]*Log[c*x^n])/2]^2/x^2,x]
[Out] Integrate[Sin[a + (Sqrt[-n^(-2)]*Log[c*x^n])/2]^2/x^2, x]
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(a+\frac{\ln(c\,x^n)\sqrt{-\frac{1}{n^2}}}{2}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+1/2*ln(c*x^n)*(-1/n^2)^(1/2))^2/x^2,x)
[Out] int(sin(a+1/2*ln(c*x^n)*(-1/n^2)^(1/2))^2/x^2,x)
```

Maxima [A]
time = 0.30, size = 48, normalized size = 0.65

$$-\frac{2\,c^{\frac{2}{n}}x^3\cos\left(2\,a\right)\log\left(x\right)+4\,c^{\left(\frac{1}{n}\right)}x^2-x\cos\left(2\,a\right)}{8\,c^{\left(\frac{1}{n}\right)}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/2*log(c*x^n)*(-1/n^2)^(1/2))^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/8*(2*c^(2/n)*x^3*cos(2*a)*log(x) + 4*c^(1/n)*x^2 - x*cos(2*a))/(c^(1/n)*
x^3)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.58, size = 62, normalized size = 0.84

$$-\frac{\left(2\,x^{2}\log\left(x\right)+4\,xe^{\left(\frac{2i\,an-\log\left(c\right)}{n}\right)}-e^{\left(\frac{2\,\left(2i\,an-\log\left(c\right)\right)}{n}\right)}\right)e^{\left(-\frac{2i\,an-\log\left(c\right)}{n}\right)}}{8\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/2*log(c*x^n)*(-1/n^2)^(1/2))^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*x^2*log(x) + 4*x*e^((2*I*a*n - log(c))/n) - e^(2*(2*I*a*n - log(c))
/n))*e^(-(2*I*a*n - log(c))/n)/x^2
```

Sympy [A]
time = 14.41, size = 117, normalized size = 1.58

$$\frac{n\sqrt{-\frac{1}{n^{2}}}\sin\left(2a+\sqrt{-\frac{1}{n^{2}}}\log\left(cx^{n}\right)\right)}{4x}+\frac{\sqrt{-\frac{1}{n^{2}}}\log\left(cx^{n}\right)\sin\left(2a+\sqrt{-\frac{1}{n^{2}}}\log\left(cx^{n}\right)\right)}{4x}-\frac{1}{2x}-\frac{\log\left(cx^{n}\right)\cos\left(2a+\sqrt{-\frac{1}{n^{2}}}\log\left(cx^{n}\right)\right)}{4nx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/2*ln(c*x**n)*(-1/n**2)**(1/2))**2/x**2,x)
```

```
[Out] n*sqrt(-1/n**2)*sin(2*a + sqrt(-1/n**2)*log(c*x**n))/(4*x) + sqrt(-1/n**2)*
log(c*x**n)*sin(2*a + sqrt(-1/n**2)*log(c*x**n))/(4*x) - 1/(2*x) - log(c*x*
*n)*cos(2*a + sqrt(-1/n**2)*log(c*x**n))/(4*n*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/2*log(c*x^n)*(-1/n^2)^(1/2))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(sin(1/2*sqrt(-1/n^2)*log(c*x^n) + a)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sin\left(a+\frac{\ln\left(c\,x^n\right)\,\sqrt{-\frac{1}{n^2}}}{2}\right)}^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + (log(c*x^n)*(-1/n^2)^(1/2))/2)^2/x^2,x)
[Out] int(sin(a + (log(c*x^n)*(-1/n^2)^(1/2))/2)^2/x^2, x)
```

3.39 $\int \frac{\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{x^3}\,dx$

Optimal. Leaf size=76

$$-\frac{1}{4x^2}+\frac{e^{2a\sqrt{-\frac{1}{n^2}}\,n}(cx^n)^{-2/n}}{16x^2}-\frac{e^{-2a\sqrt{-\frac{1}{n^2}}\,n}(cx^n)^{2/n}\log(x)}{4x^2}$$

```
[Out] -1/4/x^2+1/16*exp(2*a*n*(-1/n^2)^(1/2))/x^2/((c*x^n)^(2/n))-1/4*(c*x^n)^(2/
n)*ln(x)/exp(2*a*n*(-1/n^2)^(1/2))/x^2
```

Rubi [A]
time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = $\{4581, 4577\}$

$$\frac{e^{2a\sqrt{-\frac{1}{n^2}}\,n}(cx^n)^{-2/n}}{16x^2}-\frac{e^{-2a\sqrt{-\frac{1}{n^2}}\,n}\log(x)\,(cx^n)^{2/n}}{4x^2}-\frac{1}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]]^2/x^3,x]
```

```
[Out] -1/4*1/x^2 + E^(2*a*Sqrt[-n^(-2)]*n)/(16*x^2*(c*x^n)^(2/n)) - ((c*x^n)^(2/n
)*Log[x])/(4*E^(2*a*Sqrt[-n^(-2)]*n)*x^2)
```

Rule 4577

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[(m + 1)^p/(2^p*b^p*d^p*p^p), Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^
2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x]
, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m
 + 1)^2, 0]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\ \log\left(cx^n\right)\right)}{x^3}\,dx = \frac{\left(cx^n\right)^{2/n}\text{Subst}\left(\int x^{-1-\frac{2}{n}}\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\ \log(x)\right)\,dx, x, cx^n\right)}{nx^2}$$

$$= -\frac{\left(cx^n\right)^{2/n}\text{Subst}\left(\int\left(\frac{e^{-2a\sqrt{-\frac{1}{n^2}}\,n}}{x}-2x^{-\frac{2+n}{n}}+e^{2a\sqrt{-\frac{1}{n^2}}\,n}x^{-\frac{4+n}{n}}\right)dx, x}{4nx^2}$$

$$= -\frac{1}{4x^2}+\frac{e^{2a\sqrt{-\frac{1}{n^2}}\,n}\left(cx^n\right)^{-2/n}}{16x^2}-\frac{e^{-2a\sqrt{-\frac{1}{n^2}}\,n}\left(cx^n\right)^{2/n}\log(x)}{4x^2}$$

Mathematica [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\ \log\left(cx^n\right)\right)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]]^2/x^3,x]
[Out] Integrate[Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]]^2/x^3, x]
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(a+\ln\left(c\,x^n\right)\sqrt{-\frac{1}{n^2}}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+ln(c*x^n)*(-1/n^2)^(1/2))^2/x^3,x)
[Out] int(sin(a+ln(c*x^n)*(-1/n^2)^(1/2))^2/x^3,x)
```

Maxima [A]
time = 0.29, size = 54, normalized size = 0.71

$$-\frac{4\,c^{\frac{4}{n}}x^6\cos\left(2\,a\right)\log\left(x\right)+4\,c^{\frac{2}{n}}x^4-x^2\cos\left(2\,a\right)}{16\,c^{\frac{2}{n}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+log(c*x^n)*(-1/n^2)^(1/2))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/16*(4*c^(4/n)*x^6*cos(2*a)*log(x) + 4*c^(2/n)*x^4 - x^2*cos(2*a))/(c^(2/
n)*x^6)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.41, size = 65, normalized size = 0.86

$$-\frac{\left(4\,x^{4}\log\left(x\right)+4\,x^{2}e^{\left(\frac{2\,\left(i\,an-\log\left(c\right)\right)}{n}\right)}-e^{\left(\frac{4\,\left(i\,an-\log\left(c\right)\right)}{n}\right)}\right)e^{\left(-\frac{2\,\left(i\,an-\log\left(c\right)\right)}{n}\right)}}{16\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+log(c*x^n)*(-1/n^2)^(1/2))^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*(4*x^4*log(x) + 4*x^2*e^(2*(I*a*n - log(c))/n) - e^(4*(I*a*n - log(c)
)/n))*e^(-2*(I*a*n - log(c))/n)/x^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(70) = 140$.
time = 6.11, size = 221, normalized size = 2.91

$$-\frac{\sin^{2}\left(a+\sqrt{-\frac{1}{n^{2}}}\log\left(cx^{n}\right)\right)}{2x^{2}}+\frac{\log\left(cx^{n}\right)\sin^{2}\left(a+\sqrt{-\frac{1}{n^{2}}}\log\left(cx^{n}\right)\right)}{4nx^{2}}-\frac{\log\left(cx^{n}\right)\cos^{2}\left(a+\sqrt{-\frac{1}{n^{2}}}\log\left(cx^{n}\right)\right)}{4nx^{2}}+\frac{\sin\left(a+\sqrt{-\frac{1}{n^{2}}}\log\left(cx^{n}\right)\right)\cos\left(a+\sqrt{-\frac{1}{n^{2}}}\log\left(cx^{n}\right)\right)}{4nx^{2}\sqrt{-\frac{1}{n^{2}}}}-\frac{\log\left(cx^{n}\right)\sin\left(a+\sqrt{-\frac{1}{n^{2}}}\log\left(cx^{n}\right)\right)\cos\left(a+\sqrt{-\frac{1}{n^{2}}}\log\left(cx^{n}\right)\right)}{2n^{2}x^{2}\sqrt{-\frac{1}{n^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+ln(c*x**n)*(-1/n**2)**(1/2))**2/x**3,x)
```

```
[Out] -sin(a + sqrt(-1/n**2)*log(c*x**n))**2/(2*x**2) + log(c*x**n)*sin(a + sqrt(
-1/n**2)*log(c*x**n))**2/(4*n*x**2) - log(c*x**n)*cos(a + sqrt(-1/n**2)*log
(c*x**n))**2/(4*n*x**2) + sin(a + sqrt(-1/n**2)*log(c*x**n))*cos(a + sqrt(-
1/n**2)*log(c*x**n))/(4*n*x**2*sqrt(-1/n**2)) - log(c*x**n)*sin(a + sqrt(-1
/n**2)*log(c*x**n))*cos(a + sqrt(-1/n**2)*log(c*x**n))/(2*n**2*x**2*sqrt(-1
/n**2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+log(c*x^n)*(-1/n^2)^(1/2))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(sin(sqrt(-1/n^2)*log(c*x^n) + a)^2/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sin\left(a+\ln\left(c\,x^n\right)\,\sqrt{-\frac{1}{n^2}}\right)}^2}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + log(c*x^n)*(-1/n^2)^(1/2))^2/x^3,x)
[Out] int(sin(a + log(c*x^n)*(-1/n^2)^(1/2))^2/x^3, x)
```

3.40 $\int x^m \sin^3\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=226

$$-\frac{4\sqrt{-\frac{(1+m)^2}{n^2}}\,nx^{1+m}\cos\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)}{5(1+m)^2}+\frac{8x^{1+m}\sin\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)}{5(1+m)}$$

```
[Out] 8/5*x^(1+m)*sin(a+1/2*ln(c*x^n)*(-(1+m)^2/n^2)^(1/2))/(1+m)-4/5*x^(1+m)*sin
(a+1/2*ln(c*x^n)*(-(1+m)^2/n^2)^(1/2))^3/(1+m)-4/5*n*x^(1+m)*cos(a+1/2*ln(c
*x^n)*(-(1+m)^2/n^2)^(1/2))*(-(1+m)^2/n^2)^(1/2)/(1+m)^2+6/5*n*x^(1+m)*cos(
a+1/2*ln(c*x^n)*(-(1+m)^2/n^2)^(1/2))*sin(a+1/2*ln(c*x^n)*(-(1+m)^2/n^2)^(1
/2))^2*(-(1+m)^2/n^2)^(1/2)/(1+m)^2
```

Rubi [A]
time = 0.05, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {4575, 4573}

$$-\frac{4x^{m+1}\sin^3\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)}{5(m+1)}+\frac{8x^{m+1}\sin\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)}{5(m+1)}-\frac{4n\sqrt{-\frac{(m+1)^2}{n^2}}\,x^{m+1}\cos\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)}{5(m+1)^2}+\frac{6n\sqrt{-\frac{(m+1)^2}{n^2}}\,x^{m+1}\sin^2\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)\cos\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)}{5(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sin[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2]^3,x]
```

```
[Out] (-4*Sqrt[-((1 + m)^2/n^2)]*n*x^(1 + m)*Cos[a + (Sqrt[-((1 + m)^2/n^2)]*Log[
c*x^n])/2])/(5*(1 + m)^2) + (8*x^(1 + m)*Sin[a + (Sqrt[-((1 + m)^2/n^2)]*Lo
g[c*x^n])/2])/(5*(1 + m)) + (6*Sqrt[-((1 + m)^2/n^2)]*n*x^(1 + m)*Cos[a + (
Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2]*Sin[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c
*x^n])/2]^2)/(5*(1 + m)^2) - (4*x^(1 + m)*Sin[a + (Sqrt[-((1 + m)^2/n^2)]*L
og[c*x^n])/2]^3)/(5*(1 + m))
```

Rule 4573

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] - Simp[b*d*n*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Rule 4575

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2
```

```
*n^2*p^2 + (m + 1)^2)), Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x],
x] - Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a + b*Log[c*x^n])]*(Sin[d*(a + b*Log
[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x]) /; FreeQ[{a, b, c
, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int x^m \sin^3\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)dx = \frac{6\sqrt{-\frac{(1+m)^2}{n^2}}\,nx^{1+m}\cos\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right.}{5(1+m)^2}$$

$$= -\frac{4\sqrt{-\frac{(1+m)^2}{n^2}}\,nx^{1+m}\cos\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^{n}\right.\right.}{5(1+m)^2}$$

Mathematica [A]
time = 1.39, size = 169, normalized size = 0.75

$$\frac{x^{1+m}\left(-5\sqrt{-\frac{(1+m)^2}{n^2}}\,n\cos\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)-3\sqrt{-\frac{(1+m)^2}{n^2}}\,n\cos\left(3a+\frac{3}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)+2(1+m)\left(5\sin\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)+\sin\left(3a+\frac{3}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\right)\right)}{10(1+m)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sin[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2]^3,x]
```

```
[Out] (x^(1 + m)*(-5*Sqrt[-((1 + m)^2/n^2)]*n*Cos[a + (Sqrt[-((1 + m)^2/n^2)]*Log
[c*x^n])/2] - 3*Sqrt[-((1 + m)^2/n^2)]*n*Cos[3*a + (3*Sqrt[-((1 + m)^2/n^2)
]*Log[c*x^n])/2] + 2*(1 + m)*(5*Sin[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])
/2] + Sin[3*a + (3*Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2])))/(10*(1 + m)^2)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int x^m\left(\sin^3\left(a+\frac{\ln\left(c\,x^n\right)\sqrt{-\frac{(1+m)^2}{n^2}}}{2}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(a+1/2*ln(c*x^n)*(-(1+m)^2/n^2)^(1/2))^3,x)
```

```
[Out] int(x^m*sin(a+1/2*ln(c*x^n)*(-(1+m)^2/n^2)^(1/2))^3,x)
```

Maxima [A]
time = 0.32, size = 195, normalized size = 0.86

$$-\frac{\left(c^{\frac{3m}{n}+\frac{3}{n}} x e^{\left(m\log(x)+\frac{3m\log(x^n)}{n}+\frac{3\log(x^n)}{n}\right)}\sin(3a) - 5c^{\frac{2m}{n}+\frac{2}{n}} x e^{\left(m\log(x)+\frac{2m\log(x^n)}{n}+\frac{2\log(x^n)}{n}\right)}\sin(a) - 15c^{\frac{m}{n}+\frac{1}{n}} x e^{\left(m\log(x)+\frac{m\log(x^n)}{n}+\frac{\log(x^n)}{n}\right)}\sin(a) - 5xx^m\sin(3a)\right)e^{\left(-\frac{3m\log(x^n)}{2n}-\frac{3\log(x^n)}{2n}\right)}}{20\left(c^{\frac{3m}{2n}+\frac{3}{2n}}m + c^{\frac{3m}{2n}+\frac{3}{2n}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+1/2*log(c*x^n)*(-(1+m)^2/n^2)^(1/2))^3,x, algorithm="ma
xima")
```

```
[Out] -1/20*(c^(3*m/n + 3/n)*x*e^(m*log(x) + 3*m*log(x^n)/n + 3*log(x^n)/n)*sin(3
*a) - 5*c^(2*m/n + 2/n)*x*e^(m*log(x) + 2*m*log(x^n)/n + 2*log(x^n)/n)*sin(
a) - 15*c^(m/n + 1/n)*x*e^(m*log(x) + m*log(x^n)/n + log(x^n)/n)*sin(a) - 5
*x*x^m*sin(3*a))*e^(-3/2*m*log(x^n)/n - 3/2*log(x^n)/n)/(c^(3/2*m/n + 3/2/n
)*m + c^(3/2*m/n + 3/2/n))
```

Fricas [C] Result contains complex when optimal does not.
time = 2.35, size = 128, normalized size = 0.57

$$\frac{\left(5i\,e^{\left(-\frac{(m+1)n\log(x)-2ian+(m+1)\log(c)}{n}\right)} - 15i\,e^{\left(-\frac{2((m+1)n\log(x)-2ian+(m+1)\log(c))}{n}\right)} - 5i\,e^{\left(-\frac{3((m+1)n\log(x)-2ian+(m+1)\log(c))}{n}\right)} - i\right)e^{\left(\frac{5((m+1)n\log(x)-2ian+(m+1)\log(c))}{2n}+\frac{2ian-(m+1)\log(c)}{n}\right)}}{20\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+1/2*log(c*x^n)*(-(1+m)^2/n^2)^(1/2))^3,x, algorithm="fr
icas")
```

```
[Out] 1/20*(5*I*e^(-((m + 1)*n*log(x) - 2*I*a*n + (m + 1)*log(c))/n) - 15*I*e^(-2
*((m + 1)*n*log(x) - 2*I*a*n + (m + 1)*log(c))/n) - 5*I*e^(-3*((m + 1)*n*lo
g(x) - 2*I*a*n + (m + 1)*log(c))/n) - I)*e^(5/2*((m + 1)*n*log(x) - 2*I*a*n
 + (m + 1)*log(c))/n + (2*I*a*n - (m + 1)*log(c))/n)/(m + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sin^3\left(a + \frac{\sqrt{-\frac{m^2}{n^2} - \frac{2m}{n^2} - \frac{1}{n^2}}\,\log(cx^n)}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sin(a+1/2*ln(c*x**n)*(-(1+m)**2/n**2)**(1/2))**3,x)
```

```
[Out] Integral(x**m*sin(a + sqrt(-m**2/n**2 - 2*m/n**2 - 1/n**2)*log(c*x**n)/2)**
3, x)
```

Giac [C] Result contains complex when optimal does not.
time = 5.78, size = 1870, normalized size = 8.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+1/2*log(c*x^n)*(-(1+m)^2/n^2)^(1/2))^3,x, algorithm="gi
ac")
```

```
[Out] 1/4*(8*I*m^3*n^4*x*x^m*e^(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)
*log(c))/n^2) - 24*I*m^3*n^4*x*x^m*e^(I*a - 1/2*(n*abs(m*n + n)*log(x) + ab
s(m*n + n)*log(c))/n^2) + 24*I*m^3*n^4*x*x^m*e^(-I*a + 1/2*(n*abs(m*n + n)*
log(x) + abs(m*n + n)*log(c))/n^2) - 8*I*m^3*n^4*x*x^m*e^(-3*I*a + 3/2*(n*a
bs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 24*I*m^2*n^4*x*x^m*e^(3*I*
a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 12*I*m^2*n^3*x
*x^m*abs(m*n + n)*e^(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(
c))/n^2) - 72*I*m^2*n^4*x*x^m*e^(I*a - 1/2*(n*abs(m*n + n)*log(x) + abs(m*n
 + n)*log(c))/n^2) - 12*I*m^2*n^3*x*x^m*abs(m*n + n)*e^(I*a - 1/2*(n*abs(m*
n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 72*I*m^2*n^4*x*x^m*e^(-I*a + 1/
2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - 12*I*m^2*n^3*x*x^m*a
bs(m*n + n)*e^(-I*a + 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2
) - 24*I*m^2*n^4*x*x^m*e^(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n
)*log(c))/n^2) + 12*I*m^2*n^3*x*x^m*abs(m*n + n)*e^(-3*I*a + 3/2*(n*abs(m*n
 + n)*log(x) + abs(m*n + n)*log(c))/n^2) - 2*I*(m*n + n)^2*m*n^2*x*x^m*e^(3
*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 24*I*m*n^4*
x*x^m*e^(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 2
4*I*m*n^3*x*x^m*abs(m*n + n)*e^(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*
n + n)*log(c))/n^2) + 54*I*(m*n + n)^2*m*n^2*x*x^m*e^(I*a - 1/2*(n*abs(m*n
+ n)*log(x) + abs(m*n + n)*log(c))/n^2) - 72*I*m*n^4*x*x^m*e^(I*a - 1/2*(n*
abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - 24*I*m*n^3*x*x^m*abs(m*n
+ n)*e^(I*a - 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - 54*I
*(m*n + n)^2*m*n^2*x*x^m*e^(-I*a + 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n
)*log(c))/n^2) + 72*I*m*n^4*x*x^m*e^(-I*a + 1/2*(n*abs(m*n + n)*log(x) + ab
s(m*n + n)*log(c))/n^2) - 24*I*m*n^3*x*x^m*abs(m*n + n)*e^(-I*a + 1/2*(n*ab
s(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 2*I*(m*n + n)^2*m*n^2*x*x^m
*e^(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - 24*I*
m*n^4*x*x^m*e^(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n
^2) + 24*I*m*n^3*x*x^m*abs(m*n + n)*e^(-3*I*a + 3/2*(n*abs(m*n + n)*log(x)
+ abs(m*n + n)*log(c))/n^2) - 2*I*(m*n + n)^2*n^2*x*x^m*e^(3*I*a - 3/2*(n*a
bs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 8*I*n^4*x*x^m*e^(3*I*a - 3
/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - 3*I*(m*n + n)^2*n*x
*x^m*abs(m*n + n)*e^(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(
c))/n^2) + 12*I*n^3*x*x^m*abs(m*n + n)*e^(3*I*a - 3/2*(n*abs(m*n + n)*log(x
) + abs(m*n + n)*log(c))/n^2) + 54*I*(m*n + n)^2*n^2*x*x^m*e^(I*a - 1/2*(n*
abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - 24*I*n^4*x*x^m*e^(I*a - 1
```

```
/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 27*I*(m*n + n)^2*n*
x*x^m*abs(m*n + n)*e^(I*a - 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c
))/n^2) - 12*I*n^3*x*x^m*abs(m*n + n)*e^(I*a - 1/2*(n*abs(m*n + n)*log(x) +
 abs(m*n + n)*log(c))/n^2) - 54*I*(m*n + n)^2*n^2*x*x^m*e^(-I*a + 1/2*(n*ab
s(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 24*I*n^4*x*x^m*e^(-I*a + 1/
2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 27*I*(m*n + n)^2*n*x
*x^m*abs(m*n + n)*e^(-I*a + 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c
))/n^2) - 12*I*n^3*x*x^m*abs(m*n + n)*e^(-I*a + 1/2*(n*abs(m*n + n)*log(x)
+ abs(m*n + n)*log(c))/n^2) + 2*I*(m*n + n)^2*n^2*x*x^m*e^(-3*I*a + 3/2*(n*
abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - 8*I*n^4*x*x^m*e^(-3*I*a +
 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - 3*I*(m*n + n)^2*n
*x*x^m*abs(m*n + n)*e^(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*l
og(c))/n^2) + 12*I*n^3*x*x^m*abs(m*n + n)*e^(-3*I*a + 3/2*(n*abs(m*n + n)*l
og(x) + abs(m*n + n)*log(c))/n^2))/(16*m^4*n^4 + 64*m^3*n^4 - 40*(m*n + n)^
2*m^2*n^2 + 96*m^2*n^4 - 80*(m*n + n)^2*m*n^2 + 64*m*n^4 + 9*(m*n + n)^4 -
40*(m*n + n)^2*n^2 + 16*n^4)
```

Mupad [B]
time = 4.71, size = 297, normalized size = 1.31

$$-\frac{x\,x^m\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\frac{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,1\mathrm{i}}{2}}}\,\left(2\,m+2+n\,\sqrt{-\frac{{\left(m+1\right)}^2}{n^2}}\,1\mathrm{i}\right)\,1\mathrm{i}}{4\,{\left(m\,1\mathrm{i}+1\mathrm{i}\right)}^2}+\frac{x\,x^m\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{\frac{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,1\mathrm{i}}{2}}\,\left(2\,m+2-n\,\sqrt{-\frac{{\left(m+1\right)}^2}{n^2}}\,1\mathrm{i}\right)\,1\mathrm{i}}{4\,{\left(m\,1\mathrm{i}+1\mathrm{i}\right)}^2}-\frac{x\,x^m\,\mathrm{e}^{-a\,3\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\frac{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,3\mathrm{i}}{2}}}\,\left(2\,m+2+n\,\sqrt{-\frac{{\left(m+1\right)}^2}{n^2}}\,3\mathrm{i}\right)\,1\mathrm{i}}{20\,{\left(m\,1\mathrm{i}+1\mathrm{i}\right)}^2}+\frac{x\,x^m\,\mathrm{e}^{a\,3\mathrm{i}}\,{\left(c\,x^n\right)}^{\frac{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,3\mathrm{i}}{2}}\,\left(2\,m+2-n\,\sqrt{-\frac{{\left(m+1\right)}^2}{n^2}}\,3\mathrm{i}\right)\,1\mathrm{i}}{20\,{\left(m\,1\mathrm{i}+1\mathrm{i}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(a + (log(c*x^n)*(-(m + 1)^2/n^2)^(1/2))/2)^3,x)
```

```
[Out] (x*x^m*exp(a*1i)*(c*x^n)^(((- (2*m)/n^2 - 1/n^2 - m^2/n^2)^(1/2)*1i)/2)*(2*
m - n*(-(m + 1)^2/n^2)^(1/2)*1i + 2)*1i)/(4*(m*1i + 1i)^2) - (x*x^m*exp(-a*
1i)/(c*x^n)^(((- (2*m)/n^2 - 1/n^2 - m^2/n^2)^(1/2)*1i)/2)*(2*m + n*(-(m +
1)^2/n^2)^(1/2)*1i + 2)*1i)/(4*(m*1i + 1i)^2) - (x*x^m*exp(-a*3i)/(c*x^n)^(
((- (2*m)/n^2 - 1/n^2 - m^2/n^2)^(1/2)*3i)/2)*(2*m + n*(-(m + 1)^2/n^2)^(1/
2)*3i + 2)*1i)/(20*(m*1i + 1i)^2) + (x*x^m*exp(a*3i)*(c*x^n)^(((- (2*m)/n^2
 - 1/n^2 - m^2/n^2)^(1/2)*3i)/2)*(2*m - n*(-(m + 1)^2/n^2)^(1/2)*3i + 2)*1i
)/(20*(m*1i + 1i)^2)
```

3.41 $\int x^2 \sin^3\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx$

Optimal. Leaf size=172

$$-\frac{3}{16}e^{a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,nx^3(cx^n)^{-1/n}+\frac{3}{32}e^{-a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,nx^3(cx^n)^{\frac{1}{n}}-\frac{1}{48}e^{-3a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,nx^3(cx^n)^{3/n}+\frac{1}{8}$$

```
[Out] -3/16*exp(a*n*(-1/n^2)^(1/2))*n*x^3*(-1/n^2)^(1/2)/((c*x^n)^(1/n))+3/32*n*x
^3*(c*x^n)^(1/n)*(-1/n^2)^(1/2)/exp(a*n*(-1/n^2)^(1/2))-1/48*n*x^3*(c*x^n)^
(3/n)*(-1/n^2)^(1/2)/exp(3*a*n*(-1/n^2)^(1/2))+1/8*exp(3*a*n*(-1/n^2)^(1/2)
)*n*x^3*ln(x)*(-1/n^2)^(1/2)/((c*x^n)^(3/n))
```

Rubi [A]
time = 0.11, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {4581, 4577}

$$-\frac{3}{16}\sqrt{-\frac{1}{n^2}}\,nx^3e^{a\sqrt{-\frac{1}{n^2}}\,n}(cx^n)^{-1/n}-\frac{1}{48}\sqrt{-\frac{1}{n^2}}\,nx^3e^{-3a\sqrt{-\frac{1}{n^2}}\,n}(cx^n)^{3/n}+\frac{3}{32}\sqrt{-\frac{1}{n^2}}\,nx^3e^{-a\sqrt{-\frac{1}{n^2}}\,n}(cx^n)^{\frac{1}{n}}+\frac{1}{8}\sqrt{-\frac{1}{n^2}}\,nx^3e^{3a\sqrt{-\frac{1}{n^2}}\,n}\log(x)\left(cx^n\right)^{-3/n}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]]^3,x]
```

```
[Out] (-3*E^(a*Sqrt[-n^(-2)]*n)*Sqrt[-n^(-2)]*n*x^3)/(16*(c*x^n)^n^(-1)) + (3*Sqr
t[-n^(-2)]*n*x^3*(c*x^n)^n^(-1))/(32*E^(a*Sqrt[-n^(-2)]*n)) - (Sqrt[-n^(-2)
]*n*x^3*(c*x^n)^(3/n))/(48*E^(3*a*Sqrt[-n^(-2)]*n)) + (E^(3*a*Sqrt[-n^(-2)]
*n)*Sqrt[-n^(-2)]*n*x^3*Log[x])/(8*(c*x^n)^(3/n))
```

Rule 4577

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[(m + 1)^p/(2^p*b^p*d^p*p^p), Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^
2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x]
, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m
 + 1)^2, 0]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^2 \sin^3\left(a+\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)dx = \frac{\left(x^3(cx^n)^{-3/n}\right)\text{Subst}\left(\int x^{-1+\frac{3}{n}}\sin^3\left(a+\sqrt{-\frac{1}{n^2}}\log(x)\right)dx, x, c\right.}{n}$$

$$= \frac{1}{8}\left(\sqrt{-\frac{1}{n^2}}x^3(cx^n)^{-3/n}\right)\text{Subst}\left(\int\left(\frac{e^{3a\sqrt{-\frac{1}{n^2}}n}}{x} - 3e^{a\sqrt{-\frac{1}{n^2}}n}x^{-1}\right.\right.$$

$$= -\frac{3}{16}e^{a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}nx^3(cx^n)^{-1/n} + \frac{3}{32}e^{-a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}nx^3(c$$

Mathematica [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int x^2 \sin^3\left(a+\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]]^3,x]
[Out] Integrate[x^2*Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]]^3, x]
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int x^2\left(\sin^3\left(a+\ln(c\,x^n)\sqrt{-\frac{1}{n^2}}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a+ln(c*x^n)*(-1/n^2)^(1/2))^3,x)
[Out] int(x^2*sin(a+ln(c*x^n)*(-1/n^2)^(1/2))^3,x)
```

Maxima [A]
time = 0.30, size = 90, normalized size = 0.52

$$\frac{18\,c^{\frac{2}{n}}x^3\sin(a) - 12\,(x^n)^{\left(\frac{1}{n}\right)}\log(x)\sin(3\,a) - \left(2\,c^{\frac{6}{n}}x^6\sin(3\,a) - 9\,c^{\frac{4}{n}}x^4\sin(a)\right)(x^n)^{\left(\frac{1}{n}\right)}}{96\,c^{\frac{3}{n}}(x^n)^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+log(c*x^n)*(-1/n^2)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] 1/96*(18*c^(2/n)*x^3*sin(a) - 12*(x^n)^(1/n)*log(x)*sin(3*a) - (2*c^(6/n)*x
^6*sin(3*a) - 9*c^(4/n)*x^4*sin(a))*(x^n)^(1/n))/(c^(3/n)*(x^n)^(1/n))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.57, size = 82, normalized size = 0.48

$$\frac{1}{96}\left(-2i\,x^{6}+9i\,x^{4}e^{\left(\frac{2\,(i\,an-\log(c))}{n}\right)}-18i\,x^{2}e^{\left(\frac{4\,(i\,an-\log(c))}{n}\right)}+12i\,e^{\left(\frac{6\,(i\,an-\log(c))}{n}\right)}\log\left(x\right)\right)e^{\left(-\frac{3\,(i\,an-\log(c))}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+log(c*x^n)*(-1/n^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/96*(-2*I*x^6 + 9*I*x^4*e^(2*(I*a*n - log(c))/n) - 18*I*x^2*e^(4*(I*a*n -
log(c))/n) + 12*I*e^(6*(I*a*n - log(c))/n)*log(x))*e^(-3*(I*a*n - log(c))/n
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(a+ln(c*x**n)*(-1/n**2)**(1/2))**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+log(c*x^n)*(-1/n^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: ((-9*
i)*sageVARn^4*sageVARx^3*exp((-3*i)*sageVARa)*exp((3*sageVARn*abs(sageVARn)
*ln(sageVARx)+3*abs(sageVARn)*ln(sageVARc))/sageVARn^2)+27*i*sageVARn^4*sag
eVARx^3*exp((-i)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\sin\left(a+\ln\left(c\,x^n\right)\,\sqrt{-\frac{1}{n^2}}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a + log(c*x^n)*(-1/n^2)^(1/2))^3,x)
```

```
[Out] int(x^2*sin(a + log(c*x^n)*(-1/n^2)^(1/2))^3, x)
```

3.42 $\int x\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\,\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=178

$$-\frac{9}{32}e^{a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,nx^2(cx^n)^{-\frac{2}{3}/n}+\frac{9}{64}e^{-a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,nx^2(cx^n)^{\frac{2}{3}/n}-\frac{1}{32}e^{-3a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,nx^2(cx^n)^{2/n}$$

```
[Out] -9/32*exp(a*n*(-1/n^2)^(1/2))*n*x^2*(-1/n^2)^(1/2)/((c*x^n)^(2/3/n))+9/64*n
*x^2*(c*x^n)^(2/3/n)*(-1/n^2)^(1/2)/exp(a*n*(-1/n^2)^(1/2))-1/32*n*x^2*(c*x
^n)^(2/n)*(-1/n^2)^(1/2)/exp(3*a*n*(-1/n^2)^(1/2))+1/8*exp(3*a*n*(-1/n^2)^(
1/2))*n*x^2*ln(x)*(-1/n^2)^(1/2)/((c*x^n)^(2/n))
```

Rubi [A]
time = 0.08, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {4581, 4577}

$$-\frac{9}{32}\sqrt{-\frac{1}{n^2}}\,nx^2e^{a\sqrt{-\frac{1}{n^2}}\,n}(cx^n)^{-\frac{2}{3}/n}+\frac{9}{64}\sqrt{-\frac{1}{n^2}}\,nx^2e^{-a\sqrt{-\frac{1}{n^2}}\,n}(cx^n)^{\frac{2}{3}/n}-\frac{1}{32}\sqrt{-\frac{1}{n^2}}\,nx^2e^{-3a\sqrt{-\frac{1}{n^2}}\,n}(cx^n)^{2/n}+\frac{1}{8}\sqrt{-\frac{1}{n^2}}\,nx^2e^{3a\sqrt{-\frac{1}{n^2}}\,n}\log(x)\,(cx^n)^{-2/n}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[a + (2*Sqrt[-n^(-2)]*Log[c*x^n])/3]^3,x]
```

```
[Out] (-9*E^(a*Sqrt[-n^(-2)]*n)*Sqrt[-n^(-2)]*n*x^2)/(32*(c*x^n)^(2/(3*n))) + (9*
Sqrt[-n^(-2)]*n*x^2*(c*x^n)^(2/(3*n)))/(64*E^(a*Sqrt[-n^(-2)]*n)) - (Sqrt[-
n^(-2)]*n*x^2*(c*x^n)^(2/n))/(32*E^(3*a*Sqrt[-n^(-2)]*n)) + (E^(3*a*Sqrt[-n
^(-2)]*n)*Sqrt[-n^(-2)]*n*x^2*Log[x])/(8*(c*x^n)^(2/n))
```

Rule 4577

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[(m + 1)^p/(2^p*b^p*d^p*p^p), Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^
2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x]
, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m
 + 1)^2, 0]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)dx = \frac{\left(x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int x^{-1+\frac{2}{n}}\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log(x)\right)dx,x,\right.}{n}$$

$$=\frac{1}{8}\left(\sqrt{-\frac{1}{n^2}}x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int\left(\frac{e^{3a\sqrt{-\frac{1}{n^2}}n}}{x}-3e^{a\sqrt{-\frac{1}{n^2}}n}x^{-1-}\right.\right.$$

$$=-\frac{9}{32}e^{a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}nx^2(cx^n)^{-\frac{2}{3}/n}+\frac{9}{64}e^{-a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}nx^2(c$$

Mathematica [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int x\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*Sin[a + (2*Sqrt[-n^(-2)]*Log[c*x^n])/3]^3,x]
```

```
[Out] Integrate[x*Sin[a + (2*Sqrt[-n^(-2)]*Log[c*x^n])/3]^3, x]
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int x\left(\sin^3\left(a+\frac{2\ln(c\,x^n)\sqrt{-\frac{1}{n^2}}}{3}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a+2/3*ln(c*x^n)*(-1/n^2)^(1/2))^3,x)
```

```
[Out] int(x*sin(a+2/3*ln(c*x^n)*(-1/n^2)^(1/2))^3,x)
```

Maxima [A]
time = 0.30, size = 112, normalized size = 0.63

$$\frac{9\,c^{\frac{10}{3\,n}}x^2(x^n)^{\frac{4}{3\,n}}\sin(a)-8\,c^{\frac{2}{3\,n}}(x^n)^{\frac{2}{3\,n}}\log(x)\sin(3\,a)+18\,c^{\frac{2}{n}}x^2\sin(a)-2\,c^{\frac{14}{3\,n}}e^{\left(\frac{2\log(x^n)}{3\,n}+4\log(x)\right)}\sin(3\,a)}{64\,c^{\frac{8}{3\,n}}(x^n)^{\frac{2}{3\,n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+2/3*log(c*x^n)*(-1/n^2)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] 1/64*(9*c^(10/3/n)*x^2*(x^n)^(4/3/n)*sin(a) - 8*c^(2/3/n)*(x^n)^(2/3/n)*log
(x)*sin(3*a) + 18*c^(2/n)*x^2*sin(a) - 2*c^(14/3/n)*e^(2/3*log(x^n)/n + 4*l
og(x))*sin(3*a))/(c^(8/3/n)*(x^n)^(2/3/n))
```

Fricas [**C**] Result contains complex when optimal does not.
time = 1.92, size = 84, normalized size = 0.47

$$\frac{1}{64}\left(-2i\,x^4+9i\,x^{\frac{8}{3}}e^{\left(\frac{2\,(3i\,an-2\,\log(c))}{3\,n}\right)}-18i\,x^{\frac{4}{3}}e^{\left(\frac{4\,(3i\,an-2\,\log(c))}{3\,n}\right)}+24i\,e^{\left(\frac{2\,(3i\,an-2\,\log(c))}{n}\right)}\log\left(x^{\frac{1}{3}}\right)\right)e^{\left(-\frac{3i\,an-2\,\log(c)}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+2/3*log(c*x^n)*(-1/n^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/64*(-2*I*x^4 + 9*I*x^(8/3)*e^(2/3*(3*I*a*n - 2*log(c))/n) - 18*I*x^(4/3)*
e^(4/3*(3*I*a*n - 2*log(c))/n) + 24*I*e^(2*(3*I*a*n - 2*log(c))/n)*log(x^(1
/3)))*e^(-(3*I*a*n - 2*log(c))/n)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sin^3\left(a+\frac{2\sqrt{-\frac{1}{n^2}}\,\log\left(cx^n\right)}{3}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+2/3*ln(c*x**n)*(-1/n**2)**(1/2))**3,x)
```

```
[Out] Integral(x*sin(a + 2*sqrt(-1/n**2)*log(c*x**n)/3)**3, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+2/3*log(c*x^n)*(-1/n^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: ((-9*
i)*sageVARn^4*sageVARx^2*exp((-3*i)*sageVARa)*exp((2*sageVARn*abs(sageVARn)
*ln(sageVARx)+2*abs(sageVARn)*ln(sageVARc))/sageVARn^2)+27*i*sageVARn^4*sag
eVARx^2*exp((-i)
```

Mupad [**B**]
time = 3.32, size = 163, normalized size = 0.92

$$-x^2\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\frac{\sqrt{-\frac{1}{n^2}}\,2\mathrm{i}}{3}}}\,\left(\frac{9\,n\,\sqrt{-\frac{1}{n^2}}}{128}-\frac{27}{128}\mathrm{i}\right)-x^2\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{\frac{\sqrt{-\frac{1}{n^2}}\,2\mathrm{i}}{3}}\,\left(\frac{9\,n\,\sqrt{-\frac{1}{n^2}}}{128}+\frac{27}{128}\mathrm{i}\right)+\frac{x^2\,\mathrm{e}^{-a\,3\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,2\mathrm{i}}}}{16\,n\,\sqrt{-\frac{1}{n^2}}+16\mathrm{i}}+\frac{x^2\,\mathrm{e}^{a\,3\mathrm{i}}\,{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,2\mathrm{i}}}{16\,n\,\sqrt{-\frac{1}{n^2}}-16\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + (2*log(c*x^n)*(-1/n^2)^(1/2))/3)^3,x)
```

```
[Out] (x^2*exp(-a*3i)/(c*x^n)^((-1/n^2)^(1/2)*2i))/(16*n*(-1/n^2)^(1/2) + 16i) -
x^2*exp(a*1i)*(c*x^n)^(((-1/n^2)^(1/2)*2i)/3)*((9*n*(-1/n^2)^(1/2))/128 + 2
7i/128) - x^2*exp(-a*1i)/(c*x^n)^(((-1/n^2)^(1/2)*2i)/3)*((9*n*(-1/n^2)^(1/
2))/128 - 27i/128) + (x^2*exp(a*3i)*(c*x^n)^((-1/n^2)^(1/2)*2i))/(16*n*(-1/
n^2)^(1/2) - 16i)
```

3.43 $\int \sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)dx$

Optimal. Leaf size=168

$$-\frac{9}{16}e^{a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}nx(cx^n)^{-\frac{1}{3}/n}+\frac{9}{32}e^{-a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}nx(cx^n)^{\frac{1}{3}/n}-\frac{1}{16}e^{-3a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}nx(cx^n)^{\frac{1}{n}}+\frac{1}{8}$$

```
[Out] -9/16*exp(a*n*(-1/n^2)^(1/2))*n*x*(-1/n^2)^(1/2)/((c*x^n)^(1/3/n))+9/32*n*x
*(c*x^n)^(1/3/n)*(-1/n^2)^(1/2)/exp(a*n*(-1/n^2)^(1/2))-1/16*n*x*(c*x^n)^(1
/n)*(-1/n^2)^(1/2)/exp(3*a*n*(-1/n^2)^(1/2))+1/8*exp(3*a*n*(-1/n^2)^(1/2))*
n*x*ln(x)*(-1/n^2)^(1/2)/((c*x^n)^(1/n))
```

Rubi [A]
time = 0.07, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4571, 4577}

$$-\frac{9}{16}\sqrt{-\frac{1}{n^2}}nxe^{a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{-\frac{1}{3}/n}+\frac{9}{32}\sqrt{-\frac{1}{n^2}}nxe^{-a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{1}{3}/n}-\frac{1}{16}\sqrt{-\frac{1}{n^2}}nxe^{-3a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{1}{n}}+\frac{1}{8}\sqrt{-\frac{1}{n^2}}nxe^{3a\sqrt{-\frac{1}{n^2}}n}\log(x)(cx^n)^{-1/n}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + (Sqrt[-n^(-2)]*Log[c*x^n])/3]^3,x]
```

```
[Out] (-9*E^(a*Sqrt[-n^(-2)]*n)*Sqrt[-n^(-2)]*n*x)/(16*(c*x^n)^(1/(3*n))) + (9*Sq
rt[-n^(-2)]*n*x*(c*x^n)^(1/(3*n)))/(32*E^(a*Sqrt[-n^(-2)]*n)) - (Sqrt[-n^(-
2)]*n*x*(c*x^n)^n^(-1))/(16*E^(3*a*Sqrt[-n^(-2)]*n)) + (E^(3*a*Sqrt[-n^(-2)
]*n)*Sqrt[-n^(-2)]*n*x*Log[x])/(8*(c*x^n)^n^(-1))
```

Rule 4571

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4577

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[(m + 1)^p/(2^p*b^p*d^p*p^p), Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^
2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x]
, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m
 + 1)^2, 0]
```

Rubi steps

$$\int \sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log(x)\right)dx, x, cx\right.}{n}$$

$$= \frac{1}{8}\left(\sqrt{-\frac{1}{n^2}}\,x(cx^n)^{-1/n}\right)\text{Subst}\left(\int\left(\frac{e^{3a\sqrt{-\frac{1}{n^2}}\,n}}{x} - 3e^{a\sqrt{-\frac{1}{n^2}}\,n}x^{-1+\frac{2}{3n}}\right.\right.$$

$$= -\frac{9}{16}e^{a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,nx(cx^n)^{-\frac{1}{3}/n} + \frac{9}{32}e^{-a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,nx(cx^n)^{\frac{1}{3}}$$

__Mathematica__ [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + (Sqrt[-n^(-2)]*Log[c*x^n])/3]^3,x]
[Out] Integrate[Sin[a + (Sqrt[-n^(-2)]*Log[c*x^n])/3]^3, x]
```

__Maple__ [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \sin^3\left(a+\frac{\ln\left(c\,x^n\right)\sqrt{-\frac{1}{n^2}}}{3}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+1/3*ln(c*x^n)*(-1/n^2)^(1/2))^3,x)
[Out] int(sin(a+1/3*ln(c*x^n)*(-1/n^2)^(1/2))^3,x)
```

__Maxima__ [A]
time = 0.29, size = 106, normalized size = 0.63

$$-\frac{4\,c^{\frac{1}{3\,n}}\left(x^n\right)^{\frac{1}{3\,n}}\log\left(x\right)\sin\left(3\,a\right) - 9\,c^{\frac{5}{3\,n}}x\left(x^n\right)^{\frac{2}{3\,n}}\sin\left(a\right) + 2\,c^{\frac{7}{3\,n}}e^{\left(\frac{\log(x^n)}{3\,n}+2\,\log(x)\right)}\sin\left(3\,a\right) - 18\,c^{\left(\frac{1}{n}\right)}x\sin\left(a\right)}{32\,c^{\frac{4}{3\,n}}\left(x^n\right)^{\frac{1}{3\,n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/3*log(c*x^n)*(-1/n^2)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] -1/32*(4*c^(1/3/n)*(x^n)^(1/3/n)*log(x)*sin(3*a) - 9*c^(5/3/n)*x*(x^n)^(2/3
/n)*sin(a) + 2*c^(7/3/n)*e^(1/3*log(x^n)/n + 2*log(x))*sin(3*a) - 18*c^(1/n
)*x*sin(a))/(c^(4/3/n)*(x^n)^(1/3/n))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.72, size = 84, normalized size = 0.50

$$\frac{1}{32}\left(9i\,x^{\frac{4}{3}}e^{\left(\frac{2\,(3i\,an-\log(c))}{3\,n}\right)}-2i\,x^{2}+12i\,e^{\left(\frac{2\,(3i\,an-\log(c))}{n}\right)}\log\left(x^{\frac{1}{3}}\right)-18i\,x^{\frac{2}{3}}e^{\left(\frac{4\,(3i\,an-\log(c))}{3\,n}\right)}\right)e^{\left(-\frac{3i\,an-\log(c)}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/3*log(c*x^n)*(-1/n^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(9*I*x^(4/3)*e^(2/3*(3*I*a*n - log(c))/n) - 2*I*x^2 + 12*I*e^(2*(3*I*a
*n - log(c))/n)*log(x^(1/3)) - 18*I*x^(2/3)*e^(4/3*(3*I*a*n - log(c))/n))*e
^(-(3*I*a*n - log(c))/n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sin^{3}\left(a+\frac{\sqrt{-\frac{1}{n^{2}}}\,\log\left(cx^{n}\right)}{3}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/3*ln(c*x**n)*(-1/n**2)**(1/2))**3,x)
```

```
[Out] Integral(sin(a + sqrt(-1/n**2)*log(c*x**n)/3)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/3*log(c*x^n)*(-1/n^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: ((-9*
i)*sageVARn^4*sageVARx*exp((-3*i)*sageVARa)*exp((sageVARn*abs(sageVARn)*ln(
sageVARx)+abs(sageVARn)*ln(sageVARc))/sageVARn^2)+27*i*sageVARn^4*sageVARx*
exp((-i)*sageVAR
```

Mupad [B]
time = 2.98, size = 155, normalized size = 0.92

$$-x\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\frac{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}{3}}}\,\left(\frac{9\,n\,\sqrt{-\frac{1}{n^2}}}{64}-\frac{27}{64}\mathrm{i}\right)-x\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{\frac{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}{3}}\,\left(\frac{9\,n\,\sqrt{-\frac{1}{n^2}}}{64}+\frac{27}{64}\mathrm{i}\right)+\frac{x\,\mathrm{e}^{-a\,3\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}}}{8\,n\,\sqrt{-\frac{1}{n^2}}+8\mathrm{i}}+\frac{x\,\mathrm{e}^{a\,3\mathrm{i}}\,{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}}{8\,n\,\sqrt{-\frac{1}{n^2}}-8\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + (log(c*x^n)*(-1/n^2)^(1/2))/3)^3,x)
```

```
[Out] (x*exp(-a*3i)/(c*x^n)^((-1/n^2)^(1/2)*1i))/(8*n*(-1/n^2)^(1/2) + 8i) - x*ex
p(a*1i)*(c*x^n)^(((-1/n^2)^(1/2)*1i)/3)*((9*n*(-1/n^2)^(1/2))/64 + 27i/64)
- x*exp(-a*1i)/(c*x^n)^(((-1/n^2)^(1/2)*1i)/3)*((9*n*(-1/n^2)^(1/2))/64 - 2
7i/64) + (x*exp(a*3i)*(c*x^n)^((-1/n^2)^(1/2)*1i))/(8*n*(-1/n^2)^(1/2) - 8i
)
```

3.44 $\int \frac{\sin^3(a)}{x} \, dx$

Optimal. Leaf size=7

$$\log(x)\sin^3(a)$$

```
[Out] ln(x)*sin(a)^3
```

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {12, 29}

$$\sin^3(a)\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a]^3/x,x]
```

```
[Out] Log[x]*Sin[a]^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned} \int \frac{\sin^3(a)}{x} \, dx &= \sin^3(a) \int \frac{1}{x} \, dx \\ &= \log(x)\sin^3(a) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$\log(x)\sin^3(a)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a]^3/x,x]
```

```
[Out] Log[x]*Sin[a]^3
```

Maple [A]
time = 0.01, size = 8, normalized size = 1.14

method	result	size
default	$\ln(x)(\sin^3(a))$	8
norman	$\ln(x)(\sin^3(a))$	8
risch	$\ln(x)(\sin^3(a))$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*sin(a)^3
```

Maxima [A]
time = 0.27, size = 7, normalized size = 1.00

$$\log(x)\sin(a)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a)^3/x,x, algorithm="maxima")
```

```
[Out] log(x)*sin(a)^3
```

Fricas [A]
time = 2.67, size = 12, normalized size = 1.71

$$-\left(\cos(a)^2-1\right)\log(x)\sin(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a)^3/x,x, algorithm="fricas")
```

```
[Out] -(cos(a)^2 - 1)*log(x)*sin(a)
```

Sympy [A]
time = 0.01, size = 7, normalized size = 1.00

$$\log(x)\sin^3(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a)**3/x,x)
```

```
[Out] log(x)*sin(a)**3
```

Giac [A]
time = 0.39, size = 8, normalized size = 1.14

$$\log(|x|)\sin(a)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a)^3/x,x, algorithm="giac")
```

```
[Out] log(abs(x))*sin(a)^3
```

Mupad [B]
time = 2.12, size = 7, normalized size = 1.00

$$\sin(a)^3 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a)^3/x,x)
```

```
[Out] sin(a)^3*log(x)
```

3.45 $\int \frac{\sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{x^2}\,dx$

Optimal. Leaf size=176

$$-\frac{e^{3a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,n(cx^n)^{-1/n}}{16x}+\frac{9e^{a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,n(cx^n)^{-\frac{1}{3}/n}}{32x}-\frac{9e^{-a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,n(cx^n)^{\frac{1}{3}/n}}{16x}-\frac{e^{-3a\sqrt{-\frac{1}{n^2}}}}{}$$

```
[Out] -1/16*exp(3*a*n*(-1/n^2)^(1/2))*n*(-1/n^2)^(1/2)/x/((c*x^n)^(1/n))+9/32*exp
(a*n*(-1/n^2)^(1/2))*n*(-1/n^2)^(1/2)/x/((c*x^n)^(1/3/n))-9/16*n*(c*x^n)^(1
/3/n)*(-1/n^2)^(1/2)/exp(a*n*(-1/n^2)^(1/2))/x-1/8*n*(c*x^n)^(1/n)*ln(x)*(-
1/n^2)^(1/2)/exp(3*a*n*(-1/n^2)^(1/2))/x
```

Rubi [A]
time = 0.09, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {4581, 4577}

$$-\frac{\sqrt{-\frac{1}{n^2}}\,ne^{3a\sqrt{-\frac{1}{n^2}}\,n}(cx^n)^{-1/n}}{16x}+\frac{9\sqrt{-\frac{1}{n^2}}\,ne^{a\sqrt{-\frac{1}{n^2}}\,n}(cx^n)^{-\frac{1}{3}/n}}{32x}-\frac{9\sqrt{-\frac{1}{n^2}}\,ne^{-a\sqrt{-\frac{1}{n^2}}\,n}(cx^n)^{\frac{1}{3}/n}}{16x}-\frac{\sqrt{-\frac{1}{n^2}}\,ne^{-3a\sqrt{-\frac{1}{n^2}}\,n}\log(x)\,(cx^n)^{\frac{1}{n}}}{8x}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + (Sqrt[-n^(-2)]*Log[c*x^n])/3]^3/x^2,x]
```

```
[Out] -1/16*(E^(3*a*Sqrt[-n^(-2)]*n)*Sqrt[-n^(-2)]*n)/(x*(c*x^n)^n^(-1)) + (9*E^(
a*Sqrt[-n^(-2)]*n)*Sqrt[-n^(-2)]*n)/(32*x*(c*x^n)^(1/(3*n))) - (9*Sqrt[-n^(
-2)]*n*(c*x^n)^(1/(3*n)))/(16*E^(a*Sqrt[-n^(-2)]*n)*x) - (Sqrt[-n^(-2)]*n*(
c*x^n)^n^(-1)*Log[x])/(8*E^(3*a*Sqrt[-n^(-2)]*n)*x)
```

Rule 4577

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[(m + 1)^p/(2^p*b^p*d^p*p^p), Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^
2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x]
, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m
 + 1)^2, 0]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx &= \frac{(cx^n)^{\frac{1}{n}}\,\text{Subst}\left(\int x^{-1-\frac{1}{n}}\sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log(x)\right)dx, x, cx^n\right)}{nx} \\
&= -\frac{\left(\sqrt{-\frac{1}{n^2}}\,(cx^n)^{\frac{1}{n}}\right)\text{Subst}\left(\int\left(\frac{e^{-3a\sqrt{-\frac{1}{n^2}}\,n}}{x}+3e^{a\sqrt{-\frac{1}{n^2}}\,n}x^{-1-\frac{4}{3n}}-\right.\right.}{8x} \\
&= -\frac{e^{3a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,n(cx^n)^{-1/n}}{16x}+\frac{9e^{a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,n(cx^n)^{-\frac{1}{3}/n}}{32x}-
\end{aligned}
$$

Mathematica [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + (Sqrt[-n^(-2)]*Log[c*x^n])/3]^3/x^2,x]
```

```
[Out] Integrate[Sin[a + (Sqrt[-n^(-2)]*Log[c*x^n])/3]^3/x^2, x]
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sin^3\left(a+\frac{\ln\left(c\,x^{n}\right)\sqrt{-\frac{1}{n^2}}}{3}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+1/3*ln(c*x^n)*(-1/n^2)^(1/2))^3/x^2,x)
```

```
[Out] int(sin(a+1/3*ln(c*x^n)*(-1/n^2)^(1/2))^3/x^2,x)
```

Maxima [A]
time = 0.30, size = 122, normalized size = 0.69

$$-\frac{\left(4\,c^{\frac{7}{3\,n}}xe^{\left(\frac{\log(x^n)}{3\,n}+2\,\log(x)\right)}\log\left(x\right)\sin\left(3\,a\right)-2\,c^{\frac{1}{3\,n}}x(x^n)^{\frac{1}{3\,n}}\sin\left(3\,a\right)+9\,c^{\left(\frac{1}{n}\right)}x^2\sin\left(a\right)+18\,c^{\frac{5}{3\,n}}e^{\left(\frac{2\,\log(x^n)}{3\,n}+2\,\log(x)\right)}\sin\left(a\right)\right)e^{\left(-\frac{\log(x^n)}{3\,n}-2\,\log(x)\right)}}{32\,c^{\frac{4}{3\,n}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/3*log(c*x^n)*(-1/n^2)^(1/2))^3/x^2,x, algorithm="maxima")
```

```
[Out] -1/32*(4*c^(7/3/n)*x*e^(1/3*log(x^n)/n + 2*log(x))*log(x)*sin(3*a) - 2*c^(1
/3/n)*x*(x^n)^(1/3/n)*sin(3*a) + 9*c^(1/n)*x^2*sin(a) + 18*c^(5/3/n)*e^(2/3
*log(x^n)/n + 2*log(x))*sin(a))*e^(-1/3*log(x^n)/n - 2*log(x))/(c^(4/3/n)*x
)
```

Fricas [C] Result contains complex when optimal does not.
time = 5.51, size = 87, normalized size = 0.49

$$\frac{\left(-12i\,x^2\log\left(x^{\frac{1}{3}}\right)-18i\,x^{\frac{4}{3}}e^{\left(\frac{2\,(3i\,an-\log(c))}{3\,n}\right)}+9i\,x^{\frac{2}{3}}e^{\left(\frac{4\,(3i\,an-\log(c))}{3\,n}\right)}-2i\,e^{\left(\frac{2\,(3i\,an-\log(c))}{n}\right)}\right)e^{\left(-\frac{3i\,an-\log(c)}{n}\right)}}{32\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/3*log(c*x^n)*(-1/n^2)^(1/2))^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/32*(-12*I*x^2*log(x^(1/3)) - 18*I*x^(4/3)*e^(2/3*(3*I*a*n - log(c))/n) +
9*I*x^(2/3)*e^(4/3*(3*I*a*n - log(c))/n) - 2*I*e^(2*(3*I*a*n - log(c))/n))*
e^(-(3*I*a*n - log(c))/n)/x^2
```

Sympy [A]
time = 52.08, size = 167, normalized size = 0.95

$$-\frac{9n\sqrt{-\frac{1}{n^2}}\cos\left(a+\frac{\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)}{3}\right)}{32x}-\frac{\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\cos\left(3a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{8x}-\frac{27\sin\left(a+\frac{\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)}{3}\right)}{32x}+\frac{\sin\left(3a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{8x}-\frac{\log\left(cx^n\right)\sin\left(3a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{8nx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/3*ln(c*x**n)*(-1/n**2)**(1/2))**3/x**2,x)
```

```
[Out] -9*n*sqrt(-1/n**2)*cos(a + sqrt(-1/n**2)*log(c*x**n)/3)/(32*x) - sqrt(-1/n*
*2)*log(c*x**n)*cos(3*a + sqrt(-1/n**2)*log(c*x**n))/(8*x) - 27*sin(a + sqr
t(-1/n**2)*log(c*x**n)/3)/(32*x) + sin(3*a + sqrt(-1/n**2)*log(c*x**n))/(8*
x) - log(c*x**n)*sin(3*a + sqrt(-1/n**2)*log(c*x**n))/(8*n*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/3*log(c*x^n)*(-1/n^2)^(1/2))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(sin(1/3*sqrt(-1/n^2)*log(c*x^n) + a)^3/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+\frac{\ln(c\,x^n)\,\sqrt{-\frac{1}{n^2}}}{3}\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + (log(c*x^n)*(-1/n^2)^(1/2))/3)^3/x^2,x)
[Out] int(sin(a + (log(c*x^n)*(-1/n^2)^(1/2))/3)^3/x^2, x)
```

3.46 $\int \frac{\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\,\log(cx^n)\right)}{x^3}\,dx$

Optimal. Leaf size=178

$$-\frac{e^{3a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,n(cx^n)^{-2/n}}{32x^2}+\frac{9e^{a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,n(cx^n)^{-\frac{2}{3}/n}}{64x^2}-\frac{9e^{-a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,n(cx^n)^{\frac{2}{3}/n}}{32x^2}-\frac{e^{-3a\sqrt{-\frac{1}{n^2}}}}{}$$

```
[Out] -1/32*exp(3*a*n*(-1/n^2)^(1/2))*n*(-1/n^2)^(1/2)/x^2/((c*x^n)^(2/n))+9/64*e
xp(a*n*(-1/n^2)^(1/2))*n*(-1/n^2)^(1/2)/x^2/((c*x^n)^(2/3/n))-9/32*n*(c*x^n
)^(2/3/n)*(-1/n^2)^(1/2)/exp(a*n*(-1/n^2)^(1/2))/x^2-1/8*n*(c*x^n)^(2/n)*ln
(x)*(-1/n^2)^(1/2)/exp(3*a*n*(-1/n^2)^(1/2))/x^2
```

Rubi [A]
time = 0.08, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {4581, 4577}

$$-\frac{\sqrt{-\frac{1}{n^2}}\,ne^{3a\sqrt{-\frac{1}{n^2}}\,n}(cx^n)^{-2/n}}{32x^2}+\frac{9\sqrt{-\frac{1}{n^2}}\,ne^{a\sqrt{-\frac{1}{n^2}}\,n}(cx^n)^{-\frac{2}{3}/n}}{64x^2}-\frac{9\sqrt{-\frac{1}{n^2}}\,ne^{-a\sqrt{-\frac{1}{n^2}}\,n}(cx^n)^{\frac{2}{3}/n}}{32x^2}-\frac{\sqrt{-\frac{1}{n^2}}\,ne^{-3a\sqrt{-\frac{1}{n^2}}\,n}\log(x)\,(cx^n)^{2/n}}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + (2*Sqrt[-n^(-2)]*Log[c*x^n])/3]^3/x^3,x]
```

```
[Out] -1/32*(E^(3*a*Sqrt[-n^(-2)]*n)*Sqrt[-n^(-2)]*n)/(x^2*(c*x^n)^(2/n)) + (9*E^
(a*Sqrt[-n^(-2)]*n)*Sqrt[-n^(-2)]*n)/(64*x^2*(c*x^n)^(2/(3*n))) - (9*Sqrt[-
n^(-2)]*n*(c*x^n)^(2/(3*n)))/(32*E^(a*Sqrt[-n^(-2)]*n)*x^2) - (Sqrt[-n^(-2)
]*n*(c*x^n)^(2/n)*Log[x])/(8*E^(3*a*Sqrt[-n^(-2)]*n)*x^2)
```

Rule 4577

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[(m + 1)^p/(2^p*b^p*d^p*p^p), Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^
2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x]
, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m
 + 1)^2, 0]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\,\log\left(cx^n\right)\right)}{x^3}\,dx = \frac{\left(cx^n\right)^{2/n}\text{Subst}\left(\int x^{-1-\frac{2}{n}}\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\,\log(x)\right)\,dx, x, cx^n\right)}{nx^2}$$

$$= -\frac{\left(\sqrt{-\frac{1}{n^2}}\left(cx^n\right)^{2/n}\right)\text{Subst}\left(\int\left(\frac{e^{-3a\sqrt{-\frac{1}{n^2}}\,n}}{x}+3e^{a\sqrt{-\frac{1}{n^2}}\,n}x^{-1-\frac{8}{3n}}-\right.\right.}{8x^2}$$

$$= -\frac{e^{3a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,n\left(cx^n\right)^{-2/n}}{32x^2}+\frac{9e^{a\sqrt{-\frac{1}{n^2}}\,n}\sqrt{-\frac{1}{n^2}}\,n\left(cx^n\right)^{-\frac{2}{3}/n}}{64x^2}-$$

Mathematica [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\,\log\left(cx^n\right)\right)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + (2*Sqrt[-n^(-2)]*Log[c*x^n])/3]^3/x^3,x]
[Out] Integrate[Sin[a + (2*Sqrt[-n^(-2)]*Log[c*x^n])/3]^3/x^3, x]
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sin^3\left(a+\frac{2\ln\left(c\,x^n\right)\sqrt{-\frac{1}{n^2}}}{3}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+2/3*ln(c*x^n)*(-1/n^2)^(1/2))^3/x^3,x)
[Out] int(sin(a+2/3*ln(c*x^n)*(-1/n^2)^(1/2))^3/x^3,x)
```

Maxima [A]
time = 0.30, size = 128, normalized size = 0.72

$$-\frac{\left(8\,c^{\frac{14}{3\,n}}x^2e^{\left(\frac{2\,\log\left(x^n\right)}{3\,n}+4\,\log\left(x\right)\right)}\log\left(x\right)\sin\left(3\,a\right)+9\,c^{\frac{2}{n}}x^4\sin\left(a\right)-2\,c^{\frac{2}{3\,n}}x^2\left(x^n\right)^{\frac{2}{3\,n}}\sin\left(3\,a\right)+18\,c^{\frac{10}{3\,n}}e^{\left(\frac{4\,\log\left(x^n\right)}{3\,n}+4\,\log\left(x\right)\right)}\sin\left(a\right)\right)e^{\left(-\frac{2\,\log\left(x^n\right)}{3\,n}-4\,\log\left(x\right)\right)}}{64\,c^{\frac{8}{3\,n}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+2/3*log(c*x^n)*(-1/n^2)^(1/2))^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/64*(8*c^(14/3/n)*x^2*e^(2/3*log(x^n)/n + 4*log(x))*log(x)*sin(3*a) + 9*c
^(2/n)*x^4*sin(a) - 2*c^(2/3/n)*x^2*(x^n)^(2/3/n)*sin(3*a) + 18*c^(10/3/n)*
e^(4/3*log(x^n)/n + 4*log(x))*sin(a))*e^(-2/3*log(x^n)/n - 4*log(x))/(c^(8/
3/n)*x^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 2.15, size = 87, normalized size = 0.49

$$\frac{\left(-24i\,x^4\log\left(x^{\frac{1}{3}}\right)-18i\,x^{\frac{8}{3}}e^{\left(\frac{2\,(3i\,an-2\,\log(c))}{3\,n}\right)}+9i\,x^{\frac{4}{3}}e^{\left(\frac{4\,(3i\,an-2\,\log(c))}{3\,n}\right)}-2i\,e^{\left(\frac{2\,(3i\,an-2\,\log(c))}{n}\right)}\right)e^{\left(-\frac{3i\,an-2\,\log(c)}{n}\right)}}{64\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+2/3*log(c*x^n)*(-1/n^2)^(1/2))^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/64*(-24*I*x^4*log(x^(1/3)) - 18*I*x^(8/3)*e^(2/3*(3*I*a*n - 2*log(c))/n)
+ 9*I*x^(4/3)*e^(4/3*(3*I*a*n - 2*log(c))/n) - 2*I*e^(2*(3*I*a*n - 2*log(c)
)/n))*e^(-(3*I*a*n - 2*log(c))/n)/x^4
```

Sympy [A]
time = 61.24, size = 184, normalized size = 1.03

$$-\frac{9n\sqrt{-\frac{1}{n^2}}\cos\left(a+\frac{2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)}{3}\right)}{64x^2}-\frac{\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\cos\left(3a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{8x^2}-\frac{27\sin\left(a+\frac{2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)}{3}\right)}{64x^2}+\frac{\sin\left(3a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{16x^2}-\frac{\log\left(cx^n\right)\sin\left(3a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{8nx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+2/3*ln(c*x**n)*(-1/n**2)**(1/2))**3/x**3,x)
```

```
[Out] -9*n*sqrt(-1/n**2)*cos(a + 2*sqrt(-1/n**2)*log(c*x**n)/3)/(64*x**2) - sqrt(
-1/n**2)*log(c*x**n)*cos(3*a + 2*sqrt(-1/n**2)*log(c*x**n))/(8*x**2) - 27*s
in(a + 2*sqrt(-1/n**2)*log(c*x**n)/3)/(64*x**2) + sin(3*a + 2*sqrt(-1/n**2)
*log(c*x**n))/(16*x**2) - log(c*x**n)*sin(3*a + 2*sqrt(-1/n**2)*log(c*x**n)
)/(8*n*x**2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+2/3*log(c*x^n)*(-1/n^2)^(1/2))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate(sin(2/3*sqrt(-1/n^2)*log(c*x^n) + a)^3/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+\frac{2\ln(c\,x^n)\sqrt{-\frac{1}{n^2}}}{3}\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + (2*log(c*x^n)*(-1/n^2)^(1/2))/3)^3/x^3,x)
[Out] int(sin(a + (2*log(c*x^n)*(-1/n^2)^(1/2))/3)^3/x^3, x)
```

3.47 $\int x^m \sin\left(a+\frac{1}{2}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right) dx$

Optimal. Leaf size=112

$$-\frac{e^{\frac{a(1+m)}{\sqrt{-(1+m)^2}}}x^{1+m}\left(cx^2\right)^{\frac{1+m}{2}}}{4\sqrt{-(1+m)^2}}+\frac{e^{\frac{a\sqrt{-(1+m)^2}}{1+m}}(1+m)x^{1+m}\left(cx^2\right)^{\frac{1}{2}(-1-m)}\log(x)}{2\sqrt{-(1+m)^2}}$$

```
[Out] -1/4*exp(a*(1+m)/(-(1+m)^2)^(1/2))*x^(1+m)*(c*x^2)^(1/2+1/2*m)/(-(1+m)^2)^(
1/2)+1/2*exp(a*(-(1+m)^2)^(1/2)/(1+m))*(1+m)*x^(1+m)*(c*x^2)^(-1/2-1/2*m)*l
n(x)/(-(1+m)^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {4581, 4577}

$$\frac{(m+1)e^{\frac{a\sqrt{-(m+1)^2}}{m+1}}x^{m+1}\log(x)\left(cx^2\right)^{\frac{1}{2}(-m-1)}}{2\sqrt{-(m+1)^2}}-\frac{e^{\frac{a(m+1)}{\sqrt{-(m+1)^2}}}x^{m+1}\left(cx^2\right)^{\frac{m+1}{2}}}{4\sqrt{-(m+1)^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sin[a + (Sqrt[-(1 + m)^2]*Log[c*x^2])/2],x]
```

```
[Out] -1/4*(E^((a*(1 + m))/Sqrt[-(1 + m)^2])*x^(1 + m)*(c*x^2)^((1 + m)/2))/Sqrt[
-(1 + m)^2] + (E^((a*Sqrt[-(1 + m)^2])/(1 + m))*(1 + m)*x^(1 + m)*(c*x^2)^(
(-1 - m)/2)*Log[x])/(2*Sqrt[-(1 + m)^2])
```

Rule 4577

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[(m + 1)^p/(2^p*b^p*d^p*p^p), Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^
2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x]
, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m
 + 1)^2, 0]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^m \sin\left(a+\frac{1}{2}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)dx = \frac{1}{2}\left(x^{1+m}\left(cx^2\right)^{\frac{1}{2}(-1-m)}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{2}}\sin\left(a+\frac{1}{2}\sqrt{-\right.\right.$$

$$= \frac{\left((1+m)x^{1+m}(cx^2)^{\frac{1}{2}(-1-m)}\right)\text{Subst}\left(\int\left(\frac{e^{\frac{a\sqrt{-(1+m)^2}}{1+m}}}{x} - e\right.\right.}{4\sqrt{-(1+m)^2}}$$

$$= -\frac{e^{\frac{a(1+m)}{\sqrt{-(1+m)^2}}}x^{1+m}(cx^2)^{\frac{1+m}{2}}}{4\sqrt{-(1+m)^2}} + \frac{e^{\frac{a\sqrt{-(1+m)^2}}{1+m}}(1+m)x^{1+m}(c}{2\sqrt{-(1+m}}$$

Mathematica [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int x^m \sin\left(a+\frac{1}{2}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*Sin[a + (Sqrt[-(1 + m)^2]*Log[c*x^2])/2],x]
[Out] Integrate[x^m*Sin[a + (Sqrt[-(1 + m)^2]*Log[c*x^2])/2], x]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^m \sin\left(a+\frac{\ln\left(c\,x^2\right)\sqrt{-\left(1+m\right)^2}}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(a+1/2*ln(c*x^2)*(-(1+m)^2)^(1/2)),x)
[Out] int(x^m*sin(a+1/2*ln(c*x^2)*(-(1+m)^2)^(1/2)),x)
```

Maxima [A]
time = 0.28, size = 48, normalized size = 0.43

$$\frac{c^{m+1}x^2x^{2m}\sin(a) + 2\left(m\sin(a)+\sin(a)\right)\log(x)}{4\left(c^{\frac{1}{2}m}m + c^{\frac{1}{2}m}\right)\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+1/2*log(c*x^2)*(-(1+m)^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*(c^(m + 1)*x^2*x^(2*m)*sin(a) + 2*(m*sin(a) + sin(a))*log(x))/((c^(1/2*
m)*m + c^(1/2*m))*sqrt(c))
```

Fricas [C] Result contains complex when optimal does not.
time = 2.72, size = 51, normalized size = 0.46

$$\frac{\left(i\,x^2x^{2\,m}-2\,(i\,m+i)e^{\left(-(m+1)\log(c)+2i\,a\right)}\log\left(x\right)\right)e^{\left(\frac{1}{2}\,(m+1)\log(c)-i\,a\right)}}{4\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+1/2*log(c*x^2)*(-(1+m)^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(I*x^2*x^(2*m) - 2*(I*m + I)*e^(-(m + 1)*log(c) + 2*I*a)*log(x))*e^(1/2
*(m + 1)*log(c) - I*a)/(m + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\sin\left(a+\frac{\sqrt{-m^2-2m-1}\,\log\left(cx^2\right)}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sin(a+1/2*ln(c*x**2)*(-(1+m)**2)**(1/2)),x)
```

```
[Out] Integral(x**m*sin(a + sqrt(-m**2 - 2*m - 1)*log(c*x**2)/2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.56, size = 189, normalized size = 1.69

$$-\frac{i\,mxx^me^{\left(\frac{1}{2}\,|m+1|\log(c)+|m+1|\log(x)-i\,a\right)}-i\,xx^m|m+1|e^{\left(\frac{1}{2}\,|m+1|\log(c)+|m+1|\log(x)-i\,a\right)}-i\,mxx^me^{\left(-\frac{1}{2}\,|m+1|\log(c)-|m+1|\log(x)+i\,a\right)}-i\,xx^m|m+1|e^{\left(-\frac{1}{2}\,|m+1|\log(c)-|m+1|\log(x)+i\,a\right)}+i\,xx^me^{\left(\frac{1}{2}\,|m+1|\log(c)+|m+1|\log(x)-i\,a\right)}-i\,xx^me^{\left(-\frac{1}{2}\,|m+1|\log(c)-|m+1|\log(x)+i\,a\right)}}{2\left((m+1)^2-m^2-2\,m-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+1/2*log(c*x^2)*(-(1+m)^2)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*(I*m*x*x^m*e^(1/2*abs(m + 1)*log(c) + abs(m + 1)*log(x) - I*a) - I*x*x
^m*abs(m + 1)*e^(1/2*abs(m + 1)*log(c) + abs(m + 1)*log(x) - I*a) - I*m*x*x
^m*e^(-1/2*abs(m + 1)*log(c) - abs(m + 1)*log(x) + I*a) - I*x*x^m*abs(m + 1
)*e^(-1/2*abs(m + 1)*log(c) - abs(m + 1)*log(x) + I*a) + I*x*x^m*e^(1/2*abs
(m + 1)*log(c) + abs(m + 1)*log(x) - I*a) - I*x*x^m*e^(-1/2*abs(m + 1)*log(
c) - abs(m + 1)*log(x) + I*a))/((m + 1)^2 - m^2 - 2*m - 1)
```

Mupad [B]
time = 3.13, size = 139, normalized size = 1.24

$$\frac{\frac{1}{c^{\frac{\sqrt{-m^2-2\,m-1}\,1\mathrm{i}}{2}}}\,x\,x^m\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{{\left(x^2\right)}^{\frac{\sqrt{-m^2-2\,m-1}\,1\mathrm{i}}{2}}}\,1\mathrm{i}}{2\,m+2-\sqrt{-{\left(m+1\right)}^2}\,2\mathrm{i}}-\frac{c^{\frac{\sqrt{-m^2-2\,m-1}\,1\mathrm{i}}{2}}\,x\,x^m\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(x^2\right)}^{\frac{\sqrt{-m^2-2\,m-1}\,1\mathrm{i}}{2}}\,1\mathrm{i}}{2\,m+2+\sqrt{-{\left(m+1\right)}^2}\,2\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(a + (log(c*x^2)*(-(m + 1)^2)^(1/2))/2),x)
```

```
[Out] (1/c^(((- 2*m - m^2 - 1)^(1/2)*1i)/2)*x*x^m*exp(-a*1i)/(x^2)^(((- 2*m - m^2
 - 1)^(1/2)*1i)/2)*1i)/(2*m - (-(m + 1)^2)^(1/2)*2i + 2) - (c^(((- 2*m - m^
2 - 1)^(1/2)*1i)/2)*x*x^m*exp(a*1i)*(x^2)^(((- 2*m - m^2 - 1)^(1/2)*1i)/2)*
1i)/(2*m + (-(m + 1)^2)^(1/2)*2i + 2)
```

3.48 $\int \sin\left(a+\frac{1}{2}i\log\left(cx^2\right)\right)\,dx$

Optimal. Leaf size=52

$$\frac{ice^{-ia}x^3}{4\sqrt{cx^2}}-\frac{ie^{ia}x\log(x)}{2\sqrt{cx^2}}$$

```
[Out] 1/4*I*c*x^3/exp(I*a)/(c*x^2)^(1/2)-1/2*I*exp(I*a)*x*ln(x)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {4571, 4577}

$$\frac{ie^{-ia}cx^3}{4\sqrt{cx^2}}-\frac{ie^{ia}x\log(x)}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + (I/2)*Log[c*x^2]],x]
```

```
[Out] ((I/4)*c*x^3)/(E^(I*a)*Sqrt[c*x^2]) - ((I/2)*E^(I*a)*x*Log[x])/Sqrt[c*x^2]
```

Rule 4571

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4577

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[(m + 1)^p/(2^p*b^p*d^p*p^p), Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^
2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x]
, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m
 + 1)^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \sin\left(a+\frac{1}{2}i\log\left(cx^2\right)\right)\,dx &= \frac{x\text{Subst}\left(\int \frac{\sin\left(a+\frac{1}{2}i\log(x)\right)}{\sqrt{x}}\,dx,x,cx^2\right)}{2\sqrt{cx^2}}\\
&=-\frac{(ix)\text{Subst}\left(\int\left(-e^{-ia}+\frac{e^{ia}}{x}\right)\,dx,x,cx^2\right)}{4\sqrt{cx^2}}\\
&=\frac{ice^{-ia}x^3}{4\sqrt{cx^2}}-\frac{ie^{ia}x\log(x)}{2\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 44, normalized size = 0.85

$$\frac{x(i\cos(a)\left(cx^2-2\log(x)\right)+\left(cx^2+2\log(x)\right)\sin(a))}{4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + (I/2)*Log[c*x^2]],x]
```

```
[Out] (x*(I*Cos[a]*(c*x^2 - 2*Log[x]) + (c*x^2 + 2*Log[x])*Sin[a]))/(4*Sqrt[c*x^2
])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(40) = 80$.
time = 0.05, size = 106, normalized size = 2.04

method	result	size
norman	$\frac{\frac{ix}{2}-\frac{ix\left(\tan^2\left(\frac{a}{2}+\frac{i\ln\left(c\,x^2\right)}{4}\right)\right)}{2}+\frac{x\ln\left(c\,x^2\right)\tan\left(\frac{a}{2}+\frac{i\ln\left(c\,x^2\right)}{4}\right)}{2}-\frac{ix\ln\left(c\,x^2\right)}{4}+\frac{ix\ln\left(c\,x^2\right)\left(\tan^2\left(\frac{a}{2}+\frac{i\ln\left(c\,x^2\right)}{4}\right)\right)}{4}}{1+\tan^2\left(\frac{a}{2}+\frac{i\ln\left(c\,x^2\right)}{4}\right)}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+1/2*I*ln(c*x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*I*x-1/2*I*x*tan(1/2*a+1/4*I*ln(c*x^2))^2+1/2*x*ln(c*x^2)*tan(1/2*a+1/4
*I*ln(c*x^2))-1/4*I*x*ln(c*x^2)+1/4*I*x*ln(c*x^2)*tan(1/2*a+1/4*I*ln(c*x^2)
)^2)/(1+tan(1/2*a+1/4*I*ln(c*x^2))^2)
```

Maxima [A]
time = 0.27, size = 31, normalized size = 0.60

$$\frac{cx^2(i\,\cos(a)+\sin(a))-2\,(i\,\cos(a)-\sin(a))\log(x)}{4\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/2*I*log(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/4*(c*x^2*(I*cos(a) + sin(a)) - 2*(I*cos(a) - sin(a))*log(x))/sqrt(c)
```

Fricas [A]
time = 3.03, size = 24, normalized size = 0.46

$$\frac{\left(i\,cx^2-2i\,e^{(2i\,a)}\log(x)\right)e^{(-i\,a)}}{4\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/2*I*log(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/4*(I*c*x^2 - 2*I*e^(2*I*a)*log(x))*e^(-I*a)/sqrt(c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(a + \frac{i\log\left(cx^2\right)}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/2*I*ln(c*x**2)),x)
```

```
[Out] Integral(sin(a + I*log(c*x**2)/2), x)
```

Giac [A]
time = 0.43, size = 29, normalized size = 0.56

$$-\frac{-i\,c^{\frac{3}{2}}x^2e^{(-i\,a)} + 2i\,\sqrt{c}\,e^{(i\,a)}\log\left(x\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/2*I*log(c*x^2)),x, algorithm="giac")
```

```
[Out] -1/4*(-I*c^(3/2)*x^2*e^(-I*a) + 2*I*sqrt(c)*e^(I*a)*log(x))/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sin\left(a + \frac{\ln\left(c\,x^2\right)\,1\mathrm{i}}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + (log(c*x^2)*1i)/2),x)
```

```
[Out] int(sin(a + (log(c*x^2)*1i)/2), x)
```

3.49 $\int x^m \sin^2\left(a+\frac{1}{4}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)dx$

Optimal. Leaf size=106

$$\frac{x^{1+m}}{2(1+m)}-\frac{e^{\frac{2a(1+m)}{\sqrt{-(1+m)^2}}}x^{1+m}\left(cx^2\right)^{\frac{1+m}{2}}}{8(1+m)}-\frac{1}{4}e^{-\frac{2a(1+m)}{\sqrt{-(1+m)^2}}}x^{1+m}\left(cx^2\right)^{\frac{1}{2}(-1-m)}\log(x)$$

```
[Out] 1/2*x^(1+m)/(1+m)-1/8*exp(2*a*(1+m)/(-(1+m)^2)^(1/2))*x^(1+m)*(c*x^2)^(1/2+
1/2*m)/(1+m)-1/4*x^(1+m)*(c*x^2)^(-1/2-1/2*m)*ln(x)/exp(2*a*(1+m)/(-(1+m)^2
)^(1/2))
```

Rubi [A]
time = 0.10, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {4581, 4577}

$$-\frac{e^{\frac{2a(m+1)}{\sqrt{-(m+1)^2}}}x^{m+1}\left(cx^2\right)^{\frac{m+1}{2}}}{8(m+1)}-\frac{1}{4}e^{-\frac{2a(m+1)}{\sqrt{-(m+1)^2}}}x^{m+1}\log(x)\left(cx^2\right)^{\frac{1}{2}(-m-1)}+\frac{x^{m+1}}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sin[a + (Sqrt[-(1 + m)^2]*Log[c*x^2])/4]^2,x]
```

```
[Out] x^(1 + m)/(2*(1 + m)) - (E^((2*a*(1 + m))/Sqrt[-(1 + m)^2])*x^(1 + m)*(c*x^
2)^((1 + m)/2))/(8*(1 + m)) - (x^(1 + m)*(c*x^2)^((-1 - m)/2)*Log[x])/(4*E^
((2*a*(1 + m))/Sqrt[-(1 + m)^2]))
```

Rule 4577

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[(m + 1)^p/(2^p*b^p*d^p*p^p), Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^
2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x]
, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m
 + 1)^2, 0]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^m \sin^2\left(a+\frac{1}{4}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)dx = \frac{1}{2}\left(x^{1+m}\left(cx^2\right)^{\frac{1}{2}(-1-m)}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{2}}\sin^2\left(a+\frac{1}{4}\sqrt{-\right.\right.$$

$$= -\left(\frac{1}{8}\left(x^{1+m}\left(cx^2\right)^{\frac{1}{2}(-1-m)}\right)\text{Subst}\left(\left(\int\left(\frac{e^{-\frac{2a(1+m)}{\sqrt{-(1+m)^2}}}}{x} - 2x\right.\right.\right.\right.$$

$$= \frac{x^{1+m}}{2(1+m)} - \frac{e^{\frac{2a(1+m)}{\sqrt{-(1+m)^2}}}x^{1+m}\left(cx^2\right)^{\frac{1+m}{2}}}{8(1+m)} - \frac{1}{4}e^{-\frac{2a(1+m)}{\sqrt{-(1+m)^2}}}x^{1+}$$

<u>**Mathematica**</u> [**F**]
time = 0.37, size = 0, normalized size = 0.00

$$\int x^m \sin^2\left(a+\frac{1}{4}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*Sin[a + (Sqrt[-(1 + m)^2]*Log[c*x^2])/4]^2,x]
[Out] Integrate[x^m*Sin[a + (Sqrt[-(1 + m)^2]*Log[c*x^2])/4]^2, x]
```

<u>**Maple**</u> [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int x^m\left(\sin^2\left(a+\frac{\ln\left(c\,x^2\right)\sqrt{-\left(1+m\right)^2}}{4}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(a+1/4*ln(c*x^2)*(-(1+m)^2)^(1/2))^2,x)
[Out] int(x^m*sin(a+1/4*ln(c*x^2)*(-(1+m)^2)^(1/2))^2,x)
```

<u>**Maxima**</u> [**A**]
time = 0.29, size = 134, normalized size = 1.26

$$-\frac{c^{m+1}x^2x^{2m}\cos\left(2a\right) - 4\left(\cos\left(2a\right)^2+\sin\left(2a\right)^2\right)c^{\frac{1}{2}m+\frac{1}{2}}xx^m + 2\left(\cos\left(2a\right)^3+\cos\left(2a\right)\sin\left(2a\right)^2+\left(\cos\left(2a\right)^3+\cos\left(2a\right)\sin\left(2a\right)^2\right)m\right)\log\left(x\right)}{8\left(\left(\cos\left(2a\right)^2+\sin\left(2a\right)^2\right)c^{\frac{1}{2}m}m+\left(\cos\left(2a\right)^2+\sin\left(2a\right)^2\right)c^{\frac{1}{2}m}\right)\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+1/4*log(c*x^2)*(-(1+m)^2)^(1/2))^2,x, algorithm="maxima
")
```

```
[Out] -1/8*(c^(m + 1)*x^2*x^(2*m)*cos(2*a) - 4*(cos(2*a)^2 + sin(2*a)^2)*c^(1/2*m
 + 1/2)*x*x^m + 2*(cos(2*a)^3 + cos(2*a)*sin(2*a)^2 + (cos(2*a)^3 + cos(2*a
)*sin(2*a)^2)*m)*log(x))/(((cos(2*a)^2 + sin(2*a)^2)*c^(1/2*m)*m + (cos(2*a
)^2 + sin(2*a)^2)*c^(1/2*m))*sqrt(c))
```

Fricas [C] Result contains complex when optimal does not.
time = 3.01, size = 75, normalized size = 0.71

$$-\frac{\left(2\,(m+1)e^{\left(-(m+1)\log(c)-2\,(m+1)\log(x)+4i\,a\right)}\log\left(x\right)-4\,e^{\left(-\frac{1}{2}\,(m+1)\log(c)-(m+1)\log(x)+2i\,a\right)}+1\right)e^{\left(\frac{1}{2}\,(m+1)\log(c)+2\,(m+1)\log(x)-2i\,a\right)}}{8\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+1/4*log(c*x^2)*(-(1+m)^2)^(1/2))^2,x, algorithm="fricas
")
```

```
[Out] -1/8*(2*(m + 1)*e^(-(m + 1)*log(c) - 2*(m + 1)*log(x) + 4*I*a)*log(x) - 4*e
^(-1/2*(m + 1)*log(c) - (m + 1)*log(x) + 2*I*a) + 1)*e^(1/2*(m + 1)*log(c)
+ 2*(m + 1)*log(x) - 2*I*a)/(m + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m}\sin^{2}\left(a+\frac{\sqrt{-m^{2}-2m-1}\,\log\left(cx^{2}\right)}{4}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sin(a+1/4*ln(c*x**2)*(-(1+m)**2)**(1/2))**2,x)
```

```
[Out] Integral(x**m*sin(a + sqrt(-m**2 - 2*m - 1)*log(c*x**2)/4)**2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.99, size = 350, normalized size = 3.30

$$\frac{m^{2}xx^{m}e^{\left(\frac{1}{2}\,|m+1|\log(c)+|m+1|\log(x)-2i\,a\right)}-mxx^{m}|m+1|e^{\left(\frac{1}{2}\,|m+1|\log(c)+|m+1|\log(x)-2i\,a\right)}+m^{2}xx^{m}e^{\left(-\frac{1}{2}\,|m+1|\log(c)-|m+1|\log(x)+2i\,a\right)}+mxx^{m}|m+1|e^{\left(-\frac{1}{2}\,|m+1|\log(c)-|m+1|\log(x)+2i\,a\right)}+2\,(m+1)^{2}xx^{m}-2\,m^{2}xx^{m}+2\,mxx^{m}e^{\left(\frac{1}{2}\,|m+1|\log(c)+|m+1|\log(x)-2i\,a\right)}-xx^{m}|m+1|e^{\left(\frac{1}{2}\,|m+1|\log(c)+|m+1|\log(x)-2i\,a\right)}+2\,mxx^{m}e^{\left(-\frac{1}{2}\,|m+1|\log(c)-|m+1|\log(x)+2i\,a\right)}+xx^{m}|m+1|e^{\left(-\frac{1}{2}\,|m+1|\log(c)-|m+1|\log(x)+2i\,a\right)}-4\,mxx^{m}+xx^{m}e^{\left(\frac{1}{2}\,|m+1|\log(c)+|m+1|\log(x)-2i\,a\right)}+xx^{m}e^{\left(-\frac{1}{2}\,|m+1|\log(c)-|m+1|\log(x)+2i\,a\right)}-2\,xx^{m}}{4\left((m+1)^{2}m-m^{3}+(m+1)^{2}-3\,m^{2}-3\,m-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+1/4*log(c*x^2)*(-(1+m)^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/4*(m^2*x*x^m*e^(1/2*abs(m + 1)*log(c) + abs(m + 1)*log(x) - 2*I*a) - m*x*
x^m*abs(m + 1)*e^(1/2*abs(m + 1)*log(c) + abs(m + 1)*log(x) - 2*I*a) + m^2*
x*x^m*e^(-1/2*abs(m + 1)*log(c) - abs(m + 1)*log(x) + 2*I*a) + m*x*x^m*abs(
m + 1)*e^(-1/2*abs(m + 1)*log(c) - abs(m + 1)*log(x) + 2*I*a) + 2*(m + 1)^2
*x*x^m - 2*m^2*x*x^m + 2*m*x*x^m*e^(1/2*abs(m + 1)*log(c) + abs(m + 1)*log(
x) - 2*I*a) - x*x^m*abs(m + 1)*e^(1/2*abs(m + 1)*log(c) + abs(m + 1)*log(x)
 - 2*I*a) + 2*m*x*x^m*e^(-1/2*abs(m + 1)*log(c) - abs(m + 1)*log(x) + 2*I*a
) + x*x^m*abs(m + 1)*e^(-1/2*abs(m + 1)*log(c) - abs(m + 1)*log(x) + 2*I*a)
```

```
 - 4*m*x*x^m + x*x^m*e^(1/2*abs(m + 1)*log(c) + abs(m + 1)*log(x) - 2*I*a)
+ x*x^m*e^(-1/2*abs(m + 1)*log(c) - abs(m + 1)*log(x) + 2*I*a) - 2*x*x^m)/(
(m + 1)^2*m - m^3 + (m + 1)^2 - 3*m^2 - 3*m - 1)
```

Mupad [B]
time = 3.04, size = 149, normalized size = 1.41

$$\frac{x\,x^m}{2\,m+2}-\frac{\frac{1}{c^{\frac{\sqrt{-m^2-2\,m-1}\,1\mathrm{i}}{2}}}\,x\,x^m\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{{\left(x^2\right)}^{\frac{\sqrt{-m^2-2\,m-1}\,1\mathrm{i}}{2}}}}{4\,m+4-\sqrt{-{\left(m+1\right)}^2}\,4\mathrm{i}}-\frac{c^{\frac{\sqrt{-m^2-2\,m-1}\,1\mathrm{i}}{2}}\,x\,x^m\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(x^2\right)}^{\frac{\sqrt{-m^2-2\,m-1}\,1\mathrm{i}}{2}}}{4\,m+4+\sqrt{-{\left(m+1\right)}^2}\,4\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(a + (log(c*x^2)*(-(m + 1)^2)^(1/2))/4)^2,x)
```

```
[Out] (x*x^m)/(2*m + 2) - (1/c^(((- 2*m - m^2 - 1)^(1/2)*1i)/2)*x*x^m*exp(-a*2i)/
(x^2)^(((- 2*m - m^2 - 1)^(1/2)*1i)/2))/(4*m - (-(m + 1)^2)^(1/2)*4i + 4) -
 (c^(((- 2*m - m^2 - 1)^(1/2)*1i)/2)*x*x^m*exp(a*2i)*(x^2)^(((- 2*m - m^2 -
 1)^(1/2)*1i)/2))/(4*m + (-(m + 1)^2)^(1/2)*4i + 4)
```

3.50 $\int \sin^2\left(a+\frac{1}{4}i\log\left(cx^2\right)\right)\,dx$

Optimal. Leaf size=53

$$\frac{x}{2}-\frac{ce^{-2ia}x^3}{8\sqrt{cx^2}}-\frac{e^{2ia}x\log(x)}{4\sqrt{cx^2}}$$

```
[Out] 1/2*x-1/8*c*x^3/exp(2*I*a)/(c*x^2)^(1/2)-1/4*exp(2*I*a)*x*ln(x)/(c*x^2)^(1/
2)
```

Rubi [A]

time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4571, 4577}

$$-\frac{e^{2ia}x\log(x)}{4\sqrt{cx^2}}-\frac{e^{-2ia}cx^3}{8\sqrt{cx^2}}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + (I/4)*Log[c*x^2]]^2,x]
```

```
[Out] x/2 - (c*x^3)/(8*E^((2*I)*a)*Sqrt[c*x^2]) - (E^((2*I)*a)*x*Log[x])/(4*Sqrt[
c*x^2])
```

Rule 4571

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4577

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[(m + 1)^p/(2^p*b^p*d^p*p^p), Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^
2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x]
, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m
 + 1)^2, 0]
```

Rubi steps

$$\int \sin^2\left(a+\frac{1}{4}i\log\left(cx^2\right)\right)dx = \frac{x\text{Subst}\left(\int \frac{\sin^2\left(a+\frac{1}{4}i\log(x)\right)}{\sqrt{x}}dx,x,cx^2\right)}{2\sqrt{cx^2}}$$
$$= -\frac{x\text{Subst}\left(\int \left(e^{-2ia}+\frac{e^{2ia}}{x}-\frac{2}{\sqrt{x}}\right)dx,x,cx^2\right)}{8\sqrt{cx^2}}$$
$$= \frac{x}{2}-\frac{ce^{-2ia}x^3}{8\sqrt{cx^2}}-\frac{e^{2ia}x\log(x)}{4\sqrt{cx^2}}$$

Mathematica [A]
time = 0.11, size = 60, normalized size = 1.13

$$\frac{x\left(4\sqrt{cx^2}-\cos(2a)\left(cx^2+2\log(x)\right)+i\left(cx^2-2\log(x)\right)\sin(2a)\right)}{8\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + (I/4)*Log[c*x^2]]^2,x]
```

```
[Out] (x*(4*Sqrt[c*x^2] - Cos[2*a]*(c*x^2 + 2*Log[x]) + I*(c*x^2 - 2*Log[x])*Sin[
2*a]))/(8*Sqrt[c*x^2])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(41) = 82$.
time = 0.12, size = 173, normalized size = 3.26

method	result
norman	$\frac{\frac{x}{4}+\frac{5x\left(\tan^2\left(\frac{a}{2}+\frac{i\ln\left(cx^2\right)}{8}\right)\right)}{2}+\frac{x\left(\tan^4\left(\frac{a}{2}+\frac{i\ln\left(cx^2\right)}{8}\right)\right)}{4}-\frac{x\ln\left(cx^2\right)}{8}+\frac{3x\ln\left(cx^2\right)\left(\tan^2\left(\frac{a}{2}+\frac{i\ln\left(cx^2\right)}{8}\right)\right)}{4}-\frac{x\ln\left(cx^2\right)\left(\tan^4\left(\frac{a}{2}+\frac{i\ln\left(c x^2\right)}{8}\right.\right.}{8}}{\left(1+\tan^2\left(\frac{a}{2}+\frac{i\ln\left(cx^2\right)}{8}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+1/4*I*ln(c*x^2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/4*x+5/2*x*tan(1/2*a+1/8*I*ln(c*x^2))^2+1/4*x*tan(1/2*a+1/8*I*ln(c*x^2))^
4-1/8*x*ln(c*x^2)+3/4*x*ln(c*x^2)*tan(1/2*a+1/8*I*ln(c*x^2))^2-1/8*x*ln(c*x
^2)*tan(1/2*a+1/8*I*ln(c*x^2))^4-1/2*I*x*ln(c*x^2)*tan(1/2*a+1/8*I*ln(c*x^2
))+1/2*I*x*ln(c*x^2)*tan(1/2*a+1/8*I*ln(c*x^2))^3)/(1+tan(1/2*a+1/8*I*ln(c*
x^2))^2)^2
```

Maxima [A]
time = 0.29, size = 47, normalized size = 0.89

$$\frac{4\,cx - \left(cx^2(\cos\left(2\,a\right) - i\,\sin\left(2\,a\right)) + 2\left(\cos\left(2\,a\right) + i\,\sin\left(2\,a\right)\right)\log\left(x\right)\right)\sqrt{c}}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/4*I*log(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(4*c*x - (c*x^2*(cos(2*a) - I*sin(2*a)) + 2*(cos(2*a) + I*sin(2*a))*log
(x))*sqrt(c))/c
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(37) = 74.
time = 3.78, size = 145, normalized size = 2.74

$$\frac{\left(4\,x^2e^{(2\,i\,a)} - \frac{xe^{(4\,i\,a)}\log\left(\frac{\left(\sqrt{cx^2}\,(x^2+1)e^{(2\,i\,a)} + \frac{(cx^3-cx)e^{(2\,i\,a)}}{\sqrt{c}}\right)e^{(-2\,i\,a)}}{8\,x^2}\right)}{\sqrt{c}} + \frac{xe^{(4\,i\,a)}\log\left(\frac{\left(\sqrt{cx^2}\,(x^2+1)e^{(2\,i\,a)} - \frac{(cx^3-cx)e^{(2\,i\,a)}}{\sqrt{c}}\right)e^{(-2\,i\,a)}}{8\,x^2}\right)}{\sqrt{c}} - \sqrt{cx^2}\,(x^2-1)\right)e^{(-2\,i\,a)}}{8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/4*I*log(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*x^2*e^(2*I*a) - x*e^(4*I*a)*log(1/8*(sqrt(c*x^2)*(x^2 + 1)*e^(2*I*a)
 + (c*x^3 - c*x)*e^(2*I*a)/sqrt(c))*e^(-2*I*a)/x^2)/sqrt(c) + x*e^(4*I*a)*l
og(1/8*(sqrt(c*x^2)*(x^2 + 1)*e^(2*I*a) - (c*x^3 - c*x)*e^(2*I*a)/sqrt(c))*
e^(-2*I*a)/x^2)/sqrt(c) - sqrt(c*x^2)*(x^2 - 1))*e^(-2*I*a)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^2\left(a + \frac{i\log\left(cx^2\right)}{4}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/4*I*ln(c*x**2))**2,x)
```

```
[Out] Integral(sin(a + I*log(c*x**2)/4)**2, x)
```

Giac [A]
time = 0.49, size = 32, normalized size = 0.60

$$\frac{1}{2}\,x - \frac{c^{\frac{3}{2}}x^2e^{(-2\,i\,a)} + 2\,\sqrt{c}\,e^{(2\,i\,a)}\log\left(x\right)}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/4*I*log(c*x^2))^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/8*(c^(3/2)*x^2*e^(-2*I*a) + 2*sqrt(c)*e^(2*I*a)*log(x))/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sin\left(a+\frac{\ln\left(c\,x^2\right)\,1\mathrm{i}}{4}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + (log(c*x^2)*1i)/4)^2,x)
```

```
[Out] int(sin(a + (log(c*x^2)*1i)/4)^2, x)
```

3.51 $\int x^m \sin^3\left(a+\frac{1}{6}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)dx$

Optimal. Leaf size=218

$$\frac{9e^{\frac{a\sqrt{-(1+m)^2}}{1+m}}x^{1+m}\left(cx^2\right)^{\frac{1}{6}(-1-m)}}{16\sqrt{-(1+m)^2}}-\frac{9e^{\frac{a(1+m)}{\sqrt{-(1+m)^2}}}x^{1+m}\left(cx^2\right)^{\frac{1+m}{6}}}{32\sqrt{-(1+m)^2}}+\frac{e^{\frac{3a(1+m)}{\sqrt{-(1+m)^2}}}x^{1+m}\left(cx^2\right)^{\frac{1+m}{2}}}{16\sqrt{-(1+m)^2}}-\frac{e^{-\frac{3a(1+m)}{\sqrt{-(1+m)^2}}}(1+}{8}$$

```
[Out] 9/16*exp(a*(-(1+m)^2)^(1/2)/(1+m))*x^(1+m)*(c*x^2)^(-1/6-1/6*m)/(-(1+m)^2)^
(1/2)-9/32*exp(a*(1+m)/(-(1+m)^2)^(1/2))*x^(1+m)*(c*x^2)^(1/6+1/6*m)/(-(1+m
)^2)^(1/2)+1/16*exp(3*a*(1+m)/(-(1+m)^2)^(1/2))*x^(1+m)*(c*x^2)^(1/2+1/2*m)
/(-(1+m)^2)^(1/2)-1/8*(1+m)*x^(1+m)*(c*x^2)^(-1/2-1/2*m)*ln(x)/exp(3*a*(1+m
)/(-(1+m)^2)^(1/2))/(-(1+m)^2)^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}=0.067$, Rules used = {4581, 4577}

$$\frac{9e^{\frac{a\sqrt{-(m+1)^2}}{m+1}}x^{m+1}\left(cx^2\right)^{\frac{1}{6}(-m-1)}}{16\sqrt{-(m+1)^2}}-\frac{9e^{\frac{a(m+1)}{\sqrt{-(m+1)^2}}}x^{m+1}\left(cx^2\right)^{\frac{m+1}{6}}}{32\sqrt{-(m+1)^2}}+\frac{e^{\frac{3a(m+1)}{\sqrt{-(m+1)^2}}}x^{m+1}\left(cx^2\right)^{\frac{m+1}{2}}}{16\sqrt{-(m+1)^2}}-\frac{(m+1)e^{-\frac{3a(m+1)}{\sqrt{-(m+1)^2}}}x^{m+1}\log(x)\left(cx^2\right)^{\frac{1}{2}(-m-1)}}{8\sqrt{-(m+1)^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sin[a + (Sqrt[-(1 + m)^2]*Log[c*x^2])/6]^3,x]
```

```
[Out] (9*E^((a*Sqrt[-(1 + m)^2])/(1 + m))*x^(1 + m)*(c*x^2)^((-1 - m)/6))/(16*Sqr
t[-(1 + m)^2]) - (9*E^((a*(1 + m))/Sqrt[-(1 + m)^2])*x^(1 + m)*(c*x^2)^((1
+ m)/6))/(32*Sqrt[-(1 + m)^2]) + (E^((3*a*(1 + m))/Sqrt[-(1 + m)^2])*x^(1 +
 m)*(c*x^2)^((1 + m)/2))/(16*Sqrt[-(1 + m)^2]) - ((1 + m)*x^(1 + m)*(c*x^2)
^((-1 - m)/2)*Log[x])/(8*E^((3*a*(1 + m))/Sqrt[-(1 + m)^2])*Sqrt[-(1 + m)^2
])
```

Rule 4577

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[(m + 1)^p/(2^p*b^p*d^p*p^p), Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^
2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x]
, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m
 + 1)^2, 0]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^m \sin^3\left(a+\frac{1}{6}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)dx = \frac{1}{2}\left(x^{1+m}\left(cx^2\right)^{\frac{1}{2}(-1-m)}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{2}}\sin^3\left(a+\frac{1}{6}\sqrt{-\right.\right.$$

$$= \frac{\left(\sqrt{-(1+m)^2}\,x^{1+m}\left(cx^2\right)^{\frac{1}{2}(-1-m)}\right)\text{Subst}\left(\int\left(\frac{e^{-\frac{3a(1+m)}{\sqrt{-(1+m)^2}}}}{x}\right.\right.}{}$$

$$= \frac{9e^{\frac{a\sqrt{-(1+m)^2}}{1+m}}x^{1+m}\left(cx^2\right)^{\frac{1}{6}(-1-m)}}{16\sqrt{-(1+m)^2}} - \frac{9e^{\frac{a(1+m)}{\sqrt{-(1+m)^2}}}x^{1+m}\left(cx^2\right)^{1}}{32\sqrt{-(1+m)^2}}$$

Mathematica [F]
time = 0.49, size = 0, normalized size = 0.00

$$\int x^m \sin^3\left(a+\frac{1}{6}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*Sin[a + (Sqrt[-(1 + m)^2]*Log[c*x^2])/6]^3,x]
```

```
[Out] Integrate[x^m*Sin[a + (Sqrt[-(1 + m)^2]*Log[c*x^2])/6]^3, x]
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int x^m\left(\sin^3\left(a+\frac{\ln\left(c\,x^2\right)\sqrt{-\left(1+m\right)^2}}{6}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(a+1/6*ln(c*x^2)*(-(1+m)^2)^(1/2))^3,x)
```

```
[Out] int(x^m*sin(a+1/6*ln(c*x^2)*(-(1+m)^2)^(1/2))^3,x)
```

Maxima [A]
time = 0.29, size = 206, normalized size = 0.94

$$\frac{9\left(\cos(2a)\sin(3a)-\cos(3a)\sin(2a)\right)c^{\frac{5}{6}m+\frac{5}{6}}x^{\frac{5}{3}}x^{\frac{4}{3}m}+18\left(\cos(3a)\sin(4a)-\cos(4a)\sin(3a)\right)c^{\frac{1}{2}m+\frac{1}{2}}xx^{\frac{2}{3}m}-2\left(c^{\frac{7}{6}m+1}x^2x^{2m}\sin(3a)+2\left(\left(\cos(3a)^2\sin(3a)+\sin(3a)^3\right)c^{\frac{1}{6}m}m+\left(\cos(3a)^2\sin(3a)+\sin(3a)^3\right)c^{\frac{1}{6}m}\right)\log(x)\right)c^{\frac{1}{6}}x^{\frac{1}{3}}}{32\left(\left(\cos(3a)^2+\sin(3a)^2\right)c^{\frac{2}{3}m}m+\left(\cos(3a)^2+\sin(3a)^2\right)c^{\frac{2}{3}m}\right)c^{\frac{2}{3}}x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+1/6*log(c*x^2)*(-(1+m)^2)^(1/2))^3,x, algorithm="maxima
")
```

```
[Out] 1/32*(9*(cos(2*a)*sin(3*a) - cos(3*a)*sin(2*a))*c^(5/6*m + 5/6)*x^(5/3)*x^(
4/3*m) + 18*(cos(3*a)*sin(4*a) - cos(4*a)*sin(3*a))*c^(1/2*m + 1/2)*x*x^(2/
3*m) - 2*(c^(7/6*m + 1)*x^2*x^(2*m)*sin(3*a) + 2*((cos(3*a)^2*sin(3*a) + si
n(3*a)^3)*c^(1/6*m)*m + (cos(3*a)^2*sin(3*a) + sin(3*a)^3)*c^(1/6*m))*log(x
))*c^(1/6)*x^(1/3))/(((cos(3*a)^2 + sin(3*a)^2)*c^(2/3*m)*m + (cos(3*a)^2 +
 sin(3*a)^2)*c^(2/3*m))*c^(2/3)*x^(1/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 3.02, size = 98, normalized size = 0.45

$$-\frac{\left(4\,(-i\,m-i)e^{\left(-(m+1)\log(c)-2\,(m+1)\log(x)+6i\,a\right)}\log\left(x\right)-9i\,e^{\left(-\frac{1}{3}\,(m+1)\log(c)-\frac{2}{3}\,(m+1)\log(x)+2i\,a\right)}+18i\,e^{\left(-\frac{2}{3}\,(m+1)\log(c)-\frac{4}{3}\,(m+1)\log(x)+4i\,a\right)}+2i\right)e^{\left(\frac{1}{2}\,(m+1)\log(c)+2\,(m+1)\log(x)-3i\,a\right)}}{32\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+1/6*log(c*x^2)*(-(1+m)^2)^(1/2))^3,x, algorithm="fricas
")
```

```
[Out] -1/32*(4*(-I*m - I)*e^(-(m + 1)*log(c) - 2*(m + 1)*log(x) + 6*I*a)*log(x) -
 9*I*e^(-1/3*(m + 1)*log(c) - 2/3*(m + 1)*log(x) + 2*I*a) + 18*I*e^(-2/3*(m
 + 1)*log(c) - 4/3*(m + 1)*log(x) + 4*I*a) + 2*I)*e^(1/2*(m + 1)*log(c) + 2
*(m + 1)*log(x) - 3*I*a)/(m + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m}\sin^{3}\left(a+\frac{\sqrt{-m^{2}-2m-1}\,\log\left(cx^{2}\right)}{6}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sin(a+1/6*ln(c*x**2)*(-(1+m)**2)**(1/2))**3,x)
```

```
[Out] Integral(x**m*sin(a + sqrt(-m**2 - 2*m - 1)*log(c*x**2)/6)**3, x)
```

Giac [C] Result contains complex when optimal does not.
time = 1.58, size = 1297, normalized size = 5.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+1/6*log(c*x^2)*(-(1+m)^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] 1/8*(I*(m + 1)^2*m*x*x^m*e^(1/2*abs(m + 1)*log(c) + abs(m + 1)*log(x) - 3*I
*a) - 9*I*m^3*x*x^m*e^(1/2*abs(m + 1)*log(c) + abs(m + 1)*log(x) - 3*I*a) -
 I*(m + 1)^2*x*x^m*abs(m + 1)*e^(1/2*abs(m + 1)*log(c) + abs(m + 1)*log(x)
- 3*I*a) + 9*I*m^2*x*x^m*abs(m + 1)*e^(1/2*abs(m + 1)*log(c) + abs(m + 1)*l
og(x) - 3*I*a) - 27*I*(m + 1)^2*m*x*x^m*e^(1/6*abs(m + 1)*log(c) + 1/3*abs(
```

```
m + 1)*log(x) - I*a) + 27*I*m^3*x*x^m*e^(1/6*abs(m + 1)*log(c) + 1/3*abs(m
+ 1)*log(x) - I*a) + 9*I*(m + 1)^2*x*x^m*abs(m + 1)*e^(1/6*abs(m + 1)*log(c
) + 1/3*abs(m + 1)*log(x) - I*a) - 9*I*m^2*x*x^m*abs(m + 1)*e^(1/6*abs(m +
1)*log(c) + 1/3*abs(m + 1)*log(x) - I*a) + 27*I*(m + 1)^2*m*x*x^m*e^(-1/6*a
bs(m + 1)*log(c) - 1/3*abs(m + 1)*log(x) + I*a) - 27*I*m^3*x*x^m*e^(-1/6*ab
s(m + 1)*log(c) - 1/3*abs(m + 1)*log(x) + I*a) + 9*I*(m + 1)^2*x*x^m*abs(m
+ 1)*e^(-1/6*abs(m + 1)*log(c) - 1/3*abs(m + 1)*log(x) + I*a) - 9*I*m^2*x*x
^m*abs(m + 1)*e^(-1/6*abs(m + 1)*log(c) - 1/3*abs(m + 1)*log(x) + I*a) - I*
(m + 1)^2*m*x*x^m*e^(-1/2*abs(m + 1)*log(c) - abs(m + 1)*log(x) + 3*I*a) +
9*I*m^3*x*x^m*e^(-1/2*abs(m + 1)*log(c) - abs(m + 1)*log(x) + 3*I*a) - I*(m
 + 1)^2*x*x^m*abs(m + 1)*e^(-1/2*abs(m + 1)*log(c) - abs(m + 1)*log(x) + 3*
I*a) + 9*I*m^2*x*x^m*abs(m + 1)*e^(-1/2*abs(m + 1)*log(c) - abs(m + 1)*log(
x) + 3*I*a) + I*(m + 1)^2*x*x^m*e^(1/2*abs(m + 1)*log(c) + abs(m + 1)*log(x
) - 3*I*a) - 27*I*m^2*x*x^m*e^(1/2*abs(m + 1)*log(c) + abs(m + 1)*log(x) -
3*I*a) + 18*I*m*x*x^m*abs(m + 1)*e^(1/2*abs(m + 1)*log(c) + abs(m + 1)*log(
x) - 3*I*a) - 27*I*(m + 1)^2*x*x^m*e^(1/6*abs(m + 1)*log(c) + 1/3*abs(m + 1
)*log(x) - I*a) + 81*I*m^2*x*x^m*e^(1/6*abs(m + 1)*log(c) + 1/3*abs(m + 1)*
log(x) - I*a) - 18*I*m*x*x^m*abs(m + 1)*e^(1/6*abs(m + 1)*log(c) + 1/3*abs(
m + 1)*log(x) - I*a) + 27*I*(m + 1)^2*x*x^m*e^(-1/6*abs(m + 1)*log(c) - 1/3
*abs(m + 1)*log(x) + I*a) - 81*I*m^2*x*x^m*e^(-1/6*abs(m + 1)*log(c) - 1/3*
abs(m + 1)*log(x) + I*a) - 18*I*m*x*x^m*abs(m + 1)*e^(-1/6*abs(m + 1)*log(c
) - 1/3*abs(m + 1)*log(x) + I*a) - I*(m + 1)^2*x*x^m*e^(-1/2*abs(m + 1)*log
(c) - abs(m + 1)*log(x) + 3*I*a) + 27*I*m^2*x*x^m*e^(-1/2*abs(m + 1)*log(c)
 - abs(m + 1)*log(x) + 3*I*a) + 18*I*m*x*x^m*abs(m + 1)*e^(-1/2*abs(m + 1)*
log(c) - abs(m + 1)*log(x) + 3*I*a) - 27*I*m*x*x^m*e^(1/2*abs(m + 1)*log(c)
 + abs(m + 1)*log(x) - 3*I*a) + 9*I*x*x^m*abs(m + 1)*e^(1/2*abs(m + 1)*log(
c) + abs(m + 1)*log(x) - 3*I*a) + 81*I*m*x*x^m*e^(1/6*abs(m + 1)*log(c) + 1
/3*abs(m + 1)*log(x) - I*a) - 9*I*x*x^m*abs(m + 1)*e^(1/6*abs(m + 1)*log(c)
 + 1/3*abs(m + 1)*log(x) - I*a) - 81*I*m*x*x^m*e^(-1/6*abs(m + 1)*log(c) -
1/3*abs(m + 1)*log(x) + I*a) - 9*I*x*x^m*abs(m + 1)*e^(-1/6*abs(m + 1)*log(
c) - 1/3*abs(m + 1)*log(x) + I*a) + 27*I*m*x*x^m*e^(-1/2*abs(m + 1)*log(c)
- abs(m + 1)*log(x) + 3*I*a) + 9*I*x*x^m*abs(m + 1)*e^(-1/2*abs(m + 1)*log(
c) - abs(m + 1)*log(x) + 3*I*a) - 9*I*x*x^m*e^(1/2*abs(m + 1)*log(c) + abs(
m + 1)*log(x) - 3*I*a) + 27*I*x*x^m*e^(1/6*abs(m + 1)*log(c) + 1/3*abs(m +
1)*log(x) - I*a) - 27*I*x*x^m*e^(-1/6*abs(m + 1)*log(c) - 1/3*abs(m + 1)*lo
g(x) + I*a) + 9*I*x*x^m*e^(-1/2*abs(m + 1)*log(c) - abs(m + 1)*log(x) + 3*I
*a))/((m + 1)^4 - 10*(m + 1)^2*m^2 + 9*m^4 - 20*(m + 1)^2*m + 36*m^3 - 10*(
m + 1)^2 + 54*m^2 + 36*m + 9)
```

Mupad [B]

time = 4.08, size = 291, normalized size = 1.33

$$-\frac{\frac{1}{c^{\frac{\sqrt{-m^2-2\,m-1}\,1\mathrm{i}}{2}}}\,x\,x^m\,\mathrm{e}^{-a\,3\mathrm{i}}\,\frac{1}{{\left(x^2\right)}^{\frac{\sqrt{-m^2-2\,m-1}\,1\mathrm{i}}{2}}}\,1\mathrm{i}}{8\,m+8-\sqrt{-{\left(m+1\right)}^2}\,8\mathrm{i}}+\frac{c^{\frac{\sqrt{-m^2-2\,m-1}\,1\mathrm{i}}{2}}\,x\,x^m\,\mathrm{e}^{a\,3\mathrm{i}}\,{\left(x^2\right)}^{\frac{\sqrt{-m^2-2\,m-1}\,1\mathrm{i}}{2}}\,1\mathrm{i}}{8\,m+8+\sqrt{-{\left(m+1\right)}^2}\,8\mathrm{i}}-\frac{\frac{1}{c^{\frac{\sqrt{-m^2-2\,m-1}\,1\mathrm{i}}{6}}}\,x\,x^m\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{{\left(x^2\right)}^{\frac{\sqrt{-m^2-2\,m-1}\,1\mathrm{i}}{6}}}\,\left(27\,m+27+\sqrt{-{\left(m+1\right)}^2}\,9\mathrm{i}\right)\,1\mathrm{i}}{64\,{\left(m\,1\mathrm{i}+1\mathrm{i}\right)}^2}+\frac{c^{\frac{\sqrt{-m^2-2\,m-1}\,1\mathrm{i}}{6}}\,x\,x^m\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(x^2\right)}^{\frac{\sqrt{-m^2-2\,m-1}\,1\mathrm{i}}{6}}\,\left(27\,m+27-\sqrt{-{\left(m+1\right)}^2}\,9\mathrm{i}\right)\,1\mathrm{i}}{64\,{\left(m\,1\mathrm{i}+1\mathrm{i}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(a + (log(c*x^2)*(-(m + 1)^2)^(1/2))/6)^3,x)
```

```
[Out] (c^(((- 2*m - m^2 - 1)^(1/2)*1i)/2)*x*x^m*exp(a*3i)*(x^2)^(((- 2*m - m^2 -
1)^(1/2)*1i)/2)*1i)/(8*m + (-(m + 1)^2)^(1/2)*8i + 8) - (1/c^(((- 2*m - m^2
 - 1)^(1/2)*1i)/2)*x*x^m*exp(-a*3i)/(x^2)^(((- 2*m - m^2 - 1)^(1/2)*1i)/2)*
1i)/(8*m - (-(m + 1)^2)^(1/2)*8i + 8) - (1/c^(((- 2*m - m^2 - 1)^(1/2)*1i)/
6)*x*x^m*exp(-a*1i)/(x^2)^(((- 2*m - m^2 - 1)^(1/2)*1i)/6)*(27*m + (-(m + 1
)^2)^(1/2)*9i + 27)*1i)/(64*(m*1i + 1i)^2) + (c^(((- 2*m - m^2 - 1)^(1/2)*1
i)/6)*x*x^m*exp(a*1i)*(x^2)^(((- 2*m - m^2 - 1)^(1/2)*1i)/6)*(27*m - (-(m +
 1)^2)^(1/2)*9i + 27)*1i)/(64*(m*1i + 1i)^2)
```

3.52 $\int \sin^3\left(a+\frac{1}{6}i\log\left(cx^2\right)\right)\,dx$

Optimal. Leaf size=98

$$-\frac{ice^{-3ia}x^3}{16\sqrt{cx^2}}-\frac{9ie^{ia}x}{16\sqrt[6]{cx^2}}+\frac{9}{32}ie^{-ia}x\sqrt[6]{cx^2}+\frac{ie^{3ia}x\log(x)}{8\sqrt{cx^2}}$$

```
[Out] -9/16*I*exp(I*a)*x/(c*x^2)^(1/6)+9/32*I*x*(c*x^2)^(1/6)/exp(I*a)-1/16*I*c*x
^3/exp(3*I*a)/(c*x^2)^(1/2)+1/8*I*exp(3*I*a)*x*ln(x)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4571, 4577}

$$\frac{9}{32}ie^{-ia}x\sqrt[6]{cx^2}-\frac{9ie^{ia}x}{16\sqrt[6]{cx^2}}+\frac{ie^{3ia}x\log(x)}{8\sqrt{cx^2}}-\frac{ie^{-3ia}cx^3}{16\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + (I/6)*Log[c*x^2]]^3,x]
```

```
[Out] ((-1/16*I)*c*x^3)/(E^((3*I)*a)*Sqrt[c*x^2]) - (((9*I)/16)*E^(I*a)*x)/(c*x^2
)^(1/6) + (((9*I)/32)*x*(c*x^2)^(1/6))/E^(I*a) + ((I/8)*E^((3*I)*a)*x*Log[x
])/Sqrt[c*x^2]
```

Rule 4571

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4577

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[(m + 1)^p/(2^p*b^p*d^p*p^p), Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^
2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x]
, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m
 + 1)^2, 0]
```

Rubi steps

$$\int \sin^3\left(a+\frac{1}{6}i\log\left(cx^2\right)\right)dx = \frac{x\text{Subst}\left(\int \frac{\sin^3\left(a+\frac{1}{6}i\log(x)\right)}{\sqrt{x}}dx,x,cx^2\right)}{2\sqrt{cx^2}}$$

$$= \frac{(ix)\text{Subst}\left(\int \left(-e^{-3ia}+\frac{e^{3ia}}{x}-\frac{3e^{ia}}{x^{2/3}}+\frac{3e^{-ia}}{\sqrt[3]{x}}\right)dx,x,cx^2\right)}{16\sqrt{cx^2}}$$

$$= -\frac{ice^{-3ia}x^3}{16\sqrt{cx^2}}-\frac{9ie^{ia}x}{16\sqrt[6]{cx^2}}+\frac{9}{32}ie^{-ia}x\sqrt[6]{cx^2}+\frac{ie^{3ia}x\log(x)}{8\sqrt{cx^2}}$$

Mathematica [A]
time = 0.14, size = 103, normalized size = 1.05

$$\frac{x\left(9i\sqrt[3]{cx^2}\left(-2+\sqrt[3]{cx^2}\right)\cos(a)-2i\cos(3a)\left(cx^2-2\log(x)\right)+18\sqrt[3]{cx^2}\sin(a)+9\left(cx^2\right)^{2/3}\sin(a)-2cx^2\sin(3a)-4\log(x)\sin(3a)\right)}{32\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + (I/6)*Log[c*x^2]]^3,x]
```

```
[Out] (x*((9*I)*(c*x^2)^(1/3)*(-2 + (c*x^2)^(1/3))*Cos[a] - (2*I)*Cos[3*a]*(c*x^2
 - 2*Log[x]) + 18*(c*x^2)^(1/3)*Sin[a] + 9*(c*x^2)^(2/3)*Sin[a] - 2*c*x^2*S
in[3*a] - 4*Log[x]*Sin[3*a]))/(32*Sqrt[c*x^2])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(74) = 148.
time = 0.13, size = 284, normalized size = 2.90

method	result
norman	$-\frac{23ix}{40}+\frac{27x\tan\left(\frac{a}{2}+\frac{i\ln\left(cx^2\right)}{12}\right)}{10}+\frac{27x\left(\tan^5\left(\frac{a}{2}+\frac{i\ln\left(cx^2\right)}{12}\right)\right)}{10}+\frac{33ix\left(\tan^2\left(\frac{a}{2}+\frac{i\ln\left(cx^2\right)}{12}\right)\right)}{8}+\frac{23ix\left(\tan^6\left(\frac{a}{2}+\frac{i\ln\left(cx^2\right)}{12}\right)\right)}{40}-\frac{33ix\left(\tan^4\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+1/6*I*ln(c*x^2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-23/40*I*x+27/10*x*tan(1/2*a+1/12*I*ln(c*x^2))+27/10*x*tan(1/2*a+1/12*I*ln
(c*x^2))^5+33/8*I*x*tan(1/2*a+1/12*I*ln(c*x^2))^2+23/40*I*x*tan(1/2*a+1/12*
I*ln(c*x^2))^6-33/8*I*x*tan(1/2*a+1/12*I*ln(c*x^2))^4-3/8*x*ln(c*x^2)*tan(1
/2*a+1/12*I*ln(c*x^2))+5/4*x*ln(c*x^2)*tan(1/2*a+1/12*I*ln(c*x^2))^3-3/8*x*
ln(c*x^2)*tan(1/2*a+1/12*I*ln(c*x^2))^5+1/16*I*x*ln(c*x^2)-15/16*I*x*ln(c*x
^2)*tan(1/2*a+1/12*I*ln(c*x^2))^2+15/16*I*x*ln(c*x^2)*tan(1/2*a+1/12*I*ln(c
```

```
*x^2))^4-1/16*I*x*ln(c*x^2)*tan(1/2*a+1/12*I*ln(c*x^2))^6)/(1+tan(1/2*a+1/1
2*I*ln(c*x^2))^2)^3
```

Maxima [A]
time = 0.28, size = 75, normalized size = 0.77

$$-\frac{9\,c^{\frac{4}{3}}x^{\frac{4}{3}}(-i\,\cos{(a)}-\sin{(a)})+18\,cx^{\frac{2}{3}}(i\,\cos{(a)}-\sin{(a)})+2\,(cx^2(i\,\cos{(3\,a)}+\sin{(3\,a)})+2\,(-i\,\cos{(3\,a)}+\sin{(3\,a)})\log{(x)})c^{\frac{2}{3}}}{32\,c^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/6*I*log(c*x^2))^3,x, algorithm="maxima")
```

```
[Out] -1/32*(9*c^(4/3)*x^(4/3)*(-I*cos(a) - sin(a)) + 18*c*x^(2/3)*(I*cos(a) - si
n(a)) + 2*(c*x^2*(I*cos(3*a) + sin(3*a)) + 2*(-I*cos(3*a) + sin(3*a))*log(x
))*c^(2/3))/c^(7/6)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(62) = 124.
time = 6.06, size = 204, normalized size = 2.08

$$\frac{\left(2\,cx\sqrt{-\frac{e^{(6i\,a)}}{c}}\,e^{(3i\,a)}\log\left(\frac{\left(\sqrt{cx^2}\,(x^2+1)e^{(3i\,a)}-(i\,cx^3-i\,cx)\sqrt{-\frac{e^{(6i\,a)}}{c}}\right)e^{(-3i\,a)}}{8\,x^2}\right)-2\,cx\sqrt{-\frac{e^{(6i\,a)}}{c}}\,e^{(3i\,a)}\log\left(\frac{\left(\sqrt{cx^2}\,(x^2+1)e^{(3i\,a)}-(-i\,cx^3+i\,cx)\sqrt{-\frac{e^{(6i\,a)}}{c}}\right)e^{(-3i\,a)}}{8\,x^2}\right)+9i\,(cx^2)^{\frac{1}{6}}\,cx^2e^{(2i\,a)}-18i\,(cx^2)^{\frac{5}{6}}\,e^{(4i\,a)}-2\,\sqrt{cx^2}\,(i\,cx^2-i\,c)\right)e^{(-3i\,a)}}{32\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/6*I*log(c*x^2))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(2*c*x*sqrt(-e^(6*I*a)/c)*e^(3*I*a)*log(1/8*(sqrt(c*x^2)*(x^2 + 1)*e^(
3*I*a) - (I*c*x^3 - I*c*x)*sqrt(-e^(6*I*a)/c))*e^(-3*I*a)/x^2) - 2*c*x*sqrt
(-e^(6*I*a)/c)*e^(3*I*a)*log(1/8*(sqrt(c*x^2)*(x^2 + 1)*e^(3*I*a) - (-I*c*x
^3 + I*c*x)*sqrt(-e^(6*I*a)/c))*e^(-3*I*a)/x^2) + 9*I*(c*x^2)^(1/6)*c*x^2*e
^(2*I*a) - 18*I*(c*x^2)^(5/6)*e^(4*I*a) - 2*sqrt(c*x^2)*(I*c*x^2 - I*c))*e^
(-3*I*a)/(c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^3\left(a+\frac{i\log{(cx^2)}}{6}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/6*I*ln(c*x**2))**3,x)
```

```
[Out] Integral(sin(a + I*log(c*x**2)/6)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/6*I*log(c*x^2))^3,x, algorithm="giac")
```

```
[Out] integrate(sin(a + 1/6*I*log(c*x^2))^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+\frac{\ln\left(c\,x^2\right)\,1\mathrm{i}}{6}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + (log(c*x^2)*1i)/6)^3,x)
```

```
[Out] int(sin(a + (log(c*x^2)*1i)/6)^3, x)
```

3.53 $\int x \sqrt{\sin(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=111

$$\frac{2x^2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4}\left(-1-\frac{4i}{bn}\right);\frac{1}{4}\left(3-\frac{4i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)\sqrt{\sin(a+b\log(cx^n))}}{(4-ibn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

```
[Out] 2*x^2*hypergeom([-1/2, -1/4-I/b/n],[3/4-I/b/n],exp(2*I*a)*(c*x^n)^(2*I*b))*
sin(a+b*ln(c*x^n))^(1/2)/(4-I*b*n)/(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4581, 4579, 371}

$$\frac{2x^2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4}\left(-1-\frac{4i}{bn}\right);\frac{1}{4}\left(3-\frac{4i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)\sqrt{\sin(a+b\log(cx^n))}}{(4-ibn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[Sin[a + b*Log[c*x^n]]],x]
```

```
[Out] (2*x^2*Hypergeometric2F1[-1/2, (-1 - (4*I)/(b*n))/4, (3 - (4*I)/(b*n))/4, E
^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[Sin[a + b*Log[c*x^n]]])/((4 - I*b*n)*Sqr
t[1 - E^((2*I)*a)*(c*x^n)^((2*I)*b)])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4579

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Dist[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
```

```
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}\,dx = \frac{\left(x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int x^{-1+\frac{2}{n}}\sqrt{\sin(a+b\log(x))}\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x^2(cx^n)^{\frac{ib}{2}-\frac{2}{n}}\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}\right)\text{Subst}\left(\int x^{-1-\frac{ib}{2}+\frac{2}{n}}\sqrt{1-e^{2ia}x^{2ib}}\,dx\right.}{n\sqrt{1-e^{2ia}\left(cx^n\right)^{2ib}}}$$

$$= \frac{2x^2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4}\left(-1-\frac{4i}{bn}\right);\frac{1}{4}\left(3-\frac{4i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}}{(4-ibn)\sqrt{1-e^{2ia}\left(cx^n\right)^{2ib}}}$$

Mathematica [A]
time = 6.84, size = 145, normalized size = 1.31

$$\frac{i\sqrt{2}\,x^2\sqrt{-ie^{-ia}\left(cx^n\right)^{-ib}\left(-1+e^{2ia}\left(cx^n\right)^{2ib}\right)}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4}-\frac{i}{bn};\frac{3}{4}-\frac{i}{bn};e^{2ia}(cx^n)^{2ib}\right)}{(4i+bn)\sqrt{1-e^{2ia}\left(cx^n\right)^{2ib}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[Sin[a + b*Log[c*x^n]]],x]
[Out] (I*Sqrt[2]*x^2*Sqrt[((-I)*(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))/(E^(I*a)*(c
*x^n)^(I*b))]*Hypergeometric2F1[-1/2, -1/4 - I/(b*n), 3/4 - I/(b*n), E^((2*
I)*a)*(c*x^n)^((2*I)*b)])/((4*I + b*n)*Sqrt[1 - E^((2*I)*a)*(c*x^n)^((2*I)*
b)])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x\left(\sqrt{\sin}\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a+b*ln(c*x^n))^(1/2),x)
[Out] int(x*sin(a+b*ln(c*x^n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(sin(b*log(c*x^n) + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\sin\left(a + b\log\left(cx^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(x*sqrt(sin(a + b*log(c*x**n))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(sin(b*log(c*x^n) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b*log(c*x^n))^(1/2),x)
```

```
[Out] int(x*sin(a + b*log(c*x^n))^(1/2), x)
```

3.54 $\int \sqrt{\sin(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=110

$$\frac{2x\,{}_2F_1\left(-\frac{1}{2},-\frac{2i+bn}{4bn};\frac{1}{4}\left(3-\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)\sqrt{\sin(a+b\log(cx^n))}}{(2-ibn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

```
[Out] 2*x*hypergeom([-1/2, 1/4*(-2*I-b*n)/b/n],[3/4-1/2*I/b/n],exp(2*I*a)*(c*x^n)
^(2*I*b))*sin(a+b*ln(c*x^n))^(1/2)/(2-I*b*n)/(1-exp(2*I*a)*(c*x^n)^(2*I*b))
^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4571, 4579, 371}

$$\frac{2x\,{}_2F_1\left(-\frac{1}{2},-\frac{bn+2i}{4bn};\frac{1}{4}\left(3-\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)\sqrt{\sin(a+b\log(cx^n))}}{(2-ibn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sin[a + b*Log[c*x^n]]],x]
```

```
[Out] (2*x*Hypergeometric2F1[-1/2, -1/4*(2*I + b*n)/(b*n), (3 - (2*I)/(b*n))/4, E
^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[Sin[a + b*Log[c*x^n]]])/((2 - I*b*n)*Sqr
t[1 - E^((2*I)*a)*(c*x^n)^((2*I)*b)])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4571

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4579

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Dist[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
```

```
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\sqrt{\sin(a+b\log(x))}\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(x(cx^n)^{\frac{ib}{2}-\frac{1}{n}}\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}\right)\text{Subst}\left(\int x^{-1-\frac{ib}{2}+\frac{1}{n}}\sqrt{1-e^{2ia}x^{2ib}}\,dx,x,\right.}{n\sqrt{1-e^{2ia}\left(cx^n\right)^{2ib}}}$$
$$= \frac{2x\,{}_2F_1\left(-\frac{1}{2},-\frac{2i+bn}{4bn};\frac{1}{4}\left(3-\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}}{(2-ibn)\sqrt{1-e^{2ia}\left(cx^n\right)^{2ib}}}$$

Mathematica [A]
time = 6.49, size = 148, normalized size = 1.35

$$\frac{i\sqrt{2}\,x\sqrt{-ie^{-ia}\left(cx^n\right)^{-ib}\left(-1+e^{2ia}\left(cx^n\right)^{2ib}\right)}\,{}_2F_1\left(-\frac{1}{2},-\frac{2i+bn}{4bn};\frac{3}{4}-\frac{i}{2bn};e^{2ia}(cx^n)^{2ib}\right)}{(2i+bn)\sqrt{1-e^{2ia}\left(cx^n\right)^{2ib}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sin[a + b*Log[c*x^n]]],x]
[Out] (I*Sqrt[2]*x*Sqrt[((-I)*(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))/(E^(I*a)*(c*x
^n)^(I*b))]*Hypergeometric2F1[-1/2, -1/4*(2*I + b*n)/(b*n), 3/4 - (I/2)/(b*
n), E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((2*I + b*n)*Sqrt[1 - E^((2*I)*a)*(c*x^
n)^((2*I)*b)])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \sqrt{\sin}\left(a+b\ln\left(c\,x^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))^(1/2),x)
[Out] int(sin(a+b*ln(c*x^n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(b*log(c*x^n) + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin \left(a + b \log \left(cx^{n}\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(sqrt(sin(a + b*log(c*x**n))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(b*log(c*x^n) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\sin \left(a+b\,\ln \left(c\,x^n\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))^(1/2),x)
```

```
[Out] int(sin(a + b*log(c*x^n))^(1/2), x)
```

3.55 $\int \frac{\sqrt{\sin (a+b \log (c x^n))}}{x} \, dx$

Optimal. Leaf size=29

$$\frac{2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}$$

```
[Out] -2*(sin(1/2*a+1/4*Pi+1/2*b*ln(c*x^n))^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*ln(c*x^n))*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*ln(c*x^n)),2^(1/2))/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{2719\}$

$$\frac{2E\left(\frac{1}{2}\left(a+b\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sin[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] (2*EllipticE[(a - Pi/2 + b*Log[c*x^n])/2, 2])/(b*n)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\sin (a+b\log (cx^n))}}{x} \, dx &= \frac{\text{Subst}\left(\int \sqrt{\sin(a+bx)} \, dx, x, \log\left(cx^n\right)\right)}{n}\\
&= \frac{2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 32, normalized size = 1.10

$$-\frac{2E\left(\frac{1}{2}\left(-a+\frac{\pi}{2}-b\log\left(cx^n\right)\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sin[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] (-2*EllipticE[(-a + Pi/2 - b*Log[c*x^n])/2, 2])/(b*n)
```

Maple [A]
time = 0.31, size = 129, normalized size = 4.45

method	result
derivativedivides	$-\frac{\sqrt{\sin\left(a+b\ln\left(c\,x^{n}\right)\right)+1}\,\sqrt{-2\sin\left(a+b\ln\left(c\,x^{n}\right)\right)+2}\,\sqrt{-\sin\left(a+b\ln\left(c\,x^{n}\right)\right)}\,\left(2\,\mathrm{E}\right.}{n\cos\left(a+b\ln\left(c\,x^{n}\right)\right)\sqrt{\sin}}$
default	$-\frac{\sqrt{\sin\left(a+b\ln\left(c\,x^{n}\right)\right)+1}\,\sqrt{-2\sin\left(a+b\ln\left(c\,x^{n}\right)\right)+2}\,\sqrt{-\sin\left(a+b\ln\left(c\,x^{n}\right)\right)}\,\left(2\,\mathrm{E}\right.}{n\cos\left(a+b\ln\left(c\,x^{n}\right)\right)\sqrt{\sin}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/n*(sin(a+b*ln(c*x^n))+1)^(1/2)*(-2*sin(a+b*ln(c*x^n))+2)^(1/2)*(-sin(a+b
*ln(c*x^n)))^(1/2)*(2*EllipticE((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2))-E
llipticF((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2)))/cos(a+b*ln(c*x^n))/sin(
a+b*ln(c*x^n))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(b*log(c*x^n) + a))/x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.50, size = 90, normalized size = 3.10

$$\frac{i\sqrt{2}\sqrt{-i}\,\mathrm{weierstrassZeta}\left(4,0,\mathrm{weierstrassPInverse}\left(4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+i\,\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\right)-i\sqrt{2}\sqrt{i}\,\mathrm{weierstrassZeta}\left(4,0,\mathrm{weierstrassPInverse}\left(4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-i\,\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] (I*sqrt(2)*sqrt(-I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*n
*log(x) + b*log(c) + a) + I*sin(b*n*log(x) + b*log(c) + a))) - I*sqrt(2)*sq
rt(I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*n*log(x) + b*lo
g(c) + a) - I*sin(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sin{\left(a + b\log{\left(cx^{n} \right)} \right)}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(sin(a + b*log(c*x**n)))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(b*log(c*x^n) + a))/x, x)
```

Mupad [B]
time = 2.32, size = 26, normalized size = 0.90

$$\frac{2\,\mathrm{E}\left(\frac{a}{2}-\frac{\pi}{4}+\frac{b\,\ln\left(c\,x^n\right)}{2}\middle|2\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))^(1/2)/x,x)
```

```
[Out] (2*ellipticE(a/2 - pi/4 + (b*log(c*x^n))/2, 2))/(b*n)
```

3.56 $\int \frac{\sqrt{\sin(a+b\log(cx^n))}}{x^2}\,dx$

Optimal. Leaf size=111

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4}\left(-1+\frac{2i}{bn}\right);\frac{1}{4}\left(3+\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)\sqrt{\sin(a+b\log(cx^n))}}{(2+ibn)x\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

```
[Out] -2*hypergeom([-1/2, -1/4+1/2*I/b/n],[3/4+1/2*I/b/n],exp(2*I*a)*(c*x^n)^(2*I
*b))*sin(a+b*ln(c*x^n))^(1/2)/(2+I*b*n)/x/(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(1
/2)
```

Rubi [A]
time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4581, 4579, 371}

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4}\left(\frac{2i}{bn}-1\right);\frac{1}{4}\left(3+\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)\sqrt{\sin(a+b\log(cx^n))}}{x(2+ibn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sin[a + b*Log[c*x^n]]]/x^2,x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, (-1 + (2*I)/(b*n))/4, (3 + (2*I)/(b*n))/4, E^((
2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[Sin[a + b*Log[c*x^n]]])/((2 + I*b*n)*x*Sqrt
[1 - E^((2*I)*a)*(c*x^n)^((2*I)*b)])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4579

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Dist[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
```

```
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}}{x^2}\,dx = \frac{\left(cx^n\right)^{\frac{1}{n}}\operatorname{Subst}\left(\int x^{-1-\frac{1}{n}}\sqrt{\sin(a+b\log(x))}\,dx, x, cx^n\right)}{nx}$$

$$= \frac{\left(\left(cx^n\right)^{\frac{ib}{2}+\frac{1}{n}}\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}\right)\operatorname{Subst}\left(\int x^{-1-\frac{ib}{2}-\frac{1}{n}}\sqrt{1-e^{2ia}x^{2ib}}\,dx, x,\right.}{nx\sqrt{1-e^{2ia}\left(cx^n\right)^{2ib}}}$$

$$= -\frac{2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4}\left(-1+\frac{2i}{bn}\right);\frac{1}{4}\left(3+\frac{2i}{bn}\right);e^{2ia}\left(cx^n\right)^{2ib}\right)\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}}{(2+ibn)x\sqrt{1-e^{2ia}\left(cx^n\right)^{2ib}}}$$

Mathematica [A]
time = 6.79, size = 149, normalized size = 1.34

$$\frac{i\sqrt{2}\sqrt{-ie^{-ia}\left(cx^n\right)^{-ib}\left(-1+e^{2ia}\left(cx^n\right)^{2ib}\right)}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4}+\frac{i}{2bn};\frac{3}{4}+\frac{i}{2bn};e^{2ia}\left(cx^n\right)^{2ib}\right)}{(-2i+bn)x\sqrt{1-e^{2ia}\left(cx^n\right)^{2ib}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sin[a + b*Log[c*x^n]]]/x^2,x]
```

```
[Out] (I*Sqrt[2]*Sqrt[((-I)*(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))/(E^(I*a)*(c*x^n
)^(I*b))]*Hypergeometric2F1[-1/2, -1/4 + (I/2)/(b*n), 3/4 + (I/2)/(b*n), E^
((2*I)*a)*(c*x^n)^((2*I)*b)])/((-2*I + b*n)*x*Sqrt[1 - E^((2*I)*a)*(c*x^n)^
((2*I)*b)])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sin}\left(a+b\ln\left(c\,x^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))^(1/2)/x^2,x)
```

```
[Out] int(sin(a+b*ln(c*x^n))^(1/2)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(sin(b*log(c*x^n) + a))/x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(1/2)/x^2,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sin\left(a + b\log\left(cx^{n}\right)\right)}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**(1/2)/x**2,x)
[Out] Integral(sqrt(sin(a + b*log(c*x**n)))/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(sin(b*log(c*x^n) + a))/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\sin\left(a + b\,\ln\left(c\,x^{n}\right)\right)}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))^(1/2)/x^2,x)
[Out] int(sin(a + b*log(c*x^n))^(1/2)/x^2, x)
```

3.57 $\int \frac{\sqrt{\sin(a+b\log(cx^n))}}{x^3}\,dx$

Optimal. Leaf size=111

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4}\left(-1+\frac{4i}{bn}\right);\frac{1}{4}\left(3+\frac{4i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)\sqrt{\sin(a+b\log(cx^n))}}{(4+ibn)x^2\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

```
[Out] -2*hypergeom([-1/2, -1/4+I/b/n],[3/4+I/b/n],exp(2*I*a)*(c*x^n)^(2*I*b))*sin
(a+b*ln(c*x^n))^(1/2)/(4+I*b*n)/x^2/(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4581, 4579, 371}

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4}\left(\frac{4i}{bn}-1\right);\frac{1}{4}\left(3+\frac{4i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)\sqrt{\sin(a+b\log(cx^n))}}{x^2(4+ibn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sin[a + b*Log[c*x^n]]]/x^3,x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, (-1 + (4*I)/(b*n))/4, (3 + (4*I)/(b*n))/4, E^((
2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[Sin[a + b*Log[c*x^n]]])/((4 + I*b*n)*x^2*Sq
rt[1 - E^((2*I)*a)*(c*x^n)^((2*I)*b)])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4579

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Dist[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
eQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
```

```
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}}{x^3}\,dx = \frac{\left(cx^n\right)^{2/n}\operatorname{Subst}\left(\int x^{-1-\frac{2}{n}}\sqrt{\sin(a+b\log(x))}\,dx, x, cx^n\right)}{nx^2}$$

$$= \frac{\left(\left(cx^n\right)^{\frac{ib}{2}+\frac{2}{n}}\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}\right)\operatorname{Subst}\left(\int x^{-1-\frac{ib}{2}-\frac{2}{n}}\sqrt{1-e^{2ia}x^{2ib}}\,dx, x\right.}{nx^2\sqrt{1-e^{2ia}\left(cx^n\right)^{2ib}}}$$

$$= -\frac{2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4}\left(-1+\frac{4i}{bn}\right);\frac{1}{4}\left(3+\frac{4i}{bn}\right);e^{2ia}\left(cx^n\right)^{2ib}\right)\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}}{(4+ibn)x^2\sqrt{1-e^{2ia}\left(cx^n\right)^{2ib}}}$$

Mathematica [A]
time = 6.82, size = 145, normalized size = 1.31

$$\frac{i\sqrt{2}\sqrt{-ie^{-ia}\left(cx^n\right)^{-ib}\left(-1+e^{2ia}\left(cx^n\right)^{2ib}\right)}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4}+\frac{i}{bn};\frac{3}{4}+\frac{i}{bn};e^{2ia}\left(cx^n\right)^{2ib}\right)}{(-4i+bn)x^2\sqrt{1-e^{2ia}\left(cx^n\right)^{2ib}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sin[a + b*Log[c*x^n]]]/x^3,x]
[Out] (I*Sqrt[2]*Sqrt[((-I)*(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))/(E^(I*a)*(c*x^n
)^(I*b))]*Hypergeometric2F1[-1/2, -1/4 + I/(b*n), 3/4 + I/(b*n), E^((2*I)*a
)*(c*x^n)^((2*I)*b)])/((-4*I + b*n)*x^2*Sqrt[1 - E^((2*I)*a)*(c*x^n)^((2*I)
*b)])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sin}\left(a+b\ln\left(c\,x^n\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))^(1/2)/x^3,x)
[Out] int(sin(a+b*ln(c*x^n))^(1/2)/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(b*log(c*x^n) + a))/x^3, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sin \left(a + b \log \left(c x^{n}\right)\right)}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(sin(a + b*log(c*x**n)))/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(b*log(c*x^n) + a))/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\sin \left(a+b\,\ln \left(c\,x^n\right)\right)}}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))^(1/2)/x^3,x)
```

```
[Out] int(sin(a + b*log(c*x^n))^(1/2)/x^3, x)
```

3.58 $\int x \sin^{\frac{3}{2}}(a + b\log(cx^n))\, dx$

Optimal. Leaf size=111

$$\frac{2x^2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{4i}{bn}\right);\frac{1}{4}\left(1-\frac{4i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)\sin^{\frac{3}{2}}(a+b\log(cx^n))}{(4-3ibn)\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

```
[Out] 2*x^2*hypergeom([-3/2, -3/4-I/b/n],[1/4-I/b/n],exp(2*I*a)*(c*x^n)^(2*I*b))*
sin(a+b*ln(c*x^n))^(3/2)/(4-3*I*b*n)/(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4581, 4579, 371}

$$\frac{2x^2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{4i}{bn}\right);\frac{1}{4}\left(1-\frac{4i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)\sin^{\frac{3}{2}}(a+b\log(cx^n))}{(4-3ibn)\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[a + b*Log[c*x^n]]^(3/2),x]
```

```
[Out] (2*x^2*Hypergeometric2F1[-3/2, (-3 - (4*I)/(b*n))/4, (1 - (4*I)/(b*n))/4, E
^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sin[a + b*Log[c*x^n]]^(3/2))/((4 - (3*I)*b*n)
*(1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4579

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Dist[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
```

```
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x\sin^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\,dx &= \frac{\left(x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int x^{-1+\frac{2}{n}}\sin^{\frac{3}{2}}(a+b\log(x))\,dx,x,cx^n\right)}{n}\\ &= \frac{\left(x^2(cx^n)^{\frac{3ib}{2}-\frac{2}{n}}\sin^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\right)\text{Subst}\left(\int x^{-1-\frac{3ib}{2}+\frac{2}{n}}\left(1-e^{2ia}x^{2ib}\right)^{3/2}\,dx,\right.}{n\left(1-e^{2ia}\left(cx^n\right)^{2ib}\right)^{3/2}}\\ &= \frac{2x^2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{4i}{bn}\right);\frac{1}{4}\left(1-\frac{4i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)\sin^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{(4-3ibn)\left(1-e^{2ia}\left(cx^n\right)^{2ib}\right)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 1.21, size = 218, normalized size = 1.96

$$-\frac{6ib^2\sqrt{2-2e^{2i(a+b\log(cx^n))}}\,n^2x^2\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}-\frac{i}{bn};\frac{5}{4}-\frac{i}{bn};e^{2i(a+b\log(cx^n))}\right)}{\sqrt{-ie^{-i(a+b\log(cx^n))}\left(-1+e^{2i(a+b\log(cx^n))}\right)}\,(-4i+bn)(-4i+3bn)(4i+3bn)}+\frac{2x^2\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}\left(-3bn\cos\left(a+b\log\left(cx^n\right)\right)+4\sin\left(a+b\log\left(cx^n\right)\right)\right)}{16+9b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[a + b*Log[c*x^n]]^(3/2),x]
```

```
[Out] ((-6*I)*b^2*Sqrt[2 - 2*E^((2*I)*(a + b*Log[c*x^n]))]*n^2*x^2*Hypergeometric
2F1[1/2, 1/4 - I/(b*n), 5/4 - I/(b*n), E^((2*I)*(a + b*Log[c*x^n]))])/(Sqrt
[((-I)*(-1 + E^((2*I)*(a + b*Log[c*x^n]))))/E^(I*(a + b*Log[c*x^n]))]*(-4*I
 + b*n)*(-4*I + 3*b*n)*(4*I + 3*b*n)) + (2*x^2*Sqrt[Sin[a + b*Log[c*x^n]]]*
(-3*b*n*Cos[a + b*Log[c*x^n]] + 4*Sin[a + b*Log[c*x^n]]))/(16 + 9*b^2*n^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x\left(\sin^{\frac{3}{2}}\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a+b*ln(c*x^n))^(3/2),x)
```

```
[Out] int(x*sin(a+b*ln(c*x^n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*log(c*x^n) + a)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*log(c*x^n) + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b*log(c*x^n))^(3/2),x)
```

```
[Out] int(x*sin(a + b*log(c*x^n))^(3/2), x)
```

3.59 $\int \sin^{\frac{3}{2}}(a+b\log(cx^n))\,dx$

Optimal. Leaf size=109

$$\frac{2x\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{2i}{bn}\right);\frac{1}{4}\left(1-\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)\sin^{\frac{3}{2}}(a+b\log(cx^n))}{(2-3ibn)\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

```
[Out] 2*x*hypergeom([-3/2, -3/4-1/2*I/b/n],[1/4-1/2*I/b/n],exp(2*I*a)*(c*x^n)^(2*
I*b))*sin(a+b*ln(c*x^n))^(3/2)/(2-3*I*b*n)/(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(
3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4571, 4579, 371}

$$\frac{2x\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{2i}{bn}\right);\frac{1}{4}\left(1-\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)\sin^{\frac{3}{2}}(a+b\log(cx^n))}{(2-3ibn)\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Log[c*x^n]]^(3/2),x]
```

```
[Out] (2*x*Hypergeometric2F1[-3/2, (-3 - (2*I)/(b*n))/4, (1 - (2*I)/(b*n))/4, E^(
(2*I)*a)*(c*x^n)^((2*I)*b)]*Sin[a + b*Log[c*x^n]]^(3/2))/((2 - (3*I)*b*n)*(
1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4571

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4579

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Dist[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
```

```
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sin^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{\left(x\left(cx^n\right)^{-1/n}\right)\mathrm{Subst}\left(\int x^{-1+\frac{1}{n}}\sin^{\frac{3}{2}}(a+b\log(x))\,dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x\left(cx^n\right)^{\frac{3ib}{2}-\frac{1}{n}}\sin^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\right)\mathrm{Subst}\left(\int x^{-1-\frac{3ib}{2}+\frac{1}{n}}\left(1-e^{2ia}x^{2ib}\right)^{3/2}\,dx, x\right.}{n\left(1-e^{2ia}\left(cx^n\right)^{2ib}\right)^{3/2}}$$

$$= \frac{2x\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{2i}{bn}\right);\frac{1}{4}\left(1-\frac{2i}{bn}\right);e^{2ia}\left(cx^n\right)^{2ib}\right)\sin^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{(2-3ibn)\left(1-e^{2ia}\left(cx^n\right)^{2ib}\right)^{3/2}}$$

Mathematica [A]
time = 1.09, size = 218, normalized size = 2.00

$$-\frac{6ib^2\sqrt{2-2e^{2i(a+b\log(cx^n))}}\,n^2x\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}-\frac{i}{2bn};\frac{5}{4}-\frac{i}{2bn};e^{2i(a+b\log(cx^n))}\right)}{\sqrt{-ie^{-i(a+b\log(cx^n))}\left(-1+e^{2i(a+b\log(cx^n))}\right)}\,(-2i+bn)(-2i+3bn)(2i+3bn)}+\frac{2x\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}\left(-3bn\cos\left(a+b\log\left(cx^n\right)\right)+2\sin\left(a+b\log\left(cx^n\right)\right)\right)}{4+9b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]^(3/2),x]
```

```
[Out] ((-6*I)*b^2*Sqrt[2 - 2*E^((2*I)*(a + b*Log[c*x^n]))]*n^2*x*Hypergeometric2F
1[1/2, 1/4 - (I/2)/(b*n), 5/4 - (I/2)/(b*n), E^((2*I)*(a + b*Log[c*x^n]))])
/(Sqrt[((-I)*(-1 + E^((2*I)*(a + b*Log[c*x^n]))))/E^(I*(a + b*Log[c*x^n]))]
*(-2*I + b*n)*(-2*I + 3*b*n)*(2*I + 3*b*n)) + (2*x*Sqrt[Sin[a + b*Log[c*x^n
]]]*(-3*b*n*Cos[a + b*Log[c*x^n]] + 2*Sin[a + b*Log[c*x^n]]))/(4 + 9*b^2*n^
2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \sin^{\frac{3}{2}}\left(a+b\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))^(3/2),x)
```

```
[Out] int(sin(a+b*ln(c*x^n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^{\frac{3}{2}}(a + b\log(cx^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Integral(sin(a + b*log(c*x**n))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a + b\,\ln\left(c\,x^n\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))^(3/2),x)
```

```
[Out] int(sin(a + b*log(c*x^n))^(3/2), x)
```

3.60 $\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=68

$$\frac{2F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b\log\left(cx^n\right)\right)\middle|2\right)}{3bn}-\frac{2\cos\left(a+b\log\left(cx^n\right)\right)\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}}{3bn}$$

```
[Out] -2/3*(sin(1/2*a+1/4*Pi+1/2*b*ln(c*x^n))^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*ln(
c*x^n))*EllipticF(cos(1/2*a+1/4*Pi+1/2*b*ln(c*x^n)),2^(1/2))/b/n-2/3*cos(a+
b*ln(c*x^n))*sin(a+b*ln(c*x^n))^(1/2)/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2715, 2720}

$$\frac{2F\left(\frac{1}{2}\left(a+b\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{3bn}-\frac{2\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}\cos\left(a+b\log\left(cx^n\right)\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (2*EllipticF[(a - Pi/2 + b*Log[c*x^n])/2, 2])/(3*b*n) - (2*Cos[a + b*Log[c*
x^n]]*Sqrt[Sin[a + b*Log[c*x^n]]])/(3*b*n)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\text{Subst}\left(\int \sin^{\frac{3}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2\cos(a+b\log(cx^n))\sqrt{\sin(a+b\log(cx^n))}}{3bn} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx,\right.}{3n}$$

$$= \frac{2F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b\log(cx^n)\right)\middle|2\right)}{3bn} - \frac{2\cos(a+b\log(cx^n))\sqrt{\sin(a+b\log(cx^n))}}{3bn}$$

Mathematica [A]
time = 0.17, size = 58, normalized size = 0.85

$$-\frac{2\left(F\left(\frac{1}{4}(-2a+\pi-2b\log(cx^n))\middle|2\right)+\cos(a+b\log(cx^n))\sqrt{\sin(a+b\log(cx^n))}\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (-2*(EllipticF[(-2*a + Pi - 2*b*Log[c*x^n])/4, 2] + Cos[a + b*Log[c*x^n]]*S
qrt[Sin[a + b*Log[c*x^n]]]))/(3*b*n)
```

Maple [A]
time = 0.18, size = 131, normalized size = 1.93

method	result
derivativedivides	$\frac{\frac{\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+2}\,\sqrt{-\sin(a+b\ln(c\,x^n))}\,\text{EllipticF}\left(\right.}{3}}{n\cos(a+b\ln(c\,x^n))\sqrt{\sin(a+b\ln(c\,x\right.}}$
default	$\frac{\frac{\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+2}\,\sqrt{-\sin(a+b\ln(c\,x^n))}\,\text{EllipticF}\left(\right.}{3}}{n\cos(a+b\ln(c\,x^n))\sqrt{\sin(a+b\ln(c\,x\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(1/3*(sin(a+b*ln(c*x^n))+1)^(1/2)*(-2*sin(a+b*ln(c*x^n))+2)^(1/2)*(-sin
(a+b*ln(c*x^n)))^(1/2)*EllipticF((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2))-
2/3*cos(a+b*ln(c*x^n))^2*sin(a+b*ln(c*x^n)))/cos(a+b*ln(c*x^n))/sin(a+b*ln(
c*x^n))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^(3/2)/x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.50, size = 111, normalized size = 1.63

$$\frac{\sqrt{2}\,\sqrt{-i}\,\mathrm{weierstrassPInverse}(4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+i\,\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right))+\sqrt{2}\,\sqrt{i}\,\mathrm{weierstrassPInverse}(4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-i\,\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right))-2\,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sqrt{\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}}{3\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*sqrt(-I)*weierstrassPInverse(4, 0, cos(b*n*log(x) + b*log(c) +
 a) + I*sin(b*n*log(x) + b*log(c) + a)) + sqrt(2)*sqrt(I)*weierstrassPInver
se(4, 0, cos(b*n*log(x) + b*log(c) + a) - I*sin(b*n*log(x) + b*log(c) + a))
 - 2*cos(b*n*log(x) + b*log(c) + a)*sqrt(sin(b*n*log(x) + b*log(c) + a)))/(
b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**(3/2)/x,x)
```

```
[Out] Integral(sin(a + b*log(c*x**n))**(3/2)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^(3/2)/x, x)
```

Mupad [B]
time = 2.53, size = 65, normalized size = 0.96

$$-\frac{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)\,\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \cos\left(a+b\,\ln\left(c\,x^n\right)\right)^2\right)}{b\,n\,{\left(\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^2\right)}^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))^(3/2)/x,x)
```

```
[Out] -(cos(a + b*log(c*x^n))*sin(a + b*log(c*x^n))^(5/2)*hypergeom([-1/4, 1/2],
3/2, cos(a + b*log(c*x^n))^2))/(b*n*(sin(a + b*log(c*x^n))^2)^(5/4))
```

3.61 $\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x^2}\,dx$

Optimal. Leaf size=111

$$-\frac{2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(-3+\frac{2i}{bn}\right);\frac{1}{4}\left(1+\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)\sin^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{(2+3ibn)x\left(1-e^{2ia}\left(cx^n\right)^{2ib}\right)^{3/2}}$$

```
[Out] -2*hypergeom([-3/2, -3/4+1/2*I/b/n],[1/4+1/2*I/b/n],exp(2*I*a)*(c*x^n)^(2*I
*b))*sin(a+b*ln(c*x^n))^(3/2)/(2+3*I*b*n)/x/(1-exp(2*I*a)*(c*x^n)^(2*I*b))^
(3/2)
```

Rubi [A]
time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4581, 4579, 371}

$$-\frac{2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(\frac{2i}{bn}-3\right);\frac{1}{4}\left(1+\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)\sin^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{x(2+3ibn)\left(1-e^{2ia}\left(cx^n\right)^{2ib}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Log[c*x^n]]^(3/2)/x^2,x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, (-3 + (2*I)/(b*n))/4, (1 + (2*I)/(b*n))/4, E^((
2*I)*a)*(c*x^n)^((2*I)*b)]*Sin[a + b*Log[c*x^n]]^(3/2))/((2 + (3*I)*b*n)*x*
(1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4579

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Dist[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
eQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
```

```
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x^2}\,dx &= \frac{(cx^n)^{\frac{1}{n}}\,\text{Subst}\left(\int x^{-1-\frac{1}{n}}\sin^{\frac{3}{2}}(a+b\log(x))\,dx,x,cx^n\right)}{nx}\\ &= \frac{\left((cx^n)^{\frac{3ib}{2}+\frac{1}{n}}\sin^{\frac{3}{2}}(a+b\log(cx^n))\right)\text{Subst}\left(\int x^{-1-\frac{3ib}{2}-\frac{1}{n}}\left(1-e^{2ia}x^{2ib}\right)^{3/2}\,dx,x,c\right.}{nx\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}}\\ &= -\frac{2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(-3+\frac{2i}{bn}\right);\frac{1}{4}\left(1+\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)\sin^{\frac{3}{2}}(a+b\log(cx^n))}{(2+3ibn)x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 1.16, size = 220, normalized size = 1.98

$$\frac{6b^2\sqrt{2-2e^{2i(a+b\log(cx^n))}}\,n^2\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}+\frac{i}{2bn};\frac{5}{4}+\frac{i}{2bn};e^{2i(a+b\log(cx^n))}\right)}{\sqrt{-ie^{-i(a+b\log(cx^n))}\left(-1+e^{2i(a+b\log(cx^n))}\right)}\,(2+3ibn)(2i+bn)(2i+3bn)x}-\frac{2\sqrt{\sin(a+b\log(cx^n))}\,(3bn\cos(a+b\log(cx^n))+2\sin(a+b\log(cx^n)))}{(4+9b^2n^2)\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]^(3/2)/x^2,x]
```

```
[Out] (6*b^2*Sqrt[2 - 2*E^((2*I)*(a + b*Log[c*x^n]))]*n^2*Hypergeometric2F1[1/2,
1/4 + (I/2)/(b*n), 5/4 + (I/2)/(b*n), E^((2*I)*(a + b*Log[c*x^n]))])/(Sqrt[
((-I)*(-1 + E^((2*I)*(a + b*Log[c*x^n]))))/E^(I*(a + b*Log[c*x^n]))]*(2 + (
3*I)*b*n)*(2*I + b*n)*(2*I + 3*b*n)*x) - (2*Sqrt[Sin[a + b*Log[c*x^n]]]*(3*
b*n*Cos[a + b*Log[c*x^n]] + 2*Sin[a + b*Log[c*x^n]]))/((4 + 9*b^2*n^2)*x)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sin^{\frac{3}{2}}(a+b\ln(c\,x^n))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))^(3/2)/x^2,x)
```

```
[Out] int(sin(a+b*ln(c*x^n))^(3/2)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(3/2)/x^2,x, algorithm="maxima")
[Out] integrate(sin(b*log(c*x^n) + a)^(3/2)/x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(3/2)/x^2,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{\frac{3}{2}}\left(a + b \log\left(cx^{n}\right)\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**(3/2)/x**2,x)
[Out] Integral(sin(a + b*log(c*x**n))**(3/2)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(3/2)/x^2,x, algorithm="giac")
[Out] integrate(sin(b*log(c*x^n) + a)^(3/2)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}^{3/2}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))^(3/2)/x^2,x)
[Out] int(sin(a + b*log(c*x^n))^(3/2)/x^2, x)
```

3.62 $\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x^3}\,dx$

Optimal. Leaf size=111

$$-\frac{2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(-3+\frac{4i}{bn}\right);\frac{1}{4}\left(1+\frac{4i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)\sin^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{(4+3ibn)x^2\left(1-e^{2ia}\left(cx^n\right)^{2ib}\right)^{3/2}}$$

```
[Out] -2*hypergeom([-3/2, -3/4+I/b/n],[1/4+I/b/n],exp(2*I*a)*(c*x^n)^(2*I*b))*sin
(a+b*ln(c*x^n))^(3/2)/(4+3*I*b*n)/x^2/(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(3/2)
```

Rubi [A]
time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4581, 4579, 371}

$$-\frac{2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(\frac{4i}{bn}-3\right);\frac{1}{4}\left(1+\frac{4i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)\sin^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{x^2(4+3ibn)\left(1-e^{2ia}\left(cx^n\right)^{2ib}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Log[c*x^n]]^(3/2)/x^3,x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, (-3 + (4*I)/(b*n))/4, (1 + (4*I)/(b*n))/4, E^((
2*I)*a)*(c*x^n)^((2*I)*b)]*Sin[a + b*Log[c*x^n]]^(3/2))/((4 + (3*I)*b*n)*x^
2*(1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4579

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Dist[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
```

```
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{x^3}\,dx &= \frac{\left(cx^n\right)^{2/n}\operatorname{Subst}\left(\int x^{-1-\frac{2}{n}}\sin^{\frac{3}{2}}(a+b\log(x))\,dx,x,cx^n\right)}{nx^2}\\
&= \frac{\left(\left(cx^n\right)^{\frac{3ib}{2}+\frac{2}{n}}\sin^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\right)\operatorname{Subst}\left(\int x^{-1-\frac{3ib}{2}-\frac{2}{n}}\left(1-e^{2ia}x^{2ib}\right)^{3/2}\,dx,x,\right.}{nx^2\left(1-e^{2ia}\left(cx^n\right)^{2ib}\right)^{3/2}}\\
&= -\frac{2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(-3+\frac{4i}{bn}\right);\frac{1}{4}\left(1+\frac{4i}{bn}\right);e^{2ia}\left(cx^n\right)^{2ib}\right)\sin^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{(4+3ibn)x^2\left(1-e^{2ia}\left(cx^n\right)^{2ib}\right)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 1.17, size = 216, normalized size = 1.95

$$\frac{6b^2\sqrt{2-2e^{2i(a+b\log(cx^n))}}\,n^2\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}+\frac{i}{bn};\frac{5}{4}+\frac{i}{bn};e^{2i(a+b\log(cx^n))}\right)}{\sqrt{-ie^{-i(a+b\log(cx^n))}\left(-1+e^{2i(a+b\log(cx^n))}\right)}\,(4+3ibn)(4i+bn)(4i+3bn)x^2}-\frac{2\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}\left(3bn\cos\left(a+b\log\left(cx^n\right)\right)+4\sin\left(a+b\log\left(cx^n\right)\right)\right)}{\left(16+9b^2n^2\right)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]^(3/2)/x^3,x]
```

```
[Out] (6*b^2*Sqrt[2 - 2*E^((2*I)*(a + b*Log[c*x^n]))]*n^2*Hypergeometric2F1[1/2,
1/4 + I/(b*n), 5/4 + I/(b*n), E^((2*I)*(a + b*Log[c*x^n]))])/(Sqrt[((-I)*(-
1 + E^((2*I)*(a + b*Log[c*x^n]))))/E^(I*(a + b*Log[c*x^n]))]*(4 + (3*I)*b*n
)*(4*I + b*n)*(4*I + 3*b*n)*x^2) - (2*Sqrt[Sin[a + b*Log[c*x^n]]]*(3*b*n*Co
s[a + b*Log[c*x^n]] + 4*Sin[a + b*Log[c*x^n]]))/((16 + 9*b^2*n^2)*x^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sin^{\frac{3}{2}}\left(a+b\ln\left(c\,x^n\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))^(3/2)/x^3,x)
```

```
[Out] int(sin(a+b*ln(c*x^n))^(3/2)/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(3/2)/x^3,x, algorithm="maxima")
[Out] integrate(sin(b*log(c*x^n) + a)^(3/2)/x^3, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(3/2)/x^3,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{\frac{3}{2}}(a + b\log(cx^n))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**(3/2)/x**3,x)
[Out] Integral(sin(a + b*log(c*x**n))**(3/2)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(3/2)/x^3,x, algorithm="giac")
[Out] integrate(sin(b*log(c*x^n) + a)^(3/2)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a + b\,\ln\left(c\,x^n\right)\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))^(3/2)/x^3,x)
[Out] int(sin(a + b*log(c*x^n))^(3/2)/x^3, x)
```

3.63 $\int \frac{1}{\sqrt{\sin(a+b\log(cx^n))}}\,dx$

Optimal. Leaf size=109

$$\frac{2x\sqrt{1-e^{2ia}(cx^n)^{2ib}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}\left(1-\frac{2i}{bn}\right);\frac{1}{4}\left(5-\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{(2+ibn)\sqrt{\sin(a+b\log(cx^n))}}$$

```
[Out] 2*x*hypergeom([1/2, 1/4-1/2*I/b/n],[5/4-1/2*I/b/n],exp(2*I*a)*(c*x^n)^(2*I*
b))*(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(1/2)/(2+I*b*n)/sin(a+b*ln(c*x^n))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4571, 4579, 371}

$$\frac{2x\sqrt{1-e^{2ia}(cx^n)^{2ib}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}\left(1-\frac{2i}{bn}\right);\frac{1}{4}\left(5-\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{(2+ibn)\sqrt{\sin(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sin[a + b*Log[c*x^n]]],x]
```

```
[Out] (2*x*Sqrt[1 - E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Hypergeometric2F1[1/2, (1 - (2
*I)/(b*n))/4, (5 - (2*I)/(b*n))/4, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((2 + I*
b*n)*Sqrt[Sin[a + b*Log[c*x^n]]])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4571

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4579

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Dist[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
```

```
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}}\,dx = \frac{\left(x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\sqrt{\sin(a+b\log(x))}}\,dx, x, cx^n\right)}{n}$$
$$= \frac{\left(x\left(cx^n\right)^{-\frac{ib}{2}-\frac{1}{n}}\sqrt{1-e^{2ia}\left(cx^n\right)^{2ib}}\right)\operatorname{Subst}\left(\int \frac{x^{-1+\frac{ib}{2}+\frac{1}{n}}}{\sqrt{1-e^{2ia}x^{2ib}}}\,dx, x, cx^n\right)}{n\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}}$$
$$= \frac{2x\sqrt{1-e^{2ia}\left(cx^n\right)^{2ib}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}\left(1-\frac{2i}{bn}\right);\frac{1}{4}\left(5-\frac{2i}{bn}\right);e^{2ia}\left(cx^n\right)^{2ib}\right)}{(2+ibn)\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}}$$

Mathematica [A]
time = 0.47, size = 132, normalized size = 1.21

$$-\frac{2i\sqrt{2-2e^{2i(a+b\log(cx^n))}}\,x\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}-\frac{i}{2bn};\frac{5}{4}-\frac{i}{2bn};e^{2i(a+b\log(cx^n))}\right)}{\sqrt{-ie^{-i(a+b\log(cx^n))}\left(-1+e^{2i(a+b\log(cx^n))}\right)}\,(-2i+bn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sin[a + b*Log[c*x^n]]],x]
[Out] ((-2*I)*Sqrt[2 - 2*E^((2*I)*(a + b*Log[c*x^n]))]*x*Hypergeometric2F1[1/2, 1
/4 - (I/2)/(b*n), 5/4 - (I/2)/(b*n), E^((2*I)*(a + b*Log[c*x^n]))])/(Sqrt[(
(-I)*(-1 + E^((2*I)*(a + b*Log[c*x^n]))))/E^(I*(a + b*Log[c*x^n]))]*(-2*I +
 b*n))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sin\left(a+b\ln\left(c\,x^n\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(a+b*ln(c*x^n))^(1/2),x)
[Out] int(1/sin(a+b*ln(c*x^n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(sin(b*log(c*x^n) + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sin\left(a+b\log\left(cx^{n}\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(1/sqrt(sin(a + b*log(c*x**n))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(sin(b*log(c*x^n) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\sin\left(a+b\,\ln\left(c\,x^{n}\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(a + b*log(c*x^n))^(1/2),x)
```

```
[Out] int(1/sin(a + b*log(c*x^n))^(1/2), x)
```

3.64 $\int \frac{1}{x\sqrt{\sin(a+b\log(cx^n))}}\,dx$

Optimal. Leaf size=29

$$\frac{2F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b\log(cx^n)\right)\middle|2\right)}{bn}$$

```
[Out] -2*(sin(1/2*a+1/4*Pi+1/2*b*ln(c*x^n))^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*ln(c*
x^n))*EllipticF(cos(1/2*a+1/4*Pi+1/2*b*ln(c*x^n)),2^(1/2))/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2720}

$$\frac{2F\left(\frac{1}{2}\left(a+b\log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Sin[a + b*Log[c*x^n]]]),x]
```

```
[Out] (2*EllipticF[(a - Pi/2 + b*Log[c*x^n])/2, 2])/(b*n)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{\sin(a+b\log(cx^n))}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx, x, \log(cx^n)\right)}{n}\\ &= \frac{2F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b\log(cx^n)\right)\middle|2\right)}{bn}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 32, normalized size = 1.10

$$-\frac{2F\left(\frac{1}{2}\left(-a+\frac{\pi}{2}-b\log(cx^n)\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[Sin[a + b*Log[c*x^n]]]),x]

[Out] (-2*EllipticF[(-a + Pi/2 - b*Log[c*x^n])/2, 2])/(b*n)

Maple [A]
time = 0.12, size = 102, normalized size = 3.52

method	result
derivativedivides	$\frac{\sqrt{\sin\left(a+b\ln\left(c\,x^{n}\right)\right)+1}\,\sqrt{-2\sin\left(a+b\ln\left(c\,x^{n}\right)\right)+2}\,\sqrt{-\sin\left(a+b\ln\left(c\,x^{n}\right)\right)}\,\mathrm{Ellipti}}{n\cos(a+b\ln(c\,x^{n}))\sqrt{\sin\left(a+b\ln\left(c\,x^{n}\right)\right)}\,b}$
default	$\frac{\sqrt{\sin\left(a+b\ln\left(c\,x^{n}\right)\right)+1}\,\sqrt{-2\sin\left(a+b\ln\left(c\,x^{n}\right)\right)+2}\,\sqrt{-\sin\left(a+b\ln\left(c\,x^{n}\right)\right)}\,\mathrm{Ellipti}}{n\cos(a+b\ln(c\,x^{n}))\sqrt{\sin\left(a+b\ln\left(c\,x^{n}\right)\right)}\,b}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/sin(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/n*(sin(a+b*ln(c*x^n))+1)^(1/2)*(-2*sin(a+b*ln(c*x^n))+2)^(1/2)*(-sin(a+b*ln(c*x^n)))^(1/2)*EllipticF((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2))/cos(a+b*ln(c*x^n))/sin(a+b*ln(c*x^n))^(1/2)/b

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/sin(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(x*sqrt(sin(b*log(c*x^n) + a))), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.40, size = 82, normalized size = 2.83

$$\frac{\sqrt{2}\,\sqrt{-i}\,\mathrm{weierstrassPInverse}(4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+i\,\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right))+\sqrt{2}\,\sqrt{i}\,\mathrm{weierstrassPInverse}(4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-i\,\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right))}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/sin(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")

[Out] (sqrt(2)*sqrt(-I)*weierstrassPInverse(4, 0, cos(b*n*log(x) + b*log(c) + a) + I*sin(b*n*log(x) + b*log(c) + a)) + sqrt(2)*sqrt(I)*weierstrassPInverse(4, 0, cos(b*n*log(x) + b*log(c) + a) - I*sin(b*n*log(x) + b*log(c) + a)))/(b*n)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sin(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(sin(a + b*log(c*x**n)))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sin(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sqrt(sin(b*log(c*x^n) + a))), x)
```

Mupad [B]
time = 2.55, size = 26, normalized size = 0.90

$$-\frac{2\,\mathrm{F}\left(\frac{\pi}{4}-\frac{a}{2}-\frac{b\,\ln\left(c\,x^n\right)}{2}\middle|2\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*sin(a + b*log(c*x^n))^(1/2)),x)
```

```
[Out] -(2*ellipticF(pi/4 - a/2 - (b*log(c*x^n))/2, 2))/(b*n)
```

3.65 $\int \frac{1}{\sin^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=109

$$\frac{2x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}{}_2F_1\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i}{bn}\right);\frac{1}{4}\left(7-\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{(2+3ibn)\sin^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

```
[Out] 2*x*(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(3/2)*hypergeom([3/2, 3/4-1/2*I/b/n],[7/
4-1/2*I/b/n],exp(2*I*a)*(c*x^n)^(2*I*b))/(2+3*I*b*n)/sin(a+b*ln(c*x^n))^(3/
2)
```

Rubi [A]
time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4571, 4579, 371}

$$\frac{2x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}{}_2F_1\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i}{bn}\right);\frac{1}{4}\left(7-\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{(2+3ibn)\sin^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Log[c*x^n]]^(-3/2),x]
```

```
[Out] (2*x*(1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2)*Hypergeometric2F1[3/2, (3 -
(2*I)/(b*n))/4, (7 - (2*I)/(b*n))/4, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((2 +
(3*I)*b*n)*Sin[a + b*Log[c*x^n]]^(3/2))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4571

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4579

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Dist[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
```

```
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\sin^{\frac{3}{2}}(a+b\log(x))}\,dx, x, cx^n\right)}{n}$$
$$= \frac{\left(x(cx^n)^{-\frac{3ib}{2}-\frac{1}{n}}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{3ib}{2}+\frac{1}{n}}}{\left(1-e^{2ia}x^{2ib}\right)^{3/2}}\,dx, x, cx^n\right)}{n\sin^{\frac{3}{2}}(a+b\log(cx^n))}$$
$$= \frac{2x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}{}_2F_1\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i}{bn}\right);\frac{1}{4}\left(7-\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{(2+3ibn)\sin^{\frac{3}{2}}(a+b\log(cx^n))}$$

Mathematica [A]
time = 6.55, size = 164, normalized size = 1.50

$$\frac{4\sqrt{2}\,e^{2ia}x(cx^n)^{2ib}\sqrt{-ie^{-ia}(cx^n)^{-ib}\left(-1+e^{2ia}(cx^n)^{2ib}\right)}\,{}_2F_1\left(\frac{3}{2},\frac{3}{4}-\frac{i}{2bn};\frac{7}{4}-\frac{i}{2bn};e^{2ia}(cx^n)^{2ib}\right)}{(-2-3ibn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]^(-3/2),x]
```

```
[Out] (4*Sqrt[2]*E^((2*I)*a)*x*(c*x^n)^((2*I)*b)*Sqrt[((-I)*(-1 + E^((2*I)*a)*(c*
x^n)^((2*I)*b)))/(E^(I*a)*(c*x^n)^(I*b))]*Hypergeometric2F1[3/2, 3/4 - (I/2
)/(b*n), 7/4 - (I/2)/(b*n), E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((-2 - (3*I)*b*
n)*Sqrt[1 - E^((2*I)*a)*(c*x^n)^((2*I)*b)])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin\left(a+b\ln\left(c\,x^n\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(a+b*ln(c*x^n))^(3/2),x)
```

```
[Out] int(1/sin(a+b*ln(c*x^n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
[Out] integrate(sin(b*log(c*x^n) + a)^(-3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin^{\frac{3}{2}}\left(a + b \log \left(cx^{n}\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(a+b*ln(c*x**n))**(3/2),x)
[Out] Integral(sin(a + b*log(c*x**n))**(-3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(a + b*log(c*x^n))^(3/2),x)
[Out] int(1/sin(a + b*log(c*x^n))^(3/2), x)
```

3.66 $\int \frac{1}{x \sin^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=64

$$-\frac{2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}-\frac{2\cos\left(a+b\log\left(cx^n\right)\right)}{bn\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] 2*(sin(1/2*a+1/4*Pi+1/2*b*ln(c*x^n))^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*ln(c*x
^n))*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*ln(c*x^n)),2^(1/2))/b/n-2*cos(a+b*ln(
c*x^n))/b/n/sin(a+b*ln(c*x^n))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2716, 2719}

$$-\frac{2E\left(\frac{1}{2}\left(a+b\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{bn}-\frac{2\cos\left(a+b\log\left(cx^n\right)\right)}{bn\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sin[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (-2*EllipticE[(a - Pi/2 + b*Log[c*x^n])/2, 2])/(b*n) - (2*Cos[a + b*Log[c*x
^n]])/(b*n*Sqrt[Sin[a + b*Log[c*x^n]]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{x\sin^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$
$$= -\frac{2\cos(a+b\log(cx^n))}{bn\sqrt{\sin(a+b\log(cx^n))}} - \frac{\text{Subst}\left(\int \sqrt{\sin(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$
$$= -\frac{2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b\log(cx^n)\right)\middle|2\right)}{bn} - \frac{2\cos(a+b\log(cx^n))}{bn\sqrt{\sin(a+b\log(cx^n))}}$$

Mathematica [A]
time = 0.25, size = 57, normalized size = 0.89

$$\frac{2\left(E\left(\frac{1}{4}(-2a+\pi-2b\log(cx^n))\middle|2\right)-\frac{\cos(a+b\log(cx^n))}{\sqrt{\sin(a+b\log(cx^n))}}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sin[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (2*(EllipticE[(-2*a + Pi - 2*b*Log[c*x^n])/4, 2] - Cos[a + b*Log[c*x^n]]/Sq
rt[Sin[a + b*Log[c*x^n]]]))/(b*n)
```

Maple [A]
time = 0.19, size = 190, normalized size = 2.97

method	result
derivativedivides	$\frac{2\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+2}\,\sqrt{-\sin(a+b\ln(c\,x^n))}\,\text{Ellipt}}{}$
default	$\frac{2\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+2}\,\sqrt{-\sin(a+b\ln(c\,x^n))}\,\text{Ellipt}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/sin(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(2*(sin(a+b*ln(c*x^n))+1)^(1/2)*(-2*sin(a+b*ln(c*x^n))+2)^(1/2)*(-sin(a
+b*ln(c*x^n)))^(1/2)*EllipticE((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2))-(s
in(a+b*ln(c*x^n))+1)^(1/2)*(-2*sin(a+b*ln(c*x^n))+2)^(1/2)*(-sin(a+b*ln(c*x
^n)))^(1/2)*EllipticF((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2))-2*cos(a+b*l
n(c*x^n))^2)/cos(a+b*ln(c*x^n))/sin(a+b*ln(c*x^n))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sin(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sin(b*log(c*x^n) + a)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.38, size = 156, normalized size = 2.44

$$\frac{-i\sqrt{2}\sqrt{-i}\,\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+i\,\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)))+i\sqrt{2}\sqrt{i}\,\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-i\,\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)))-2\,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sqrt{\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}}{bn\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sin(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*sqrt(-I)*sin(b*n*log(x) + b*log(c) + a)*weierstrassZeta(4, 0, w
eierstrassPInverse(4, 0, cos(b*n*log(x) + b*log(c) + a) + I*sin(b*n*log(x)
+ b*log(c) + a))) + I*sqrt(2)*sqrt(I)*sin(b*n*log(x) + b*log(c) + a)*weiers
trassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*n*log(x) + b*log(c) + a) -
I*sin(b*n*log(x) + b*log(c) + a))) - 2*cos(b*n*log(x) + b*log(c) + a)*sqrt(
sin(b*n*log(x) + b*log(c) + a)))/(b*n*sin(b*n*log(x) + b*log(c) + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sin^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sin(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Integral(1/(x*sin(a + b*log(c*x**n))**(3/2)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sin(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 2.73, size = 65, normalized size = 1.02

$$-\frac{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)\,{\left({\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}^2\right)}^{1/4}\,{{}}_2{\mathrm{F}}_1\left(\frac{1}{2},\frac{5}{4};\ \frac{3}{2};\ {\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}^2\right)}{b\,n\,\sqrt{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*sin(a + b*log(c*x^n))^(3/2)),x)
```

```
[Out] -(cos(a + b*log(c*x^n))*(sin(a + b*log(c*x^n))^2)^(1/4)*hypergeom([1/2, 5/4
], 3/2, cos(a + b*log(c*x^n))^2))/(b*n*sin(a + b*log(c*x^n))^(1/2))
```

3.67 $\int \frac{1}{\sin^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=109

$$\frac{2x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i}{bn}\right);\frac{1}{4}\left(9-\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{(2+5ibn)\sin^{\frac{5}{2}}(a+b\log(cx^n))}$$

```
[Out] 2*x*(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(5/2)*hypergeom([5/2, 5/4-1/2*I/b/n],[9/
4-1/2*I/b/n],exp(2*I*a)*(c*x^n)^(2*I*b))/(2+5*I*b*n)/sin(a+b*ln(c*x^n))^(5/
2)
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4571, 4579, 371}

$$\frac{2x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i}{bn}\right);\frac{1}{4}\left(9-\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{(2+5ibn)\sin^{\frac{5}{2}}(a+b\log(cx^n))}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Log[c*x^n]]^(-5/2),x]
```

```
[Out] (2*x*(1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^(5/2)*Hypergeometric2F1[5/2, (5 -
(2*I)/(b*n))/4, (9 - (2*I)/(b*n))/4, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((2 +
(5*I)*b*n)*Sin[a + b*Log[c*x^n]]^(5/2))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4571

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4579

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Dist[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
```

```
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sin^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\sin^{\frac{5}{2}}(a+b\log(x))}\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(x(cx^n)^{-\frac{5ib}{2}-\frac{1}{n}}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{5/2}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{5ib}{2}+\frac{1}{n}}}{\left(1-e^{2ia}x^{2ib}\right)^{5/2}}\,dx,x,cx^n\right)}{n\sin^{\frac{5}{2}}(a+b\log(cx^n))}$$
$$= \frac{2x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{5/2}{}_2F_1\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i}{bn}\right);\frac{1}{4}\left(9-\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{(2+5ibn)\sin^{\frac{5}{2}}(a+b\log(cx^n))}$$

Mathematica [A]
time = 3.66, size = 191, normalized size = 1.75

$$\frac{2x\left(\frac{(2-ibn)\sqrt{2-2e^{2ia}(cx^n)^{2ib}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}-\frac{i}{2bn};\frac{5}{4}-\frac{i}{2bn};e^{2i(a+b\log(cx^n))}\right)}{\sqrt{-ie^{-ia}(cx^n)^{-ib}\left(-1+e^{2ia}(cx^n)^{2ib}\right)}}-\frac{bn\cos(a+b\log(cx^n))+2\sin(a+b\log(cx^n))}{\sin^{\frac{3}{2}}(a+b\log(cx^n))}\right)}{3b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]^(-5/2),x]
[Out] (2*x*(((2 - I*b*n)*Sqrt[2 - 2*E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Hypergeometric
2F1[1/2, 1/4 - (I/2)/(b*n), 5/4 - (I/2)/(b*n), E^((2*I)*(a + b*Log[c*x^n]))
])/Sqrt[((-I)*(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))/(E^(I*a)*(c*x^n)^(I*b))
] - (b*n*Cos[a + b*Log[c*x^n]] + 2*Sin[a + b*Log[c*x^n]])/Sin[a + b*Log[c*x
^n]]^(3/2)))/(3*b^2*n^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin\left(a+b\ln\left(c\,x^n\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(a+b*ln(c*x^n))^(5/2),x)
[Out] int(1/sin(a+b*ln(c*x^n))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
[Out] integrate(sin(b*log(c*x^n) + a)^(-5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(a+b*ln(c*x**n))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(a + b*log(c*x^n))^(5/2),x)
[Out] int(1/sin(a + b*log(c*x^n))^(5/2), x)
```

3.68 $\int \frac{1}{x\sin^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=68

$$\frac{2F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b\log\left(cx^n\right)\right)\middle|2\right)}{3bn}-\frac{2\cos\left(a+b\log\left(cx^n\right)\right)}{3bn\sin^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

```
[Out] -2/3*(sin(1/2*a+1/4*Pi+1/2*b*ln(c*x^n))^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*ln(
c*x^n))*EllipticF(cos(1/2*a+1/4*Pi+1/2*b*ln(c*x^n)),2^(1/2))/b/n-2/3*cos(a+
b*ln(c*x^n))/b/n/sin(a+b*ln(c*x^n))^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2716, 2720}

$$\frac{2F\left(\frac{1}{2}\left(a+b\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{3bn}-\frac{2\cos\left(a+b\log\left(cx^n\right)\right)}{3bn\sin^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sin[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (2*EllipticF[(a - Pi/2 + b*Log[c*x^n])/2, 2])/(3*b*n) - (2*Cos[a + b*Log[c*
x^n]])/(3*b*n*Sin[a + b*Log[c*x^n]]^(3/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{x\sin^{\frac{5}{2}}(a+b\log(cx^n))}dx = \frac{\text{Subst}\left(\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}dx, x, \log(cx^n)\right)}{n}$$
$$= -\frac{2\cos(a+b\log(cx^n))}{3bn\sin^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\sin(a+bx)}}dx, x, \log(cx^n)\right)}{3n}$$
$$= \frac{2F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b\log(cx^n)\right)\middle|2\right)}{3bn} - \frac{2\cos(a+b\log(cx^n))}{3bn\sin^{\frac{3}{2}}(a+b\log(cx^n))}$$

Mathematica [A]
time = 0.24, size = 61, normalized size = 0.90

$$\frac{2\left(F\left(\frac{1}{4}(2a-\pi+2b\log(cx^n))\middle|2\right)-\frac{\cos(a+b\log(cx^n))}{\sin^{\frac{3}{2}}(a+b\log(cx^n))}\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sin[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (2*(EllipticF[(2*a - Pi + 2*b*Log[c*x^n])/4, 2] - Cos[a + b*Log[c*x^n]]/Sin
[a + b*Log[c*x^n]]^(3/2)))/(3*b*n)
```

Maple [A]
time = 0.18, size = 131, normalized size = 1.93

method	result
derivativedivides	$\frac{\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+2}\,\sqrt{-\sin(a+b\ln(c\,x^n))}\ \text{EllipticF}}{3n\sin(a+b\ln(c\,x^n))^{\frac{3}{2}}\cos(a+b\ln(c\,x}$
default	$\frac{\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+2}\,\sqrt{-\sin(a+b\ln(c\,x^n))}\ \text{EllipticF}}{3n\sin(a+b\ln(c\,x^n))^{\frac{3}{2}}\cos(a+b\ln(c\,x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/sin(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/n/sin(a+b*ln(c*x^n))^(3/2)*((sin(a+b*ln(c*x^n))+1)^(1/2)*(-2*sin(a+b*ln
(c*x^n))+2)^(1/2)*(-sin(a+b*ln(c*x^n)))^(1/2)*EllipticF((sin(a+b*ln(c*x^n))
+1)^(1/2),1/2*2^(1/2))*sin(a+b*ln(c*x^n))-2*cos(a+b*ln(c*x^n))^2)/cos(a+b*l
n(c*x^n))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sin(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sin(b*log(c*x^n) + a)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.46, size = 177, normalized size = 2.60

$$\frac{\left(\sqrt{2}\,\sqrt{-i}\,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-\sqrt{2}\,\sqrt{-i}\right)\mathrm{weierstrassPInverse}(4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+i\,\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right))+\left(\sqrt{2}\,\sqrt{i}\,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-\sqrt{2}\,\sqrt{i}\right)\mathrm{weierstrassPInverse}(4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-i\,\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right))+2\,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sqrt{\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}}{3\left(bn\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-bn\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sin(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((sqrt(2)*sqrt(-I)*cos(b*n*log(x) + b*log(c) + a)^2 - sqrt(2)*sqrt(-I))
*weierstrassPInverse(4, 0, cos(b*n*log(x) + b*log(c) + a) + I*sin(b*n*log(x
) + b*log(c) + a)) + (sqrt(2)*sqrt(I)*cos(b*n*log(x) + b*log(c) + a)^2 - sq
rt(2)*sqrt(I))*weierstrassPInverse(4, 0, cos(b*n*log(x) + b*log(c) + a) - I
*sin(b*n*log(x) + b*log(c) + a)) + 2*cos(b*n*log(x) + b*log(c) + a)*sqrt(si
n(b*n*log(x) + b*log(c) + a)))/(b*n*cos(b*n*log(x) + b*log(c) + a)^2 - b*n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sin(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sin(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 2.96, size = 65, normalized size = 0.96

$$-\frac{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)\,{\left({\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}^2\right)}^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{3}{2};\ {\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}^2\right)}{b\,n\,{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*sin(a + b*log(c*x^n))^(5/2)),x)
```

```
[Out] -(cos(a + b*log(c*x^n))*(sin(a + b*log(c*x^n))^2)^(3/4)*hypergeom([1/2, 7/4
], 3/2, cos(a + b*log(c*x^n))^2))/(b*n*sin(a + b*log(c*x^n))^(3/2))
```

3.69 $\int \frac{1}{\sin^{\frac{3}{2}}(a-2i\log(cx))}\,dx$

Optimal. Leaf size=49

$$\frac{e^{-2ia}(1-c^4e^{2ia}x^4)}{2c^4x^3\sin^{\frac{3}{2}}(a-2i\log(cx))}$$

```
[Out] 1/2*(1-c^4*exp(2*I*a)*x^4)/c^4/exp(2*I*a)/x^3/sin(a-2*I*ln(c*x))^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4571, 4569, 267}

$$\frac{e^{-2ia}(1-e^{2ia}c^4x^4)}{2c^4x^3\sin^{\frac{3}{2}}(a-2i\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a - (2*I)*Log[c*x]]^(-3/2),x]
```

```
[Out] (1 - c^4*E^((2*I)*a)*x^4)/(2*c^4*E^((2*I)*a)*x^3*Sin[a - (2*I)*Log[c*x]]^(3
/2))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4569

```
Int[Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :> Dist[Sin[d*(a + b
*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), Int[(1 - E^(2*I*
a*d)*x^(2*I*b*d))^p/x^(I*b*d*p), x], x] /; FreeQ[{a, b, d, p}, x] &&  !Inte
gerQ[p]
```

Rule 4571

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{1}{\sin^{\frac{3}{2}}(a-2i\log(cx))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sin^{\frac{3}{2}}(a-2i\log(x))}\,dx, x, cx\right)}{c}$$

$$= \frac{(1-c^4e^{2ia}x^4)^{3/2}\,\text{Subst}\left(\int \frac{x^3}{(1-e^{2ia}x^4)^{3/2}}\,dx, x, cx\right)}{c^4x^3\sin^{\frac{3}{2}}(a-2i\log(cx))}$$

$$= \frac{e^{-2ia}(1-c^4e^{2ia}x^4)}{2c^4x^3\sin^{\frac{3}{2}}(a-2i\log(cx))}$$

Mathematica [A]
time = 0.14, size = 81, normalized size = 1.65

$$\frac{x(\cos(a)-i\sin(a))\sqrt{\frac{-2i\left(-1+c^4x^4\right)\cos(a)+2\left(1+c^4x^4\right)\sin(a)}{c^2x^2}}}{\left(-1+c^4x^4\right)\cos(a)+i\left(1+c^4x^4\right)\sin(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a - (2*I)*Log[c*x]]^(-3/2),x]
```

```
[Out] (x*(Cos[a] - I*Sin[a])*Sqrt[((-2*I)*(-1 + c^4*x^4)*Cos[a] + 2*(1 + c^4*x^4)
*Sin[a])/(c^2*x^2)])/((-1 + c^4*x^4)*Cos[a] + I*(1 + c^4*x^4)*Sin[a])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin\left(a-2i\ln\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(a-2*I*ln(c*x))^(3/2),x)
```

```
[Out] int(1/sin(a-2*I*ln(c*x))^(3/2),x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(36) = 72$.
time = 0.62, size = 402, normalized size = 8.20

$$\frac{\left((\cos(a)^2+\sin(a)^2)c^4x^4+2c^2x^2\cos(a)+1\right)^{\frac{1}{4}}\left((\cos(a)^2+\sin(a)^2)c^4x^4-2c^2x^2\cos(a)+1\right)^{\frac{1}{4}}\left(\left(\left(c^4x^4\left((i+1)\cos\left(\frac{3}{2}a\right)+(i-1)\sin\left(\frac{3}{2}a\right)\right)-(i+1)\cos\left(\frac{1}{2}a\right)+(i-1)\sin\left(\frac{1}{2}a\right)\right)\cos\left(\frac{3}{2}\arctan\left(c^2x^2\sin(a),-c^2x^2\cos(a)+1\right)\right)+\left(c^4x^4\left((i-1)\cos\left(\frac{3}{2}a\right)-(i+1)\sin\left(\frac{3}{2}a\right)\right)-(i-1)\cos\left(\frac{1}{2}a\right)-(i+1)\sin\left(\frac{1}{2}a\right)\right)\sin\left(\frac{3}{2}\arctan\left(c^2x^2\sin(a),-c^2x^2\cos(a)+1\right)\right)\right)\cos\left(\frac{3}{2}\arctan\left(c^2x^2\sin(a),c^2x^2\cos(a)+1\right)\right)+\left(\left(c^4x^4\left(-(i-1)\cos\left(\frac{3}{2}a\right)+(i+1)\sin\left(\frac{3}{2}a\right)\right)+(i-1)\cos\left(\frac{1}{2}a\right)+(i+1)\sin\left(\frac{1}{2}a\right)\right)\cos\left(\frac{3}{2}\arctan\left(c^2x^2\sin(a),-c^2x^2\cos(a)+1\right)\right)+\left(c^4x^4\left((i+1)\cos\left(\frac{3}{2}a\right)+(i-1)\sin\left(\frac{3}{2}a\right)\right)-(i+1)\cos\left(\frac{1}{2}a\right)+(i-1)\sin\left(\frac{1}{2}a\right)\right)\sin\left(\frac{3}{2}\arctan\left(c^2x^2\sin(a),-c^2x^2\cos(a)+1\right)\right)\right)\sin\left(\frac{3}{2}\arctan\left(c^2x^2\sin(a),c^2x^2\cos(a)+1\right)\right)\right)}{\left((\cos(a)^4+2\cos(a)^2\sin(a)^2+\sin(a)^4)c^8x^8-2(\cos(a)^2-\sin(a)^2)c^4x^4+1\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(a-2*I*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] ((cos(a)^2 + sin(a)^2)*c^4*x^4 + 2*c^2*x^2*cos(a) + 1)^(1/4)*((cos(a)^2 + s
in(a)^2)*c^4*x^4 - 2*c^2*x^2*cos(a) + 1)^(1/4)*(((c^4*x^4*((I + 1)*cos(3/2*
a) + (I - 1)*sin(3/2*a)) - (I + 1)*cos(1/2*a) + (I - 1)*sin(1/2*a))*cos(3/2
*arctan2(c^2*x^2*sin(a), -c^2*x^2*cos(a) + 1)) + (c^4*x^4*((I - 1)*cos(3/2*
a) - (I + 1)*sin(3/2*a)) - (I - 1)*cos(1/2*a) - (I + 1)*sin(1/2*a))*sin(3/2
*arctan2(c^2*x^2*sin(a), -c^2*x^2*cos(a) + 1)))*cos(3/2*arctan2(c^2*x^2*sin
(a), c^2*x^2*cos(a) + 1)) + ((c^4*x^4*(-(I - 1)*cos(3/2*a) + (I + 1)*sin(3/
2*a)) + (I - 1)*cos(1/2*a) + (I + 1)*sin(1/2*a))*cos(3/2*arctan2(c^2*x^2*si
n(a), -c^2*x^2*cos(a) + 1)) + (c^4*x^4*((I + 1)*cos(3/2*a) + (I - 1)*sin(3/
2*a)) - (I + 1)*cos(1/2*a) + (I - 1)*sin(1/2*a))*sin(3/2*arctan2(c^2*x^2*si
n(a), -c^2*x^2*cos(a) + 1)))*sin(3/2*arctan2(c^2*x^2*sin(a), c^2*x^2*cos(a)
 + 1)))/(((cos(a)^4 + 2*cos(a)^2*sin(a)^2 + sin(a)^4)*c^8*x^8 - 2*(cos(a)^2
 - sin(a)^2)*c^4*x^4 + 1)*c)
```

Fricas [A]
time = 0.80, size = 43, normalized size = 0.88

$$\frac{2\sqrt{\frac{1}{2}}\sqrt{-i\,c^4x^4 + i\,e^{(-2i\,a)}}\,e^{\left(-\frac{3}{2}i\,a\right)}}{c^5x^4 - ce^{(-2i\,a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(a-2*I*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(1/2)*sqrt(-I*c^4*x^4 + I*e^(-2*I*a))*e^(-3/2*I*a)/(c^5*x^4 - c*e^(-2
*I*a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin^{\frac{3}{2}}\left(a - 2i\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(a-2*I*ln(c*x))**(3/2),x)
```

```
[Out] Integral(sin(a - 2*I*log(c*x))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(a-2*I*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(a - 2*I*log(c*x))^(-3/2), x)
```

Mupad [B]
time = 2.94, size = 50, normalized size = 1.02

$$\frac{2\,x\,\sqrt{\frac{\mathrm{e}^{-a\,1\mathrm{i}}\,1\mathrm{i}}{2\,c^2\,x^2}-\frac{c^2\,x^2\,\mathrm{e}^{a\,1\mathrm{i}}\,1\mathrm{i}}{2}}}{c^4\,x^4\,\mathrm{e}^{a\,2\mathrm{i}}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(a - log(c*x)*2i)^(3/2),x)
```

```
[Out] (2*x*((exp(-a*1i)*1i)/(2*c^2*x^2) - (c^2*x^2*exp(a*1i)*1i)/2)^(1/2))/(c^4*x
^4*exp(a*2i) - 1)
```

3.70 $\int (ex)^m \sin^4 (d(a + b \log (cx^n))) \, dx$

Optimal. Leaf size=337

$$\frac{24b^4d^4n^4(ex)^{1+m}}{e(1+m)\left((1+m)^2+4b^2d^2n^2\right)\left((1+m)^2+16b^2d^2n^2\right)}-\frac{24b^3d^3n^3(ex)^{1+m}\cos\left(d(a+b\log\left(cx^n\right))\right)\sin\left(d(a+\right.}{e\left((1+m)^2+4b^2d^2n^2\right)\left((1+m)^2+16b^2d^2\right.}$$

```
[Out] 24*b^4*d^4*n^4*(e*x)^(1+m)/e/(1+m)/((1+m)^2+4*b^2*d^2*n^2)/((1+m)^2+16*b^2*
d^2*n^2)-24*b^3*d^3*n^3*(e*x)^(1+m)*cos(d*(a+b*ln(c*x^n)))*sin(d*(a+b*ln(c*
x^n)))/e/((1+m)^2+4*b^2*d^2*n^2)/((1+m)^2+16*b^2*d^2*n^2)+12*b^2*d^2*(1+m)*
n^2*(e*x)^(1+m)*sin(d*(a+b*ln(c*x^n)))^2/e/((1+m)^2+4*b^2*d^2*n^2)/((1+m)^2
+16*b^2*d^2*n^2)-4*b*d*n*(e*x)^(1+m)*cos(d*(a+b*ln(c*x^n)))*sin(d*(a+b*ln(c
*x^n)))^3/e/((1+m)^2+16*b^2*d^2*n^2)+(1+m)*(e*x)^(1+m)*sin(d*(a+b*ln(c*x^n)
))^4/e/((1+m)^2+16*b^2*d^2*n^2)
```

Rubi [A]
time = 0.12, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4575, 32}

$$\frac{(m+1)(ex)^{m+1}\sin^4\left(d(a+b\log\left(cx^n\right))\right)}{e\left(16b^2d^2n^2+(m+1)^2\right)}+\frac{12b^2d^2(m+1)n^2(ex)^{m+1}\sin^2\left(d(a+b\log\left(cx^n\right))\right)}{e\left(4b^2d^2n^2+(m+1)^2\right)\left(16b^2d^2n^2+(m+1)^2\right)}-\frac{4bdn(ex)^{m+1}\sin^3\left(d(a+b\log\left(cx^n\right))\right)\cos\left(d(a+b\log\left(cx^n\right))\right)}{e\left(16b^2d^2n^2+(m+1)^2\right)}-\frac{24b^3d^3n^3(ex)^{m+1}\sin\left(d(a+b\log\left(cx^n\right))\right)\cos\left(d(a+b\log\left(cx^n\right))\right)}{e\left(4b^2d^2n^2+(m+1)^2\right)\left(16b^2d^2n^2+(m+1)^2\right)}+\frac{24b^4d^4n^4(ex)^{m+1}}{e(m+1)\left(4b^2d^2n^2+(m+1)^2\right)\left(16b^2d^2n^2+(m+1)^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^4,x]
```

```
[Out] (24*b^4*d^4*n^4*(e*x)^(1 + m))/(e*(1 + m)*((1 + m)^2 + 4*b^2*d^2*n^2)*((1 +
 m)^2 + 16*b^2*d^2*n^2)) - (24*b^3*d^3*n^3*(e*x)^(1 + m)*Cos[d*(a + b*Log[c
*x^n])]*Sin[d*(a + b*Log[c*x^n])])/(e*((1 + m)^2 + 4*b^2*d^2*n^2)*((1 + m)^
2 + 16*b^2*d^2*n^2)) + (12*b^2*d^2*(1 + m)*n^2*(e*x)^(1 + m)*Sin[d*(a + b*L
og[c*x^n])]^2)/(e*((1 + m)^2 + 4*b^2*d^2*n^2)*((1 + m)^2 + 16*b^2*d^2*n^2))
 - (4*b*d*n*(e*x)^(1 + m)*Cos[d*(a + b*Log[c*x^n])]*Sin[d*(a + b*Log[c*x^n]
)]^3)/(e*((1 + m)^2 + 16*b^2*d^2*n^2)) + ((1 + m)*(e*x)^(1 + m)*Sin[d*(a +
b*Log[c*x^n])]^4)/(e*((1 + m)^2 + 16*b^2*d^2*n^2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 4575

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2
*n^2*p^2 + (m + 1)^2)), Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x],
x] - Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a + b*Log[c*x^n])]*(Sin[d*(a + b*Log
```

```
[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x]) /; FreeQ[{a, b, c
, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int (ex)^m \sin^4\left(d(a+b\log\left(cx^n\right))\right)\,dx = -\frac{4bdn(ex)^{1+m}\cos\left(d(a+b\log\left(cx^n\right))\right)\sin^3\left(d(a+b\log\left(cx^n\right))\right)}{e\left((1+m)^2+16b^2d^2n^2\right)} + \frac{(1+\cdots}{\cdots}$$

$$= -\frac{24b^3d^3n^3(ex)^{1+m}\cos\left(d(a+b\log\left(cx^n\right))\right)\sin\left(d(a+b\log\left(cx^n\right))\right)}{e\left((1+m)^2+4b^2d^2n^2\right)\left((1+m)^2+16b^2d^2n^2\right)} + \frac{12\cdots}{\cdots}$$

$$= \frac{24b^4d^4n^4(ex)^{1+m}}{e(1+m)\left((1+m)^4+20b^2d^2(1+m)^2n^2+64b^4d^4n^4\right)} - \frac{24b^3d^3n^3(ex)^{1\cdots}}{e\left((1\cdots\right.}$$

Mathematica [A]
time = 2.06, size = 341, normalized size = 1.01

$$\frac{1}{8}x(ex)^m\left(\frac{3}{1+m}+\frac{4\sin(2bdn\log(x))\left(-2bdn\cos\left(2d(a-bn\log(x)+b\log\left(cx^n\right))\right)+(1+m)\sin\left(2d(a-bn\log(x)+b\log\left(cx^n\right))\right)\right)}{1+2m+m^2+4b^2d^2n^2}-\frac{4\cos(2bdn\log(x))\left((1+m)\cos\left(2d(a-bn\log(x)+b\log\left(cx^n\right))\right)+2bdn\sin\left(2d(a-bn\log(x)+b\log\left(cx^n\right))\right)\right)}{1+2m+m^2+4b^2d^2n^2}-\frac{\sin(4bdn\log(x))\left(-4bdn\cos\left(4d(a-bn\log(x)+b\log\left(cx^n\right))\right)+(1+m)\sin\left(4d(a-bn\log(x)+b\log\left(cx^n\right))\right)\right)}{1+2m+m^2+16b^2d^2n^2}+\frac{\cos(4bdn\log(x))\left((1+m)\cos\left(4d(a-bn\log(x)+b\log\left(cx^n\right))\right)+4bdn\sin\left(4d(a-bn\log(x)+b\log\left(cx^n\right))\right)\right)}{1+2m+m^2+16b^2d^2n^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^4,x]
```

```
[Out] (x*(e*x)^m*(3/(1 + m) + (4*Sin[2*b*d*n*Log[x]]*(-2*b*d*n*Cos[2*d*(a - b*n*L
og[x] + b*Log[c*x^n])] + (1 + m)*Sin[2*d*(a - b*n*Log[x] + b*Log[c*x^n])]))
/(1 + 2*m + m^2 + 4*b^2*d^2*n^2) - (4*Cos[2*b*d*n*Log[x]]*((1 + m)*Cos[2*d*
(a - b*n*Log[x] + b*Log[c*x^n])] + 2*b*d*n*Sin[2*d*(a - b*n*Log[x] + b*Log[
c*x^n])]))/(1 + 2*m + m^2 + 4*b^2*d^2*n^2) - (Sin[4*b*d*n*Log[x]]*(-4*b*d*n
*Cos[4*d*(a - b*n*Log[x] + b*Log[c*x^n])] + (1 + m)*Sin[4*d*(a - b*n*Log[x]
 + b*Log[c*x^n])]))/(1 + 2*m + m^2 + 16*b^2*d^2*n^2) + (Cos[4*b*d*n*Log[x]]
*((1 + m)*Cos[4*d*(a - b*n*Log[x] + b*Log[c*x^n])] + 4*b*d*n*Sin[4*d*(a - b
*n*Log[x] + b*Log[c*x^n])]))/(1 + 2*m + m^2 + 16*b^2*d^2*n^2)))/8
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\sin^4\left(d(a+b\ln\left(c\,x^n\right))\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sin(d*(a+b*ln(c*x^n)))^4,x)
```

```
[Out] int((e*x)^m*sin(d*(a+b*ln(c*x^n)))^4,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 16750 vs. $2(337) = 674$.
time = 1.00, size = 16750, normalized size = 49.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^4,x, algorithm="maxima")
```

```
[Out] 1/16*((16*(b^3*d^3*cos(4*a*d)*sin(4*b*d*log(c)) + b^3*d^3*cos(4*b*d*log(c))
*sin(4*a*d) + (b^3*d^3*cos(4*a*d)*sin(4*b*d*log(c)) + b^3*d^3*cos(4*b*d*log
(c))*sin(4*a*d) + ((b^3*d^3*cos(4*a*d)*sin(8*a*d) - b^3*d^3*cos(8*a*d)*sin(
4*a*d))*cos(4*b*d*log(c)) - (b^3*d^3*cos(8*a*d)*cos(4*a*d) + b^3*d^3*sin(8*
a*d)*sin(4*a*d))*sin(4*b*d*log(c)))*cos(8*b*d*log(c)) + ((b^3*d^3*cos(8*a*d
)*cos(4*a*d) + b^3*d^3*sin(8*a*d)*sin(4*a*d))*cos(4*b*d*log(c)) + (b^3*d^3*
cos(4*a*d)*sin(8*a*d) - b^3*d^3*cos(8*a*d)*sin(4*a*d))*sin(4*b*d*log(c)))*s
in(8*b*d*log(c)))*m + ((b^3*d^3*cos(4*a*d)*sin(8*a*d) - b^3*d^3*cos(8*a*d)*
sin(4*a*d))*cos(4*b*d*log(c)) - (b^3*d^3*cos(8*a*d)*cos(4*a*d) + b^3*d^3*si
n(8*a*d)*sin(4*a*d))*sin(4*b*d*log(c)))*cos(8*b*d*log(c)) + ((b^3*d^3*cos(8
*a*d)*cos(4*a*d) + b^3*d^3*sin(8*a*d)*sin(4*a*d))*cos(4*b*d*log(c)) + (b^3*
d^3*cos(4*a*d)*sin(8*a*d) - b^3*d^3*cos(8*a*d)*sin(4*a*d))*sin(4*b*d*log(c)
))*sin(8*b*d*log(c)))*n^3*e^m + 4*(b^2*d^2*cos(4*b*d*log(c))*cos(4*a*d) - b
^2*d^2*sin(4*b*d*log(c))*sin(4*a*d) + (b^2*d^2*cos(4*b*d*log(c))*cos(4*a*d)
 - b^2*d^2*sin(4*b*d*log(c))*sin(4*a*d) + ((b^2*d^2*cos(8*a*d)*cos(4*a*d) +
 b^2*d^2*sin(8*a*d)*sin(4*a*d))*cos(4*b*d*log(c)) + (b^2*d^2*cos(4*a*d)*sin
(8*a*d) - b^2*d^2*cos(8*a*d)*sin(4*a*d))*sin(4*b*d*log(c)))*cos(8*b*d*log(c
)) - ((b^2*d^2*cos(4*a*d)*sin(8*a*d) - b^2*d^2*cos(8*a*d)*sin(4*a*d))*cos(4
*b*d*log(c)) - (b^2*d^2*cos(8*a*d)*cos(4*a*d) + b^2*d^2*sin(8*a*d)*sin(4*a*
d))*sin(4*b*d*log(c)))*sin(8*b*d*log(c)))*m^2 + 2*(b^2*d^2*cos(4*b*d*log(c)
)*cos(4*a*d) - b^2*d^2*sin(4*b*d*log(c))*sin(4*a*d) + ((b^2*d^2*cos(8*a*d)*
cos(4*a*d) + b^2*d^2*sin(8*a*d)*sin(4*a*d))*cos(4*b*d*log(c)) + (b^2*d^2*co
s(4*a*d)*sin(8*a*d) - b^2*d^2*cos(8*a*d)*sin(4*a*d))*sin(4*b*d*log(c)))*cos
(8*b*d*log(c)) - ((b^2*d^2*cos(4*a*d)*sin(8*a*d) - b^2*d^2*cos(8*a*d)*sin(4
*a*d))*cos(4*b*d*log(c)) - (b^2*d^2*cos(8*a*d)*cos(4*a*d) + b^2*d^2*sin(8*a
*d)*sin(4*a*d))*sin(4*b*d*log(c)))*sin(8*b*d*log(c)))*m + ((b^2*d^2*cos(8*a
*d)*cos(4*a*d) + b^2*d^2*sin(8*a*d)*sin(4*a*d))*cos(4*b*d*log(c)) + (b^2*d^
2*cos(4*a*d)*sin(8*a*d) - b^2*d^2*cos(8*a*d)*sin(4*a*d))*sin(4*b*d*log(c)))
*cos(8*b*d*log(c)) - ((b^2*d^2*cos(4*a*d)*sin(8*a*d) - b^2*d^2*cos(8*a*d)*s
in(4*a*d))*cos(4*b*d*log(c)) - (b^2*d^2*cos(8*a*d)*cos(4*a*d) + b^2*d^2*sin
(8*a*d)*sin(4*a*d))*sin(4*b*d*log(c)))*sin(8*b*d*log(c)))*n^2*e^m + 4*((b*d
*cos(4*a*d)*sin(4*b*d*log(c)) + b*d*cos(4*b*d*log(c))*sin(4*a*d) + ((b*d*co
s(4*a*d)*sin(8*a*d) - b*d*cos(8*a*d)*sin(4*a*d))*cos(4*b*d*log(c)) - (b*d*c
os(8*a*d)*cos(4*a*d) + b*d*sin(8*a*d)*sin(4*a*d))*sin(4*b*d*log(c)))*cos(8*
b*d*log(c)) + ((b*d*cos(8*a*d)*cos(4*a*d) + b*d*sin(8*a*d)*sin(4*a*d))*cos(
4*b*d*log(c)) + (b*d*cos(4*a*d)*sin(8*a*d) - b*d*cos(8*a*d)*sin(4*a*d))*sin
```

```
(4*b*d*log(c)))*sin(8*b*d*log(c)))*m^3 + b*d*cos(4*a*d)*sin(4*b*d*log(c)) +
 b*d*cos(4*b*d*log(c))*sin(4*a*d) + 3*(b*d*cos(4*a*d)*sin(4*b*d*log(c)) + b
*d*cos(4*b*d*log(c))*sin(4*a*d) + ((b*d*cos(4*a*d)*sin(8*a*d) - b*d*cos(8*a
*d)*sin(4*a*d))*cos(4*b*d*log(c)) - (b*d*cos(8*a*d)*cos(4*a*d) + b*d*sin(8*
a*d)*sin(4*a*d))*sin(4*b*d*log(c)))*cos(8*b*d*log(c)) + ((b*d*cos(8*a*d)*co
s(4*a*d) + b*d*sin(8*a*d)*sin(4*a*d))*cos(4*b*d*log(c)) + (b*d*cos(4*a*d)*s
in(8*a*d) - b*d*cos(8*a*d)*sin(4*a*d))*sin(4*b*d*log(c)))*sin(8*b*d*log(c))
)*m^2 + 3*(b*d*cos(4*a*d)*sin(4*b*d*log(c)) + b*d*cos(4*b*d*log(c))*sin(4*a
*d) + ((b*d*cos(4*a*d)*sin(8*a*d) - b*d*cos(8*a*d)*sin(4*a*d))*cos(4*b*d*lo
g(c)) - (b*d*cos(8*a*d)*cos(4*a*d) + b*d*sin(8*a*d)*sin(4*a*d))*sin(4*b*d*l
og(c)))*cos(8*b*d*log(c)) + ((b*d*cos(8*a*d)*cos(4*a*d) + b*d*sin(8*a*d)*si
n(4*a*d))*cos(4*b*d*log(c)) + (b*d*cos(4*a*d)*sin(8*a*d) - b*d*cos(8*a*d)*s
in(4*a*d))*sin(4*b*d*log(c)))*sin(8*b*d*log(c)))*m + ((b*d*cos(4*a*d)*sin(8
*a*d) - b*d*cos(8*a*d)*sin(4*a*d))*cos(4*b*d*log(c)) - (b*d*cos(8*a*d)*cos(
4*a*d) + b*d*sin(8*a*d)*sin(4*a*d))*sin(4*b*d*log(c)))*cos(8*b*d*log(c)) +
((b*d*cos(8*a*d)*cos(4*a*d) + b*d*sin(8*a*d)*sin(4*a*d))*cos(4*b*d*log(c))
+ (b*d*cos(4*a*d)*sin(8*a*d) - b*d*cos(8*a*d)*sin(4*a*d))*sin(4*b*d*log(c))
)*sin(8*b*d*log(c)))*n*e^m + ((((cos(8*a*d)*cos(4*a*d) + sin(8*a*d)*sin(4*a
*d))*cos(4*b*d*log(c)) + (cos(4*a*d)*sin(8*a*d) - cos(8*a*d)*sin(4*a*d))*si
n(4*b*d*log(c)))*cos(8*b*d*log(c)) + cos(4*b*d*log(c))*cos(4*a*d) - ((cos(4
*a*d)*sin(8*a*d) - cos(8*a*d)*sin(4*a*d))*cos(4*b*d*log(c)) - (cos(8*a*d)*c
os(4*a*d) + sin(8*a*d)*sin(4*a*d))*sin(4*b*d*log(c)))*sin(8*b*d*log(c)) - s
in(4*b*d*log(c))*sin(4*a*d))*m^4 + 4*(((cos(8*a*d)*cos(4*a*d) + sin(8*a*d)*
sin(4*a*d))*cos(4*b*d*log(c)) + (cos(4*a*d)*sin(8*a*d) - cos(8*a*d)*sin(4*a
*d))*sin(4*b*d*log(c)))*cos(8*b*d*log(c)) + cos(4*b*d*log(c))*cos(4*a*d) -
((cos(4*a*d)*sin(8*a*d) - cos(8*a*d)*sin(4*a*d))*cos(4*b*d*log(c)) - (cos(8
*a*d)*cos(4*a*d) + sin(8*a*d)*sin(4*a*d))*sin(4*b*d*log(c)))*sin(8*b*d*log(
c)) - sin(4*b*d*log(c))*sin(4*a*d))*m^3 + 6*(((cos(8*a*d)*cos(4*a*d) + sin(
8*a*d)*sin(4*a*d))*cos(4*b*d*log(c)) + (cos(4*a*d)*sin(8*a*d) - cos(8*a*d)*
sin(4*a*d))*sin(4*b*d*log(c)))*cos(8*b*d*log(c)) + cos(4*b*d*log(c))*cos(4*
a*d) - ((cos(4*a*d)*sin(8*a*d) - cos(8*a*d)*sin...
```

Fricas [A]
time = 1.35, size = 461, normalized size = 1.37

$$\frac{4\left(\left(4\left(b^3d^3m+b^3d^3\right)n^3+\left(bdm^3+3\,bdm^2+3\,bdm+bd\right)n\right)x\cos\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)^3-\left(10\left(b^3d^3m+b^3d^3\right)n^3+\left(bdm^3+3\,bdm^2+3\,bdm+bd\right)n\right)x\cos\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)\right)e^{\left(m\log\left(x\right)+m\right)}\sin\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)+\left(\left(m^4+4\,m^3+4\left(b^2d^2m^2+2\,b^2d^2m+b^2d^2\right)n^2+6\,m^2+4\,m+1\right)x\cos\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)^4-2\left(m^4+4\,m^3+10\left(b^2d^2m^2+2\,b^2d^2m+b^2d^2\right)n^2+6\,m^2+4\,m+1\right)x\cos\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)^2+\left(24\,b^4d^4n^4+m^4+4\,m^3+16\left(b^2d^2m^2+2\,b^2d^2m+b^2d^2\right)n^2+6\,m^2+4\,m+1\right)x\right)e^{\left(m\log\left(x\right)+m\right)}}{m^5+64\left(b^4d^4m+b^4d^4\right)n^4+5\,m^4+10\,m^3+20\left(b^2d^2m^3+3\,b^2d^2m^2+3\,b^2d^2m+b^2d^2\right)n^2+10\,m^2+5\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^4,x, algorithm="fricas")
```

```
[Out] (4*((4*(b^3*d^3*m + b^3*d^3)*n^3 + (b*d*m^3 + 3*b*d*m^2 + 3*b*d*m + b*d)*n)
*x*cos(b*d*n*log(x) + b*d*log(c) + a*d)^3 - (10*(b^3*d^3*m + b^3*d^3)*n^3 +
 (b*d*m^3 + 3*b*d*m^2 + 3*b*d*m + b*d)*n)*x*cos(b*d*n*log(x) + b*d*log(c) +
 a*d))*e^(m*log(x) + m)*sin(b*d*n*log(x) + b*d*log(c) + a*d) + ((m^4 + 4*m^
3 + 4*(b^2*d^2*m^2 + 2*b^2*d^2*m + b^2*d^2)*n^2 + 6*m^2 + 4*m + 1)*x*cos(b*
```

```
d*n*log(x) + b*d*log(c) + a*d)^4 - 2*(m^4 + 4*m^3 + 10*(b^2*d^2*m^2 + 2*b^2
*d^2*m + b^2*d^2)*n^2 + 6*m^2 + 4*m + 1)*x*cos(b*d*n*log(x) + b*d*log(c) +
a*d)^2 + (24*b^4*d^4*n^4 + m^4 + 4*m^3 + 16*(b^2*d^2*m^2 + 2*b^2*d^2*m + b^
2*d^2)*n^2 + 6*m^2 + 4*m + 1)*x)*e^(m*log(x) + m))/(m^5 + 64*(b^4*d^4*m + b
^4*d^4)*n^4 + 5*m^4 + 10*m^3 + 20*(b^2*d^2*m^3 + 3*b^2*d^2*m^2 + 3*b^2*d^2*
m + b^2*d^2)*n^2 + 10*m^2 + 5*m + 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\begin{cases}\frac{\log(x)\cos(2ad)}{e} & \text{for } b=0 \wedge m=-1\\ \int (ex)^m \cos\left(-2ad+\frac{im\log(cx^n)}{n}+\frac{i\log(cx^n)}{n}\right)dx & \text{for } b=\frac{i(-m-1)}{2dn}\\ \int (ex)^m \cos\left(2ad+\frac{im\log(cx^n)}{n}+\frac{i\log(cx^n)}{n}\right)dx & \text{for } b=\frac{i(m+1)}{2dn}\\ \frac{2bdnx(ex)^m\sin(2ad+2bd\log(cx^n))}{4b^2d^2n^2+m^2+2m+1}+\frac{mx(ex)^m\cos(2ad+2bd\log(cx^n))}{4b^2d^2n^2+m^2+2m+1}+\frac{x(ex)^m\cos(2ad+2bd\log(cx^n))}{4b^2d^2n^2+m^2+2m+1} & \text{otherwise}\end{cases}}{2}+\frac{\begin{cases}\frac{\log(x)\cos(4ad)}{e} & \text{for } b=0 \wedge m=-1\\ \int (ex)^m \cos\left(-4ad+\frac{im\log(cx^n)}{n}+\frac{i\log(cx^n)}{n}\right)dx & \text{for } b=\frac{i(-m-1)}{4dn}\\ \int (ex)^m \cos\left(4ad+\frac{im\log(cx^n)}{n}+\frac{i\log(cx^n)}{n}\right)dx & \text{for } b=\frac{i(m+1)}{4dn}\\ \frac{4bdnx(ex)^m\sin(4ad+4bd\log(cx^n))}{16b^2d^2n^2+m^2+2m+1}+\frac{mx(ex)^m\cos(4ad+4bd\log(cx^n))}{16b^2d^2n^2+m^2+2m+1}+\frac{x(ex)^m\cos(4ad+4bd\log(cx^n))}{16b^2d^2n^2+m^2+2m+1} & \text{otherwise}\end{cases}}{8}+\frac{3\left(\begin{cases}\frac{(ex)^{m+1}}{m+1} & \text{for } m\neq -1\\ \log(ex) & \text{otherwise}\end{cases}\right)}{8e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sin(d*(a+b*ln(c*x**n)))**4,x)
```

```
[Out] -Piecewise((log(x)*cos(2*a*d)/e, Eq(b, 0) & Eq(m, -1)), (Integral((e*x)**m*
cos(-2*a*d + I*m*log(c*x**n)/n + I*log(c*x**n)/n), x), Eq(b, I*(-m - 1)/(2*
d*n))), (Integral((e*x)**m*cos(2*a*d + I*m*log(c*x**n)/n + I*log(c*x**n)/n)
, x), Eq(b, I*(m + 1)/(2*d*n))), (2*b*d*n*x*(e*x)**m*sin(2*a*d + 2*b*d*log(
c*x**n))/(4*b**2*d**2*n**2 + m**2 + 2*m + 1) + m*x*(e*x)**m*cos(2*a*d + 2*b
*d*log(c*x**n))/(4*b**2*d**2*n**2 + m**2 + 2*m + 1) + x*(e*x)**m*cos(2*a*d
+ 2*b*d*log(c*x**n))/(4*b**2*d**2*n**2 + m**2 + 2*m + 1), True))/2 + Piecew
ise((log(x)*cos(4*a*d)/e, Eq(b, 0) & Eq(m, -1)), (Integral((e*x)**m*cos(-4*
a*d + I*m*log(c*x**n)/n + I*log(c*x**n)/n), x), Eq(b, I*(-m - 1)/(4*d*n))),
 (Integral((e*x)**m*cos(4*a*d + I*m*log(c*x**n)/n + I*log(c*x**n)/n), x), E
q(b, I*(m + 1)/(4*d*n))), (4*b*d*n*x*(e*x)**m*sin(4*a*d + 4*b*d*log(c*x**n)
)/(16*b**2*d**2*n**2 + m**2 + 2*m + 1) + m*x*(e*x)**m*cos(4*a*d + 4*b*d*log
(c*x**n))/(16*b**2*d**2*n**2 + m**2 + 2*m + 1) + x*(e*x)**m*cos(4*a*d + 4*b
*d*log(c*x**n))/(16*b**2*d**2*n**2 + m**2 + 2*m + 1), True))/8 + 3*Piecewis
e(((e*x)**(m + 1)/(m + 1), Ne(m, -1)), (log(e*x), True))/(8*e)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 706991 vs. $2(337) = 674$.
time = 16.64, size = 706991, normalized size = 2097.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^4,x, algorithm="giac")
```

```
[Out] -1/16*(384*(abs(e)*abs(x))^m*b^4*d^4*n^4*x*tan(2*b*d*n*log(abs(x)) + 2*b*d*
log(abs(c)))^2*tan(b*d*n*log(abs(x)) + b*d*log(abs(c)))^2*tan(pi*m*floor(-1
/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)
^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(2*a*d)^2*tan(a*d
```

)^2 + 384*(abs(e)*abs(x))^m*b^4*d^4*n^4*x*tan(2*b*d*n*log(abs(x)) + 2*b*d*l
og(abs(c)))^2*tan(b*d*n*log(abs(x)) + b*d*log(abs(c)))^2*tan(pi*m*floor(-1/
4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^
2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(2*a*d)^2 + 384*(a
bs(e)*abs(x))^m*b^4*d^4*n^4*x*tan(2*b*d*n*log(abs(x)) + 2*b*d*log(abs(c)))^
2*tan(b*d*n*log(abs(x)) + b*d*log(abs(c)))^2*tan(pi*m*floor(-1/4*sgn(e) - 1
/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(1/4*pi
*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(a*d)^2 + 384*(abs(e)*abs(x))^
m*b^4*d^4*n^4*x*tan(2*b*d*n*log(abs(x)) + 2*b*d*log(abs(c)))^2*tan(b*d*n*lo
g(abs(x)) + b*d*log(abs(c)))^2*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1)
 + 1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(2*a*d)^2*tan(a*d)^2
 - 384*(abs(e)*abs(x))^m*b^4*d^4*n^4*x*tan(2*b*d*n*log(abs(x)) + 2*b*d*log(a
bs(c)))^2*tan(b*d*n*log(abs(x)) + b*d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) +
1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(2*a*d)^2*tan(a*d)^2 + 384*(abs(e)*abs(x))
^m*b^4*d^4*n^4*x*tan(2*b*d*n*log(abs(x)) + 2*b*d*log(abs(c)))^2*tan(pi*m*fl
oor(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2
*pi*m)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(2*a*d)^2*t
an(a*d)^2 + 384*(abs(e)*abs(x))^m*b^4*d^4*n^4*x*tan(b*d*n*log(abs(x)) + b*d
*log(abs(c)))^2*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn
(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) -
 1/2*pi*m)^2*tan(2*a*d)^2*tan(a*d)^2 + 256*b^3*d^3*m*n^3*x*e^(pi*b*d*n*sgn(
x) - pi*b*d*n + pi*b*d*sgn(c) - pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan
(2*b*d*n*log(abs(x)) + 2*b*d*log(abs(c)))^2*tan(b*d*n*log(abs(x)) + b*d*log
(abs(c)))^2*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e)
+ 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2
*pi*m)^2*tan(2*a*d)^2*tan(a*d) + 256*b^3*d^3*m*n^3*x*e^(-pi*b*d*n*sgn(x) +
pi*b*d*n - pi*b*d*sgn(c) + pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(2*b*
d*n*log(abs(x)) + 2*b*d*log(abs(c)))^2*tan(b*d*n*log(abs(x)) + b*d*log(abs(
c)))^2*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) + 1/4
*pi*m*sgn(x) - 1/2*pi*m)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m
)^2*tan(2*a*d)^2*tan(a*d) - 32*b^3*d^3*m*n^3*x*e^(2*pi*b*d*n*sgn(x) - 2*pi*
b*d*n + 2*pi*b*d*sgn(c) - 2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(2*b
*d*n*log(abs(x)) + 2*b*d*log(abs(c)))^2*tan(b*d*n*log(abs(x)) + b*d*log(abs
(c)))^2*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) + 1/
4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*
m)^2*tan(2*a*d)*tan(a*d)^2 - 32*b^3*d^3*m*n^3*x*e^(-2*pi*b*d*n*sgn(x) + 2*p
i*b*d*n - 2*pi*b*d*sgn(c) + 2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(2
*b*d*n*log(abs(x)) + 2*b*d*log(abs(c)))^2*tan(b*d*n*log(abs(x)) + b*d*log(a
bs(c)))^2*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) +
1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*p
i*m)^2*tan(2*a*d)*tan(a*d)^2 + 32*b^3*d^3*m*n^3*x*e^(2*pi*b*d*n*sgn(x) - 2*
pi*b*d*n + 2*pi*b*d*sgn(c) - 2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(
2*b*d*n*log(abs(x)) + 2*b*d*log(abs(c)))^2*tan(b*d*n*log(abs(x)) + b*d*log(
abs(c)))^2*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) +
 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*

```
pi*m)*tan(2*a*d)^2*tan(a*d)^2 - 256*b^3*d^3*m*n^3*x*e^(pi*b*d*n*sgn(x) - pi
*b*d*n + pi*b*d*sgn(c) - pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(2*b*d*
n*log(abs(x)) + 2*b*d*log(abs(c)))^2*tan(b*d*n*log(abs(x)) + b*d*log(abs(c)
))^2*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) + 1/4*p
i*m*sgn(x) - 1/2*pi*m)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)*
tan(2*a*d)^2*tan(a*d)^2 + 256*b^3*d^3*m*n^3*x*e^(-pi*b*d*n*sgn(x) + pi*b*d*
n - pi*b*d*sgn(c) + pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(2*b*d*n*log
(abs(x)) + 2*b*d*log(abs(c)))^2*tan(b*d*n*log(abs(x)) + b*d*log(abs(c)))^2*
tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) + 1/4*pi*m*s
gn(x) - 1/2*pi*m)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)*tan(2
*a*d)^2*tan(a*d)^2 - 32*b^3*d^3*m*n^3*x*e^(-2*pi*b*d*n*sgn(x) + 2*pi*b*d*n
- 2*pi*b*d*sgn(c) + 2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(2*b*d*n*l
og(abs(x)) + 2*b*d*log(abs(c)))^2*tan(b*d*n*log(abs(x)) + b*d*log(abs(c)))^
2*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) + 1/4*pi*m
*sgn(x) - 1/2*pi*m)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)*tan
(2*a*d)^2*tan(a*d)^2 + 256*b^3*d^3*m*n^3*x*e^(pi*b*d*n*sgn(x) - pi*b*d*n +
pi*b*d*sgn(c) - pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(2*b*d*n*log(abs
(x)) + 2*b*d*log(abs(c)))^2*tan(b*d*n*log(abs(x...
```

Mupad [B]
time = 4.04, size = 175, normalized size = 0.52

$$\frac{3\,x\,{\left(e\,x\right)}^m}{8\,m+8}-\frac{x\,{\mathrm{e}}^{a\,d\,2{}\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,d\,2{}\mathrm{i}}\,{\left(e\,x\right)}^m}{4\,m+4+b\,d\,n\,8{}\mathrm{i}}-\frac{x\,{\mathrm{e}}^{-a\,d\,2{}\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,d\,2{}\mathrm{i}}}\,{\left(e\,x\right)}^m\,1{}\mathrm{i}}{m\,4{}\mathrm{i}+8\,b\,d\,n+4{}\mathrm{i}}+\frac{x\,{\mathrm{e}}^{a\,d\,4{}\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,d\,4{}\mathrm{i}}\,{\left(e\,x\right)}^m}{16\,m+16+b\,d\,n\,64{}\mathrm{i}}+\frac{x\,{\mathrm{e}}^{-a\,d\,4{}\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,d\,4{}\mathrm{i}}}\,{\left(e\,x\right)}^m\,1{}\mathrm{i}}{m\,16{}\mathrm{i}+64\,b\,d\,n+16{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*(a + b*log(c*x^n)))^4*(e*x)^m,x)
```

```
[Out] (3*x*(e*x)^m)/(8*m + 8) - (x*exp(a*d*2i)*(c*x^n)^(b*d*2i)*(e*x)^m)/(4*m + b
*d*n*8i + 4) - (x*exp(-a*d*2i)/(c*x^n)^(b*d*2i)*(e*x)^m*1i)/(m*4i + 8*b*d*n
 + 4i) + (x*exp(a*d*4i)*(c*x^n)^(b*d*4i)*(e*x)^m)/(16*m + b*d*n*64i + 16) +
 (x*exp(-a*d*4i)/(c*x^n)^(b*d*4i)*(e*x)^m*1i)/(m*16i + 64*b*d*n + 16i)
```

3.71 $\int (ex)^m \sin^3(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=256

$$-\frac{6b^3d^3n^3(ex)^{1+m}\cos(d(a+b\log(cx^n)))}{e((1+m)^2+b^2d^2n^2)((1+m)^2+9b^2d^2n^2)}+\frac{6b^2d^2(1+m)n^2(ex)^{1+m}\sin(d(a+b\log(cx^n)))}{e((1+m)^2+b^2d^2n^2)((1+m)^2+9b^2d^2n^2)}-\frac{3bdn(ex)^{1+m}}{}$$

```
[Out] -6*b^3*d^3*n^3*(e*x)^(1+m)*cos(d*(a+b*ln(c*x^n)))/e/((1+m)^2+b^2*d^2*n^2)/(
(1+m)^2+9*b^2*d^2*n^2)+6*b^2*d^2*(1+m)*n^2*(e*x)^(1+m)*sin(d*(a+b*ln(c*x^n)
))/e/((1+m)^2+b^2*d^2*n^2)/((1+m)^2+9*b^2*d^2*n^2)-3*b*d*n*(e*x)^(1+m)*cos(
d*(a+b*ln(c*x^n)))*sin(d*(a+b*ln(c*x^n)))^2/e/((1+m)^2+9*b^2*d^2*n^2)+(1+m)
*(e*x)^(1+m)*sin(d*(a+b*ln(c*x^n)))^3/e/((1+m)^2+9*b^2*d^2*n^2)
```

Rubi [A]
time = 0.08, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4575, 4573}

$$\frac{(m+1)(ex)^{m+1}\sin^3(d(a+b\log(cx^n)))}{e(9b^2d^2n^2+(m+1)^2)}+\frac{6b^2d^2(m+1)n^2(ex)^{m+1}\sin(d(a+b\log(cx^n)))}{e(b^2d^2n^2+(m+1)^2)(9b^2d^2n^2+(m+1)^2)}-\frac{3bdn(ex)^{m+1}\sin^2(d(a+b\log(cx^n)))\cos(d(a+b\log(cx^n)))}{e(9b^2d^2n^2+(m+1)^2)}-\frac{6b^3d^3n^3(ex)^{m+1}\cos(d(a+b\log(cx^n)))}{e(b^2d^2n^2+(m+1)^2)(9b^2d^2n^2+(m+1)^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^3,x]
```

```
[Out] (-6*b^3*d^3*n^3*(e*x)^(1 + m)*Cos[d*(a + b*Log[c*x^n])])/(e*((1 + m)^2 + b^
2*d^2*n^2)*((1 + m)^2 + 9*b^2*d^2*n^2)) + (6*b^2*d^2*(1 + m)*n^2*(e*x)^(1 +
 m)*Sin[d*(a + b*Log[c*x^n])])/(e*((1 + m)^2 + b^2*d^2*n^2)*((1 + m)^2 + 9*
b^2*d^2*n^2)) - (3*b*d*n*(e*x)^(1 + m)*Cos[d*(a + b*Log[c*x^n])]*Sin[d*(a +
 b*Log[c*x^n])]^2)/(e*((1 + m)^2 + 9*b^2*d^2*n^2)) + ((1 + m)*(e*x)^(1 + m)
*Sin[d*(a + b*Log[c*x^n])]^3)/(e*((1 + m)^2 + 9*b^2*d^2*n^2))
```

Rule 4573

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] - Simp[b*d*n*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Rule 4575

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2
*n^2*p^2 + (m + 1)^2)), Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x],
x] - Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a + b*Log[c*x^n])]*(Sin[d*(a + b*Log
[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x]) /; FreeQ[{a, b, c
```

```
, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int (ex)^m \sin^3\left(d(a+b\log\left(cx^n\right))\right)\,dx = -\frac{3bdn(ex)^{1+m}\cos\left(d(a+b\log\left(cx^n\right))\right)\sin^2\left(d(a+b\log\left(cx^n\right))\right)}{e\left((1+m)^2+9b^2d^2n^2\right)} + \frac{(1 \cdots}{}$$

$$= -\frac{6b^3d^3n^3(ex)^{1+m}\cos\left(d(a+b\log\left(cx^n\right))\right)}{e\left((1+m)^2+b^2d^2n^2\right)\left((1+m)^2+9b^2d^2n^2\right)} + \frac{6b^2d^2(1+m)n^2(ex)\cdots}{e\left((1+m)^2+b^2d^2\cdots\right.}$$

Mathematica [A]
time = 1.37, size = 326, normalized size = 1.27

$$\frac{1}{4}x(ex)^m\left(\frac{3\cos(bdn\log(x))\left(-bdn\cos\left(d\left(a-bn\log(x)+b\log\left(cx^n\right)\right)\right)+(1+m)\sin\left(d\left(a-bn\log(x)+b\log\left(cx^n\right)\right)\right)\right)}{1+2m+m^2+b^2d^2n^2}+\frac{3\sin(bdn\log(x))\left((1+m)\cos\left(d\left(a-bn\log(x)+b\log\left(cx^n\right)\right)\right)+bdn\sin\left(d\left(a-bn\log(x)+b\log\left(cx^n\right)\right)\right)\right)}{1+2m+m^2+b^2d^2n^2}-\frac{\cos(3bdn\log(x))\left(-3bdn\cos\left(3d\left(a-bn\log(x)+b\log\left(cx^n\right)\right)\right)+(1+m)\sin\left(3d\left(a-bn\log(x)+b\log\left(cx^n\right)\right)\right)\right)}{1+2m+m^2+9b^2d^2n^2}-\frac{\sin(3bdn\log(x))\left((1+m)\cos\left(3d\left(a-bn\log(x)+b\log\left(cx^n\right)\right)\right)+3bdn\sin\left(3d\left(a-bn\log(x)+b\log\left(cx^n\right)\right)\right)\right)}{1+2m+m^2+9b^2d^2n^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^3,x]
```

```
[Out] (x*(e*x)^m*((3*Cos[b*d*n*Log[x]]*(-(b*d*n*Cos[d*(a - b*n*Log[x] + b*Log[c*x
^n])]) + (1 + m)*Sin[d*(a - b*n*Log[x] + b*Log[c*x^n])]))/(1 + 2*m + m^2 +
b^2*d^2*n^2) + (3*Sin[b*d*n*Log[x]]*((1 + m)*Cos[d*(a - b*n*Log[x] + b*Log[
c*x^n])] + b*d*n*Sin[d*(a - b*n*Log[x] + b*Log[c*x^n])]))/(1 + 2*m + m^2 +
b^2*d^2*n^2) - (Cos[3*b*d*n*Log[x]]*(-3*b*d*n*Cos[3*d*(a - b*n*Log[x] + b*L
og[c*x^n])] + (1 + m)*Sin[3*d*(a - b*n*Log[x] + b*Log[c*x^n])]))/(1 + 2*m +
 m^2 + 9*b^2*d^2*n^2) - (Sin[3*b*d*n*Log[x]]*((1 + m)*Cos[3*d*(a - b*n*Log[
x] + b*Log[c*x^n])] + 3*b*d*n*Sin[3*d*(a - b*n*Log[x] + b*Log[c*x^n])]))/(1
 + 2*m + m^2 + 9*b^2*d^2*n^2)))/4
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\sin^3\left(d(a+b\ln\left(c\,x^n\right))\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sin(d*(a+b*ln(c*x^n)))^3,x)
```

```
[Out] int((e*x)^m*sin(d*(a+b*ln(c*x^n)))^3,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 11390 vs. $2(256) = 512$.
time = 0.63, size = 11390, normalized size = 44.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*(b^3*d^3*cos(3*b*d*log(c))*cos(3*a*d) - b^3*d^3*sin(3*b*d*log(c))*s
in(3*a*d) + ((b^3*d^3*cos(6*a*d)*cos(3*a*d) + b^3*d^3*sin(6*a*d)*sin(3*a*d)
)*cos(3*b*d*log(c)) + (b^3*d^3*cos(3*a*d)*sin(6*a*d) - b^3*d^3*cos(6*a*d)*s
in(3*a*d))*sin(3*b*d*log(c)))*cos(6*b*d*log(c)) - ((b^3*d^3*cos(3*a*d)*sin(
6*a*d) - b^3*d^3*cos(6*a*d)*sin(3*a*d))*cos(3*b*d*log(c)) - (b^3*d^3*cos(6*
a*d)*cos(3*a*d) + b^3*d^3*sin(6*a*d)*sin(3*a*d))*sin(3*b*d*log(c)))*sin(6*b
*d*log(c)))*n^3*e^m - (b^2*d^2*cos(3*a*d)*sin(3*b*d*log(c)) + b^2*d^2*cos(3
*b*d*log(c))*sin(3*a*d) + (b^2*d^2*cos(3*a*d)*sin(3*b*d*log(c)) + b^2*d^2*c
os(3*b*d*log(c))*sin(3*a*d) + ((b^2*d^2*cos(3*a*d)*sin(6*a*d) - b^2*d^2*cos
(6*a*d)*sin(3*a*d))*cos(3*b*d*log(c)) - (b^2*d^2*cos(6*a*d)*cos(3*a*d) + b^
2*d^2*sin(6*a*d)*sin(3*a*d))*sin(3*b*d*log(c)))*cos(6*b*d*log(c)) + ((b^2*d
^2*cos(6*a*d)*cos(3*a*d) + b^2*d^2*sin(6*a*d)*sin(3*a*d))*cos(3*b*d*log(c))
 + (b^2*d^2*cos(3*a*d)*sin(6*a*d) - b^2*d^2*cos(6*a*d)*sin(3*a*d))*sin(3*b*
d*log(c)))*sin(6*b*d*log(c)))*m + ((b^2*d^2*cos(3*a*d)*sin(6*a*d) - b^2*d^2
*cos(6*a*d)*sin(3*a*d))*cos(3*b*d*log(c)) - (b^2*d^2*cos(6*a*d)*cos(3*a*d)
+ b^2*d^2*sin(6*a*d)*sin(3*a*d))*sin(3*b*d*log(c)))*cos(6*b*d*log(c)) + ((b
^2*d^2*cos(6*a*d)*cos(3*a*d) + b^2*d^2*sin(6*a*d)*sin(3*a*d))*cos(3*b*d*log
(c)) + (b^2*d^2*cos(3*a*d)*sin(6*a*d) - b^2*d^2*cos(6*a*d)*sin(3*a*d))*sin(
3*b*d*log(c)))*sin(6*b*d*log(c)))*n^2*e^m + 3*(b*d*cos(3*b*d*log(c))*cos(3*
a*d) - b*d*sin(3*b*d*log(c))*sin(3*a*d) + (b*d*cos(3*b*d*log(c))*cos(3*a*d)
 - b*d*sin(3*b*d*log(c))*sin(3*a*d) + ((b*d*cos(6*a*d)*cos(3*a*d) + b*d*sin
(6*a*d)*sin(3*a*d))*cos(3*b*d*log(c)) + (b*d*cos(3*a*d)*sin(6*a*d) - b*d*co
s(6*a*d)*sin(3*a*d))*sin(3*b*d*log(c)))*cos(6*b*d*log(c)) - ((b*d*cos(3*a*d
)*sin(6*a*d) - b*d*cos(6*a*d)*sin(3*a*d))*cos(3*b*d*log(c)) - (b*d*cos(6*a*
d)*cos(3*a*d) + b*d*sin(6*a*d)*sin(3*a*d))*sin(3*b*d*log(c)))*sin(6*b*d*log
(c)))*m^2 + 2*(b*d*cos(3*b*d*log(c))*cos(3*a*d) - b*d*sin(3*b*d*log(c))*sin
(3*a*d) + ((b*d*cos(6*a*d)*cos(3*a*d) + b*d*sin(6*a*d)*sin(3*a*d))*cos(3*b*
d*log(c)) + (b*d*cos(3*a*d)*sin(6*a*d) - b*d*cos(6*a*d)*sin(3*a*d))*sin(3*b
*d*log(c)))*cos(6*b*d*log(c)) - ((b*d*cos(3*a*d)*sin(6*a*d) - b*d*cos(6*a*d
)*sin(3*a*d))*cos(3*b*d*log(c)) - (b*d*cos(6*a*d)*cos(3*a*d) + b*d*sin(6*a*
d)*sin(3*a*d))*sin(3*b*d*log(c)))*sin(6*b*d*log(c)))*m + ((b*d*cos(6*a*d)*c
os(3*a*d) + b*d*sin(6*a*d)*sin(3*a*d))*cos(3*b*d*log(c)) + (b*d*cos(3*a*d)*
sin(6*a*d) - b*d*cos(6*a*d)*sin(3*a*d))*sin(3*b*d*log(c)))*cos(6*b*d*log(c)
) - ((b*d*cos(3*a*d)*sin(6*a*d) - b*d*cos(6*a*d)*sin(3*a*d))*cos(3*b*d*log(
c)) - (b*d*cos(6*a*d)*cos(3*a*d) + b*d*sin(6*a*d)*sin(3*a*d))*sin(3*b*d*log
(c)))*sin(6*b*d*log(c)))*n*e^m - ((((cos(3*a*d)*sin(6*a*d) - cos(6*a*d)*sin
(3*a*d))*cos(3*b*d*log(c)) - (cos(6*a*d)*cos(3*a*d) + sin(6*a*d)*sin(3*a*d)
)*sin(3*b*d*log(c)))*cos(6*b*d*log(c)) + ((cos(6*a*d)*cos(3*a*d) + sin(6*a*
d)*sin(3*a*d))*cos(3*b*d*log(c)) + (cos(3*a*d)*sin(6*a*d) - cos(6*a*d)*sin(
3*a*d))*sin(3*b*d*log(c)))*sin(6*b*d*log(c)) + cos(3*a*d)*sin(3*b*d*log(c))
 + cos(3*b*d*log(c))*sin(3*a*d))*m^3 + 3*(((cos(3*a*d)*sin(6*a*d) - cos(6*a
```

```
*d)*sin(3*a*d))*cos(3*b*d*log(c)) - (cos(6*a*d)*cos(3*a*d) + sin(6*a*d)*sin
(3*a*d))*sin(3*b*d*log(c)))*cos(6*b*d*log(c)) + ((cos(6*a*d)*cos(3*a*d) + s
in(6*a*d)*sin(3*a*d))*cos(3*b*d*log(c)) + (cos(3*a*d)*sin(6*a*d) - cos(6*a*
d)*sin(3*a*d))*sin(3*b*d*log(c)))*sin(6*b*d*log(c)) + cos(3*a*d)*sin(3*b*d*
log(c)) + cos(3*b*d*log(c))*sin(3*a*d))*m^2 + 3*(((cos(3*a*d)*sin(6*a*d) -
cos(6*a*d)*sin(3*a*d))*cos(3*b*d*log(c)) - (cos(6*a*d)*cos(3*a*d) + sin(6*a
*d)*sin(3*a*d))*sin(3*b*d*log(c)))*cos(6*b*d*log(c)) + ((cos(6*a*d)*cos(3*a
*d) + sin(6*a*d)*sin(3*a*d))*cos(3*b*d*log(c)) + (cos(3*a*d)*sin(6*a*d) - c
os(6*a*d)*sin(3*a*d))*sin(3*b*d*log(c)))*sin(6*b*d*log(c)) + cos(3*a*d)*sin
(3*b*d*log(c)) + cos(3*b*d*log(c))*sin(3*a*d))*m + ((cos(3*a*d)*sin(6*a*d)
- cos(6*a*d)*sin(3*a*d))*cos(3*b*d*log(c)) - (cos(6*a*d)*cos(3*a*d) + sin(6
*a*d)*sin(3*a*d))*sin(3*b*d*log(c)))*cos(6*b*d*log(c)) + ((cos(6*a*d)*cos(3
*a*d) + sin(6*a*d)*sin(3*a*d))*cos(3*b*d*log(c)) + (cos(3*a*d)*sin(6*a*d) -
 cos(6*a*d)*sin(3*a*d))*sin(3*b*d*log(c)))*sin(6*b*d*log(c)) + cos(3*a*d)*s
in(3*b*d*log(c)) + cos(3*b*d*log(c))*sin(3*a*d))*e^m)*x*x^m*cos(3*b*d*log(x
^n)) - 3*(9*(((b^3*d^3*cos(4*a*d)*cos(3*a*d) + b^3*d^3*sin(4*a*d)*sin(3*a*d
))*cos(3*b*d*log(c)) + (b^3*d^3*cos(3*a*d)*sin(4*a*d) - b^3*d^3*cos(4*a*d)*
sin(3*a*d))*sin(3*b*d*log(c)))*cos(4*b*d*log(c)) + ((b^3*d^3*cos(3*a*d)*cos
(2*a*d) + b^3*d^3*sin(3*a*d)*sin(2*a*d))*cos(2*b*d*log(c)) + (b^3*d^3*cos(2
*a*d)*sin(3*a*d) - b^3*d^3*cos(3*a*d)*sin(2*a*d))*sin(2*b*d*log(c)))*cos(3*
b*d*log(c)) - ((b^3*d^3*cos(3*a*d)*sin(4*a*d) - b^3*d^3*cos(4*a*d)*sin(3*a*
d))*cos(3*b*d*log(c)) - (b^3*d^3*cos(4*a*d)*cos(3*a*d) + b^3*d^3*sin(4*a*d)
*sin(3*a*d))*sin(3*b*d*log(c)))*sin(4*b*d*log(c)) - ((b^3*d^3*cos(2*a*d)*si
n(3*a*d) - b^3*d^3*cos(3*a*d)*sin(2*a*d))*cos(2*b*d*log(c)) - (b^3*d^3*cos(
3*a*d)*cos(2*a*d) + b^3*d^3*sin(3*a*d)*sin(2*a*d))*sin(2*b*d*log(c)))*sin(3
*b*d*log(c)))*n^3*e^m - 9*((((b^2*d^2*cos(3*a*d...
```

Fricas [A]
time = 1.16, size = 287, normalized size = 1.12

$$-\frac{\left((m^3 + (b^2d^2m + b^2d^2)n^2 + 3\,m^2 + 3\,m + 1)x\cos\left(bdn\log\left(x\right) + bd\log\left(c\right) + ad\right)^2 - (m^3 + 7\,(b^2d^2m + b^2d^2)n^2 + 3\,m^2 + 3\,m + 1)x\right)e^{(m\log(x)+m)}\sin\left(bdn\log\left(x\right) + bd\log\left(c\right) + ad\right) - 3\,\left((b^3d^3n^3 + (bdm^2 + 2\,bdm + bd)n)x\cos\left(bdn\log\left(x\right) + bd\log\left(c\right) + ad\right)^3 - (3\,b^3d^3n^3 + (bdm^2 + 2\,bdm + bd)n)x\cos\left(bdn\log\left(x\right) + bd\log\left(c\right) + ad\right)\right)e^{(m\log(x)+m)}}{9\,b^4d^4n^4 + m^4 + 4\,m^3 + 10\,(b^2d^2m^2 + 2\,b^2d^2m + b^2d^2)n^2 + 6\,m^2 + 4\,m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^3,x, algorithm="fricas")
```

```
[Out] -(((m^3 + (b^2*d^2*m + b^2*d^2)*n^2 + 3*m^2 + 3*m + 1)*x*cos(b*d*n*log(x) +
 b*d*log(c) + a*d)^2 - (m^3 + 7*(b^2*d^2*m + b^2*d^2)*n^2 + 3*m^2 + 3*m + 1
)*x)*e^(m*log(x) + m)*sin(b*d*n*log(x) + b*d*log(c) + a*d) - 3*((b^3*d^3*n^
3 + (b*d*m^2 + 2*b*d*m + b*d)*n)*x*cos(b*d*n*log(x) + b*d*log(c) + a*d)^3 -
 (3*b^3*d^3*n^3 + (b*d*m^2 + 2*b*d*m + b*d)*n)*x*cos(b*d*n*log(x) + b*d*log
(c) + a*d))*e^(m*log(x) + m))/(9*b^4*d^4*n^4 + m^4 + 4*m^3 + 10*(b^2*d^2*m^
2 + 2*b^2*d^2*m + b^2*d^2)*n^2 + 6*m^2 + 4*m + 1)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{3\left(\begin{cases} \frac{\log(x)\sin(ad)}{e} & \text{for } b = 0 \wedge m = -1 \\ -\int (ex)^m \sin\left(-ad + \frac{im\log(cx^n)}{n} + \frac{i\log(cx^n)}{n}\right) dx & \text{for } b = \frac{i(-m-1)}{dn} \\ \int (ex)^m \sin\left(ad + \frac{im\log(cx^n)}{n} + \frac{i\log(cx^n)}{n}\right) dx & \text{for } b = \frac{i(m+1)}{dn} \\ -\frac{bdnx(ex)^m \cos(ad+bd\log(cx^n))}{b^2d^2n^2+m^2+2m+1} + \frac{mx(ex)^m \sin(ad+bd\log(cx^n))}{b^2d^2n^2+m^2+2m+1} + \frac{x(ex)^m \sin(ad+bd\log(cx^n))}{b^2d^2n^2+m^2+2m+1} & \text{otherwise} \end{cases}\right)}{4} - \frac{\begin{cases} \frac{\log(x)\sin(3ad)}{e} & \text{for } b = 0 \wedge m = -1 \\ -\int (ex)^m \sin\left(-3ad + \frac{im\log(cx^n)}{n} + \frac{i\log(cx^n)}{n}\right) dx & \text{for } b = \frac{i(-m-1)}{3dn} \\ \int (ex)^m \sin\left(3ad + \frac{im\log(cx^n)}{n} + \frac{i\log(cx^n)}{n}\right) dx & \text{for } b = \frac{i(m+1)}{3dn} \\ -\frac{3bdnx(ex)^m \cos(3ad+3bd\log(cx^n))}{9b^2d^2n^2+m^2+2m+1} + \frac{mx(ex)^m \sin(3ad+3bd\log(cx^n))}{9b^2d^2n^2+m^2+2m+1} + \frac{x(ex)^m \sin(3ad+3bd\log(cx^n))}{9b^2d^2n^2+m^2+2m+1} & \text{otherwise} \end{cases}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sin(d*(a+b*ln(c*x**n)))**3,x)
```

```
[Out] 3*Piecewise((log(x)*sin(a*d)/e, Eq(b, 0) & Eq(m, -1)), (-Integral((e*x)**m*
sin(-a*d + I*m*log(c*x**n)/n + I*log(c*x**n)/n), x), Eq(b, I*(-m - 1)/(d*n)
)), (Integral((e*x)**m*sin(a*d + I*m*log(c*x**n)/n + I*log(c*x**n)/n), x),
Eq(b, I*(m + 1)/(d*n))), (-b*d*n*x*(e*x)**m*cos(a*d + b*d*log(c*x**n))/(b**
2*d**2*n**2 + m**2 + 2*m + 1) + m*x*(e*x)**m*sin(a*d + b*d*log(c*x**n))/(b*
*2*d**2*n**2 + m**2 + 2*m + 1) + x*(e*x)**m*sin(a*d + b*d*log(c*x**n))/(b**
2*d**2*n**2 + m**2 + 2*m + 1), True))/4 - Piecewise((log(x)*sin(3*a*d)/e, E
q(b, 0) & Eq(m, -1)), (-Integral((e*x)**m*sin(-3*a*d + I*m*log(c*x**n)/n +
I*log(c*x**n)/n), x), Eq(b, I*(-m - 1)/(3*d*n))), (Integral((e*x)**m*sin(3*
a*d + I*m*log(c*x**n)/n + I*log(c*x**n)/n), x), Eq(b, I*(m + 1)/(3*d*n))),
(-3*b*d*n*x*(e*x)**m*cos(3*a*d + 3*b*d*log(c*x**n))/(9*b**2*d**2*n**2 + m**
2 + 2*m + 1) + m*x*(e*x)**m*sin(3*a*d + 3*b*d*log(c*x**n))/(9*b**2*d**2*n**
2 + m**2 + 2*m + 1) + x*(e*x)**m*sin(3*a*d + 3*b*d*log(c*x**n))/(9*b**2*d**
2*n**2 + m**2 + 2*m + 1), True))/4
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 200416 vs. $2(256) = 512$.
time = 5.29, size = 200416, normalized size = 782.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*b^3*d^3*n^3*x*e^(3/2*pi*b*d*n*sgn(x) - 3/2*pi*b*d*n + 3/2*pi*b*d*sg
n(c) - 3/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(3/2*b*d*n*log(abs(x)
) + 3/2*b*d*log(abs(c)))^2*tan(1/2*b*d*n*log(abs(x)) + 1/2*b*d*log(abs(c)))
^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(3/2*a*d)^2*tan(1
/2*a*d)^2 - 27*b^3*d^3*n^3*x*e^(1/2*pi*b*d*n*sgn(x) - 1/2*pi*b*d*n + 1/2*pi
*b*d*sgn(c) - 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(3/2*b*d*n*log
(abs(x)) + 3/2*b*d*log(abs(c)))^2*tan(1/2*b*d*n*log(abs(x)) + 1/2*b*d*log(a
bs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(3/2*a*d)^
2*tan(1/2*a*d)^2 - 27*b^3*d^3*n^3*x*e^(-1/2*pi*b*d*n*sgn(x) + 1/2*pi*b*d*n
- 1/2*pi*b*d*sgn(c) + 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(3/2*b
*d*n*log(abs(x)) + 3/2*b*d*log(abs(c)))^2*tan(1/2*b*d*n*log(abs(x)) + 1/2*b
*d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(3
```

/2*a*d)^2*tan(1/2*a*d)^2 + 3*b^3*d^3*n^3*x*e^(-3/2*pi*b*d*n*sgn(x) + 3/2*pi
*b*d*n - 3/2*pi*b*d*sgn(c) + 3/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*ta
n(3/2*b*d*n*log(abs(x)) + 3/2*b*d*log(abs(c)))^2*tan(1/2*b*d*n*log(abs(x))
+ 1/2*b*d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^
2*tan(3/2*a*d)^2*tan(1/2*a*d)^2 + 3*b^3*d^3*n^3*x*e^(3/2*pi*b*d*n*sgn(x) -
3/2*pi*b*d*n + 3/2*pi*b*d*sgn(c) - 3/2*pi*b*d + m*log(abs(e)) + m*log(abs(x
)))*tan(3/2*b*d*n*log(abs(x)) + 3/2*b*d*log(abs(c)))^2*tan(1/2*b*d*n*log(ab
s(x)) + 1/2*b*d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*
pi*m)^2*tan(3/2*a*d)^2 + 27*b^3*d^3*n^3*x*e^(1/2*pi*b*d*n*sgn(x) - 1/2*pi*b
*d*n + 1/2*pi*b*d*sgn(c) - 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(
3/2*b*d*n*log(abs(x)) + 3/2*b*d*log(abs(c)))^2*tan(1/2*b*d*n*log(abs(x)) +
1/2*b*d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*
tan(3/2*a*d)^2 + 27*b^3*d^3*n^3*x*e^(-1/2*pi*b*d*n*sgn(x) + 1/2*pi*b*d*n -
1/2*pi*b*d*sgn(c) + 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(3/2*b*d
*n*log(abs(x)) + 3/2*b*d*log(abs(c)))^2*tan(1/2*b*d*n*log(abs(x)) + 1/2*b*d
*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(3/2
*a*d)^2 + 3*b^3*d^3*n^3*x*e^(-3/2*pi*b*d*n*sgn(x) + 3/2*pi*b*d*n - 3/2*pi*b
*d*sgn(c) + 3/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(3/2*b*d*n*log(a
bs(x)) + 3/2*b*d*log(abs(c)))^2*tan(1/2*b*d*n*log(abs(x)) + 1/2*b*d*log(abs
(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(3/2*a*d)^2
- 108*b^3*d^3*n^3*x*e^(1/2*pi*b*d*n*sgn(x) - 1/2*pi*b*d*n + 1/2*pi*b*d*sgn(
c) - 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(3/2*b*d*n*log(abs(x))
+ 3/2*b*d*log(abs(c)))^2*tan(1/2*b*d*n*log(abs(x)) + 1/2*b*d*log(abs(c)))^2
*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)*tan(3/2*a*d)^2*tan(1/2*a
*d) + 108*b^3*d^3*n^3*x*e^(-1/2*pi*b*d*n*sgn(x) + 1/2*pi*b*d*n - 1/2*pi*b*d
*sgn(c) + 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(3/2*b*d*n*log(abs
(x)) + 3/2*b*d*log(abs(c)))^2*tan(1/2*b*d*n*log(abs(x)) + 1/2*b*d*log(abs(c
)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)*tan(3/2*a*d)^2*tan(
1/2*a*d) + 108*b^3*d^3*n^3*x*e^(1/2*pi*b*d*n*sgn(x) - 1/2*pi*b*d*n + 1/2*pi
*b*d*sgn(c) - 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(3/2*b*d*n*log
(abs(x)) + 3/2*b*d*log(abs(c)))^2*tan(1/2*b*d*n*log(abs(x)) + 1/2*b*d*log(a
bs(c)))*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(3/2*a*d)^2*
tan(1/2*a*d) + 108*b^3*d^3*n^3*x*e^(-1/2*pi*b*d*n*sgn(x) + 1/2*pi*b*d*n - 1
/2*pi*b*d*sgn(c) + 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(3/2*b*d*
n*log(abs(x)) + 3/2*b*d*log(abs(c)))^2*tan(1/2*b*d*n*log(abs(x)) + 1/2*b*d*
log(abs(c)))*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(3/2*a*
d)^2*tan(1/2*a*d) - 3*b^3*d^3*n^3*x*e^(3/2*pi*b*d*n*sgn(x) - 3/2*pi*b*d*n +
3/2*pi*b*d*sgn(c) - 3/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(3/2*b*
d*n*log(abs(x)) + 3/2*b*d*log(abs(c)))^2*tan(1/2*b*d*n*log(abs(x)) + 1/2*b*
d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(1/
2*a*d)^2 - 27*b^3*d^3*n^3*x*e^(1/2*pi*b*d*n*sgn(x) - 1/2*pi*b*d*n + 1/2*pi*
b*d*sgn(c) - 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(3/2*b*d*n*log(
abs(x)) + 3/2*b*d*log(abs(c)))^2*tan(1/2*b*d*n*log(abs(x)) + 1/2*b*d*log(ab
s(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(1/2*a*d)^2
- 27*b^3*d^3*n^3*x*e^(-1/2*pi*b*d*n*sgn(x) + 1/2*pi*b*d*n - 1/2*pi*b*d*sgn

```
(c) + 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(3/2*b*d*n*log(abs(x))
 + 3/2*b*d*log(abs(c)))^2*tan(1/2*b*d*n*log(abs(x)) + 1/2*b*d*log(abs(c)))^
2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(1/2*a*d)^2 - 3*b^
3*d^3*n^3*x*e^(-3/2*pi*b*d*n*sgn(x) + 3/2*pi*b*d*n - 3/2*pi*b*d*sgn(c) + 3/
2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(3/2*b*d*n*log(abs(x)) + 3/2*b
*d*log(abs(c)))^2*tan(1/2*b*d*n*log(abs(x)) + 1/2*b*d*log(abs(c)))^2*tan(1/
4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(1/2*a*d)^2 + 12*b^3*d^3*n
^3*x*e^(3/2*pi*b*d*n*sgn(x) - 3/2*pi*b*d*n + 3/2*pi*b*d*sgn(c) - 3/2*pi*b*d
 + m*log(abs(e)) + m*log(abs(x)))*tan(3/2*b*d*n*log(abs(x)) + 3/2*b*d*log(a
bs(c)))^2*tan(1/2*b*d*n*log(abs(x)) + 1/2*b*d*l...
```

Mupad [B]
time = 3.93, size = 161, normalized size = 0.63

$$\frac{x\,\mathrm{e}^{-a\,d\,1\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,d\,1\mathrm{i}}}\,{\left(e\,x\right)}^m\,3\mathrm{i}}{8\,m+8-b\,d\,n\,8\mathrm{i}}+\frac{3\,x\,\mathrm{e}^{a\,d\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,d\,1\mathrm{i}}\,{\left(e\,x\right)}^m}{m\,8\mathrm{i}-8\,b\,d\,n+8\mathrm{i}}-\frac{x\,\mathrm{e}^{-a\,d\,3\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,d\,3\mathrm{i}}}\,{\left(e\,x\right)}^m\,1\mathrm{i}}{8\,m+8-b\,d\,n\,24\mathrm{i}}-\frac{x\,\mathrm{e}^{a\,d\,3\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,d\,3\mathrm{i}}\,{\left(e\,x\right)}^m}{m\,8\mathrm{i}-24\,b\,d\,n+8\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*(a + b*log(c*x^n)))^3*(e*x)^m,x)
```

```
[Out] (x*exp(-a*d*1i)/(c*x^n)^(b*d*1i)*(e*x)^m*3i)/(8*m - b*d*n*8i + 8) + (3*x*ex
p(a*d*1i)*(c*x^n)^(b*d*1i)*(e*x)^m)/(m*8i - 8*b*d*n + 8i) - (x*exp(-a*d*3i)
/(c*x^n)^(b*d*3i)*(e*x)^m*1i)/(8*m - b*d*n*24i + 8) - (x*exp(a*d*3i)*(c*x^n
)^(b*d*3i)*(e*x)^m)/(m*8i - 24*b*d*n + 8i)
```

3.72 $\int (ex)^m \sin^2(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=154

$$\frac{2b^2d^2n^2(ex)^{1+m}}{e(1+m)\left((1+m)^2+4b^2d^2n^2\right)}-\frac{2bdn(ex)^{1+m}\cos\left(d(a+b\log\left(cx^n\right))\right)\sin\left(d(a+b\log\left(cx^n\right))\right)}{e\left((1+m)^2+4b^2d^2n^2\right)}+\frac{(1+m)(ex)^{1}}{e((1}$$

```
[Out] 2*b^2*d^2*n^2*(e*x)^(1+m)/e/(1+m)/((1+m)^2+4*b^2*d^2*n^2)-2*b*d*n*(e*x)^(1+
m)*cos(d*(a+b*ln(c*x^n)))*sin(d*(a+b*ln(c*x^n)))/e/((1+m)^2+4*b^2*d^2*n^2)+
(1+m)*(e*x)^(1+m)*sin(d*(a+b*ln(c*x^n)))^2/e/((1+m)^2+4*b^2*d^2*n^2)
```

Rubi [A]
time = 0.03, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4575, 32}

$$\frac{(m+1)(ex)^{m+1}\sin^2\left(d(a+b\log\left(cx^n\right))\right)}{e\left(4b^2d^2n^2+(m+1)^2\right)}-\frac{2bdn(ex)^{m+1}\sin\left(d(a+b\log\left(cx^n\right))\right)\cos\left(d(a+b\log\left(cx^n\right))\right)}{e\left(4b^2d^2n^2+(m+1)^2\right)}+\frac{2b^2d^2n^2(ex)^{m+1}}{e(m+1)\left(4b^2d^2n^2+(m+1)^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (2*b^2*d^2*n^2*(e*x)^(1 + m))/(e*(1 + m)*((1 + m)^2 + 4*b^2*d^2*n^2)) - (2*
b*d*n*(e*x)^(1 + m)*Cos[d*(a + b*Log[c*x^n])]*Sin[d*(a + b*Log[c*x^n])])/(e
*((1 + m)^2 + 4*b^2*d^2*n^2)) + ((1 + m)*(e*x)^(1 + m)*Sin[d*(a + b*Log[c*x
^n])]^2)/(e*((1 + m)^2 + 4*b^2*d^2*n^2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 4575

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2
*n^2*p^2 + (m + 1)^2)), Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x],
x] - Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a + b*Log[c*x^n])]*(Sin[d*(a + b*Log
[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x]) /; FreeQ[{a, b, c
, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int (ex)^m \sin^2 \left(d(a+b\log \left(cx^n\right))\right) \, dx = -\frac{2bdn(ex)^{1+m} \cos \left(d(a+b\log \left(cx^n\right))\right) \sin \left(d(a+b\log \left(cx^n\right))\right)}{e\left((1+m)^2+4b^2d^2n^2\right)} + \frac{(1+
$$
$$
= \frac{2b^2d^2n^2(ex)^{1+m}}{e(1+m)\left((1+m)^2+4b^2d^2n^2\right)} - \frac{2bdn(ex)^{1+m} \cos \left(d(a+b\log \left(cx^n\right))\right)}{e\left((1+m)^2+4b^2\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.32, size = 102, normalized size = 0.66

$$-\frac{x(ex)^m \left(-1-2m-m^2-4b^2d^2n^2+(1+m)^2 \cos \left(2d(a+b\log \left(cx^n\right))\right)+2bd(1+m)n \sin \left(2d(a+b\log \left(cx^n\right))\right)\right)}{2(1+m)(1+m-2ibdn)(1+m+2ibdn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] -1/2*(x*(e*x)^m*(-1 - 2*m - m^2 - 4*b^2*d^2*n^2 + (1 + m)^2*Cos[2*d*(a + b*
Log[c*x^n])] + 2*b*d*(1 + m)*n*Sin[2*d*(a + b*Log[c*x^n])]))/((1 + m)*(1 +
m - (2*I)*b*d*n)*(1 + m + (2*I)*b*d*n))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\sin^2 \left(d(a+b\ln \left(c\,x^n\right))\right)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sin(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int((e*x)^m*sin(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2517 vs. 2(154) = 308.
time = 0.38, size = 2517, normalized size = 16.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] -1/4*((2*(b*d*cos(2*a*d)*sin(2*b*d*log(c)) + b*d*cos(2*b*d*log(c))*sin(2*a*
d) + (b*d*cos(2*a*d)*sin(2*b*d*log(c)) + b*d*cos(2*b*d*log(c))*sin(2*a*d) +
 ((b*d*cos(2*a*d)*sin(4*a*d) - b*d*cos(4*a*d)*sin(2*a*d))*cos(2*b*d*log(c))
 - (b*d*cos(4*a*d)*cos(2*a*d) + b*d*sin(4*a*d)*sin(2*a*d))*sin(2*b*d*log(c)
))*cos(4*b*d*log(c)) + ((b*d*cos(4*a*d)*cos(2*a*d) + b*d*sin(4*a*d)*sin(2*a
```

```
*d))*cos(2*b*d*log(c)) + (b*d*cos(2*a*d)*sin(4*a*d) - b*d*cos(4*a*d)*sin(2*
a*d))*sin(2*b*d*log(c)))*sin(4*b*d*log(c)))*m + ((b*d*cos(2*a*d)*sin(4*a*d)
 - b*d*cos(4*a*d)*sin(2*a*d))*cos(2*b*d*log(c)) - (b*d*cos(4*a*d)*cos(2*a*d
) + b*d*sin(4*a*d)*sin(2*a*d))*sin(2*b*d*log(c)))*cos(4*b*d*log(c)) + ((b*d
*cos(4*a*d)*cos(2*a*d) + b*d*sin(4*a*d)*sin(2*a*d))*cos(2*b*d*log(c)) + (b*
d*cos(2*a*d)*sin(4*a*d) - b*d*cos(4*a*d)*sin(2*a*d))*sin(2*b*d*log(c)))*sin
(4*b*d*log(c)))*n*e^m + ((((cos(4*a*d)*cos(2*a*d) + sin(4*a*d)*sin(2*a*d))*
cos(2*b*d*log(c)) + (cos(2*a*d)*sin(4*a*d) - cos(4*a*d)*sin(2*a*d))*sin(2*b
*d*log(c)))*cos(4*b*d*log(c)) + cos(2*b*d*log(c))*cos(2*a*d) - ((cos(2*a*d)
*sin(4*a*d) - cos(4*a*d)*sin(2*a*d))*cos(2*b*d*log(c)) - (cos(4*a*d)*cos(2*
a*d) + sin(4*a*d)*sin(2*a*d))*sin(2*b*d*log(c)))*sin(4*b*d*log(c)) - sin(2*
b*d*log(c))*sin(2*a*d))*m^2 + 2*(((cos(4*a*d)*cos(2*a*d) + sin(4*a*d)*sin(2
*a*d))*cos(2*b*d*log(c)) + (cos(2*a*d)*sin(4*a*d) - cos(4*a*d)*sin(2*a*d))*
sin(2*b*d*log(c)))*cos(4*b*d*log(c)) + cos(2*b*d*log(c))*cos(2*a*d) - ((cos
(2*a*d)*sin(4*a*d) - cos(4*a*d)*sin(2*a*d))*cos(2*b*d*log(c)) - (cos(4*a*d)
*cos(2*a*d) + sin(4*a*d)*sin(2*a*d))*sin(2*b*d*log(c)))*sin(4*b*d*log(c)) -
 sin(2*b*d*log(c))*sin(2*a*d))*m + ((cos(4*a*d)*cos(2*a*d) + sin(4*a*d)*sin
(2*a*d))*cos(2*b*d*log(c)) + (cos(2*a*d)*sin(4*a*d) - cos(4*a*d)*sin(2*a*d)
)*sin(2*b*d*log(c)))*cos(4*b*d*log(c)) + cos(2*b*d*log(c))*cos(2*a*d) - ((c
os(2*a*d)*sin(4*a*d) - cos(4*a*d)*sin(2*a*d))*cos(2*b*d*log(c)) - (cos(4*a*
d)*cos(2*a*d) + sin(4*a*d)*sin(2*a*d))*sin(2*b*d*log(c)))*sin(4*b*d*log(c))
 - sin(2*b*d*log(c))*sin(2*a*d))*e^m)*x*x^m*cos(2*b*d*log(x^n)) + (2*(b*d*c
os(2*b*d*log(c))*cos(2*a*d) - b*d*sin(2*b*d*log(c))*sin(2*a*d) + (b*d*cos(2
*b*d*log(c))*cos(2*a*d) - b*d*sin(2*b*d*log(c))*sin(2*a*d) + ((b*d*cos(4*a*
d)*cos(2*a*d) + b*d*sin(4*a*d)*sin(2*a*d))*cos(2*b*d*log(c)) + (b*d*cos(2*a
*d)*sin(4*a*d) - b*d*cos(4*a*d)*sin(2*a*d))*sin(2*b*d*log(c)))*cos(4*b*d*lo
g(c)) - ((b*d*cos(2*a*d)*sin(4*a*d) - b*d*cos(4*a*d)*sin(2*a*d))*cos(2*b*d*
log(c)) - (b*d*cos(4*a*d)*cos(2*a*d) + b*d*sin(4*a*d)*sin(2*a*d))*sin(2*b*d
*log(c)))*sin(4*b*d*log(c)))*m + ((b*d*cos(4*a*d)*cos(2*a*d) + b*d*sin(4*a*
d)*sin(2*a*d))*cos(2*b*d*log(c)) + (b*d*cos(2*a*d)*sin(4*a*d) - b*d*cos(4*a
*d)*sin(2*a*d))*sin(2*b*d*log(c)))*cos(4*b*d*log(c)) - ((b*d*cos(2*a*d)*sin
(4*a*d) - b*d*cos(4*a*d)*sin(2*a*d))*cos(2*b*d*log(c)) - (b*d*cos(4*a*d)*co
s(2*a*d) + b*d*sin(4*a*d)*sin(2*a*d))*sin(2*b*d*log(c)))*sin(4*b*d*log(c)))
*n*e^m - ((((cos(2*a*d)*sin(4*a*d) - cos(4*a*d)*sin(2*a*d))*cos(2*b*d*log(c
)) - (cos(4*a*d)*cos(2*a*d) + sin(4*a*d)*sin(2*a*d))*sin(2*b*d*log(c)))*cos
(4*b*d*log(c)) + ((cos(4*a*d)*cos(2*a*d) + sin(4*a*d)*sin(2*a*d))*cos(2*b*d
*log(c)) + (cos(2*a*d)*sin(4*a*d) - cos(4*a*d)*sin(2*a*d))*sin(2*b*d*log(c)
))*sin(4*b*d*log(c)) + cos(2*a*d)*sin(2*b*d*log(c)) + cos(2*b*d*log(c))*sin
(2*a*d))*m^2 + 2*(((cos(2*a*d)*sin(4*a*d) - cos(4*a*d)*sin(2*a*d))*cos(2*b*
d*log(c)) - (cos(4*a*d)*cos(2*a*d) + sin(4*a*d)*sin(2*a*d))*sin(2*b*d*log(c
)))*cos(4*b*d*log(c)) + ((cos(4*a*d)*cos(2*a*d) + sin(4*a*d)*sin(2*a*d))*co
s(2*b*d*log(c)) + (cos(2*a*d)*sin(4*a*d) - cos(4*a*d)*sin(2*a*d))*sin(2*b*d
*log(c)))*sin(4*b*d*log(c)) + cos(2*a*d)*sin(2*b*d*log(c)) + cos(2*b*d*log(
c))*sin(2*a*d))*m + ((cos(2*a*d)*sin(4*a*d) - cos(4*a*d)*sin(2*a*d))*cos(2*
b*d*log(c)) - (cos(4*a*d)*cos(2*a*d) + sin(4*a*d)*sin(2*a*d))*sin(2*b*d*log
```

```
(c)))*cos(4*b*d*log(c)) + ((cos(4*a*d)*cos(2*a*d) + sin(4*a*d)*sin(2*a*d))*
cos(2*b*d*log(c)) + (cos(2*a*d)*sin(4*a*d) - cos(4*a*d)*sin(2*a*d))*sin(2*b
*d*log(c)))*sin(4*b*d*log(c)) + cos(2*a*d)*sin(2*b*d*log(c)) + cos(2*b*d*lo
g(c))*sin(2*a*d))*e^m)*x*x^m*sin(2*b*d*log(x^n)) - 2*(4*((b^2*d^2*cos(2*a*d
)^2 + b^2*d^2*sin(2*a*d)^2)*cos(2*b*d*log(c))^2 + (b^2*d^2*cos(2*a*d)^2 + b
^2*d^2*sin(2*a*d)^2)*sin(2*b*d*log(c))^2)*n^2*e^m + (((cos(2*a*d)^2 + sin(2
*a*d)^2)*cos(2*b*d*log(c))^2 + (cos(2*a*d)^2 + sin(2*a*d)^2)*sin(2*b*d*log(
c))^2)*m^2 + (cos(2*a*d)^2 + sin(2*a*d)^2)*cos(2*b*d*log(c))^2 + (cos(2*a*d
)^2 + sin(2*a*d)^2)*sin(2*b*d*log(c))^2 + 2*((cos(2*a*d)^2 + sin(2*a*d)^2)*
cos(2*b*d*log(c))^2 + (cos(2*a*d)^2 + sin(2*a*d)^2)*sin(2*b*d*log(c))^2)*m)
*e^m)*x*x^m)/(((cos(2*a*d)^2 + sin(2*a*d)^2)*cos(2*b*d*log(c))^2 + (cos(2*a
*d)^2 + sin(2*a*d)^2)*sin(2*b*d*log(c))^2)*m^3 + 3*((cos(2*a*d)^2 + sin(2*a
*d)^2)*cos(2*b*d*log(c))^2 + (cos(2*a*d)^2 + sin(2*a*d)^2)*sin(2*b*d*log(c)
)^2)*m^2 + 4*((b^2*d^2*cos(2*a*d)^2 + b^2*d^2*sin(2*a*d)^2)*cos(2*b*d*log(c
))^2 + (b^2*d^2*cos(2*a*d)^2 + b^2*d^2*sin(2*a*d)^2)*sin(2*b*d*log(c))^2 +
((b^2*d^2*cos(2*a*d)^2 + b^2*d^2*sin(2*a*d)^2)*cos(2*b*d*log(c))^2 + (b^2*d
^2*cos(2*a*d)^2 + b^2*d^2*sin(2*a*d)^2)*sin(2*b...
```

Fricas [A]
time = 1.83, size = 149, normalized size = 0.97

$$-\frac{2\,(bdm+bd)nx\cos\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)e^{\left(m\log\left(x\right)+m\right)}\sin\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)+\left(\left(m^{2}+2\,m+1\right)x\cos\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)^{2}-\left(2\,b^{2}d^{2}n^{2}+m^{2}+2\,m+1\right)x\right)e^{\left(m\log\left(x\right)+m\right)}}{m^{3}+4\,(b^{2}d^{2}m+b^{2}d^{2})n^{2}+3\,m^{2}+3\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] -(2*(b*d*m + b*d)*n*x*cos(b*d*n*log(x) + b*d*log(c) + a*d)*e^(m*log(x) + m)
*sin(b*d*n*log(x) + b*d*log(c) + a*d) + ((m^2 + 2*m + 1)*x*cos(b*d*n*log(x)
 + b*d*log(c) + a*d)^2 - (2*b^2*d^2*n^2 + m^2 + 2*m + 1)*x)*e^(m*log(x) + m
))/(m^3 + 4*(b^2*d^2*m + b^2*d^2)*n^2 + 3*m^2 + 3*m + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\begin{cases}\frac{\log\left(x\right)\cos\left(2ad\right)}{e} & \text{for}\: b=0\wedge m=-1\\\int\left(ex\right)^{m}\cos\left(-2ad+\frac{im\log\left(cx^{n}\right)}{n}+\frac{i\log\left(cx^{n}\right)}{n}\right)\,dx & \text{for}\: b=\frac{i\left(-m-1\right)}{2dn}\\\int\left(ex\right)^{m}\cos\left(2ad+\frac{im\log\left(cx^{n}\right)}{n}+\frac{i\log\left(cx^{n}\right)}{n}\right)\,dx & \text{for}\: b=\frac{i\left(m+1\right)}{2dn}\\\frac{2bdnx\left(ex\right)^{m}\sin\left(2ad+2bd\log\left(cx^{n}\right)\right)}{4b^{2}d^{2}n^{2}+m^{2}+2m+1}+\frac{mx\left(ex\right)^{m}\cos\left(2ad+2bd\log\left(cx^{n}\right)\right)}{4b^{2}d^{2}n^{2}+m^{2}+2m+1}+\frac{x\left(ex\right)^{m}\cos\left(2ad+2bd\log\left(cx^{n}\right)\right)}{4b^{2}d^{2}n^{2}+m^{2}+2m+1} & \text{otherwise}\end{cases}}{2}+\frac{\begin{cases}\frac{\left(ex\right)^{m+1}}{m+1} & \text{for}\: m\neq-1\\\log\left(ex\right) & \text{otherwise}\end{cases}}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sin(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] -Piecewise((log(x)*cos(2*a*d)/e, Eq(b, 0) & Eq(m, -1)), (Integral((e*x)**m*
cos(-2*a*d + I*m*log(c*x**n)/n + I*log(c*x**n)/n), x), Eq(b, I*(-m - 1)/(2*
```

```
d*n))), (Integral((e*x)**m*cos(2*a*d + I*m*log(c*x**n)/n + I*log(c*x**n)/n)
, x), Eq(b, I*(m + 1)/(2*d*n))), (2*b*d*n*x*(e*x)**m*sin(2*a*d + 2*b*d*log(
c*x**n))/(4*b**2*d**2*n**2 + m**2 + 2*m + 1) + m*x*(e*x)**m*cos(2*a*d + 2*b
*d*log(c*x**n))/(4*b**2*d**2*n**2 + m**2 + 2*m + 1) + x*(e*x)**m*cos(2*a*d
+ 2*b*d*log(c*x**n))/(4*b**2*d**2*n**2 + m**2 + 2*m + 1), True))/2 + Piecew
ise(((e*x)**(m + 1)/(m + 1), Ne(m, -1)), (log(e*x), True))/(2*e)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 30585 vs. $2(154) = 308$.
time = 1.23, size = 30585, normalized size = 198.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] -1/4*(8*(abs(e)*abs(x))^m*b^2*d^2*n^2*x*tan(b*d*n*log(abs(x)) + b*d*log(abs
(c)))^2*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) + 1/
4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*
m)^2*tan(a*d)^2 + 8*(abs(e)*abs(x))^m*b^2*d^2*n^2*x*tan(b*d*n*log(abs(x)) +
 b*d*log(abs(c)))^2*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m
*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(
x) - 1/2*pi*m)^2 + 8*(abs(e)*abs(x))^m*b^2*d^2*n^2*x*tan(b*d*n*log(abs(x))
+ b*d*log(abs(c)))^2*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*
m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(a*d)^2 - 8*(abs(e)*abs(x))^m*b
^2*d^2*n^2*x*tan(b*d*n*log(abs(x)) + b*d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e)
 + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(a*d)^2 + 8*(abs(e)*abs(x))^m*b^2*d^2*n
^2*x*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) + 1/4*p
i*m*sgn(x) - 1/2*pi*m)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^
2*tan(a*d)^2 + 4*b*d*m*n*x*e^(pi*b*d*n*sgn(x) - pi*b*d*n + pi*b*d*sgn(c) -
pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(b*d*n*log(abs(x)) + b*d*log(abs
(c)))^2*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) + 1/
4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*
m)^2*tan(a*d) + 4*b*d*m*n*x*e^(-pi*b*d*n*sgn(x) + pi*b*d*n - pi*b*d*sgn(c)
+ pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(b*d*n*log(abs(x)) + b*d*log(a
bs(c)))^2*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) +
1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*p
i*m)^2*tan(a*d) - 4*b*d*m*n*x*e^(pi*b*d*n*sgn(x) - pi*b*d*n + pi*b*d*sgn(c)
 - pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(b*d*n*log(abs(x)) + b*d*log(
abs(c)))^2*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) +
 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*
pi*m)*tan(a*d)^2 + 4*b*d*m*n*x*e^(-pi*b*d*n*sgn(x) + pi*b*d*n - pi*b*d*sgn(
c) + pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(b*d*n*log(abs(x)) + b*d*lo
g(abs(c)))^2*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e)
 + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/
```

```
2*pi*m)*tan(a*d)^2 + 4*b*d*m*n*x*e^(pi*b*d*n*sgn(x) - pi*b*d*n + pi*b*d*sgn
(c) - pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(b*d*n*log(abs(x)) + b*d*l
og(abs(c)))*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e)
+ 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2
*pi*m)^2*tan(a*d)^2 + 4*b*d*m*n*x*e^(-pi*b*d*n*sgn(x) + pi*b*d*n - pi*b*d*s
gn(c) + pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(b*d*n*log(abs(x)) + b*d
*log(abs(c)))*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e
) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1
/2*pi*m)^2*tan(a*d)^2 + 8*(abs(e)*abs(x))^m*b^2*d^2*n^2*x*tan(b*d*n*log(abs
(x)) + b*d*log(abs(c)))^2*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/
4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2 - 8*(abs(e)*abs(x))^m*b^2*d^2
*n^2*x*tan(b*d*n*log(abs(x)) + b*d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4
*pi*m*sgn(x) - 1/2*pi*m)^2 + 8*(abs(e)*abs(x))^m*b^2*d^2*n^2*x*tan(pi*m*flo
or(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*
pi*m)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2 + 4*b*d*n*x*e^(
pi*b*d*n*sgn(x) - pi*b*d*n + pi*b*d*sgn(c) - pi*b*d + m*log(abs(e)) + m*log
(abs(x)))*tan(b*d*n*log(abs(x)) + b*d*log(abs(c)))^2*tan(pi*m*floor(-1/4*sg
n(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*ta
n(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(a*d) + 4*b*d*n*x*e^(-
pi*b*d*n*sgn(x) + pi*b*d*n - pi*b*d*sgn(c) + pi*b*d + m*log(abs(e)) + m*log
(abs(x)))*tan(b*d*n*log(abs(x)) + b*d*log(abs(c)))^2*tan(pi*m*floor(-1/4*sg
n(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*ta
n(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(a*d) - 8*(abs(e)*abs(
x))^m*b^2*d^2*n^2*x*tan(b*d*n*log(abs(x)) + b*d*log(abs(c)))^2*tan(a*d)^2 +
 8*(abs(e)*abs(x))^m*b^2*d^2*n^2*x*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x)
+ 1) + 1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(a*d)^2 - 4*b*d*n
*x*e^(pi*b*d*n*sgn(x) - pi*b*d*n + pi*b*d*sgn(c) - pi*b*d + m*log(abs(e)) +
 m*log(abs(x)))*tan(b*d*n*log(abs(x)) + b*d*log(abs(c)))^2*tan(pi*m*floor(-
1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m
)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)*tan(a*d)^2 + 4*b*d*n*
x*e^(-pi*b*d*n*sgn(x) + pi*b*d*n - pi*b*d*sgn(c) + pi*b*d + m*log(abs(e)) +
 m*log(abs(x)))*tan(b*d*n*log(abs(x)) + b*d*log(abs(c)))^2*tan(pi*m*floor(-
1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m
)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)*tan(a*d)^2 - 8*(abs(e
)*abs(x))^m*b^2*d^2*n^2*x*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)
^2*tan(a*d)^2 + 4*b*d*n*x*e^(pi*b*d*n*sgn(x) - pi*b*d*n + pi*b*d*sgn(c) - p
i*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(b*d*n*log(abs(x)) + b*d*log(abs(
c)))*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + ...
```

Mupad [B]
time = 3.05, size = 95, normalized size = 0.62

$$\frac{x\,{\left(e\,x\right)}^m}{2\,m+2}-\frac{x\,{\mathrm{e}}^{a\,d\,2{}\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,d\,2{}\mathrm{i}}\,{\left(e\,x\right)}^m}{4\,m+4+b\,d\,n\,8{}\mathrm{i}}-\frac{x\,{\mathrm{e}}^{-a\,d\,2{}\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,d\,2{}\mathrm{i}}}\,{\left(e\,x\right)}^m\,1{}\mathrm{i}}{m\,4{}\mathrm{i}+8\,b\,d\,n+4{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*(a + b*log(c*x^n)))^2*(e*x)^m,x)
```

```
[Out] (x*(e*x)^m)/(2*m + 2) - (x*exp(a*d*2i)*(c*x^n)^(b*d*2i)*(e*x)^m)/(4*m + b*d
*n*8i + 4) - (x*exp(-a*d*2i)/(c*x^n)^(b*d*2i)*(e*x)^m*1i)/(m*4i + 8*b*d*n +
 4i)
```

3.73 $\int (ex)^m \sin(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=92

$$-\frac{bdn(ex)^{1+m}\cos(d(a+b\log(cx^n)))}{e\left((1+m)^2+b^2d^2n^2\right)}+\frac{(1+m)(ex)^{1+m}\sin(d(a+b\log(cx^n)))}{e\left((1+m)^2+b^2d^2n^2\right)}$$

```
[Out] -b*d*n*(e*x)^(1+m)*cos(d*(a+b*ln(c*x^n)))/e/((1+m)^2+b^2*d^2*n^2)+(1+m)*(e*
x)^(1+m)*sin(d*(a+b*ln(c*x^n)))/e/((1+m)^2+b^2*d^2*n^2)
```

Rubi [A]
time = 0.01, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {4573}

$$\frac{(m+1)(ex)^{m+1}\sin(d(a+b\log(cx^n)))}{e\left(b^2d^2n^2+(m+1)^2\right)}-\frac{bdn(ex)^{m+1}\cos(d(a+b\log(cx^n)))}{e\left(b^2d^2n^2+(m+1)^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])],x]
```

```
[Out] -((b*d*n*(e*x)^(1 + m)*Cos[d*(a + b*Log[c*x^n])])/(e*((1 + m)^2 + b^2*d^2*n
^2))) + ((1 + m)*(e*x)^(1 + m)*Sin[d*(a + b*Log[c*x^n])])/(e*((1 + m)^2 + b
^2*d^2*n^2))
```

Rule 4573

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] - Simp[b*d*n*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int (ex)^m\sin(d(a+b\log(cx^n)))\,dx=-\frac{bdn(ex)^{1+m}\cos(d(a+b\log(cx^n)))}{e\left((1+m)^2+b^2d^2n^2\right)}+\frac{(1+m)(ex)^{1+m}\sin(d(a+b\log(cx^n)))}{e\left((1+m)^2+b^2d^2n^2\right)}$$

Mathematica [A]
time = 0.15, size = 63, normalized size = 0.68

$$\frac{x(ex)^m\left(-bdn\cos(d(a+b\log(cx^n)))+(1+m)\sin(d(a+b\log(cx^n)))\right)}{1+2m+m^2+b^2d^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sin[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x*(e*x)^m*(-(b*d*n*Cos[d*(a + b*Log[c*x^n])]) + (1 + m)*Sin[d*(a + b*Log[c
*x^n])]))/(1 + 2*m + m^2 + b^2*d^2*n^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (ex)^m \sin \left(d(a + b \ln \left(c\, x^n\right))\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sin(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int((e*x)^m*sin(d*(a+b*ln(c*x^n))),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1254 vs. 2(92) = 184.
time = 0.33, size = 1254, normalized size = 13.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] -1/2*(((b*d*cos(b*d*log(c))*cos(a*d) - b*d*sin(b*d*log(c))*sin(a*d) + ((b*d
*cos(2*a*d)*cos(a*d) + b*d*sin(2*a*d)*sin(a*d))*cos(b*d*log(c)) + (b*d*cos(
a*d)*sin(2*a*d) - b*d*cos(2*a*d)*sin(a*d))*sin(b*d*log(c)))*cos(2*b*d*log(c
)) - ((b*d*cos(a*d)*sin(2*a*d) - b*d*cos(2*a*d)*sin(a*d))*cos(b*d*log(c)) -
 (b*d*cos(2*a*d)*cos(a*d) + b*d*sin(2*a*d)*sin(a*d))*sin(b*d*log(c)))*sin(2
*b*d*log(c)))*n*e^m - ((((cos(a*d)*sin(2*a*d) - cos(2*a*d)*sin(a*d))*cos(b*
d*log(c)) - (cos(2*a*d)*cos(a*d) + sin(2*a*d)*sin(a*d))*sin(b*d*log(c)))*co
s(2*b*d*log(c)) + ((cos(2*a*d)*cos(a*d) + sin(2*a*d)*sin(a*d))*cos(b*d*log(
c)) + (cos(a*d)*sin(2*a*d) - cos(2*a*d)*sin(a*d))*sin(b*d*log(c)))*sin(2*b*
d*log(c)) + cos(a*d)*sin(b*d*log(c)) + cos(b*d*log(c))*sin(a*d))*m + ((cos(
a*d)*sin(2*a*d) - cos(2*a*d)*sin(a*d))*cos(b*d*log(c)) - (cos(2*a*d)*cos(a*
d) + sin(2*a*d)*sin(a*d))*sin(b*d*log(c)))*cos(2*b*d*log(c)) + ((cos(2*a*d)
*cos(a*d) + sin(2*a*d)*sin(a*d))*cos(b*d*log(c)) + (cos(a*d)*sin(2*a*d) - c
os(2*a*d)*sin(a*d))*sin(b*d*log(c)))*sin(2*b*d*log(c)) + cos(a*d)*sin(b*d*l
og(c)) + cos(b*d*log(c))*sin(a*d))*e^m)*x*x^m*cos(b*d*log(x^n)) - ((b*d*cos
(a*d)*sin(b*d*log(c)) + b*d*cos(b*d*log(c))*sin(a*d) + ((b*d*cos(a*d)*sin(2
*a*d) - b*d*cos(2*a*d)*sin(a*d))*cos(b*d*log(c)) - (b*d*cos(2*a*d)*cos(a*d)
 + b*d*sin(2*a*d)*sin(a*d))*sin(b*d*log(c)))*cos(2*b*d*log(c)) + ((b*d*cos(
2*a*d)*cos(a*d) + b*d*sin(2*a*d)*sin(a*d))*cos(b*d*log(c)) + (b*d*cos(a*d)*
sin(2*a*d) - b*d*cos(2*a*d)*sin(a*d))*sin(b*d*log(c)))*sin(2*b*d*log(c)))*n
```

```
*e^m + ((((cos(2*a*d)*cos(a*d) + sin(2*a*d)*sin(a*d))*cos(b*d*log(c)) + (co
s(a*d)*sin(2*a*d) - cos(2*a*d)*sin(a*d))*sin(b*d*log(c)))*cos(2*b*d*log(c))
 + cos(b*d*log(c))*cos(a*d) - ((cos(a*d)*sin(2*a*d) - cos(2*a*d)*sin(a*d))*
cos(b*d*log(c)) - (cos(2*a*d)*cos(a*d) + sin(2*a*d)*sin(a*d))*sin(b*d*log(c
)))*sin(2*b*d*log(c)) - sin(b*d*log(c))*sin(a*d))*m + ((cos(2*a*d)*cos(a*d)
 + sin(2*a*d)*sin(a*d))*cos(b*d*log(c)) + (cos(a*d)*sin(2*a*d) - cos(2*a*d)
*sin(a*d))*sin(b*d*log(c)))*cos(2*b*d*log(c)) + cos(b*d*log(c))*cos(a*d) -
((cos(a*d)*sin(2*a*d) - cos(2*a*d)*sin(a*d))*cos(b*d*log(c)) - (cos(2*a*d)*
cos(a*d) + sin(2*a*d)*sin(a*d))*sin(b*d*log(c)))*sin(2*b*d*log(c)) - sin(b*
d*log(c))*sin(a*d))*e^m)*x*x^m*sin(b*d*log(x^n)))/(((cos(a*d)^2 + sin(a*d)^
2)*cos(b*d*log(c))^2 + (cos(a*d)^2 + sin(a*d)^2)*sin(b*d*log(c))^2)*m^2 + (
(b^2*d^2*cos(a*d)^2 + b^2*d^2*sin(a*d)^2)*cos(b*d*log(c))^2 + (b^2*d^2*cos(
a*d)^2 + b^2*d^2*sin(a*d)^2)*sin(b*d*log(c))^2)*n^2 + (cos(a*d)^2 + sin(a*d
)^2)*cos(b*d*log(c))^2 + (cos(a*d)^2 + sin(a*d)^2)*sin(b*d*log(c))^2 + 2*((
cos(a*d)^2 + sin(a*d)^2)*cos(b*d*log(c))^2 + (cos(a*d)^2 + sin(a*d)^2)*sin(
b*d*log(c))^2)*m)
```

Fricas [A]
time = 1.42, size = 80, normalized size = 0.87

$$-\frac{bdnx\cos\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)e^{\left(m\log\left(x\right)+m\right)}-(m+1)xe^{\left(m\log\left(x\right)+m\right)}\sin\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)}{b^{2}d^{2}n^{2}+m^{2}+2\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] -(b*d*n*x*cos(b*d*n*log(x) + b*d*log(c) + a*d)*e^(m*log(x) + m) - (m + 1)*x
*e^(m*log(x) + m)*sin(b*d*n*log(x) + b*d*log(c) + a*d))/(b^2*d^2*n^2 + m^2
+ 2*m + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ex\right)^{m}\sin\left(ad+bd\log\left(cx^{n}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sin(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral((e*x)**m*sin(a*d + b*d*log(c*x**n)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 6580 vs. 2(92) = 184.
time = 0.59, size = 6580, normalized size = 71.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] 1/2*(b*d*n*x*e^(1/2*pi*b*d*n*sgn(x) - 1/2*pi*b*d*n + 1/2*pi*b*d*sgn(c) - 1/
2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(1/2*b*d*n*log(abs(x)) + 1/2*b
*d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(1
/2*a*d)^2 + b*d*n*x*e^(-1/2*pi*b*d*n*sgn(x) + 1/2*pi*b*d*n - 1/2*pi*b*d*sgn
(c) + 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(1/2*b*d*n*log(abs(x))
 + 1/2*b*d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)
^2*tan(1/2*a*d)^2 - b*d*n*x*e^(1/2*pi*b*d*n*sgn(x) - 1/2*pi*b*d*n + 1/2*pi*
b*d*sgn(c) - 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(1/2*b*d*n*log(
abs(x)) + 1/2*b*d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/
2*pi*m)^2 - b*d*n*x*e^(-1/2*pi*b*d*n*sgn(x) + 1/2*pi*b*d*n - 1/2*pi*b*d*sgn
(c) + 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(1/2*b*d*n*log(abs(x))
 + 1/2*b*d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)
^2 + 4*b*d*n*x*e^(1/2*pi*b*d*n*sgn(x) - 1/2*pi*b*d*n + 1/2*pi*b*d*sgn(c) -
1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(1/2*b*d*n*log(abs(x)) + 1/2
*b*d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)*tan(1
/2*a*d) - 4*b*d*n*x*e^(-1/2*pi*b*d*n*sgn(x) + 1/2*pi*b*d*n - 1/2*pi*b*d*sgn
(c) + 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(1/2*b*d*n*log(abs(x))
 + 1/2*b*d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)
*tan(1/2*a*d) - 4*b*d*n*x*e^(1/2*pi*b*d*n*sgn(x) - 1/2*pi*b*d*n + 1/2*pi*b*
d*sgn(c) - 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(1/2*b*d*n*log(ab
s(x)) + 1/2*b*d*log(abs(c)))*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi
*m)^2*tan(1/2*a*d) - 4*b*d*n*x*e^(-1/2*pi*b*d*n*sgn(x) + 1/2*pi*b*d*n - 1/2
*pi*b*d*sgn(c) + 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(1/2*b*d*n*
log(abs(x)) + 1/2*b*d*log(abs(c)))*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) -
1/2*pi*m)^2*tan(1/2*a*d) - b*d*n*x*e^(1/2*pi*b*d*n*sgn(x) - 1/2*pi*b*d*n +
1/2*pi*b*d*sgn(c) - 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(1/2*b*d
*n*log(abs(x)) + 1/2*b*d*log(abs(c)))^2*tan(1/2*a*d)^2 - b*d*n*x*e^(-1/2*pi
*b*d*n*sgn(x) + 1/2*pi*b*d*n - 1/2*pi*b*d*sgn(c) + 1/2*pi*b*d + m*log(abs(e
)) + m*log(abs(x)))*tan(1/2*b*d*n*log(abs(x)) + 1/2*b*d*log(abs(c)))^2*tan(
1/2*a*d)^2 + 4*b*d*n*x*e^(1/2*pi*b*d*n*sgn(x) - 1/2*pi*b*d*n + 1/2*pi*b*d*s
gn(c) - 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(1/2*b*d*n*log(abs(x
)) + 1/2*b*d*log(abs(c)))*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)
*tan(1/2*a*d)^2 - 4*b*d*n*x*e^(-1/2*pi*b*d*n*sgn(x) + 1/2*pi*b*d*n - 1/2*pi
*b*d*sgn(c) + 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(1/2*b*d*n*log
(abs(x)) + 1/2*b*d*log(abs(c)))*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2
*pi*m)*tan(1/2*a*d)^2 - b*d*n*x*e^(1/2*pi*b*d*n*sgn(x) - 1/2*pi*b*d*n + 1/2
*pi*b*d*sgn(c) - 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(1/4*pi*m*s
gn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(1/2*a*d)^2 - b*d*n*x*e^(-1/2*pi*b
*d*n*sgn(x) + 1/2*pi*b*d*n - 1/2*pi*b*d*sgn(c) + 1/2*pi*b*d + m*log(abs(e))
 + m*log(abs(x)))*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(1
/2*a*d)^2 + 2*m*x*e^(1/2*pi*b*d*n*sgn(x) - 1/2*pi*b*d*n + 1/2*pi*b*d*sgn(c)
 - 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(1/2*b*d*n*log(abs(x)) +
1/2*b*d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*
```

```
tan(1/2*a*d) + 2*m*x*e^(-1/2*pi*b*d*n*sgn(x) + 1/2*pi*b*d*n - 1/2*pi*b*d*sg
n(c) + 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(1/2*b*d*n*log(abs(x)
) + 1/2*b*d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m
)^2*tan(1/2*a*d) - 2*m*x*e^(1/2*pi*b*d*n*sgn(x) - 1/2*pi*b*d*n + 1/2*pi*b*d
*sgn(c) - 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(1/2*b*d*n*log(abs
(x)) + 1/2*b*d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*p
i*m)*tan(1/2*a*d)^2 + 2*m*x*e^(-1/2*pi*b*d*n*sgn(x) + 1/2*pi*b*d*n - 1/2*pi
*b*d*sgn(c) + 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(1/2*b*d*n*log
(abs(x)) + 1/2*b*d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1
/2*pi*m)*tan(1/2*a*d)^2 + 2*m*x*e^(1/2*pi*b*d*n*sgn(x) - 1/2*pi*b*d*n + 1/2
*pi*b*d*sgn(c) - 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(1/2*b*d*n*
log(abs(x)) + 1/2*b*d*log(abs(c)))*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) -
1/2*pi*m)^2*tan(1/2*a*d)^2 + 2*m*x*e^(-1/2*pi*b*d*n*sgn(x) + 1/2*pi*b*d*n -
 1/2*pi*b*d*sgn(c) + 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(1/2*b*
d*n*log(abs(x)) + 1/2*b*d*log(abs(c)))*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x
) - 1/2*pi*m)^2*tan(1/2*a*d)^2 + b*d*n*x*e^(1/2*pi*b*d*n*sgn(x) - 1/2*pi*b*
d*n + 1/2*pi*b*d*sgn(c) - 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(1
/2*b*d*n*log(abs(x)) + 1/2*b*d*log(abs(c)))^2 + b*d*n*x*e^(-1/2*pi*b*d*n*sg
n(x) + 1/2*pi*b*d*n - 1/2*pi*b*d*sgn(c) + 1/2*pi*b*d + m*log(abs(e)) + m*lo
g(abs(x)))*tan(1/2*b*d*n*log(abs(x)) + 1/2*b*d*log(abs(c)))^2 - 4*b*d*n*x*e
^(1/2*pi*b*d*n*sgn(x) - 1/2*pi*b*d*n + 1/2*pi*b*d*sgn(c) - 1/2*pi*b*d + m*l
og(abs(e)) + m*log(abs(x)))*tan(1/2*b*d*n*log(abs(x)) + 1/2*b*d*log(abs(c))
)*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m) + 4*b*d*n*x*e^(-1/2*pi*
b*d*n*sgn(x) + 1/2*pi*b*d*n - 1/2*pi*b*d*sgn(c)...
```

Mupad [B]
time = 2.86, size = 80, normalized size = 0.87

$$\frac{x\,\mathrm{e}^{-a\,d\,1\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,d\,1\mathrm{i}}}\,{\left(e\,x\right)}^m\,1\mathrm{i}}{2\,m+2-b\,d\,n\,2\mathrm{i}}+\frac{x\,\mathrm{e}^{a\,d\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,d\,1\mathrm{i}}\,{\left(e\,x\right)}^m}{m\,2\mathrm{i}-2\,b\,d\,n+2\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*(a + b*log(c*x^n)))*(e*x)^m,x)
```

```
[Out] (x*exp(-a*d*1i)/(c*x^n)^(b*d*1i)*(e*x)^m*1i)/(2*m - b*d*n*2i + 2) + (x*exp(
a*d*1i)*(c*x^n)^(b*d*1i)*(e*x)^m)/(m*2i - 2*b*d*n + 2i)
```

3.74 $\int (ex)^m \sin^{\frac{3}{2}}(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=150

$$\frac{2(ex)^{1+m}\,{}_2F_1\left(-\frac{3}{2},-\frac{2i+2im+3bdn}{4bdn};-\frac{2i+2im-bdn}{4bdn};e^{2iad}(cx^n)^{2ibd}\right)\sin^{\frac{3}{2}}(d(a+b\log(cx^n)))}{e(2+2m-3ibdn)\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{3/2}}$$

```
[Out] 2*(e*x)^(1+m)*hypergeom([-3/2, 1/4*(-2*I-2*I*m-3*b*d*n)/b/d/n],[1/4*(-2*I-2
*I*m+b*d*n)/b/d/n],exp(2*I*a*d)*(c*x^n)^(2*I*b*d))*sin(d*(a+b*ln(c*x^n)))^(
3/2)/e/(2+2*m-3*I*b*d*n)/(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^(3/2)
```

Rubi [A]
time = 0.09, antiderivative size = 145, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4581, 4579, 371}

$$\frac{2(ex)^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(-\frac{2i(m+1)}{bdn}-3\right);-\frac{2im-bdn+2i}{4bdn};e^{2iad}(cx^n)^{2ibd}\right)\sin^{\frac{3}{2}}(d(a+b\log(cx^n)))}{e(-3ibdn+2m+2)\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(3/2),x]
```

```
[Out] (2*(e*x)^(1 + m)*Hypergeometric2F1[-3/2, (-3 - ((2*I)*(1 + m))/(b*d*n))/4,
-1/4*(2*I + (2*I)*m - b*d*n)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)]*Si
n[d*(a + b*Log[c*x^n])]^(3/2))/(e*(2 + 2*m - (3*I)*b*d*n)*(1 - E^((2*I)*a*d
)*(c*x^n)^((2*I)*b*d))^(3/2))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4579

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Dist[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
eQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int (ex)^m \sin^{\frac{3}{2}}\left(d(a+b\log(cx^n))\right)\,dx = \frac{\left((ex)^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{n}}\sin^{\frac{3}{2}}(d(a+b\log(x)))\,dx, x, cx\right.}{en}$$

$$= \frac{\left((ex)^{1+m}(cx^n)^{\frac{3ibd}{2}-\frac{1+m}{n}}\sin^{\frac{3}{2}}\left(d(a+b\log(cx^n))\right)\right)\text{Subst}\left(\int x^{-1-\frac{3ibd}{2}+\frac{1}{}}\right.}{en\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{3/2}}$$

$$= \frac{2(ex)^{1+m}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{2i(1+m)}{bdn}\right);-\frac{2i+2im-bdn}{4bdn};e^{2iad}(cx^n)^{2ibd}\right)\sin^{\frac{3}{2}}}{e(2+2m-3ibdn)\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{3/2}}$$

Mathematica [A]
time = 1.81, size = 256, normalized size = 1.71

$$\frac{2(ex)^m\left(\frac{3b^2d^2\sqrt{2-2e^{2id(a+b\log(cx^n))}}\,n^2x\,{}_2F_1\left(\frac{1}{2},\frac{-2i-2im+bdn}{4bdn};-\frac{2i+2im-5bdn}{4bdn};e^{2id(a+b\log(cx^n))}\right)}{\sqrt{-ie^{-id(a+b\log(cx^n))}\left(-1+e^{2id(a+b\log(cx^n))}\right)}\,(2+2m+ibdn)}+x\sqrt{\sin\left(d\left(a+b\log\left(cx^n\right)\right)\right)}\left(-3bdn\cos\left(d(a+b\log\left(cx^n\right))\right)+2(1+m)\sin\left(d(a+b\log\left(cx^n\right))\right)\right)\right)}{4+8m+4m^2+9b^2d^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(3/2),x]
```

```
[Out] (2*(e*x)^m*((3*b^2*d^2*Sqrt[2 - 2*E^((2*I)*d*(a + b*Log[c*x^n]))]*n^2*x*Hyp
ergeometric2F1[1/2, (-2*I - (2*I)*m + b*d*n)/(4*b*d*n), -1/4*(2*I + (2*I)*m
 - 5*b*d*n)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))])/(Sqrt[((-I)*(-1 + E^(
(2*I)*d*(a + b*Log[c*x^n]))))/E^(I*d*(a + b*Log[c*x^n]))]*(2 + 2*m + I*b*d*
n)) + x*Sqrt[Sin[d*(a + b*Log[c*x^n])]]*(-3*b*d*n*Cos[d*(a + b*Log[c*x^n])]
 + 2*(1 + m)*Sin[d*(a + b*Log[c*x^n])])))/(4 + 8*m + 4*m^2 + 9*b^2*d^2*n^2)
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int (ex)^m\left(\sin^{\frac{3}{2}}\left(d(a+b\ln(c\,x^n))\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sin(d*(a+b*ln(c*x^n)))^(3/2),x)
```

```
[Out] int((e*x)^m*sin(d*(a+b*ln(c*x^n)))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^(3/2),x, algorithm="maxima")
[Out] integrate((x*e)^m*sin((b*log(c*x^n) + a)*d)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sin(d*(a+b*ln(c*x**n)))**(3/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^(3/2),x, algorithm="giac")
[Out] integrate((e*x)^m*sin((b*log(c*x^n) + a)*d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(d\left(a+b\,\ln\left(c\,x^n\right)\right)\right)^{3/2}\,{\left(e\,x\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*(a + b*log(c*x^n)))^(3/2)*(e*x)^m,x)
[Out] int(sin(d*(a + b*log(c*x^n)))^(3/2)*(e*x)^m, x)
```

3.75 $\int (ex)^m \sqrt{\sin(d(a+b\log(cx^n)))}\,dx$

Optimal. Leaf size=149

$$\frac{2(ex)^{1+m}\,{}_2F_1\left(-\frac{1}{2},-\frac{2i+2im+bdn}{4bdn};-\frac{2i+2im-3bdn}{4bdn};e^{2iad}(cx^n)^{2ibd}\right)\sqrt{\sin(d(a+b\log(cx^n)))}}{e(2+2m-ibdn)\sqrt{1-e^{2iad}(cx^n)^{2ibd}}}$$

```
[Out] 2*(e*x)^(1+m)*hypergeom([-1/2, 1/4*(-2*I-2*I*m-b*d*n)/b/d/n],[1/4*(-2*I-2*I
*m+3*b*d*n)/b/d/n],exp(2*I*a*d)*(c*x^n)^(2*I*b*d))*sin(d*(a+b*ln(c*x^n)))^(
1/2)/e/(2+2*m-I*b*d*n)/(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 145, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4581, 4579, 371}

$$\frac{2(ex)^{m+1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4}\left(-\frac{2i(m+1)}{bdn}-1\right);-\frac{2im-3bdn+2i}{4bdn};e^{2iad}(cx^n)^{2ibd}\right)\sqrt{\sin(d(a+b\log(cx^n)))}}{e(-ibdn+2m+2)\sqrt{1-e^{2iad}(cx^n)^{2ibd}}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sqrt[Sin[d*(a + b*Log[c*x^n])]],x]
```

```
[Out] (2*(e*x)^(1 + m)*Hypergeometric2F1[-1/2, (-1 - ((2*I)*(1 + m))/(b*d*n))/4,
-1/4*(2*I + (2*I)*m - 3*b*d*n)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)]*
Sqrt[Sin[d*(a + b*Log[c*x^n])]])/(e*(2 + 2*m - I*b*d*n)*Sqrt[1 - E^((2*I)*a
*d)*(c*x^n)^((2*I)*b*d)])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4579

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Dist[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
```

```
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int (ex)^m \sqrt{\sin\left(d\left(a+b\log\left(cx^n\right)\right)\right)}\,dx = \frac{\left((ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{n}}\sqrt{\sin(d(a+b\log(x)))}\,dx,\right.}{en}$$

$$= \frac{\left((ex)^{1+m}\left(cx^n\right)^{\frac{ibd}{2}-\frac{1+m}{n}}\sqrt{\sin\left(d\left(a+b\log\left(cx^n\right)\right)\right)}\right)\text{Subst}\left(\int x^{-1-\right.}{en\sqrt{1-e^{2iad}\left(cx^n\right)^{2ibd}}}$$

$$= \frac{2(ex)^{1+m}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4}\left(-1-\frac{2i(1+m)}{bdn}\right);-\frac{2i+2im-3bdn}{4bdn};e^{2iad}\left(cx^n\right)^{2ibd}\right)}{e(2+2m-ibdn)\sqrt{1-e^{2iad}\left(cx^n\right)^{2ibd}}}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 488 vs. $2(149) = 298$.
time = 6.71, size = 488, normalized size = 3.28

$$2x(ex)^m\left(-\frac{bde^{id(a-bn\log(x)+b\log(cx^n))}nx^{-ibdn}\sqrt{2-2e^{2iad}\left(cx^n\right)^{2ibd}}\left((2i+2im+bdn)x^{2ibdn}\,{}_2F_1\left(\frac{1}{2},-\frac{2i+2im-3bdn}{4bdn};-\frac{2i+2im-7bdn}{4bdn};e^{2iad}(cx^n)^{2ibd}\right)+(-2i-2im+3bdn)\,{}_2F_1\left(\frac{1}{2},-\frac{2i+2im+bdn}{4bdn};-\frac{2i+2im-3bdn}{4bdn};e^{2iad}(cx^n)^{2ibd}\right)\right)}{(2+2m-ibdn)(2+2m+3ibdn)\left(2i+2im+bdn+e^{2id(a-bn\log(x)+b\log(cx^n))}(-2i-2im+bdn)\right)\sqrt{-ie^{-iad}\left(cx^n\right)^{-ibd}\left(-1+e^{2iad}\left(cx^n\right)^{2ibd}\right)}}+\frac{\sqrt{\sin\left(d\left(a+b\log\left(cx^n\right)\right)\right)}\sin\left(d(a-bn\log(x)+b\log\left(cx^n\right))\right)}{bdn\cos\left(d\left(a-bn\log(x)+b\log\left(cx^n\right)\right)\right)+2(1+m)\sin\left(d\left(a-bn\log(x)+b\log\left(cx^n\right)\right)\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sqrt[Sin[d*(a + b*Log[c*x^n])]],x]
```

```
[Out] 2*x*(e*x)^m*(-((b*d*E^(I*d*(a - b*n*Log[x] + b*Log[c*x^n]))*n*Sqrt[2 - 2*E^
((2*I)*a*d)*(c*x^n)^((2*I)*b*d)]*((2*I + (2*I)*m + b*d*n)*x^((2*I)*b*d*n)*H
ypergeometric2F1[1/2, -1/4*(2*I + (2*I)*m - 3*b*d*n)/(b*d*n), -1/4*(2*I + (
2*I)*m - 7*b*d*n)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)] + (-2*I - (2*
I)*m + 3*b*d*n)*Hypergeometric2F1[1/2, -1/4*(2*I + (2*I)*m + b*d*n)/(b*d*n)
, -1/4*(2*I + (2*I)*m - 3*b*d*n)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)
]))/((2 + 2*m - I*b*d*n)*(2 + 2*m + (3*I)*b*d*n)*(2*I + (2*I)*m + b*d*n + E
^((2*I)*d*(a - b*n*Log[x] + b*Log[c*x^n]))*(-2*I - (2*I)*m + b*d*n))*x^(I*b
*d*n)*Sqrt[((-I)*(-1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(E^(I*a*d)*(c*x^
n)^(I*b*d))])) + (Sqrt[Sin[d*(a + b*Log[c*x^n])]]*Sin[d*(a - b*n*Log[x] + b
*Log[c*x^n])])/(b*d*n*Cos[d*(a - b*n*Log[x] + b*Log[c*x^n])] + 2*(1 + m)*Si
n[d*(a - b*n*Log[x] + b*Log[c*x^n])]))
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int (ex)^m\left(\sqrt{\sin}\left(d(a+b\ln\left(c\,x^n\right))\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sin(d*(a+b*ln(c*x^n)))^(1/2),x)
```

```
[Out] int((e*x)^m*sin(d*(a+b*ln(c*x^n)))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e)^m*sqrt(sin((b*log(c*x^n) + a)*d)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sqrt{\sin (ad + bd \log (cx^n))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sin(d*(a+b*ln(c*x**n)))**(1/2),x)
```

```
[Out] Integral((e*x)**m*sqrt(sin(a*d + b*d*log(c*x**n))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sqrt(sin((b*log(c*x^n) + a)*d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\sin\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)}\,\left(e\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*(a + b*log(c*x^n)))^(1/2)*(e*x)^m,x)
[Out] int(sin(d*(a + b*log(c*x^n)))^(1/2)*(e*x)^m, x)
```

3.76 $\int \frac{(ex)^m}{\sqrt{\sin(d(a+b\log(cx^n)))}}\,dx$

Optimal. Leaf size=150

$$\frac{2(ex)^{1+m}\sqrt{1-e^{2iad}\left(cx^n\right)^{2ibd}}\;{}_2F_1\left(\frac{1}{2},-\frac{2i+2im-bdn}{4bdn};-\frac{2i+2im-5bdn}{4bdn};e^{2iad}\left(cx^n\right)^{2ibd}\right)}{e(2+2m+ibdn)\sqrt{\sin\left(d\left(a+b\log\left(cx^n\right)\right)\right)}}$$

```
[Out] 2*(e*x)^(1+m)*hypergeom([1/2, 1/4*(-2*I-2*I*m+b*d*n)/b/d/n],[1/4*(-2*I-2*I*
m+5*b*d*n)/b/d/n],exp(2*I*a*d)*(c*x^n)^(2*I*b*d))*(1-exp(2*I*a*d)*(c*x^n)^(
2*I*b*d))^(1/2)/e/(2+2*m+I*b*d*n)/sin(d*(a+b*ln(c*x^n)))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4581, 4579, 371}

$$\frac{2(ex)^{m+1}\sqrt{1-e^{2iad}\left(cx^n\right)^{2ibd}}\;{}_2F_1\left(\frac{1}{2},-\frac{2im-bdn+2i}{4bdn};-\frac{2im-5bdn+2i}{4bdn};e^{2iad}\left(cx^n\right)^{2ibd}\right)}{e(ibdn+2m+2)\sqrt{\sin\left(d\left(a+b\log\left(cx^n\right)\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/Sqrt[Sin[d*(a + b*Log[c*x^n])]],x]
```

```
[Out] (2*(e*x)^(1 + m)*Sqrt[1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)]*Hypergeometric
2F1[1/2, -1/4*(2*I + (2*I)*m - b*d*n)/(b*d*n), -1/4*(2*I + (2*I)*m - 5*b*d*
n)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(e*(2 + 2*m + I*b*d*n)*Sqrt
[Sin[d*(a + b*Log[c*x^n])]])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4579

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Dist[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{(ex)^m}{\sqrt{\sin\left(d\left(a+b\log\left(cx^n\right)\right)\right)}}\,dx = \frac{\left((ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}}{\sqrt{\sin(d(a+b\log(x)))}}\,dx, x, cx^n\right)}{en}$$

$$= \frac{\left((ex)^{1+m}\left(cx^n\right)^{-\frac{1}{2}ibd-\frac{1+m}{n}}\sqrt{1-e^{2iad}\left(cx^n\right)^{2ibd}}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{ibd}{2}+\frac{1+n}{n}}}{\sqrt{1-e^{2iad}x^{2}}}\right.}{en\sqrt{\sin\left(d\left(a+b\log\left(cx^n\right)\right)\right)}}$$

$$= \frac{2(ex)^{1+m}\sqrt{1-e^{2iad}\left(cx^n\right)^{2ibd}}\,{}_2F_1\left(\frac{1}{2},-\frac{2i+2im-bdn}{4bdn};-\frac{2i+2im-5bdn}{4bdn};e^{2iad}(c\right.}{e(2+2m+ibdn)\sqrt{\sin\left(d\left(a+b\log\left(cx^n\right)\right)\right)}}$$

Mathematica [A]
time = 0.71, size = 165, normalized size = 1.10

$$\frac{2\sqrt{2-2e^{2id(a+b\log(cx^n))}}\,x(ex)^m\,{}_2F_1\left(\frac{1}{2},\frac{-2i-2im+bdn}{4bdn};-\frac{2i+2im-5bdn}{4bdn};e^{2id(a+b\log(cx^n))}\right)}{\sqrt{-ie^{-id(a+b\log(cx^n))}\left(-1+e^{2id(a+b\log(cx^n))}\right)}\,(2+2m+ibdn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/Sqrt[Sin[d*(a + b*Log[c*x^n])]],x]
[Out] (2*Sqrt[2 - 2*E^((2*I)*d*(a + b*Log[c*x^n]))]*x*(e*x)^m*Hypergeometric2F1[1
/2, (-2*I - (2*I)*m + b*d*n)/(4*b*d*n), -1/4*(2*I + (2*I)*m - 5*b*d*n)/(b*d
*n), E^((2*I)*d*(a + b*Log[c*x^n]))])/(Sqrt[((-I)*(-1 + E^((2*I)*d*(a + b*L
og[c*x^n]))))/E^(I*d*(a + b*Log[c*x^n]))]*(2 + 2*m + I*b*d*n))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\sqrt{\sin\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/sin(d*(a+b*ln(c*x^n)))^(1/2),x)
```

```
[Out] int((e*x)^m/sin(d*(a+b*ln(c*x^n)))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sin(d*(a+b*log(c*x^n)))^(1/2),x, algorithm="maxima")
[Out] integrate((x*e)^m/sqrt(sin((b*log(c*x^n) + a)*d)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sin(d*(a+b*log(c*x^n)))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\sqrt{\sin(ad + bd\log(cx^n))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/sin(d*(a+b*ln(c*x**n)))**(1/2),x)
[Out] Integral((e*x)**m/sqrt(sin(a*d + b*d*log(c*x**n))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sin(d*(a+b*log(c*x^n)))^(1/2),x, algorithm="giac")
[Out] integrate((e*x)^m/sqrt(sin((b*log(c*x^n) + a)*d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m}{\sqrt{\sin\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/sin(d*(a + b*log(c*x^n)))^(1/2),x)
[Out] int((e*x)^m/sin(d*(a + b*log(c*x^n)))^(1/2), x)
```

3.77 $\int \frac{(ex)^m}{\sin^{\frac{3}{2}}(d(a+b\log(cx^n)))}\,dx$

Optimal. Leaf size=150

$$\frac{2(ex)^{1+m}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{3/2}{}_2F_1\left(\frac{3}{2},-\frac{2i+2im-3bdn}{4bdn};-\frac{2i+2im-7bdn}{4bdn};e^{2iad}(cx^n)^{2ibd}\right)}{e(2+2m+3ibdn)\sin^{\frac{3}{2}}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}$$

```
[Out] 2*(e*x)^(1+m)*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^(3/2)*hypergeom([3/2, 1/4*
(-2*I-2*I*m+3*b*d*n)/b/d/n],[1/4*(-2*I-2*I*m+7*b*d*n)/b/d/n],exp(2*I*a*d)*(
c*x^n)^(2*I*b*d))/e/(2+2*m+3*I*b*d*n)/sin(d*(a+b*ln(c*x^n)))^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 145, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4581, 4579, 371}

$$\frac{2(ex)^{m+1}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{3/2}{}_2F_1\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i(m+1)}{bdn}\right);-\frac{2im-7bdn+2i}{4bdn};e^{2iad}(cx^n)^{2ibd}\right)}{e(3ibdn+2m+2)\sin^{\frac{3}{2}}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/Sin[d*(a + b*Log[c*x^n])]^(3/2),x]
```

```
[Out] (2*(e*x)^(1 + m)*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^(3/2)*Hypergeometr
ic2F1[3/2, (3 - ((2*I)*(1 + m))/(b*d*n))/4, -1/4*(2*I + (2*I)*m - 7*b*d*n)/
(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(e*(2 + 2*m + (3*I)*b*d*n)*Sin
[d*(a + b*Log[c*x^n])]^(3/2))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4579

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Dist[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
```

```
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{(ex)^m}{\sin^{\frac{3}{2}}(d(a+b\log(cx^n)))}\,dx = \frac{\left((ex)^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\operatorname{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}}{\sin^{\frac{3}{2}}(d(a+b\log(x)))}\,dx, x, cx^n\right)}{en}$$
$$= \frac{\left((ex)^{1+m}(cx^n)^{-\frac{3}{2}ibd-\frac{1+m}{n}}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{3/2}\right)\operatorname{Subst}\left(\int \frac{x^{-1+\frac{3ibd}{2}+\frac{1+m}{n}}}{\left(1-e^{2iad}x^{2ibd}\right)^{3/2}}\right.}{en\sin^{\frac{3}{2}}(d(a+b\log(cx^n)))}$$
$$= \frac{2(ex)^{1+m}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{3/2}{}_2F_1\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i(1+m)}{bdn}\right);-\frac{2i+2im-7bdn}{4bdn};e^{2i}\right.}{e(2+2m+3ibdn)\sin^{\frac{3}{2}}(d(a+b\log(cx^n)))}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 544 vs. 2(150) = 300.
time = 5.62, size = 544, normalized size = 3.63

$$\frac{(4+8m+4m^2+b^2d^2n^2)\,x^{1+ibdn}(ex)^m\sqrt{2-2e^{2iad}(cx^n)^{2ibd}}\;{}_2F_1\left(\frac{1}{2},-\frac{2i+2im-3bdn}{4bdn};-\frac{2i+2im-7bdn}{4bdn};e^{2iad}(cx^n)^{2ibd}\right)+\frac{(-2i-2im+3bdn)x^{1-ibdn}(ex)^m\left(-2x^{ibdn}\sqrt{-ie^{-iad}(cx^n)^{-ibd}\left(-1+e^{2iad}(cx^n)^{2ibd}\right)}\,(bdn\cos(bdn\log(x))-2(1+m)\sin(bdn\log(x)))+(-2i-2im+bdn)\sqrt{2-2e^{2iad}(cx^n)^{2ibd}}\;{}_2F_1\left(\frac{1}{2},-\frac{2i+2im+bdn}{4bdn};-\frac{2i+2im-3bdn}{4bdn};e^{2iad}(cx^n)^{2ibd}\right)\sqrt{\sin(d(a+b\log(cx^n)))}\right)}{\sqrt{\sin(d(a+b\log(cx^n)))}}}{bdn(-2i-2im+3bdn)\sqrt{-ie^{-iad}(cx^n)^{-ibd}\left(-1+e^{2iad}(cx^n)^{2ibd}\right)}\,(bdn\cos(d(a-bn\log(x)+b\log(cx^n)))+2(1+m)\sin(d(a-bn\log(x)+b\log(cx^n))))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/Sin[d*(a + b*Log[c*x^n])]^(3/2),x]
```

```
[Out] ((4 + 8*m + 4*m^2 + b^2*d^2*n^2)*x^(1 + I*b*d*n)*(e*x)^m*Sqrt[2 - 2*E^((2*I
)*a*d)*(c*x^n)^((2*I)*b*d)]*Hypergeometric2F1[1/2, -1/4*(2*I + (2*I)*m - 3*
b*d*n)/(b*d*n), -1/4*(2*I + (2*I)*m - 7*b*d*n)/(b*d*n), E^((2*I)*a*d)*(c*x^
n)^((2*I)*b*d)] + ((-2*I - (2*I)*m + 3*b*d*n)*x^(1 - I*b*d*n)*(e*x)^m*(-2*x
^(I*b*d*n)*Sqrt[((-I)*(-1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(E^(I*a*d)*
(c*x^n)^(I*b*d))]*(b*d*n*Cos[b*d*n*Log[x]] - 2*(1 + m)*Sin[b*d*n*Log[x]]) +
 (-2*I - (2*I)*m + b*d*n)*Sqrt[2 - 2*E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)]*Hyp
ergeometric2F1[1/2, -1/4*(2*I + (2*I)*m + b*d*n)/(b*d*n), -1/4*(2*I + (2*I)
*m - 3*b*d*n)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)]*Sqrt[Sin[d*(a + b
*Log[c*x^n])]]))/Sqrt[Sin[d*(a + b*Log[c*x^n])]])/(b*d*n*(-2*I - (2*I)*m +
3*b*d*n)*Sqrt[((-I)*(-1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(E^(I*a*d)*(c
*x^n)^(I*b*d))]*(b*d*n*Cos[d*(a - b*n*Log[x] + b*Log[c*x^n])] + 2*(1 + m)*S
in[d*(a - b*n*Log[x] + b*Log[c*x^n])]))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\sin(d(a+b\ln(c\,x^n)))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/sin(d*(a+b*ln(c*x^n)))^(3/2),x)
```

```
[Out] int((e*x)^m/sin(d*(a+b*ln(c*x^n)))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sin(d*(a+b*log(c*x^n)))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e)^m/sin((b*log(c*x^n) + a)*d)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sin(d*(a+b*log(c*x^n)))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\sin^{\frac{3}{2}}\left(ad + bd\log\left(cx^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/sin(d*(a+b*ln(c*x**n)))**(3/2),x)
```

```
[Out] Integral((e*x)**m/sin(a*d + b*d*log(c*x**n))**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sin(d*(a+b*log(c*x^n)))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m}{\sin\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/sin(d*(a + b*log(c*x^n)))^(3/2),x)
[Out] int((e*x)^m/sin(d*(a + b*log(c*x^n)))^(3/2), x)
```

3.78 $\int \frac{(ex)^m}{\sin^{\frac{5}{2}}(d(a+b\log(cx^n)))}\,dx$

Optimal. Leaf size=150

$$\frac{2(ex)^{1+m}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{5/2}{}_2F_1\left(\frac{5}{2},-\frac{2i+2im-5bdn}{4bdn};-\frac{2i+2im-9bdn}{4bdn};e^{2iad}(cx^n)^{2ibd}\right)}{e(2+2m+5ibdn)\sin^{\frac{5}{2}}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}$$

```
[Out] 2*(e*x)^(1+m)*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^(5/2)*hypergeom([5/2, 1/4*
(-2*I-2*I*m+5*b*d*n)/b/d/n],[1/4*(-2*I-2*I*m+9*b*d*n)/b/d/n],exp(2*I*a*d)*(
c*x^n)^(2*I*b*d))/e/(2+2*m+5*I*b*d*n)/sin(d*(a+b*ln(c*x^n)))^(5/2)
```

Rubi [A]

time = 0.08, antiderivative size = 145, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4581, 4579, 371}

$$\frac{2(ex)^{m+1}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{5/2}{}_2F_1\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i(m+1)}{bdn}\right);-\frac{2im-9bdn+2i}{4bdn};e^{2iad}(cx^n)^{2ibd}\right)}{e(5ibdn+2m+2)\sin^{\frac{5}{2}}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/Sin[d*(a + b*Log[c*x^n])]^(5/2),x]
```

```
[Out] (2*(e*x)^(1 + m)*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^(5/2)*Hypergeometr
ic2F1[5/2, (5 - ((2*I)*(1 + m))/(b*d*n))/4, -1/4*(2*I + (2*I)*m - 9*b*d*n)/
(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(e*(2 + 2*m + (5*I)*b*d*n)*Sin
[d*(a + b*Log[c*x^n])]^(5/2))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4579

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Dist[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
```

```
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(ex)^m}{\sin^{\frac{5}{2}}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}\,dx &= \frac{\left((ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\operatorname{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}}{\sin^{\frac{5}{2}}(d(a+b\log(x)))}\,dx,x,cx^n\right)}{en}\\
&= \frac{\left((ex)^{1+m}\left(cx^n\right)^{-\frac{5}{2}ibd-\frac{1+m}{n}}\left(1-e^{2iad}\left(cx^n\right)^{2ibd}\right)^{5/2}\right)\operatorname{Subst}\left(\int \frac{x^{-1+\frac{5ibd}{2}+\frac{1+m}{n}}}{\left(1-e^{2iad}x^{2ibd}\right)^{5/2}}\right.}{en\sin^{\frac{5}{2}}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}\\
&= \frac{2(ex)^{1+m}\left(1-e^{2iad}\left(cx^n\right)^{2ibd}\right)^{5/2}{}_2F_1\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i(1+m)}{bdn}\right);-\frac{2i+2im-9bdn}{4bdn};e^{2i}\right.}{e(2+2m+5ibdn)\sin^{\frac{5}{2}}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}\end{aligned}$$

Mathematica [A]
time = 2.12, size = 205, normalized size = 1.37

$$\frac{x(ex)^m\left(-2bdn\cos\left(d(a+b\log\left(cx^n\right))\right)+ie^{-id(a+b\log(cx^n))}\left(1-e^{2id(a+b\log(cx^n))}\right)^{3/2}(2+2m-ibdn)\,{}_2F_1\left(\frac{1}{2},\frac{-2i-2im+bdn}{4bdn};-\frac{2i+2im-5bdn}{4bdn};e^{2id(a+b\log(cx^n))}\right)-4(1+m)\sin\left(d(a+b\log\left(cx^n\right))\right)\right)}{3b^2d^2n^2\sin^{\frac{3}{2}}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/Sin[d*(a + b*Log[c*x^n])]^(5/2),x]
```

```
[Out] (x*(e*x)^m*(-2*b*d*n*Cos[d*(a + b*Log[c*x^n])] + (I*(1 - E^((2*I)*d*(a + b*
Log[c*x^n])))^(3/2)*(2 + 2*m - I*b*d*n)*Hypergeometric2F1[1/2, (-2*I - (2*I
)*m + b*d*n)/(4*b*d*n), -1/4*(2*I + (2*I)*m - 5*b*d*n)/(b*d*n), E^((2*I)*d*
(a + b*Log[c*x^n]))])/E^(I*d*(a + b*Log[c*x^n])) - 4*(1 + m)*Sin[d*(a + b*L
og[c*x^n])]))/(3*b^2*d^2*n^2*Sin[d*(a + b*Log[c*x^n])]^(3/2))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\sin\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/sin(d*(a+b*ln(c*x^n)))^(5/2),x)
```

```
[Out] int((e*x)^m/sin(d*(a+b*ln(c*x^n)))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sin(d*(a+b*log(c*x^n)))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e)^m/sin((b*log(c*x^n) + a)*d)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sin(d*(a+b*log(c*x^n)))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/sin(d*(a+b*ln(c*x**n)))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sin(d*(a+b*log(c*x^n)))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m}{\sin\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/sin(d*(a + b*log(c*x^n)))^(5/2),x)
```

```
[Out] int((e*x)^m/sin(d*(a + b*log(c*x^n)))^(5/2), x)
```

3.79 $\int (ex)^m \sin^p(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=144

$$\frac{(ex)^{1+m}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{-p}\,{}_2F_1\left(-p,-\frac{i+im+bdnp}{2bdn};\frac{1}{2}\left(2-\frac{i(1+m)}{bdn}-p\right);e^{2iad}(cx^n)^{2ibd}\right)\sin^p(d(a+b\log(cx^n)))}{e(1+m-ibdnp)}$$

```
[Out] (e*x)^(1+m)*hypergeom([-p, 1/2*(-I-I*m-b*d*n*p)/b/d/n],[1-1/2*I*(1+m)/b/d/n
-1/2*p],exp(2*I*a*d)*(c*x^n)^(2*I*b*d))*sin(d*(a+b*ln(c*x^n)))^p/e/(1+m-I*b
*d*n*p)/((1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^p)
```

Rubi [A]
time = 0.08, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4581, 4579, 371}

$$\frac{(ex)^{m+1}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{-p}\,{}_2F_1\left(-p,-\frac{im+bdnp+i}{2bdn};\frac{1}{2}\left(-\frac{i(m+1)}{bdn}-p+2\right);e^{2iad}(cx^n)^{2ibd}\right)\sin^p(d(a+b\log(cx^n)))}{e(-ibdnp+m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] ((e*x)^(1 + m)*Hypergeometric2F1[-p, -1/2*(I + I*m + b*d*n*p)/(b*d*n), (2 -
 (I*(1 + m))/(b*d*n) - p)/2, E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)]*Sin[d*(a +
b*Log[c*x^n])]^p)/(e*(1 + m - I*b*d*n*p)*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*
b*d))^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4579

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Dist[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
```

```
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int (ex)^m \sin^p\left(d(a+b\log\left(cx^n\right))\right)\,dx = \frac{\left((ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{n}}\sin^p(d(a+b\log(x)))\,dx, x, cx\right.}{en}$$

$$= \frac{\left((ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}+ibdp}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{-p}\sin^p\left(d(a+b\log\left(cx^n\right)\right.\right.}{en}$$

$$= \frac{(ex)^{1+m}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{-p}\,{}_2F_1\left(-p,-\frac{i+im+bdnp}{2bdn};\frac{1}{2}\left(2-\frac{i(1+m)}{bdn}-p\right.\right.}{e(1+m-ibdnp)}$$

Mathematica [A]
time = 1.41, size = 174, normalized size = 1.21

$$\frac{x(ex)^m\left(2-2e^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(-ie^{-iad}(cx^n)^{-ibd}\left(-1+e^{2iad}(cx^n)^{2ibd}\right)\right)^p{}_2F_1\left(-p,-\frac{i+im+bdnp}{2bdn};1-\frac{i(1+m)}{2bdn}-\frac{p}{2};e^{2iad}(cx^n)^{2ibd}\right)}{1+m-ibdnp}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] (x*(e*x)^m*(((-I)*(-1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(E^(I*a*d)*(c*x
^n)^(I*b*d)))^p*Hypergeometric2F1[-p, -1/2*(I + I*m + b*d*n*p)/(b*d*n), 1 -
 ((I/2)*(1 + m))/(b*d*n) - p/2, E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/((1 + m
 - I*b*d*n*p)*(2 - 2*E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (ex)^m\left(\sin^p\left(d(a+b\ln\left(c\,x^n\right))\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sin(d*(a+b*ln(c*x^n)))^p,x)
```

```
[Out] int((e*x)^m*sin(d*(a+b*ln(c*x^n)))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e)^m*sin((b*log(c*x^n) + a)*d)^p, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

```
[Out] integral((x*e)^m*sin(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sin(d*(a+b*ln(c*x**n)))**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sin((b*log(c*x^n) + a)*d)^p, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(d\left(a+b\,\ln\left(c\,x^n\right)\right)\right)^p\,{\left(e\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*(a + b*log(c*x^n)))^p*(e*x)^m,x)
```

```
[Out] int(sin(d*(a + b*log(c*x^n)))^p*(e*x)^m, x)
```

3.80 $\int x^2 \sin^p(a+b\log(cx^n))\,dx$

Optimal. Leaf size=114

$$\frac{x^3\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\,{}_2F_1\left(-p,-\frac{3i+bnp}{2bn};\frac{1}{2}\left(2-\frac{3i}{bn}-p\right);e^{2ia}(cx^n)^{2ib}\right)\sin^p(a+b\log(cx^n))}{3-ibnp}$$

```
[Out] x^3*hypergeom([-p, 1/2*(-3*I-b*n*p)/b/n],[1-3/2*I/b/n-1/2*p],exp(2*I*a)*(c*
x^n)^(2*I*b))*sin(a+b*ln(c*x^n))^p/(3-I*b*n*p)/((1-exp(2*I*a)*(c*x^n)^(2*I*
b))^p)
```

Rubi [A]
time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4581, 4579, 371}

$$\frac{x^3\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\,{}_2F_1\left(-p,-\frac{bnp+3i}{2bn};\frac{1}{2}\left(-p-\frac{3i}{bn}+2\right);e^{2ia}(cx^n)^{2ib}\right)\sin^p(a+b\log(cx^n))}{3-ibnp}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[a + b*Log[c*x^n]]^p,x]
```

```
[Out] (x^3*Hypergeometric2F1[-p, -1/2*(3*I + b*n*p)/(b*n), (2 - (3*I)/(b*n) - p)/
2, E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sin[a + b*Log[c*x^n]]^p)/((3 - I*b*n*p)*(
1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4579

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Dist[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
```

```
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^2 \sin^p(a+b\log(cx^n))\,dx = \frac{\left(x^3(cx^n)^{-3/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{3}{n}}\sin^p(a+b\log(x))\,dx, x, cx^n\right)}{n}$$
$$= \frac{\left(x^3(cx^n)^{-\frac{3}{n}+ibp}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\sin^p(a+b\log(cx^n))\right)\operatorname{Subst}\left(\int x^{-1+\frac{3}{n}}\right.}{n}$$
$$= \frac{x^3\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}{}_2F_1\left(-p,-\frac{3i+bnp}{2bn};\frac{1}{2}\left(2-\frac{3i}{bn}-p\right);e^{2ia}(cx^n)^{2ib}\right)\sin^p(}{3-ibnp}$$

Mathematica [**A**]
time = 0.98, size = 148, normalized size = 1.30

$$\frac{ix^3\left(2-2e^{2ia}(cx^n)^{2ib}\right)^{-p}\left(-ie^{-ia}(cx^n)^{-ib}\left(-1+e^{2ia}(cx^n)^{2ib}\right)\right)^p{}_2F_1\left(-p,-\frac{3i+bnp}{2bn};1-\frac{3i}{2bn}-\frac{p}{2};e^{2ia}(cx^n)^{2ib}\right)}{3i+bnp}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[a + b*Log[c*x^n]]^p,x]
```

```
[Out] (I*x^3*(((-I)*(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))/(E^(I*a)*(c*x^n)^(I*b))
)^p*Hypergeometric2F1[-p, -1/2*(3*I + b*n*p)/(b*n), 1 - ((3*I)/2)/(b*n) - p
/2, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((3*I + b*n*p)*(2 - 2*E^((2*I)*a)*(c*x^
n)^((2*I)*b))^p)
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^2\left(\sin^p(a+b\ln(c\,x^n))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a+b*ln(c*x^n))^p,x)
```

```
[Out] int(x^2*sin(a+b*ln(c*x^n))^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b*log(c*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate(x^2*sin(b*log(c*x^n) + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b*log(c*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral(x^2*sin(b*log(c*x^n) + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sin^p\left(a + b\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(a+b*ln(c*x**n))**p,x)
```

```
[Out] Integral(x**2*sin(a + b*log(c*x**n))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b*log(c*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate(x^2*sin(b*log(c*x^n) + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \sin\left(a + b\,\ln\left(c\,x^n\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a + b*log(c*x^n))^p,x)
```

```
[Out] int(x^2*sin(a + b*log(c*x^n))^p, x)
```

3.81 $\int x \sin^p(a + b\log(cx^n))\, dx$

Optimal. Leaf size=114

$$\frac{x^2 \left(1 - e^{2ia}(cx^n)^{2ib}\right)^{-p} {}_2F_1\left(\frac{1}{2}\left(-\frac{2i}{bn} - p\right), -p; \frac{1}{2}\left(2 - \frac{2i}{bn} - p\right); e^{2ia}(cx^n)^{2ib}\right) \sin^p(a + b\log(cx^n))}{2 - ibnp}$$

```
[Out] x^2*hypergeom([-p, -I/b/n-1/2*p],[1-I/b/n-1/2*p],exp(2*I*a)*(c*x^n)^(2*I*b)
)*sin(a+b*ln(c*x^n))^p/(2-I*b*n*p)/((1-exp(2*I*a)*(c*x^n)^(2*I*b))^p)
```

Rubi [A]
time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4581, 4579, 371}

$$\frac{x^2 \left(1 - e^{2ia}(cx^n)^{2ib}\right)^{-p} {}_2F_1\left(\frac{1}{2}\left(-p - \frac{2i}{bn}\right), -p; \frac{1}{2}\left(-p - \frac{2i}{bn} + 2\right); e^{2ia}(cx^n)^{2ib}\right) \sin^p(a + b\log(cx^n))}{2 - ibnp}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[a + b*Log[c*x^n]]^p,x]
```

```
[Out] (x^2*Hypergeometric2F1[((-2*I)/(b*n) - p)/2, -p, (2 - (2*I)/(b*n) - p)/2, E
^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sin[a + b*Log[c*x^n]]^p)/((2 - I*b*n*p)*(1 -
E^((2*I)*a)*(c*x^n)^((2*I)*b))^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4579

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Dist[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
eQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int x\sin^p(a+b\log(cx^n))\,dx &= \frac{\left(x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int x^{-1+\frac{2}{n}}\sin^p(a+b\log(x))\,dx,x,cx^n\right)}{n}\\
&= \frac{\left(x^2(cx^n)^{-\frac{2}{n}+ibp}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\sin^p(a+b\log(cx^n))\right)\text{Subst}\left(\int x^{-1+\frac{2}{n}-i\right.}{n}\\
&= \frac{x^2\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\,{}_2F_1\left(\frac{1}{2}\left(-\frac{2i}{bn}-p\right),-p;\frac{1}{2}\left(2-\frac{2i}{bn}-p\right);e^{2ia}(cx^n)^{2ib}\right)\sin}{2-ibnp}
\end{aligned}
$$

Mathematica [A]
time = 0.80, size = 144, normalized size = 1.26

$$
\frac{ix^2\left(2-2e^{2ia}(cx^n)^{2ib}\right)^{-p}\left(-ie^{-ia}(cx^n)^{-ib}\left(-1+e^{2ia}(cx^n)^{2ib}\right)\right)^p\,{}_2F_1\left(-\frac{i}{bn}-\frac{p}{2},-p;1-\frac{i}{bn}-\frac{p}{2};e^{2ia}(cx^n)^{2ib}\right)}{2i+bnp}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[a + b*Log[c*x^n]]^p,x]
[Out] (I*x^2*(((-I)*(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))/(E^(I*a)*(c*x^n)^(I*b))
)^p*Hypergeometric2F1[(-I)/(b*n) - p/2, -p, 1 - I/(b*n) - p/2, E^((2*I)*a)*
(c*x^n)^((2*I)*b)])/((2*I + b*n*p)*(2 - 2*E^((2*I)*a)*(c*x^n)^((2*I)*b))^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$
\int x\left(\sin^p\left(a+b\ln\left(c\,x^n\right)\right)\right)dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a+b*ln(c*x^n))^p,x)
[Out] int(x*sin(a+b*ln(c*x^n))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*log(c*x^n) + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n))^p,x, algorithm="fricas")
[Out] integral(x*sin(b*log(c*x^n) + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sin^p\left(a+b\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*ln(c*x**n))**p,x)
[Out] Integral(x*sin(a + b*log(c*x**n))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n))^p,x, algorithm="giac")
[Out] integrate(x*sin(b*log(c*x^n) + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b*log(c*x^n))^p,x)
[Out] int(x*sin(a + b*log(c*x^n))^p, x)
```

3.82 $\int \sin^p(a+b\log(cx^n))\,dx$

Optimal. Leaf size=112

$$\frac{x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\,{}_2F_1\left(-p,-\frac{i+bnp}{2bn};\frac{1}{2}\left(2-\frac{i}{bn}-p\right);e^{2ia}(cx^n)^{2ib}\right)\sin^p(a+b\log(cx^n))}{1-ibnp}$$

```
[Out] x*hypergeom([-p, 1/2*(-I-b*n*p)/b/n],[1-1/2*I/b/n-1/2*p],exp(2*I*a)*(c*x^n)
^(2*I*b))*sin(a+b*ln(c*x^n))^p/(1-I*b*n*p)/((1-exp(2*I*a)*(c*x^n)^(2*I*b))^
p)
```

Rubi [A]
time = 0.05, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4571, 4579, 371}

$$\frac{x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\,{}_2F_1\left(-p,-\frac{bnp+i}{2bn};\frac{1}{2}\left(-p-\frac{i}{bn}+2\right);e^{2ia}(cx^n)^{2ib}\right)\sin^p(a+b\log(cx^n))}{1-ibnp}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Log[c*x^n]]^p,x]
```

```
[Out] (x*Hypergeometric2F1[-p, -1/2*(I + b*n*p)/(b*n), (2 - I/(b*n) - p)/2, E^((2
*I)*a)*(c*x^n)^((2*I)*b)]*Sin[a + b*Log[c*x^n]]^p)/((1 - I*b*n*p)*(1 - E^((
2*I)*a)*(c*x^n)^((2*I)*b))^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4571

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4579

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Dist[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
eQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \sin^p(a+b\log(cx^n))\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\sin^p(a+b\log(x))\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-\frac{1}{n}+ibp}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\sin^p(a+b\log(cx^n))\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}-ibp}\right.}{n}$$

$$= \frac{x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}{}_2F_1\left(-p,-\frac{i+bnp}{2bn};\frac{1}{2}\left(2-\frac{i}{bn}-p\right);e^{2ia}(cx^n)^{2ib}\right)\sin^p(a+}{1-ibnp}$$

Mathematica [A]
time = 0.73, size = 146, normalized size = 1.30

$$\frac{ix\left(2-2e^{2ia}(cx^n)^{2ib}\right)^{-p}\left(-ie^{-ia}(cx^n)^{-ib}\left(-1+e^{2ia}(cx^n)^{2ib}\right)\right)^p{}_2F_1\left(-p,-\frac{i+bnp}{2bn};1-\frac{i}{2bn}-\frac{p}{2};e^{2ia}(cx^n)^{2ib}\right)}{i+bnp}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]^p,x]
```

```
[Out] (I*x*(((-I)*(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))/(E^(I*a)*(c*x^n)^(I*b)))^
p*Hypergeometric2F1[-p, -1/2*(I + b*n*p)/(b*n), 1 - (I/2)/(b*n) - p/2, E^((
2*I)*a)*(c*x^n)^((2*I)*b)])/((I + b*n*p)*(2 - 2*E^((2*I)*a)*(c*x^n)^((2*I)*
b))^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \sin^p(a+b\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))^p,x)
```

```
[Out] int(sin(a+b*ln(c*x^n))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral(sin(b*log(c*x^n) + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^p\left(a + b\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**p,x)
```

```
[Out] Integral(sin(a + b*log(c*x**n))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a + b\,\ln\left(c\,x^n\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))^p,x)
```

```
[Out] int(sin(a + b*log(c*x^n))^p, x)
```

3.83 $\int \frac{\sin^p(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=86

$$\frac{\cos\left(a+b\log\left(cx^n\right)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1+p}{2};\frac{3+p}{2};\sin^2\left(a+b\log\left(cx^n\right)\right)\right)\sin^{1+p}\left(a+b\log\left(cx^n\right)\right)}{bn(1+p)\sqrt{\cos^2\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] cos(a+b*ln(c*x^n))*hypergeom([1/2, 1/2+1/2*p],[3/2+1/2*p],sin(a+b*ln(c*x^n)
)^2)*sin(a+b*ln(c*x^n))^(1+p)/b/n/(1+p)/(cos(a+b*ln(c*x^n))^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{2722\}$

$$\frac{\cos\left(a+b\log\left(cx^n\right)\right)\sin^{p+1}\left(a+b\log\left(cx^n\right)\right)\,{}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\sin^2\left(a+b\log\left(cx^n\right)\right)\right)}{bn(p+1)\sqrt{\cos^2\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Log[c*x^n]]^p/x,x]
```

```
[Out] (Cos[a + b*Log[c*x^n]]*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Sin[a +
 b*Log[c*x^n]]^2]*Sin[a + b*Log[c*x^n]]^(1 + p))/(b*n*(1 + p)*Sqrt[Cos[a +
b*Log[c*x^n]]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^p\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\mathrm{Subst}\left(\int \sin^p(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\ &= \frac{\cos\left(a+b\log\left(cx^n\right)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1+p}{2};\frac{3+p}{2};\sin^2\left(a+b\log\left(cx^n\right)\right)\right)\sin^{1+p}\left(a+b\log\left(cx^n\right)\right)}{bn(1+p)\sqrt{\cos^2\left(a+b\log\left(cx^n\right)\right)}}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 86, normalized size = 1.00

$$\frac{\sqrt{\cos^2\left(a+b\log\left(cx^n\right)\right)}\,{}_2F_1\left(\frac{1}{2},\frac{1+p}{2};\frac{3+p}{2};\sin^2\left(a+b\log\left(cx^n\right)\right)\right)\sec\left(a+b\log\left(cx^n\right)\right)\sin^{1+p}\left(a+b\log\left(cx^n\right)\right)}{bn(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]^p/x,x]
```

```
[Out] (Sqrt[Cos[a + b*Log[c*x^n]]^2]*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2,
 Sin[a + b*Log[c*x^n]]^2]*Sec[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^(1 +
p))/(b*n*(1 + p))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sin^{p}\left(a+b\ln\left(c\,x^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))^p/x,x)
```

```
[Out] int(sin(a+b*ln(c*x^n))^p/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^p/x,x, algorithm="maxima")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^p/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^p/x,x, algorithm="fricas")
```

```
[Out] integral(sin(b*log(c*x^n) + a)^p/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{p}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**p/x,x)
```

```
[Out] Integral(sin(a + b*log(c*x**n))**p/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^p/x,x, algorithm="giac")
[Out] integrate(sin(b*log(c*x^n) + a)^p/x, x)
```

Mupad [B]
time = 2.72, size = 77, normalized size = 0.90

$$-\frac{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)\,\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^{p+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{1}{2}-\frac{p}{2};\ \frac{3}{2};\ \cos\left(a+b\,\ln\left(c\,x^n\right)\right)^2\right)}{b\,n\,{\left({\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}^2\right)}^{\frac{p}{2}+\frac{1}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))^p/x,x)
[Out] -(cos(a + b*log(c*x^n))*sin(a + b*log(c*x^n))^(p + 1)*hypergeom([1/2, 1/2 -
 p/2], 3/2, cos(a + b*log(c*x^n))^2))/(b*n*(sin(a + b*log(c*x^n))^2)^(p/2 +
 1/2))
```

3.84 $\int \frac{\sin^p(a+b\log(cx^n))}{x^2}\,dx$

Optimal. Leaf size=115

$$-\frac{\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p} {}_2F_1\left(\frac{1}{2}\left(\frac{i}{bn}-p\right),-p;\frac{1}{2}\left(2+\frac{i}{bn}-p\right);e^{2ia}(cx^n)^{2ib}\right)\sin^p\left(a+b\log\left(cx^n\right)\right)}{(1+ibnp)x}$$

```
[Out] -hypergeom([-p, 1/2*I/b/n-1/2*p],[1+1/2*I/b/n-1/2*p],exp(2*I*a)*(c*x^n)^(2*
I*b))*sin(a+b*ln(c*x^n))^p/(1+I*b*n*p)/x/((1-exp(2*I*a)*(c*x^n)^(2*I*b))^p)
```

Rubi [A]
time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4581, 4579, 371}

$$-\frac{\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p} {}_2F_1\left(\frac{1}{2}\left(\frac{i}{bn}-p\right),-p;\frac{1}{2}\left(-p+\frac{i}{bn}+2\right);e^{2ia}(cx^n)^{2ib}\right)\sin^p\left(a+b\log\left(cx^n\right)\right)}{x(1+ibnp)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Log[c*x^n]]^p/x^2,x]
```

```
[Out] -((Hypergeometric2F1[(I/(b*n) - p)/2, -p, (2 + I/(b*n) - p)/2, E^((2*I)*a)*
(c*x^n)^((2*I)*b)]*Sin[a + b*Log[c*x^n]]^p)/((1 + I*b*n*p)*x*(1 - E^((2*I)*
a)*(c*x^n)^((2*I)*b))^p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4579

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Dist[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\sin^p\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx = \frac{(cx^n)^{\frac{1}{n}}\,\text{Subst}\left(\int x^{-1-\frac{1}{n}}\sin^p(a+b\log(x))\,dx,x,cx^n\right)}{nx}$$

$$= \frac{\left((cx^n)^{\frac{1}{n}+ibp}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\sin^p\left(a+b\log\left(cx^n\right)\right)\right)\text{Subst}\left(\int x^{-1-\frac{1}{n}-ibp}(1\right.}{nx}$$

$$= -\frac{\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\,{}_2F_1\left(\frac{1}{2}\left(\frac{i}{bn}-p\right),-p;\frac{1}{2}\left(2+\frac{i}{bn}-p\right);e^{2ia}(cx^n)^{2ib}\right)\sin^p(}{(1+ibnp)x}$$

Mathematica [A]
time = 0.81, size = 146, normalized size = 1.27

$$\frac{\left(2-2e^{2ia}(cx^n)^{2ib}\right)^{-p}\left(-ie^{-ia}(cx^n)^{-ib}\left(-1+e^{2ia}(cx^n)^{2ib}\right)\right)^p\,{}_2F_1\left(\frac{i}{2bn}-\frac{p}{2},-p;1+\frac{i}{2bn}-\frac{p}{2};e^{2ia}(cx^n)^{2ib}\right)}{(-1-ibnp)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]^p/x^2,x]
```

```
[Out] ((((-I)*(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))/(E^(I*a)*(c*x^n)^(I*b)))^p*Hy
pergeometric2F1[(I/2)/(b*n) - p/2, -p, 1 + (I/2)/(b*n) - p/2, E^((2*I)*a)*(
c*x^n)^((2*I)*b)])/((-1 - I*b*n*p)*x*(2 - 2*E^((2*I)*a)*(c*x^n)^((2*I)*b))^
p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sin^p\left(a+b\ln\left(c\,x^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))^p/x^2,x)
```

```
[Out] int(sin(a+b*ln(c*x^n))^p/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^p/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral(sin(b*log(c*x^n) + a)^p/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{p}\left(a + b\log\left(cx^{n}\right)\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**p/x**2,x)
```

```
[Out] Integral(sin(a + b*log(c*x**n))**p/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^p/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}^p}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))^p/x^2,x)
```

```
[Out] int(sin(a + b*log(c*x^n))^p/x^2, x)
```

3.85 $\int \frac{\sin^p(a+b\log(cx^n))}{x^3}\,dx$

Optimal. Leaf size=115

$$-\frac{\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\,{}_2F_1\left(\frac{1}{2}\left(\frac{2i}{bn}-p\right),-p;\frac{1}{2}\left(2+\frac{2i}{bn}-p\right);e^{2ia}(cx^n)^{2ib}\right)\sin^p\left(a+b\log\left(cx^n\right)\right)}{(2+ibnp)x^2}$$

```
[Out] -hypergeom([-p, I/b/n-1/2*p],[1+I/b/n-1/2*p],exp(2*I*a)*(c*x^n)^(2*I*b))*si
n(a+b*ln(c*x^n))^p/(2+I*b*n*p)/x^2/((1-exp(2*I*a)*(c*x^n)^(2*I*b))^p)
```

Rubi [A]
time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4581, 4579, 371}

$$-\frac{\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\,{}_2F_1\left(\frac{1}{2}\left(\frac{2i}{bn}-p\right),-p;\frac{1}{2}\left(-p+\frac{2i}{bn}+2\right);e^{2ia}(cx^n)^{2ib}\right)\sin^p\left(a+b\log\left(cx^n\right)\right)}{x^2(2+ibnp)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Log[c*x^n]]^p/x^3,x]
```

```
[Out] -((Hypergeometric2F1[((2*I)/(b*n) - p)/2, -p, (2 + (2*I)/(b*n) - p)/2, E^((
2*I)*a)*(c*x^n)^((2*I)*b)]*Sin[a + b*Log[c*x^n]]^p)/((2 + I*b*n*p)*x^2*(1 -
 E^((2*I)*a)*(c*x^n)^((2*I)*b))^p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4579

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Dist[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
eQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rule 4581

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\sin^p\left(a+b\log\left(cx^n\right)\right)}{x^3}\,dx = \frac{\left(cx^n\right)^{2/n}\operatorname{Subst}\left(\int x^{-1-\frac{2}{n}}\sin^p(a+b\log(x))\,dx,x,cx^n\right)}{nx^2}$$

$$= \frac{\left(\left(cx^n\right)^{\frac{2}{n}+ibp}\left(1-e^{2ia}\left(cx^n\right)^{2ib}\right)^{-p}\sin^p\left(a+b\log\left(cx^n\right)\right)\right)\operatorname{Subst}\left(\int x^{-1-\frac{2}{n}-ibp}(1-\right.}{nx^2}$$

$$= -\frac{\left(1-e^{2ia}\left(cx^n\right)^{2ib}\right)^{-p}\,{}_2F_1\left(\frac{1}{2}\left(\frac{2i}{bn}-p\right),-p;\frac{1}{2}\left(2+\frac{2i}{bn}-p\right);e^{2ia}\left(cx^n\right)^{2ib}\right)\sin^p(a}{(2+ibnp)x^2}$$

Mathematica [A]
time = 0.79, size = 142, normalized size = 1.23

$$\frac{\left(2-2e^{2ia}\left(cx^n\right)^{2ib}\right)^{-p}\left(-ie^{-ia}\left(cx^n\right)^{-ib}\left(-1+e^{2ia}\left(cx^n\right)^{2ib}\right)\right)^p\,{}_2F_1\left(\frac{i}{bn}-\frac{p}{2},-p;1+\frac{i}{bn}-\frac{p}{2};e^{2ia}\left(cx^n\right)^{2ib}\right)}{(-2-ibnp)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]^p/x^3,x]
[Out] ((((-I)*(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))/(E^(I*a)*(c*x^n)^(I*b)))^p*Hy
pergeometric2F1[I/(b*n) - p/2, -p, 1 + I/(b*n) - p/2, E^((2*I)*a)*(c*x^n)^(
(2*I)*b)])/((-2 - I*b*n*p)*x^2*(2 - 2*E^((2*I)*a)*(c*x^n)^((2*I)*b))^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sin^p\left(a+b\ln\left(c\,x^n\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))^p/x^3,x)
[Out] int(sin(a+b*ln(c*x^n))^p/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^p/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^p/x^3,x, algorithm="fricas")
[Out] integral(sin(b*log(c*x^n) + a)^p/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{p}\left(a + b\log\left(cx^{n}\right)\right)}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**p/x**3,x)
[Out] Integral(sin(a + b*log(c*x**n))**p/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^p/x^3,x, algorithm="giac")
[Out] integrate(sin(b*log(c*x^n) + a)^p/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}^p}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*log(c*x^n))^p/x^3,x)
[Out] int(sin(a + b*log(c*x^n))^p/x^3, x)
```

3.86 $\int x^2 \cos(a+b\log(cx^n))\,dx$

Optimal. Leaf size=56

$$\frac{3x^3\cos(a+b\log(cx^n))}{9+b^2n^2}+\frac{bnx^3\sin(a+b\log(cx^n))}{9+b^2n^2}$$

```
[Out] 3*x^3*cos(a+b*ln(c*x^n))/(b^2*n^2+9)+b*n*x^3*sin(a+b*ln(c*x^n))/(b^2*n^2+9)
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {4574}

$$\frac{bnx^3\sin(a+b\log(cx^n))}{b^2n^2+9}+\frac{3x^3\cos(a+b\log(cx^n))}{b^2n^2+9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[a + b*Log[c*x^n]],x]
```

```
[Out] (3*x^3*Cos[a + b*Log[c*x^n]])/(9 + b^2*n^2) + (b*n*x^3*Sin[a + b*Log[c*x^n]
])/(9 + b^2*n^2)
```

Rule 4574

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int x^2\cos(a+b\log(cx^n))\,dx=\frac{3x^3\cos(a+b\log(cx^n))}{9+b^2n^2}+\frac{bnx^3\sin(a+b\log(cx^n))}{9+b^2n^2}$$

Mathematica [A]
time = 0.09, size = 43, normalized size = 0.77

$$\frac{x^3(3\cos(a+b\log(cx^n))+bn\sin(a+b\log(cx^n)))}{9+b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[a + b*Log[c*x^n]],x]
```

```
[Out] (x^3*(3*Cos[a + b*Log[c*x^n]] + b*n*Sin[a + b*Log[c*x^n]]))/(9 + b^2*n^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^{2} \cos \left(a + b \ln \left(c\,x^{n}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(a+b*ln(c*x^n)),x)
```

```
[Out] int(x^2*cos(a+b*ln(c*x^n)),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(56) = 112$.
time = 0.30, size = 218, normalized size = 3.89

$$\frac{\left(\left(b \cos \left(b \log \left(c\right)\right) \sin \left(2\, b \log \left(c\right)\right) - b \cos \left(2\, b \log \left(c\right)\right) \sin \left(b \log \left(c\right)\right) + b \sin \left(b \log \left(c\right)\right)\right)n + 3\, \cos \left(2\, b \log \left(c\right)\right) \cos \left(b \log \left(c\right)\right) + 3\, \sin \left(2\, b \log \left(c\right)\right) \sin \left(b \log \left(c\right)\right) + 3\, \cos \left(b \log \left(c\right)\right)\right)x^{3} \cos \left(b \log \left(x^{n}\right) + a\right) + \left(\left(b \cos \left(2\, b \log \left(c\right)\right) \cos \left(b \log \left(c\right)\right) + b \sin \left(2\, b \log \left(c\right)\right) \sin \left(b \log \left(c\right)\right) + b \cos \left(b \log \left(c\right)\right)\right)n - 3\, \cos \left(b \log \left(c\right)\right) \sin \left(2\, b \log \left(c\right)\right) + 3\, \cos \left(2\, b \log \left(c\right)\right) \sin \left(b \log \left(c\right)\right) - 3\, \sin \left(b \log \left(c\right)\right)\right)x^{3} \sin \left(b \log \left(x^{n}\right) + a\right)}{2\, \left(\left(b^{2} \cos \left(b \log \left(c\right)\right)^{2} + b^{2} \sin \left(b \log \left(c\right)\right)^{2}\right)n^{2} + 9\, \cos \left(b \log \left(c\right)\right)^{2} + 9\, \sin \left(b \log \left(c\right)\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] 1/2*(((b*cos(b*log(c))*sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)) +
b*sin(b*log(c)))*n + 3*cos(2*b*log(c))*cos(b*log(c)) + 3*sin(2*b*log(c))*si
n(b*log(c)) + 3*cos(b*log(c)))*x^3*cos(b*log(x^n) + a) + ((b*cos(2*b*log(c)
)*cos(b*log(c)) + b*sin(2*b*log(c))*sin(b*log(c)) + b*cos(b*log(c)))*n - 3*
cos(b*log(c))*sin(2*b*log(c)) + 3*cos(2*b*log(c))*sin(b*log(c)) - 3*sin(b*l
og(c)))*x^3*sin(b*log(x^n) + a))/((b^2*cos(b*log(c))^2 + b^2*sin(b*log(c))^
2)*n^2 + 9*cos(b*log(c))^2 + 9*sin(b*log(c))^2)
```

Fricas [A]
time = 3.07, size = 48, normalized size = 0.86

$$\frac{b n x^{3} \sin \left(b n \log \left(x\right) + b \log \left(c\right) + a\right) + 3 \, x^{3} \cos \left(b n \log \left(x\right) + b \log \left(c\right) + a\right)}{b^{2} n^{2} + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] (b*n*x^3*sin(b*n*log(x) + b*log(c) + a) + 3*x^3*cos(b*n*log(x) + b*log(c) +
 a))/(b^2*n^2 + 9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int x^{2} \cos{\left(a - \frac{3 i \log{\left(c x^{n} \right)}}{n} \right)}\, dx & \text{for}\: b = - \frac{3 i}{n} \\ \int x^{2} \cos{\left(a + \frac{3 i \log{\left(c x^{n} \right)}}{n} \right)}\, dx & \text{for}\: b = \frac{3 i}{n} \\ \frac{b n x^{3} \sin{\left(a + b \log{\left(c x^{n} \right)} \right)}}{b^{2} n^{2} + 9} + \frac{3 x^{3} \cos{\left(a + b \log{\left(c x^{n} \right)} \right)}}{b^{2} n^{2} + 9} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((Integral(x**2*cos(a - 3*I*log(c*x**n)/n), x), Eq(b, -3*I/n)), (I
ntegral(x**2*cos(a + 3*I*log(c*x**n)/n), x), Eq(b, 3*I/n)), (b*n*x**3*sin(a
 + b*log(c*x**n))/(b**2*n**2 + 9) + 3*x**3*cos(a + b*log(c*x**n))/(b**2*n**
2 + 9), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 923 vs. $2(56) = 112$.
time = 0.45, size = 923, normalized size = 16.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*n*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*p
i*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a) + 2*b*n*x^3*
e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*
n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a) + 2*b*n*x^3*e^(1/2*pi*b*n*s
gn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) +
1/2*b*log(abs(c)))*tan(1/2*a)^2 + 2*b*n*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*
b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c
)))*tan(1/2*a)^2 - 3*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c
) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 -
 3*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan
(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 - 2*b*n*x^3*e^(1/2
*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(a
bs(x)) + 1/2*b*log(abs(c))) - 2*b*n*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n
- 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))
- 2*b*n*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)
*tan(1/2*a) - 2*b*n*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c
) + 1/2*pi*b)*tan(1/2*a) + 3*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi
*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2 + 3*x^
3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*
b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2 + 12*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*
pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(ab
s(c)))*tan(1/2*a) + 12*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sg
n(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a) +
3*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1
/2*a)^2 + 3*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*
pi*b)*tan(1/2*a)^2 - 3*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn
(c) - 1/2*pi*b) - 3*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c
) + 1/2*pi*b))/(b^2*n^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(
```

```
1/2*a)^2 + b^2*n^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2 + b^2*n^2
*tan(1/2*a)^2 + b^2*n^2 + 9*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*
tan(1/2*a)^2 + 9*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2 + 9*tan(1/2
*a)^2 + 9)
```

Mupad [B]
time = 2.45, size = 43, normalized size = 0.77

$$\frac{x^3\,\left(3\,\cos\left(a+b\,\ln\left(c\,x^n\right)\right)+b\,n\,\sin\left(a+b\,\ln\left(c\,x^n\right)\right)\right)}{b^2\,n^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(a + b*log(c*x^n)),x)
[Out] (x^3*(3*cos(a + b*log(c*x^n)) + b*n*sin(a + b*log(c*x^n))))/(b^2*n^2 + 9)
```

3.87 $\int x \cos(a + b \log(cx^n)) \, dx$

Optimal. Leaf size=56

$$\frac{2x^2 \cos(a + b \log(cx^n))}{4 + b^2n^2} + \frac{bnx^2 \sin(a + b \log(cx^n))}{4 + b^2n^2}$$

```
[Out] 2*x^2*cos(a+b*ln(c*x^n))/(b^2*n^2+4)+b*n*x^2*sin(a+b*ln(c*x^n))/(b^2*n^2+4)
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{4574\}$

$$\frac{bnx^2 \sin(a + b \log(cx^n))}{b^2n^2 + 4} + \frac{2x^2 \cos(a + b \log(cx^n))}{b^2n^2 + 4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*Log[c*x^n]],x]
```

```
[Out] (2*x^2*Cos[a + b*Log[c*x^n]])/(4 + b^2*n^2) + (b*n*x^2*Sin[a + b*Log[c*x^n]
])/(4 + b^2*n^2)
```

Rule 4574

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int x \cos(a + b \log(cx^n)) \, dx = \frac{2x^2 \cos(a + b \log(cx^n))}{4 + b^2n^2} + \frac{bnx^2 \sin(a + b \log(cx^n))}{4 + b^2n^2}$$

Mathematica [A]
time = 0.08, size = 43, normalized size = 0.77

$$\frac{x^2(2 \cos(a + b \log(cx^n)) + bn \sin(a + b \log(cx^n)))}{4 + b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*Log[c*x^n]],x]
```

```
[Out] (x^2*(2*Cos[a + b*Log[c*x^n]] + b*n*Sin[a + b*Log[c*x^n]]))/(4 + b^2*n^2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x\cos\left(a+b\ln\left(c\,x^{n}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a+b*ln(c*x^n)),x)
```

```
[Out] int(x*cos(a+b*ln(c*x^n)),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(56) = 112$.
time = 0.31, size = 218, normalized size = 3.89

$$\frac{\left(\left(b\cos\left(b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)-b\cos\left(2\,b\log\left(c\right)\right)\sin\left(b\log\left(c\right)\right)+b\sin\left(b\log\left(c\right)\right)\right)n+2\,\cos\left(2\,b\log\left(c\right)\right)\cos\left(b\log\left(c\right)\right)+2\,\sin\left(2\,b\log\left(c\right)\right)\sin\left(b\log\left(c\right)\right)+2\,\cos\left(b\log\left(c\right)\right)\right)x^{2}\cos\left(b\log\left(x^{n}\right)+a\right)+\left(\left(b\cos\left(2\,b\log\left(c\right)\right)\cos\left(b\log\left(c\right)\right)+b\sin\left(2\,b\log\left(c\right)\right)\sin\left(b\log\left(c\right)\right)+b\cos\left(b\log\left(c\right)\right)\right)n-2\,\cos\left(b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)+2\,\cos\left(2\,b\log\left(c\right)\right)\sin\left(b\log\left(c\right)\right)-2\,\sin\left(b\log\left(c\right)\right)\right)x^{2}\sin\left(b\log\left(x^{n}\right)+a\right)}{2\,\left(\left(b^{2}\cos\left(b\log\left(c\right)\right)^{2}+b^{2}\sin\left(b\log\left(c\right)\right)^{2}\right)n^{2}+4\,\cos\left(b\log\left(c\right)\right)^{2}+4\,\sin\left(b\log\left(c\right)\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] 1/2*(((b*cos(b*log(c))*sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)) +
b*sin(b*log(c)))*n + 2*cos(2*b*log(c))*cos(b*log(c)) + 2*sin(2*b*log(c))*si
n(b*log(c)) + 2*cos(b*log(c)))*x^2*cos(b*log(x^n) + a) + ((b*cos(2*b*log(c)
)*cos(b*log(c)) + b*sin(2*b*log(c))*sin(b*log(c)) + b*cos(b*log(c)))*n - 2*
cos(b*log(c))*sin(2*b*log(c)) + 2*cos(2*b*log(c))*sin(b*log(c)) - 2*sin(b*l
og(c)))*x^2*sin(b*log(x^n) + a))/((b^2*cos(b*log(c))^2 + b^2*sin(b*log(c))^
2)*n^2 + 4*cos(b*log(c))^2 + 4*sin(b*log(c))^2)
```

Fricas [A]
time = 2.88, size = 48, normalized size = 0.86

$$\frac{bnx^{2}\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+2\,x^{2}\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{b^{2}n^{2}+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] (b*n*x^2*sin(b*n*log(x) + b*log(c) + a) + 2*x^2*cos(b*n*log(x) + b*log(c) +
 a))/(b^2*n^2 + 4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int x\cos\left(a-\frac{2i\log\left(cx^{n}\right)}{n}\right)dx & \text{for}\: b=-\frac{2i}{n} \\ \int x\cos\left(a+\frac{2i\log\left(cx^{n}\right)}{n}\right)dx & \text{for}\: b=\frac{2i}{n} \\ \frac{bnx^{2}\sin\left(a+b\log\left(cx^{n}\right)\right)}{b^{2}n^{2}+4}+\frac{2x^{2}\cos\left(a+b\log\left(cx^{n}\right)\right)}{b^{2}n^{2}+4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((Integral(x*cos(a - 2*I*log(c*x**n)/n), x), Eq(b, -2*I/n)), (Inte
gral(x*cos(a + 2*I*log(c*x**n)/n), x), Eq(b, 2*I/n)), (b*n*x**2*sin(a + b*l
og(c*x**n))/(b**2*n**2 + 4) + 2*x**2*cos(a + b*log(c*x**n))/(b**2*n**2 + 4)
, True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 915 vs. 2(56) = 112.
time = 0.45, size = 915, normalized size = 16.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] -(b*n*x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*t
an(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a) + b*n*x^2*e^(-1/2*
pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(ab
s(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a) + b*n*x^2*e^(1/2*pi*b*n*sgn(x) - 1/
2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(
abs(c)))*tan(1/2*a)^2 + b*n*x^2*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi
*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*
a)^2 - x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*
tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 - x^2*e^(-1/2*p
i*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs
(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 - b*n*x^2*e^(1/2*pi*b*n*sgn(x) - 1
/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log
(abs(c))) - b*n*x^2*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) +
1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c))) - b*n*x^2*e^(1/2*pi*
b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*a) - b*n*x^2*
e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*a)
 + x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(
1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2 + x^2*e^(-1/2*pi*b*n*sgn(x) + 1/
2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(
abs(c)))^2 + 4*x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/
2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a) + 4*x^2*e^(
-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*l
og(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a) + x^2*e^(1/2*pi*b*n*sgn(x) - 1/2
*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*a)^2 + x^2*e^(-1/2*pi*b*n*sgn
(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*a)^2 - x^2*e^(1/2*pi
*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b) - x^2*e^(-1/2*pi*b*n
*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b))/(b^2*n^2*tan(1/2*b*n*lo
g(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 + b^2*n^2*tan(1/2*b*n*log(abs
```

```
(x)) + 1/2*b*log(abs(c)))^2 + b^2*n^2*tan(1/2*a)^2 + b^2*n^2 + 4*tan(1/2*b*
n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 + 4*tan(1/2*b*n*log(abs(x
)) + 1/2*b*log(abs(c)))^2 + 4*tan(1/2*a)^2 + 4)
```

Mupad [B]
time = 2.43, size = 43, normalized size = 0.77

$$\frac{x^2\,\left(2\,\cos\left(a+b\,\ln\left(c\,x^n\right)\right)+b\,n\,\sin\left(a+b\,\ln\left(c\,x^n\right)\right)\right)}{b^2\,n^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*log(c*x^n)),x)
[Out] (x^2*(2*cos(a + b*log(c*x^n)) + b*n*sin(a + b*log(c*x^n))))/(b^2*n^2 + 4)
```

3.88 $\int \cos(a+b\log(cx^n))\,dx$

Optimal. Leaf size=51

$$\frac{x\cos(a+b\log(cx^n))}{1+b^2n^2}+\frac{bnx\sin(a+b\log(cx^n))}{1+b^2n^2}$$

```
[Out] x*cos(a+b*ln(c*x^n))/(b^2*n^2+1)+b*n*x*sin(a+b*ln(c*x^n))/(b^2*n^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = $\{4564\}$

$$\frac{bnx\sin(a+b\log(cx^n))}{b^2n^2+1}+\frac{x\cos(a+b\log(cx^n))}{b^2n^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*Log[c*x^n]],x]
```

```
[Out] (x*Cos[a + b*Log[c*x^n]])/(1 + b^2*n^2) + (b*n*x*Sin[a + b*Log[c*x^n]])/(1
+ b^2*n^2)
```

Rule 4564

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*(
Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] + Simp[b*d*n*x*(Sin[d*(a +
 b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

Rubi steps

$$\int \cos(a+b\log(cx^n))\,dx=\frac{x\cos(a+b\log(cx^n))}{1+b^2n^2}+\frac{bnx\sin(a+b\log(cx^n))}{1+b^2n^2}$$

Mathematica [A]
time = 0.06, size = 39, normalized size = 0.76

$$\frac{x(\cos(a+b\log(cx^n))+bn\sin(a+b\log(cx^n)))}{1+b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*Log[c*x^n]],x]
```

```
[Out] (x*(Cos[a + b*Log[c*x^n]] + b*n*Sin[a + b*Log[c*x^n]]))/(1 + b^2*n^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \cos\left(a+b\ln\left(c\,x^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*ln(c*x^n)),x)
```

```
[Out] int(cos(a+b*ln(c*x^n)),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(51) = 102.
time = 0.30, size = 205, normalized size = 4.02

$$\frac{\left(\left(b\cos\left(b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)-b\cos\left(2\,b\log\left(c\right)\right)\sin\left(b\log\left(c\right)\right)+b\sin\left(b\log\left(c\right)\right)\right)n+\cos\left(2\,b\log\left(c\right)\right)\cos\left(b\log\left(c\right)\right)+\sin\left(2\,b\log\left(c\right)\right)\sin\left(b\log\left(c\right)\right)+\cos\left(b\log\left(c\right)\right)\right)x\cos\left(b\log\left(x^{n}\right)+a\right)+\left(\left(b\cos\left(2\,b\log\left(c\right)\right)\cos\left(b\log\left(c\right)\right)+b\sin\left(2\,b\log\left(c\right)\right)\sin\left(b\log\left(c\right)\right)+b\cos\left(b\log\left(c\right)\right)\right)n-\cos\left(b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)+\cos\left(2\,b\log\left(c\right)\right)\sin\left(b\log\left(c\right)\right)-\sin\left(b\log\left(c\right)\right)\right)x\sin\left(b\log\left(x^{n}\right)+a\right)}{2\left(\left(b^{2}\cos\left(b\log\left(c\right)\right)^{2}+b^{2}\sin\left(b\log\left(c\right)\right)^{2}\right)n^{2}+\cos\left(b\log\left(c\right)\right)^{2}+\sin\left(b\log\left(c\right)\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] 1/2*(((b*cos(b*log(c))*sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)) +
b*sin(b*log(c)))*n + cos(2*b*log(c))*cos(b*log(c)) + sin(2*b*log(c))*sin(b*
log(c)) + cos(b*log(c)))*x*cos(b*log(x^n) + a) + ((b*cos(2*b*log(c))*cos(b*
log(c)) + b*sin(2*b*log(c))*sin(b*log(c)) + b*cos(b*log(c)))*n - cos(b*log(
c))*sin(2*b*log(c)) + cos(2*b*log(c))*sin(b*log(c)) - sin(b*log(c)))*x*sin(
b*log(x^n) + a))/((b^2*cos(b*log(c))^2 + b^2*sin(b*log(c))^2)*n^2 + cos(b*l
og(c))^2 + sin(b*log(c))^2)
```

Fricas [A]
time = 2.60, size = 43, normalized size = 0.84

$$\frac{bnx\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{b^{2}n^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] (b*n*x*sin(b*n*log(x) + b*log(c) + a) + x*cos(b*n*log(x) + b*log(c) + a))/(
b^2*n^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \cos\left(a-\frac{i\log\left(cx^{n}\right)}{n}\right)dx & \text{for}\: b=-\frac{i}{n} \\ \int \cos\left(a+\frac{i\log\left(cx^{n}\right)}{n}\right)dx & \text{for}\: b=\frac{i}{n} \\ \frac{bnx\sin\left(a+b\log\left(cx^{n}\right)\right)}{b^{2}n^{2}+1}+\frac{x\cos\left(a+b\log\left(cx^{n}\right)\right)}{b^{2}n^{2}+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((Integral(cos(a - I*log(c*x**n)/n), x), Eq(b, -I/n)), (Integral(c
os(a + I*log(c*x**n)/n), x), Eq(b, I/n)), (b*n*x*sin(a + b*log(c*x**n))/(b*
*2*n**2 + 1) + x*cos(a + b*log(c*x**n))/(b**2*n**2 + 1), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 878 vs. 2(51) = 102.
time = 0.46, size = 878, normalized size = 17.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*n*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*
b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a) + 2*b*n*x*e^(-
1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*lo
g(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a) + 2*b*n*x*e^(1/2*pi*b*n*sgn(x)
- 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*
log(abs(c)))*tan(1/2*a)^2 + 2*b*n*x*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/
2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(
1/2*a)^2 - x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b
)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 - x*e^(-1/2*p
i*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs
(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 - 2*b*n*x*e^(1/2*pi*b*n*sgn(x) - 1
/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log
(abs(c))) - 2*b*n*x*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) +
1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c))) - 2*b*n*x*e^(1/2*pi*
b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*a) - 2*b*n*x*
e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*a)
 + x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/
2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2 + x*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi
*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(
c)))^2 + 4*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b
)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a) + 4*x*e^(-1/2*pi*
b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x
)) + 1/2*b*log(abs(c)))*tan(1/2*a) + x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n +
1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*a)^2 + x*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi
*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*a)^2 - x*e^(1/2*pi*b*n*sgn(x) -
1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b) - x*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi
*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b))/(b^2*n^2*tan(1/2*b*n*log(abs(x)) + 1/2*
b*log(abs(c)))^2*tan(1/2*a)^2 + b^2*n^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log
(abs(c)))^2 + b^2*n^2*tan(1/2*a)^2 + b^2*n^2 + tan(1/2*b*n*log(abs(x)) + 1/
```

```
2*b*log(abs(c)))^2*tan(1/2*a)^2 + tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c
)))^2 + tan(1/2*a)^2 + 1)
```

Mupad [B]
time = 2.35, size = 39, normalized size = 0.76

$$\frac{x\,\left(\cos\left(a+b\,\ln\left(c\,x^n\right)\right)+b\,n\,\sin\left(a+b\,\ln\left(c\,x^n\right)\right)\right)}{b^2\,n^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*log(c*x^n)),x)
[Out] (x*(cos(a + b*log(c*x^n)) + b*n*sin(a + b*log(c*x^n))))/(b^2*n^2 + 1)
```

3.89 $\int \frac{\cos(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=18

$$\frac{\sin\left(a+b\log\left(cx^n\right)\right)}{bn}$$

```
[Out] sin(a+b*ln(c*x^n))/b/n
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{2717\}$

$$\frac{\sin\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*Log[c*x^n]]/x,x]
[Out] Sin[a + b*Log[c*x^n]]/(b*n)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \cos(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= \frac{\sin\left(a+b\log\left(cx^n\right)\right)}{bn}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(18) = 36.
time = 0.04, size = 37, normalized size = 2.06

$$\frac{\cos\left(b\log\left(cx^n\right)\right)\sin(a)}{bn}+\frac{\cos(a)\sin\left(b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*Log[c*x^n]]/x,x]
[Out] (Cos[b*Log[c*x^n]]*Sin[a])/(b*n) + (Cos[a]*Sin[b*Log[c*x^n]])/(b*n)
```

Maple [A]
time = 0.03, size = 19, normalized size = 1.06

method	result	size
derivativedivides	$\frac{\sin(a+b\ln(c\,x^n))}{bn}$	19
default	$\frac{\sin(a+b\ln(c\,x^n))}{bn}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
[Out] sin(a+b*ln(c*x^n))/b/n
```

Maxima [A]
time = 0.27, size = 18, normalized size = 1.00

$$\frac{\sin\left(b\log\left(cx^n\right)+a\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))/x,x, algorithm="maxima")
[Out] sin(b*log(c*x^n) + a)/(b*n)
```

Fricas [A]
time = 2.31, size = 19, normalized size = 1.06

$$\frac{\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))/x,x, algorithm="fricas")
[Out] sin(b*n*log(x) + b*log(c) + a)/(b*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(14) = 28$.
time = 0.26, size = 34, normalized size = 1.89

$$\begin{cases}\log\left(x\right)\cos\left(a\right) & \text{for}\ b=0\wedge\left(b=0\vee n=0\right)\\ \log\left(x\right)\cos\left(a+b\log\left(c\right)\right) & \text{for}\ n=0\\ \frac{\sin\left(a+b\log\left(cx^n\right)\right)}{bn} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*ln(c*x**n))/x,x)
```

```
[Out] Piecewise((log(x)*cos(a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*cos(a
+ b*log(c)), Eq(n, 0)), (sin(a + b*log(c*x**n))/(b*n), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))/x,x, algorithm="giac")
```

```
[Out] integrate(cos(b*log(c*x^n) + a)/x, x)
```

Mupad [B]
time = 2.28, size = 18, normalized size = 1.00

$$\frac{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*log(c*x^n))/x,x)
```

```
[Out] sin(a + b*log(c*x^n))/(b*n)
```

3.90 $\int \frac{\cos(a+b\log(cx^n))}{x^2}\,dx$

Optimal. Leaf size=56

$$-\frac{\cos\left(a+b\log\left(cx^n\right)\right)}{\left(1+b^2n^2\right)x}+\frac{bn\sin\left(a+b\log\left(cx^n\right)\right)}{\left(1+b^2n^2\right)x}$$

```
[Out] -cos(a+b*ln(c*x^n))/(b^2*n^2+1)/x+b*n*sin(a+b*ln(c*x^n))/(b^2*n^2+1)/x
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{4574\}$

$$\frac{bn\sin\left(a+b\log\left(cx^n\right)\right)}{x\left(b^2n^2+1\right)}-\frac{\cos\left(a+b\log\left(cx^n\right)\right)}{x\left(b^2n^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*Log[c*x^n]]/x^2,x]
```

```
[Out] -(Cos[a + b*Log[c*x^n]]/((1 + b^2*n^2)*x)) + (b*n*Sin[a + b*Log[c*x^n]])/((
1 + b^2*n^2)*x)
```

Rule 4574

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int \frac{\cos\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx=-\frac{\cos\left(a+b\log\left(cx^n\right)\right)}{\left(1+b^2n^2\right)x}+\frac{bn\sin\left(a+b\log\left(cx^n\right)\right)}{\left(1+b^2n^2\right)x}$$

Mathematica [A]
time = 0.07, size = 41, normalized size = 0.73

$$\frac{-\cos\left(a+b\log\left(cx^n\right)\right)+bn\sin\left(a+b\log\left(cx^n\right)\right)}{x+b^2n^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*Log[c*x^n]]/x^2,x]
```

```
[Out] (-Cos[a + b*Log[c*x^n]] + b*n*Sin[a + b*Log[c*x^n]])/(x + b^2*n^2*x)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\cos \left(a +b \ln \left(c \,x^{n}\right)\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*ln(c*x^n))/x^2,x)
```

```
[Out] int(cos(a+b*ln(c*x^n))/x^2,x)
```

MaximA [B] Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(56) = 112$.
time = 0.30, size = 208, normalized size = 3.71

$$\frac{\left(\left(b\cos\left(b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)-b\cos\left(2\,b\log\left(c\right)\right)\sin\left(b\log\left(c\right)\right)+b\sin\left(b\log\left(c\right)\right)\right)n-\cos\left(2\,b\log\left(c\right)\right)\cos\left(b\log\left(c\right)\right)-\sin\left(2\,b\log\left(c\right)\right)\sin\left(b\log\left(c\right)\right)-\cos\left(b\log\left(c\right)\right)\right)\cos\left(b\log\left(x^{n}\right)+a\right)+\left(\left(b\cos\left(2\,b\log\left(c\right)\right)\cos\left(b\log\left(c\right)\right)+b\sin\left(2\,b\log\left(c\right)\right)\sin\left(b\log\left(c\right)\right)+b\cos\left(b\log\left(c\right)\right)\right)n+\cos\left(b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)-\cos\left(2\,b\log\left(c\right)\right)\sin\left(b\log\left(c\right)\right)+\sin\left(b\log\left(c\right)\right)\right)\sin\left(b\log\left(x^{n}\right)+a\right)}{2\,\left(\left(b^{2}\cos\left(b\log\left(c\right)\right)^{2}+b^{2}\sin\left(b\log\left(c\right)\right)^{2}\right)n^{2}+\cos\left(b\log\left(c\right)\right)^{2}+\sin\left(b\log\left(c\right)\right)^{2}\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(((b*cos(b*log(c))*sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)) +
b*sin(b*log(c)))*n - cos(2*b*log(c))*cos(b*log(c)) - sin(2*b*log(c))*sin(b*
log(c)) - cos(b*log(c)))*cos(b*log(x^n) + a) + ((b*cos(2*b*log(c))*cos(b*lo
g(c)) + b*sin(2*b*log(c))*sin(b*log(c)) + b*cos(b*log(c)))*n + cos(b*log(c)
)*sin(2*b*log(c)) - cos(2*b*log(c))*sin(b*log(c)) + sin(b*log(c)))*sin(b*lo
g(x^n) + a))/(((b^2*cos(b*log(c))^2 + b^2*sin(b*log(c))^2)*n^2 + cos(b*log(
c))^2 + sin(b*log(c))^2)*x)
```

Fricas [A]
time = 4.62, size = 45, normalized size = 0.80

$$\frac{bn \sin \left(bn \log \left(x\right) + b \log \left(c\right) + a\right) - \cos \left(bn \log \left(x\right) + b \log \left(c\right) + a\right)}{{\left(b^{2} n^{2} + 1\right)} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))/x^2,x, algorithm="fricas")
```

```
[Out] (b*n*sin(b*n*log(x) + b*log(c) + a) - cos(b*n*log(x) + b*log(c) + a))/((b^2
*n^2 + 1)*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.30, size = 190, normalized size = 3.39

$$\begin{cases} \frac{i\sin\left(a-\frac{i\log(cx^n)}{n}\right)}{2x}+\frac{i\log(cx^n)\sin\left(a-\frac{i\log(cx^n)}{n}\right)}{2nx}+\frac{\log(cx^n)\cos\left(a-\frac{i\log(cx^n)}{n}\right)}{2nx} & \text{for } b=-\frac{i}{n} \\ -\frac{\cos\left(a+\frac{i\log(cx^n)}{n}\right)}{2x}-\frac{i\log(cx^n)\sin\left(a+\frac{i\log(cx^n)}{n}\right)}{2nx}+\frac{\log(cx^n)\cos\left(a+\frac{i\log(cx^n)}{n}\right)}{2nx} & \text{for } b=\frac{i}{n} \\ \frac{bn\sin(a+b\log(cx^n))}{b^2n^2x+x}-\frac{\cos(a+b\log(cx^n))}{b^2n^2x+x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*ln(c*x**n))/x**2,x)
```

```
[Out] Piecewise((I*sin(a - I*log(c*x**n)/n)/(2*x) + I*log(c*x**n)*sin(a - I*log(c
*x**n)/n)/(2*n*x) + log(c*x**n)*cos(a - I*log(c*x**n)/n)/(2*n*x), Eq(b, -I/
n)), (-cos(a + I*log(c*x**n)/n)/(2*x) - I*log(c*x**n)*sin(a + I*log(c*x**n)
/n)/(2*n*x) + log(c*x**n)*cos(a + I*log(c*x**n)/n)/(2*n*x), Eq(b, I/n)), (b
*n*sin(a + b*log(c*x**n))/(b**2*n**2*x + x) - cos(a + b*log(c*x**n))/(b**2*
n**2*x + x), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))/x^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*log(c*x^n) + a)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*log(c*x^n))/x^2,x)
```

```
[Out] int(cos(a + b*log(c*x^n))/x^2, x)
```

3.91 $\int x^2 \cos^2(a+b\log(cx^n))\,dx$

Optimal. Leaf size=97

$$\frac{2b^2n^2x^3}{3(9+4b^2n^2)}+\frac{3x^3\cos^2(a+b\log(cx^n))}{9+4b^2n^2}+\frac{2bnx^3\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{9+4b^2n^2}$$

```
[Out] 2/3*b^2*n^2*x^3/(4*b^2*n^2+9)+3*x^3*cos(a+b*ln(c*x^n))^2/(4*b^2*n^2+9)+2*b*
n*x^3*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))/(4*b^2*n^2+9)
```

Rubi [A]
time = 0.02, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.118$, Rules used = {4576, 30}

$$\frac{3x^3\cos^2(a+b\log(cx^n))}{4b^2n^2+9}+\frac{2bnx^3\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+9}+\frac{2b^2n^2x^3}{3(4b^2n^2+9)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (2*b^2*n^2*x^3)/(3*(9 + 4*b^2*n^2)) + (3*x^3*Cos[a + b*Log[c*x^n]]^2)/(9 +
4*b^2*n^2) + (2*b*n*x^3*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/(9 + 4
*b^2*n^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4576

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2
*n^2*p^2 + (m + 1)^2)), Int[(e*x)^m*Cos[d*(a + b*Log[c*x^n])]^(p - 2), x],
x] + Simp[b*d*n*p*(e*x)^(m + 1)*Sin[d*(a + b*Log[c*x^n])]*(Cos[d*(a + b*Log
[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x]) /; FreeQ[{a, b, c
, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\cos^2(a+b\log(cx^n))\,dx&=\frac{3x^3\cos^2(a+b\log(cx^n))}{9+4b^2n^2}+\frac{2bnx^3\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{9+4b^2n^2}+\\&=\frac{2b^2n^2x^3}{3(9+4b^2n^2)}+\frac{3x^3\cos^2(a+b\log(cx^n))}{9+4b^2n^2}+\frac{2bnx^3\cos(a+b\log(cx^n))\sin(a}{9+4b^2n^2}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 61, normalized size = 0.63

$$\frac{x^3(9+4b^2n^2+9\cos(2(a+b\log(cx^n)))+6bn\sin(2(a+b\log(cx^n))))}{6(9+4b^2n^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x^3*(9 + 4*b^2*n^2 + 9*Cos[2*(a + b*Log[c*x^n])] + 6*b*n*Sin[2*(a + b*Log[
c*x^n])]))/(6*(9 + 4*b^2*n^2))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int x^2\left(\cos^2\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(x^2*cos(a+b*ln(c*x^n))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(95) = 190.
time = 0.30, size = 301, normalized size = 3.10

$$\frac{3\,\left(2\,\left(b\cos\left(2\,b\log\left(c\right)\right)\sin\left(4\,b\log\left(c\right)\right)-b\cos\left(4\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)+b\sin\left(2\,b\log\left(c\right)\right)\right)n+3\,\cos\left(4\,b\log\left(c\right)\right)\cos\left(2\,b\log\left(c\right)\right)+3\,\sin\left(4\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)+3\,\cos\left(2\,b\log\left(c\right)\right)\right)x^3\cos\left(2\,b\log\left(x^n\right)+2\,a\right)+3\,\left(2\,\left(b\cos\left(4\,b\log\left(c\right)\right)\cos\left(2\,b\log\left(c\right)\right)+b\sin\left(4\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)+b\cos\left(2\,b\log\left(c\right)\right)\right)n-3\,\cos\left(2\,b\log\left(c\right)\right)\sin\left(4\,b\log\left(c\right)\right)+3\,\cos\left(4\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)-3\,\sin\left(2\,b\log\left(c\right)\right)\right)x^3\sin\left(2\,b\log\left(x^n\right)+2\,a\right)+2\,\left(4\,\left(b^2\cos\left(2\,b\log\left(c\right)\right)^2+b^2\sin\left(2\,b\log\left(c\right)\right)^2\right)n^2+9\,\cos\left(2\,b\log\left(c\right)\right)^2+9\,\sin\left(2\,b\log\left(c\right)\right)^2\right)x^3}{12\,\left(4\,\left(b^2\cos\left(2\,b\log\left(c\right)\right)^2+b^2\sin\left(2\,b\log\left(c\right)\right)^2\right)n^2+9\,\cos\left(2\,b\log\left(c\right)\right)^2+9\,\sin\left(2\,b\log\left(c\right)\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*(b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*l
og(c)) + b*sin(2*b*log(c)))*n + 3*cos(4*b*log(c))*cos(2*b*log(c)) + 3*sin(4
*b*log(c))*sin(2*b*log(c)) + 3*cos(2*b*log(c)))*x^3*cos(2*b*log(x^n) + 2*a)
 + 3*(2*(b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log(
c)) + b*cos(2*b*log(c)))*n - 3*cos(2*b*log(c))*sin(4*b*log(c)) + 3*cos(4*b*
log(c))*sin(2*b*log(c)) - 3*sin(2*b*log(c)))*x^3*sin(2*b*log(x^n) + 2*a) +
2*(4*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + 9*cos(2*b*log(c)
)^2 + 9*sin(2*b*log(c))^2)*x^3)/(4*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log
(c))^2)*n^2 + 9*cos(2*b*log(c))^2 + 9*sin(2*b*log(c))^2)
```

Fricas [A]
time = 2.27, size = 76, normalized size = 0.78

$$\frac{2\,b^2n^2x^3+6\,bnx^3\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+9\,x^3\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2}{3\,\left(4\,b^2n^2+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*b^2*n^2*x^3 + 6*b*n*x^3*cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(x
) + b*log(c) + a) + 9*x^3*cos(b*n*log(x) + b*log(c) + a)^2)/(4*b^2*n^2 + 9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int x^{2}\cos^{2}\left(a - \frac{3i\log\left(cx^{n}\right)}{2n}\right)\,dx & \text{for}\: b = -\frac{3i}{2n} \\ \int x^{2}\cos^{2}\left(a + \frac{3i\log\left(cx^{n}\right)}{2n}\right)\,dx & \text{for}\: b = \frac{3i}{2n} \\ \frac{2b^{2}n^{2}x^{3}\sin^{2}\left(a+b\log\left(cx^{n}\right)\right)}{12b^{2}n^{2}+27} + \frac{2b^{2}n^{2}x^{3}\cos^{2}\left(a+b\log\left(cx^{n}\right)\right)}{12b^{2}n^{2}+27} + \frac{6bnx^{3}\sin\left(a+b\log\left(cx^{n}\right)\right)\cos\left(a+b\log\left(cx^{n}\right)\right)}{12b^{2}n^{2}+27} + \frac{9x^{3}\cos^{2}\left(a+b\log\left(cx^{n}\right)\right)}{12b^{2}n^{2}+27} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(a+b*ln(c*x**n))**2,x)
```

```
[Out] Piecewise((Integral(x**2*cos(a - 3*I*log(c*x**n)/(2*n))**2, x), Eq(b, -3*I/
(2*n))), (Integral(x**2*cos(a + 3*I*log(c*x**n)/(2*n))**2, x), Eq(b, 3*I/(2
*n))), (2*b**2*n**2*x**3*sin(a + b*log(c*x**n))**2/(12*b**2*n**2 + 27) + 2*
b**2*n**2*x**3*cos(a + b*log(c*x**n))**2/(12*b**2*n**2 + 27) + 6*b*n*x**3*s
in(a + b*log(c*x**n))*cos(a + b*log(c*x**n))/(12*b**2*n**2 + 27) + 9*x**3*c
os(a + b*log(c*x**n))**2/(12*b**2*n**2 + 27), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 833 vs. 2(95) = 190.
time = 0.58, size = 833, normalized size = 8.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3 - 1/4*(4*b*n*x^3*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*ta
n(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 4*b*n*x^3*e^(-pi*b*n*sgn(x) +
 pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)
 + 4*b*n*x^3*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(ab
s(x)) + b*log(abs(c)))*tan(a)^2 + 4*b*n*x^3*e^(-pi*b*n*sgn(x) + pi*b*n - pi
*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a)^2 - 3*x^3*e^(
pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(ab
s(c)))^2*tan(a)^2 - 3*x^3*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*
tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 - 4*b*n*x^3*e^(pi*b*n*sgn(x
) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c))) - 4*b
*n*x^3*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x))
 + b*log(abs(c))) - 4*b*n*x^3*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*
```

```
b)*tan(a) - 4*b*n*x^3*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(
a) + 3*x^3*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(
x)) + b*log(abs(c)))^2 + 3*x^3*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + p
i*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + 12*x^3*e^(pi*b*n*sgn(x) - pi*
b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a) + 12*
x^3*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) +
b*log(abs(c)))*tan(a) + 3*x^3*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*
b)*tan(a)^2 + 3*x^3*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(a)
^2 - 3*x^3*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b) - 3*x^3*e^(-pi*b
*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b))/(4*b^2*n^2*tan(b*n*log(abs(x)) +
b*log(abs(c)))^2*tan(a)^2 + 4*b^2*n^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^
2 + 4*b^2*n^2*tan(a)^2 + 4*b^2*n^2 + 9*tan(b*n*log(abs(x)) + b*log(abs(c)))
^2*tan(a)^2 + 9*tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + 9*tan(a)^2 + 9)
```

Mupad [B]
time = 2.70, size = 66, normalized size = 0.68

$$\frac{x^3}{6}+\frac{x^3\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,2\mathrm{i}}}\,1\mathrm{i}}{8\,b\,n+12\mathrm{i}}+\frac{x^3\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}}{12+b\,n\,8\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(a + b*log(c*x^n))^2,x)
```

```
[Out] x^3/6 + (x^3*exp(-a*2i)/(c*x^n)^(b*2i)*1i)/(8*b*n + 12i) + (x^3*exp(a*2i)*(
c*x^n)^(b*2i))/(b*n*8i + 12)
```

3.92 $\int x \cos^2(a+b\log(cx^n))\,dx$

Optimal. Leaf size=98

$$\frac{b^2n^2x^2}{4(1+b^2n^2)}+\frac{x^2\cos^2(a+b\log(cx^n))}{2(1+b^2n^2)}+\frac{bnx^2\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{2(1+b^2n^2)}$$

```
[Out] 1/4*b^2*n^2*x^2/(b^2*n^2+1)+1/2*x^2*cos(a+b*ln(c*x^n))^2/(b^2*n^2+1)+1/2*b*
n*x^2*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))/(b^2*n^2+1)
```

Rubi [A]
time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4576, 30}

$$\frac{x^2\cos^2(a+b\log(cx^n))}{2(b^2n^2+1)}+\frac{bnx^2\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{2(b^2n^2+1)}+\frac{b^2n^2x^2}{4(b^2n^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (b^2*n^2*x^2)/(4*(1 + b^2*n^2)) + (x^2*Cos[a + b*Log[c*x^n]]^2)/(2*(1 + b^2
*n^2)) + (b*n*x^2*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/(2*(1 + b^2*
n^2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4576

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2
*n^2*p^2 + (m + 1)^2)), Int[(e*x)^m*Cos[d*(a + b*Log[c*x^n])]^(p - 2), x],
x] + Simp[b*d*n*p*(e*x)^(m + 1)*Sin[d*(a + b*Log[c*x^n])]*(Cos[d*(a + b*Log
[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x]) /; FreeQ[{a, b, c
, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$\begin{aligned}\int x\cos^2(a+b\log(cx^n))\,dx&=\frac{x^2\cos^2(a+b\log(cx^n))}{2(1+b^2n^2)}+\frac{bnx^2\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{2(1+b^2n^2)}+\frac{(b}{2}\\&=\frac{b^2n^2x^2}{4(1+b^2n^2)}+\frac{x^2\cos^2(a+b\log(cx^n))}{2(1+b^2n^2)}+\frac{bnx^2\cos(a+b\log(cx^n))\sin(a+b}{2(1+b^2n^2)}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 54, normalized size = 0.55

$$\frac{x^2(1+b^2n^2+\cos(2(a+b\log(cx^n)))+bn\sin(2(a+b\log(cx^n))))}{4+4b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x^2*(1 + b^2*n^2 + Cos[2*(a + b*Log[c*x^n])] + b*n*Sin[2*(a + b*Log[c*x^n]
)]))/(4 + 4*b^2*n^2)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x\left(\cos^2\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(x*cos(a+b*ln(c*x^n))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(92) = 184.
time = 0.29, size = 282, normalized size = 2.88

$$\frac{((b\cos(2\,b\log(c))\sin(4\,b\log(c))-b\cos(4\,b\log(c))\sin(2\,b\log(c))+b\sin(2\,b\log(c)))n+\cos(4\,b\log(c))\cos(2\,b\log(c))+\sin(4\,b\log(c))\sin(2\,b\log(c))+\cos(2\,b\log(c)))x^2\cos(2\,b\log(x^n)+2\,a)+((b\cos(4\,b\log(c))\cos(2\,b\log(c))+b\sin(4\,b\log(c))\sin(2\,b\log(c))+b\cos(2\,b\log(c)))n-\cos(2\,b\log(c))\sin(4\,b\log(c))+\cos(4\,b\log(c))\sin(2\,b\log(c))-\sin(2\,b\log(c)))x^2\sin(2\,b\log(x^n)+2\,a)+2\left((b^2\cos(2\,b\log(c))^2+b^2\sin(2\,b\log(c))^2)n^2+\cos(2\,b\log(c))^2+\sin(2\,b\log(c))^2\right)x^2}{8\left((b^2\cos(2\,b\log(c))^2+b^2\sin(2\,b\log(c))^2)n^2+\cos(2\,b\log(c))^2+\sin(2\,b\log(c))^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(((b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)
) + b*sin(2*b*log(c)))*n + cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c)
)*sin(2*b*log(c)) + cos(2*b*log(c)))*x^2*cos(2*b*log(x^n) + 2*a) + ((b*cos(
4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log(c)) + b*cos(2*b
*log(c)))*n - cos(2*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c))*sin(2*b*log
(c)) - sin(2*b*log(c)))*x^2*sin(2*b*log(x^n) + 2*a) + 2*((b^2*cos(2*b*log(c
))^2 + b^2*sin(2*b*log(c))^2)*n^2 + cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*
x^2)/((b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + cos(2*b*log(c))
^2 + sin(2*b*log(c))^2)
```

Fricas [A]
time = 2.29, size = 74, normalized size = 0.76

$$\frac{b^2n^2x^2+2\,bnx^2\cos(bn\log(x)+b\log(c)+a)\sin(bn\log(x)+b\log(c)+a)+2\,x^2\cos(bn\log(x)+b\log(c)+a)^2}{4\,(b^2n^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*n^2*x^2 + 2*b*n*x^2*cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(x)
+ b*log(c) + a) + 2*x^2*cos(b*n*log(x) + b*log(c) + a)^2)/(b^2*n^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int x\cos^2\left(a - \frac{i\log(cx^n)}{n}\right)dx & \text{for } b = -\frac{i}{n} \\ \int x\cos^2\left(a + \frac{i\log(cx^n)}{n}\right)dx & \text{for } b = \frac{i}{n} \\ \frac{b^2n^2x^2\sin^2(a+b\log(cx^n))}{4b^2n^2+4} + \frac{b^2n^2x^2\cos^2(a+b\log(cx^n))}{4b^2n^2+4} + \frac{2bnx^2\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+4} + \frac{2x^2\cos^2(a+b\log(cx^n))}{4b^2n^2+4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*ln(c*x**n))**2,x)
```

```
[Out] Piecewise((Integral(x*cos(a - I*log(c*x**n)/n)**2, x), Eq(b, -I/n)), (Integ
ral(x*cos(a + I*log(c*x**n)/n)**2, x), Eq(b, I/n)), (b**2*n**2*x**2*sin(a +
 b*log(c*x**n))**2/(4*b**2*n**2 + 4) + b**2*n**2*x**2*cos(a + b*log(c*x**n)
)**2/(4*b**2*n**2 + 4) + 2*b*n*x**2*sin(a + b*log(c*x**n))*cos(a + b*log(c*
x**n))/(4*b**2*n**2 + 4) + 2*x**2*cos(a + b*log(c*x**n))**2/(4*b**2*n**2 +
4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 820 vs. 2(92) = 184.
time = 0.55, size = 820, normalized size = 8.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/4*x^2 - 1/8*(2*b*n*x^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*ta
n(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 2*b*n*x^2*e^(-pi*b*n*sgn(x) +
 pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)
 + 2*b*n*x^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(ab
s(x)) + b*log(abs(c)))*tan(a)^2 + 2*b*n*x^2*e^(-pi*b*n*sgn(x) + pi*b*n - pi
*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a)^2 - x^2*e^(pi
*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(
c)))^2*tan(a)^2 - x^2*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(
b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 - 2*b*n*x^2*e^(pi*b*n*sgn(x) -
pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c))) - 2*b*n*x
^2*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b
*log(abs(c))) - 2*b*n*x^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*t
```

```
an(a) - 2*b*n*x^2*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(a) +
 x^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) +
b*log(abs(c)))^2 + x^2*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan
(b*n*log(abs(x)) + b*log(abs(c)))^2 + 4*x^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*
b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a) + 4*x^2*e^(-pi
*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(
c)))*tan(a) + x^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(a)^2
+ x^2*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(a)^2 - x^2*e^(pi
*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b) - x^2*e^(-pi*b*n*sgn(x) + pi*b*n
 - pi*b*sgn(c) + pi*b))/(b^2*n^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan
(a)^2 + b^2*n^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + b^2*n^2*tan(a)^2 +
 b^2*n^2 + tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 + tan(b*n*log(ab
s(x)) + b*log(abs(c)))^2 + tan(a)^2 + 1)
```

Mupad [B]
time = 2.63, size = 66, normalized size = 0.67

$$\frac{x^2}{4}+\frac{x^2\,{\mathrm{e}}^{-a\,2{}\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,2{}\mathrm{i}}}\,1{}\mathrm{i}}{8\,b\,n+8{}\mathrm{i}}+\frac{x^2\,{\mathrm{e}}^{a\,2{}\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2{}\mathrm{i}}}{8+b\,n\,8{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*log(c*x^n))^2,x)
```

```
[Out] x^2/4 + (x^2*exp(-a*2i)/(c*x^n)^(b*2i)*1i)/(8*b*n + 8i) + (x^2*exp(a*2i)*(c
*x^n)^(b*2i))/(b*n*8i + 8)
```

3.93 $\int \cos^2(a+b\log(cx^n))\,dx$

Optimal. Leaf size=88

$$\frac{2b^2n^2x}{1+4b^2n^2}+\frac{x\cos^2(a+b\log(cx^n))}{1+4b^2n^2}+\frac{2bnx\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{1+4b^2n^2}$$

```
[Out] 2*b^2*n^2*x/(4*b^2*n^2+1)+x*cos(a+b*ln(c*x^n))^2/(4*b^2*n^2+1)+2*b*n*x*cos(
a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))/(4*b^2*n^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4566, 8}

$$\frac{x\cos^2(a+b\log(cx^n))}{4b^2n^2+1}+\frac{2bnx\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+1}+\frac{2b^2n^2x}{4b^2n^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (2*b^2*n^2*x)/(1 + 4*b^2*n^2) + (x*Cos[a + b*Log[c*x^n]]^2)/(1 + 4*b^2*n^2)
 + (2*b*n*x*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/(1 + 4*b^2*n^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4566

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Sim
p[x*(Cos[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 + 1)), x] + (Dist[b^2*d^2
*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + 1)), Int[Cos[d*(a + b*Log[c*x^n])]^(p -
2), x], x] + Simp[b*d*n*p*x*Cos[d*(a + b*Log[c*x^n])]^(p - 1)*(Sin[d*(a + b
*Log[c*x^n])]/(b^2*d^2*n^2*p^2 + 1)), x]) /; FreeQ[{a, b, c, d, n}, x] && I
GtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+b\log(cx^n))\,dx &= \frac{x\cos^2(a+b\log(cx^n))}{1+4b^2n^2}+\frac{2bnx\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{1+4b^2n^2}+\frac{(2b^2\\ &= \frac{2b^2n^2x}{1+4b^2n^2}+\frac{x\cos^2(a+b\log(cx^n))}{1+4b^2n^2}+\frac{2bnx\cos(a+b\log(cx^n))\sin(a+b\log(\end{aligned}$$

Mathematica [A]
time = 0.10, size = 54, normalized size = 0.61

$$\frac{x(1+4b^2n^2+\cos(2(a+b\log(cx^n)))+2bn\sin(2(a+b\log(cx^n))))}{2+8b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x*(1 + 4*b^2*n^2 + Cos[2*(a + b*Log[c*x^n])] + 2*b*n*Sin[2*(a + b*Log[c*x^
n])]))/(2 + 8*b^2*n^2)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \cos^2(a+b\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(cos(a+b*ln(c*x^n))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(88) = 176.
time = 0.32, size = 280, normalized size = 3.18

$$\frac{\left(2\left(b\cos(2b\log(c))\sin(4b\log(c))-b\cos(4b\log(c))\sin(2b\log(c))+b\sin(2b\log(c))\right)n+\cos(4b\log(c))\cos(2b\log(c))+\sin(4b\log(c))\sin(2b\log(c))+\cos(2b\log(c))\right)x\cos(2b\log(x^n)+2a)+\left(2\left(b\cos(4b\log(c))\cos(2b\log(c))+b\sin(4b\log(c))\sin(2b\log(c))+b\cos(2b\log(c))\right)n-\cos(2b\log(c))\sin(4b\log(c))+\cos(4b\log(c))\sin(2b\log(c))-\sin(2b\log(c))\right)x\sin(2b\log(x^n)+2a)+2\left(4\left(b^2\cos(2b\log(c))^2+b^2\sin(2b\log(c))^2\right)n^2+\cos(2b\log(c))^2+\sin(2b\log(c))^2\right)x}{4\left(4\left(b^2\cos(2b\log(c))^2+b^2\sin(2b\log(c))^2\right)n^2+\cos(2b\log(c))^2+\sin(2b\log(c))^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*(b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(
c)) + b*sin(2*b*log(c)))*n + cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(
c))*sin(2*b*log(c)) + cos(2*b*log(c)))*x*cos(2*b*log(x^n) + 2*a) + (2*(b*co
s(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log(c)) + b*cos(2
*b*log(c)))*n - cos(2*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c))*sin(2*b*l
og(c)) - sin(2*b*log(c)))*x*sin(2*b*log(x^n) + 2*a) + 2*(4*(b^2*cos(2*b*log
(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + cos(2*b*log(c))^2 + sin(2*b*log(c))^2
)*x)/(4*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + cos(2*b*log(c
))^2 + sin(2*b*log(c))^2)
```

Fricas [A]
time = 3.07, size = 68, normalized size = 0.77

$$\frac{2\,b^2n^2x+2\,bnx\cos(bn\log(x)+b\log(c)+a)\sin(bn\log(x)+b\log(c)+a)+x\cos(bn\log(x)+b\log(c)+a)^2}{4\,b^2n^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b*log(c*x^n))^2,x, algorithm="fricas")

[Out] (2*b^2*n^2*x + 2*b*n*x*cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(x) + b*lo
g(c) + a) + x*cos(b*n*log(x) + b*log(c) + a)^2)/(4*b^2*n^2 + 1)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \cos^2\left(a - \frac{i\log\left(cx^n\right)}{2n}\right) dx & \text{for}\ b = -\frac{i}{2n} \\ \int \cos^2\left(a + \frac{i\log\left(cx^n\right)}{2n}\right) dx & \text{for}\ b = \frac{i}{2n} \\ \frac{2b^2n^2x\sin^2\left(a+b\log\left(cx^n\right)\right)}{4b^2n^2+1} + \frac{2b^2n^2x\cos^2\left(a+b\log\left(cx^n\right)\right)}{4b^2n^2+1} + \frac{2bnx\sin\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{4b^2n^2+1} + \frac{x\cos^2\left(a+b\log\left(cx^n\right)\right)}{4b^2n^2+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b*ln(c*x**n))**2,x)

[Out] Piecewise((Integral(cos(a - I*log(c*x**n)/(2*n))**2, x), Eq(b, -I/(2*n))),
(Integral(cos(a + I*log(c*x**n)/(2*n))**2, x), Eq(b, I/(2*n))), (2*b**2*n**
2*x*sin(a + b*log(c*x**n))**2/(4*b**2*n**2 + 1) + 2*b**2*n**2*x*cos(a + b*l
og(c*x**n))**2/(4*b**2*n**2 + 1) + 2*b*n*x*sin(a + b*log(c*x**n))*cos(a + b
*log(c*x**n))/(4*b**2*n**2 + 1) + x*cos(a + b*log(c*x**n))**2/(4*b**2*n**2
+ 1), True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 786 vs. 2(88) = 176.
time = 0.53, size = 786, normalized size = 8.93

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b*log(c*x^n))^2,x, algorithm="giac")

[Out] 1/2*x - 1/4*(4*b*n*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*
n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 4*b*n*x*e^(-pi*b*n*sgn(x) + pi*b*
n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 4*b
*n*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) +
b*log(abs(c)))*tan(a)^2 + 4*b*n*x*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c)
+ pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a)^2 - x*e^(pi*b*n*sgn(x)
- pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a
)^2 - x*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)
) + b*log(abs(c)))^2*tan(a)^2 - 4*b*n*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sg
n(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c))) - 4*b*n*x*e^(-pi*b*n*sgn(
x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c))) - 4*
b*n*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(a) - 4*b*n*x*e^(-

```
pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(a) + x*e^(pi*b*n*sgn(x) -
pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + x*e^(
-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(a
bs(c)))^2 + 4*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log
(abs(x)) + b*log(abs(c)))*tan(a) + 4*x*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sg
n(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a) + x*e^(pi*b*n*sgn(
x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(a)^2 + x*e^(-pi*b*n*sgn(x) + pi*b*n -
 pi*b*sgn(c) + pi*b)*tan(a)^2 - x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) -
 pi*b) - x*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b))/(4*b^2*n^2*tan
(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 + 4*b^2*n^2*tan(b*n*log(abs(x)
) + b*log(abs(c)))^2 + 4*b^2*n^2*tan(a)^2 + 4*b^2*n^2 + tan(b*n*log(abs(x))
 + b*log(abs(c)))^2*tan(a)^2 + tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + tan
(a)^2 + 1)
```

Mupad [B]
time = 2.53, size = 56, normalized size = 0.64

$$\frac{x\left(2\cos\left(a+b\,\ln\left(c\,x^n\right)\right)^2+4\,b^2\,n^2+2\,b\,n\,\sin\left(2\,a+2\,b\,\ln\left(c\,x^n\right)\right)\right)}{8\,b^2\,n^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*log(c*x^n))^2,x)
```

```
[Out] (x*(2*cos(a + b*log(c*x^n))^2 + 4*b^2*n^2 + 2*b*n*sin(2*a + 2*b*log(c*x^n))
))/(8*b^2*n^2 + 2)
```

3.94 $\int \frac{\cos^2(a+b\log(cx^n))}{x} dx$

Optimal. Leaf size=39

$$\frac{\log(x)}{2} + \frac{\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{2bn}$$

```
[Out] 1/2*ln(x)+1/2*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2715, 8}

$$\frac{\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{2bn} + \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] Log[x]/2 + (Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/(2*b*n)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a+b\log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \cos^2(a+bx)\,dx, x, \log(cx^n)\right)}{n} \\
&= \frac{\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{2bn} + \frac{\text{Subst}\left(\int 1\,dx, x, \log(cx^n)\right)}{2n} \\
&= \frac{\log(x)}{2} + \frac{\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{2bn}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 36, normalized size = 0.92

$$\frac{2(a+b\log(cx^n)) + \sin(2(a+b\log(cx^n)))}{4bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] (2*(a + b*Log[c*x^n]) + Sin[2*(a + b*Log[c*x^n])])/(4*b*n)
```

Maple [A]
time = 0.06, size = 45, normalized size = 1.15

method	result	size
derivativedivides	$\frac{\frac{\cos(a+b\ln(c\,x^n))\sin(a+b\ln(c\,x^n))}{2}+\frac{b\ln(c\,x^n)}{2}+\frac{a}{2}}{nb}$	45
default	$\frac{\frac{\cos(a+b\ln(c\,x^n))\sin(a+b\ln(c\,x^n))}{2}+\frac{b\ln(c\,x^n)}{2}+\frac{a}{2}}{nb}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*ln(c*x^n))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(1/2*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))+1/2*b*ln(c*x^n)+1/2*a)
```

Maxima [A]
time = 0.30, size = 53, normalized size = 1.36

$$\frac{2\,bn\log\left(x\right)+\cos\left(2\,b\log\left(x^n\right)+2\,a\right)\sin\left(2\,b\log\left(c\right)\right)+\cos\left(2\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(x^n\right)+2\,a\right)}{4\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*n*log(x) + cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + cos(2*b*log(c
))*sin(2*b*log(x^n) + 2*a))/(b*n)
```

Fricas [A]
time = 1.95, size = 39, normalized size = 1.00

$$\frac{bn\log\left(x\right)+\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

```
[Out] 1/2*(b*n*log(x) + cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(x) + b*log(c)
+ a))/(b*n)
```

Sympy [A]
time = 1.38, size = 51, normalized size = 1.31

$$\frac{\begin{cases}\log\left(x\right)\cos\left(2a\right) & \text{for}\: b=0 \wedge \left(b=0 \vee n=0\right)\\ \log\left(x\right)\cos\left(2a+2b\log\left(c\right)\right) & \text{for}\: n=0\\ \frac{\sin\left(2a+2b\log\left(cx^n\right)\right)}{2bn} & \text{otherwise}\end{cases}}{2}+\frac{\log\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*ln(c*x**n))**2/x,x)
```

```
[Out] Piecewise((log(x)*cos(2*a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*cos(
2*a + 2*b*log(c)), Eq(n, 0)), (sin(2*a + 2*b*log(c*x**n))/(2*b*n), True))/2
 + log(x)/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate(cos(b*log(c*x^n) + a)^2/x, x)
```

Mupad [B]
time = 2.44, size = 32, normalized size = 0.82

$$\frac{\ln\left(x^n\right)}{2\,n}+\frac{\sin\left(2\,a+2\,b\,\ln\left(c\,x^n\right)\right)}{4\,b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*log(c*x^n))^2/x,x)
```

```
[Out] log(x^n)/(2*n) + sin(2*a + 2*b*log(c*x^n))/(4*b*n)
```

3.95 $\int \frac{\cos^2(a+b\log(cx^n))}{x^2}\,dx$

Optimal. Leaf size=95

$$-\frac{2b^2n^2}{(1+4b^2n^2)\,x}-\frac{\cos^2(a+b\log(cx^n))}{(1+4b^2n^2)\,x}+\frac{2bn\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{(1+4b^2n^2)\,x}$$

```
[Out] -2*b^2*n^2/(4*b^2*n^2+1)/x-cos(a+b*ln(c*x^n))^2/(4*b^2*n^2+1)/x+2*b*n*cos(a
+b*ln(c*x^n))*sin(a+b*ln(c*x^n))/(4*b^2*n^2+1)/x
```

Rubi [A]
time = 0.02, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4576, 30}

$$-\frac{\cos^2(a+b\log(cx^n))}{x(4b^2n^2+1)}+\frac{2bn\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{x(4b^2n^2+1)}-\frac{2b^2n^2}{x(4b^2n^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*Log[c*x^n]]^2/x^2,x]
```

```
[Out] (-2*b^2*n^2)/((1 + 4*b^2*n^2)*x) - Cos[a + b*Log[c*x^n]]^2/((1 + 4*b^2*n^2)
*x) + (2*b*n*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/((1 + 4*b^2*n^2)*
x)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4576

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2
*n^2*p^2 + (m + 1)^2)), Int[(e*x)^m*Cos[d*(a + b*Log[c*x^n])]^(p - 2), x],
x] + Simp[b*d*n*p*(e*x)^(m + 1)*Sin[d*(a + b*Log[c*x^n])]*(Cos[d*(a + b*Log
[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x]) /; FreeQ[{a, b, c
, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(a+b\log(cx^n))}{x^2}\,dx &= -\frac{\cos^2(a+b\log(cx^n))}{(1+4b^2n^2)\,x}+\frac{2bn\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{(1+4b^2n^2)\,x}+\frac{(2b^2\\ &= -\frac{2b^2n^2}{(1+4b^2n^2)\,x}-\frac{\cos^2(a+b\log(cx^n))}{(1+4b^2n^2)\,x}+\frac{2bn\cos(a+b\log(cx^n))\sin(a+b\log\end{aligned}$$

Mathematica [A]
time = 0.15, size = 57, normalized size = 0.60

$$-\frac{1+4b^2n^2+\cos(2(a+b\log(cx^n)))-2bn\sin(2(a+b\log(cx^n)))}{2(x+4b^2n^2x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*Log[c*x^n]]^2/x^2,x]
```

```
[Out] -1/2*(1 + 4*b^2*n^2 + Cos[2*(a + b*Log[c*x^n])] - 2*b*n*Sin[2*(a + b*Log[c*
x^n])])/(x + 4*b^2*n^2*x)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(a+b\ln(c\,x^n))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*ln(c*x^n))^2/x^2,x)
```

```
[Out] int(cos(a+b*ln(c*x^n))^2/x^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(95) = 190.
time = 0.31, size = 285, normalized size = 3.00

$$-\frac{8\left(b^2\cos(2\,b\log(c))^2+b^2\sin(2\,b\log(c))^2\right)n^2+2\cos(2\,b\log(c))^2-\left(2\left(b\cos(2\,b\log(c))\sin(4\,b\log(c))-b\cos(4\,b\log(c))\sin(2\,b\log(c))+b\sin(2\,b\log(c))\right)n-\cos(4\,b\log(c))\cos(2\,b\log(c))-\sin(4\,b\log(c))\sin(2\,b\log(c))-\cos(2\,b\log(c))\right)\cos(2\,b\log(x^n)+2\,a)+2\sin(2\,b\log(c))^2-\left(2\left(b\cos(4\,b\log(c))\cos(2\,b\log(c))+b\sin(4\,b\log(c))\sin(2\,b\log(c))+b\cos(2\,b\log(c))\right)n+\cos(2\,b\log(c))\sin(4\,b\log(c))-\cos(4\,b\log(c))\sin(2\,b\log(c))+\sin(2\,b\log(c))\right)\sin(2\,b\log(x^n)+2\,a)}{4\left(4\left(b^2\cos(2\,b\log(c))^2+b^2\sin(2\,b\log(c))^2\right)n^2+\cos(2\,b\log(c))^2+\sin(2\,b\log(c))^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*(8*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + 2*cos(2*b*log
(c))^2 - (2*(b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*
log(c)) + b*sin(2*b*log(c)))*n - cos(4*b*log(c))*cos(2*b*log(c)) - sin(4*b*
log(c))*sin(2*b*log(c)) - cos(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 2*sin(
2*b*log(c))^2 - (2*(b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*s
in(2*b*log(c)) + b*cos(2*b*log(c)))*n + cos(2*b*log(c))*sin(4*b*log(c)) - c
os(4*b*log(c))*sin(2*b*log(c)) + sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a))/
((4*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + cos(2*b*log(c))^2
 + sin(2*b*log(c))^2)*x)
```

Fricas [A]
time = 3.63, size = 68, normalized size = 0.72

$$-\frac{2\,b^2n^2-2\,bn\cos(bn\log(x)+b\log(c)+a)\sin(bn\log(x)+b\log(c)+a)+\cos(bn\log(x)+b\log(c)+a)^2}{(4\,b^2n^2+1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] -(2*b^2*n^2 - 2*b*n*cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(x) + b*log(c
) + a) + cos(b*n*log(x) + b*log(c) + a)^2)/((4*b^2*n^2 + 1)*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.11, size = 299, normalized size = 3.15

$$\begin{cases} -\frac{\cos\left(2a-\frac{i\log\left(cx^n\right)}{n}\right)}{4x}-\frac{1}{2x}+\frac{i\log\left(cx^n\right)\sin\left(2a-\frac{i\log\left(cx^n\right)}{n}\right)}{4nx}+\frac{\log\left(cx^n\right)\cos\left(2a-\frac{i\log\left(cx^n\right)}{n}\right)}{4nx} & \text{for}\ b=-\frac{i}{2n} \\ -\frac{\cos\left(2a+\frac{i\log\left(cx^n\right)}{n}\right)}{4x}-\frac{1}{2x}-\frac{i\log\left(cx^n\right)\sin\left(2a+\frac{i\log\left(cx^n\right)}{n}\right)}{4nx}+\frac{\log\left(cx^n\right)\cos\left(2a+\frac{i\log\left(cx^n\right)}{n}\right)}{4nx} & \text{for}\ b=\frac{i}{2n} \\ -\frac{2b^2n^2\sin^2\left(a+b\log\left(cx^n\right)\right)}{4b^2n^2x+x}-\frac{2b^2n^2\cos^2\left(a+b\log\left(cx^n\right)\right)}{4b^2n^2x+x}+\frac{2bn\sin\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{4b^2n^2x+x}-\frac{\cos^2\left(a+b\log\left(cx^n\right)\right)}{4b^2n^2x+x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*ln(c*x**n))**2/x**2,x)
```

```
[Out] Piecewise((-cos(2*a - I*log(c*x**n)/n)/(4*x) - 1/(2*x) + I*log(c*x**n)*sin(
2*a - I*log(c*x**n)/n)/(4*n*x) + log(c*x**n)*cos(2*a - I*log(c*x**n)/n)/(4*
n*x), Eq(b, -I/(2*n))), (-cos(2*a + I*log(c*x**n)/n)/(4*x) - 1/(2*x) - I*lo
g(c*x**n)*sin(2*a + I*log(c*x**n)/n)/(4*n*x) + log(c*x**n)*cos(2*a + I*log(
c*x**n)/n)/(4*n*x), Eq(b, I/(2*n))), (-2*b**2*n**2*sin(a + b*log(c*x**n))**
2/(4*b**2*n**2*x + x) - 2*b**2*n**2*cos(a + b*log(c*x**n))**2/(4*b**2*n**2*
x + x) + 2*b*n*sin(a + b*log(c*x**n))*cos(a + b*log(c*x**n))/(4*b**2*n**2*x
 + x) - cos(a + b*log(c*x**n))**2/(4*b**2*n**2*x + x), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*log(c*x^n) + a)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*log(c*x^n))^2/x^2,x)
```

```
[Out] int(cos(a + b*log(c*x^n))^2/x^2, x)
```

3.96 $\int x^2 \cos^3 (a + b \log (cx^n)) \, dx$

Optimal. Leaf size=160

$$\frac{2b^2n^2x^3 \cos (a + b \log (cx^n))}{9 + 10b^2n^2 + b^4n^4} + \frac{x^3 \cos^3 (a + b \log (cx^n))}{3 (1 + b^2n^2)} + \frac{2b^3n^3x^3 \sin (a + b \log (cx^n))}{3 (9 + 10b^2n^2 + b^4n^4)} + \frac{bnx^3 \cos^2 (a + b \log (cx$$

```
[Out] 2*b^2*n^2*x^3*cos(a+b*ln(c*x^n))/(b^4*n^4+10*b^2*n^2+9)+1/3*x^3*cos(a+b*ln(
c*x^n))^3/(b^2*n^2+1)+2/3*b^3*n^3*x^3*sin(a+b*ln(c*x^n))/(b^4*n^4+10*b^2*n^
2+9)+1/3*b*n*x^3*cos(a+b*ln(c*x^n))^2*sin(a+b*ln(c*x^n))/(b^2*n^2+1)
```

Rubi [A]
time = 0.04, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4576, 4574}

$$\frac{x^3 \cos^3 (a + b \log (cx^n))}{3 (b^2n^2 + 1)} + \frac{bnx^3 \sin (a + b \log (cx^n)) \cos^2 (a + b \log (cx^n))}{3 (b^2n^2 + 1)} + \frac{2b^2n^2x^3 \cos (a + b \log (cx^n))}{b^4n^4 + 10b^2n^2 + 9} + \frac{2b^3n^3x^3 \sin (a + b \log (cx^n))}{3 (b^4n^4 + 10b^2n^2 + 9)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (2*b^2*n^2*x^3*Cos[a + b*Log[c*x^n]])/(9 + 10*b^2*n^2 + b^4*n^4) + (x^3*Cos
[a + b*Log[c*x^n]]^3)/(3*(1 + b^2*n^2)) + (2*b^3*n^3*x^3*Sin[a + b*Log[c*x^
n]])/(3*(9 + 10*b^2*n^2 + b^4*n^4)) + (b*n*x^3*Cos[a + b*Log[c*x^n]]^2*Sin[
a + b*Log[c*x^n]])/(3*(1 + b^2*n^2))
```

Rule 4574

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Rule 4576

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2
*n^2*p^2 + (m + 1)^2)), Int[(e*x)^m*Cos[d*(a + b*Log[c*x^n])]^(p - 2), x],
x] + Simp[b*d*n*p*(e*x)^(m + 1)*Sin[d*(a + b*Log[c*x^n])]*(Cos[d*(a + b*Log
[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x]) /; FreeQ[{a, b, c
, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int x^2\cos^3(a+b\log(cx^n))\,dx = \frac{x^3\cos^3(a+b\log(cx^n))}{3(1+b^2n^2)} + \frac{bnx^3\cos^2(a+b\log(cx^n))\sin(a+b\log(cx^n))}{3(1+b^2n^2)} +$$
$$= \frac{2b^2n^2x^3\cos(a+b\log(cx^n))}{9+10b^2n^2+b^4n^4} + \frac{x^3\cos^3(a+b\log(cx^n))}{3(1+b^2n^2)} + \frac{2b^3n^3x^3\sin(a+}{3(9+10b^2n^2}$$

Mathematica [A]
time = 0.59, size = 120, normalized size = 0.75

$$\frac{x^3(27(1+b^2n^2)\cos(a+b\log(cx^n)) + (9+b^2n^2)\cos(3(a+b\log(cx^n))) + 2bn(9+5b^2n^2+(9+b^2n^2)\cos(2(a+b\log(cx^n))))\sin(a+b\log(cx^n)))}{12(9+10b^2n^2+b^4n^4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (x^3*(27*(1 + b^2*n^2)*Cos[a + b*Log[c*x^n]] + (9 + b^2*n^2)*Cos[3*(a + b*L
og[c*x^n])] + 2*b*n*(9 + 5*b^2*n^2 + (9 + b^2*n^2)*Cos[2*(a + b*Log[c*x^n])
])*Sin[a + b*Log[c*x^n]]))/(12*(9 + 10*b^2*n^2 + b^4*n^4))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int x^2\left(\cos^3(a+b\ln(c\,x^n))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(x^2*cos(a+b*ln(c*x^n))^3,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1007 vs. 2(154) = 308.
time = 0.34, size = 1007, normalized size = 6.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 1/24*(((b^3*cos(3*b*log(c))*sin(6*b*log(c)) - b^3*cos(6*b*log(c))*sin(3*b*l
og(c)) + b^3*sin(3*b*log(c)))*n^3 + (b^2*cos(6*b*log(c))*cos(3*b*log(c)) +
b^2*sin(6*b*log(c))*sin(3*b*log(c)) + b^2*cos(3*b*log(c)))*n^2 + 9*(b*cos(3
*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(3*b*log(c)) + b*sin(3*b*
log(c)))*n + 9*cos(6*b*log(c))*cos(3*b*log(c)) + 9*sin(6*b*log(c))*sin(3*b*
```

```
log(c)) + 9*cos(3*b*log(c)))*x^3*cos(3*b*log(x^n) + 3*a) + 9*((b^3*cos(3*b*
log(c))*sin(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(3*b*log(c)) + b^3*cos(2*b
*log(c))*sin(3*b*log(c)) - b^3*cos(3*b*log(c))*sin(2*b*log(c)))*n^3 + 3*(b^
2*cos(4*b*log(c))*cos(3*b*log(c)) + b^2*cos(3*b*log(c))*cos(2*b*log(c)) + b
^2*sin(4*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c))*sin(2*b*log(c)))*n
^2 + (b*cos(3*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(3*b*log(c))
 + b*cos(2*b*log(c))*sin(3*b*log(c)) - b*cos(3*b*log(c))*sin(2*b*log(c)))*n
 + 3*cos(4*b*log(c))*cos(3*b*log(c)) + 3*cos(3*b*log(c))*cos(2*b*log(c)) +
3*sin(4*b*log(c))*sin(3*b*log(c)) + 3*sin(3*b*log(c))*sin(2*b*log(c)))*x^3*
cos(b*log(x^n) + a) + ((b^3*cos(6*b*log(c))*cos(3*b*log(c)) + b^3*sin(6*b*l
og(c))*sin(3*b*log(c)) + b^3*cos(3*b*log(c)))*n^3 - (b^2*cos(3*b*log(c))*si
n(6*b*log(c)) - b^2*cos(6*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c)))*
n^2 + 9*(b*cos(6*b*log(c))*cos(3*b*log(c)) + b*sin(6*b*log(c))*sin(3*b*log(
c)) + b*cos(3*b*log(c)))*n - 9*cos(3*b*log(c))*sin(6*b*log(c)) + 9*cos(6*b*
log(c))*sin(3*b*log(c)) - 9*sin(3*b*log(c)))*x^3*sin(3*b*log(x^n) + 3*a) +
9*((b^3*cos(4*b*log(c))*cos(3*b*log(c)) + b^3*cos(3*b*log(c))*cos(2*b*log(c
)) + b^3*sin(4*b*log(c))*sin(3*b*log(c)) + b^3*sin(3*b*log(c))*sin(2*b*log(
c)))*n^3 - 3*(b^2*cos(3*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin
(3*b*log(c)) + b^2*cos(2*b*log(c))*sin(3*b*log(c)) - b^2*cos(3*b*log(c))*si
n(2*b*log(c)))*n^2 + (b*cos(4*b*log(c))*cos(3*b*log(c)) + b*cos(3*b*log(c))
*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(3*b*log(c)) + b*sin(3*b*log(c))*si
n(2*b*log(c)))*n - 3*cos(3*b*log(c))*sin(4*b*log(c)) + 3*cos(4*b*log(c))*si
n(3*b*log(c)) - 3*cos(2*b*log(c))*sin(3*b*log(c)) + 3*cos(3*b*log(c))*sin(2
*b*log(c)))*x^3*sin(b*log(x^n) + a))/((b^4*cos(3*b*log(c))^2 + b^4*sin(3*b*
log(c))^2)*n^4 + 10*(b^2*cos(3*b*log(c))^2 + b^2*sin(3*b*log(c))^2)*n^2 + 9
*cos(3*b*log(c))^2 + 9*sin(3*b*log(c))^2)
```

Fricas [**A**]
time = 4.18, size = 127, normalized size = 0.79

$$\frac{6\,b^{2}n^{2}x^{3}\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\left(b^{2}n^{2}+9\right)x^{3}\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+\left(2\,b^{3}n^{3}x^{3}+\left(b^{3}n^{3}+9\,bn\right)x^{3}\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{3\,\left(b^{4}n^{4}+10\,b^{2}n^{2}+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] 1/3*(6*b^2*n^2*x^3*cos(b*n*log(x) + b*log(c) + a) + (b^2*n^2 + 9)*x^3*cos(b
*n*log(x) + b*log(c) + a)^3 + (2*b^3*n^3*x^3 + (b^3*n^3 + 9*b*n)*x^3*cos(b*
n*log(x) + b*log(c) + a)^2)*sin(b*n*log(x) + b*log(c) + a))/(b^4*n^4 + 10*b
^2*n^2 + 9)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(a+b*ln(c*x**n))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 18053 vs. $2(154) = 308$.

time = 1.35, size = 18053, normalized size = 112.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] -1/24*(18*b^3*n^3*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) -
 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(a
bs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a) + 18*b^3*n^3*x^3*e^(-
1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*lo
g(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)
))^2*tan(3/2*a)^2*tan(1/2*a) + 2*b^3*n^3*x^3*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*
b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c
)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)*tan(1/2*a)^
2 + 2*b^3*n^3*x^3*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/
2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(
x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)*tan(1/2*a)^2 + 18*b^3*n^3*x^3*e^(1/2*
pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(ab
s(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*t
an(3/2*a)^2*tan(1/2*a)^2 + 18*b^3*n^3*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*
n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c))
)^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(3/2*a)^2*tan(1/2*a)^2
 + 2*b^3*n^3*x^3*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*p
i*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))*tan(1/2*b*n*log(abs(x)) +
 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 + 2*b^3*n^3*x^3*e^(-3/2*pi*
b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x
)) + 3/2*b*log(abs(c)))*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(
3/2*a)^2*tan(1/2*a)^2 - b^2*n^2*x^3*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2
*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan
(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 - 27*
b^2*n^2*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)
*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1
/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 - 27*b^2*n^2*x^3*e^(-1/2*pi*b
*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)
) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan
(3/2*a)^2*tan(1/2*a)^2 - b^2*n^2*x^3*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3
/2*pi*b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*t
an(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 + 2
*b^3*n^3*x^3*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b
```

```
)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) +
1/2*b*log(abs(c)))^2*tan(3/2*a) + 2*b^3*n^3*x^3*e^(-3/2*pi*b*n*sgn(x) + 3/2
*pi*b*n - 3/2*pi*b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(a
bs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a) - 18*b^
3*n^3*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*t
an(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2
*b*log(abs(c)))*tan(3/2*a)^2 - 18*b^3*n^3*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*p
i*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs
(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(3/2*a)^2 + 2*b^3*n
^3*x^3*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(
3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))*tan(1/2*b*n*log(abs(x)) + 1/2*b*lo
g(abs(c)))^2*tan(3/2*a)^2 + 2*b^3*n^3*x^3*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*
n - 3/2*pi*b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c))
)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2 + 18*b^3*n^3*
x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2
*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log
(abs(c)))^2*tan(1/2*a) + 18*b^3*n^3*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n
- 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^
2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a) - 18*b^3*n^3*x^
3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b
*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a) - 18*b^3*n^3*
x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/
2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a) + 18*b^3*n
^3*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(
1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a) + 18*b^3
*n^3*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*t
an(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a) + 18*
b^3*n^3*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)
*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1
/2*b*log(abs(c)))*tan(1/2*a)^2 + 18*b^3*n^3*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2
*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(a
bs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a)^2 - 2*b^3
*n^3*x^3*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*ta
n(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))*tan(...
```

Mupad [B]
time = 3.06, size = 122, normalized size = 0.76

$$\frac{x^3\,{\mathrm{e}}^{-a\,1{}\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,1{}\mathrm{i}}}\,3{}\mathrm{i}}{8\,b\,n+24{}\mathrm{i}}+\frac{3\,x^3\,{\mathrm{e}}^{a\,1{}\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,1{}\mathrm{i}}}{24+b\,n\,8{}\mathrm{i}}+\frac{x^3\,{\mathrm{e}}^{-a\,3{}\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,3{}\mathrm{i}}}\,1{}\mathrm{i}}{24\,b\,n+24{}\mathrm{i}}+\frac{x^3\,{\mathrm{e}}^{a\,3{}\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,3{}\mathrm{i}}}{24+b\,n\,24{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(a + b*log(c*x^n))^3,x)
[Out] (x^3*exp(-a*1i)/(c*x^n)^(b*1i)*3i)/(8*b*n + 24i) + (3*x^3*exp(a*1i)*(c*x^n)
```

```
^(b*1i))/(b*n*8i + 24) + (x^3*exp(-a*3i)/(c*x^n)^(b*3i)*1i)/(24*b*n + 24i)
+ (x^3*exp(a*3i)*(c*x^n)^(b*3i))/(b*n*24i + 24)
```

3.97 $\int x \cos^3(a + b\log(cx^n))\, dx$

Optimal. Leaf size=158

$$\frac{12b^2n^2x^2\cos(a+b\log(cx^n))}{16+40b^2n^2+9b^4n^4}+\frac{2x^2\cos^3(a+b\log(cx^n))}{4+9b^2n^2}+\frac{6b^3n^3x^2\sin(a+b\log(cx^n))}{16+40b^2n^2+9b^4n^4}+\frac{3bnx^2\cos^2(a+b\log}{4}$$

```
[Out] 12*b^2*n^2*x^2*cos(a+b*ln(c*x^n))/(9*b^4*n^4+40*b^2*n^2+16)+2*x^2*cos(a+b*l
n(c*x^n))^3/(9*b^2*n^2+4)+6*b^3*n^3*x^2*sin(a+b*ln(c*x^n))/(9*b^4*n^4+40*b^
2*n^2+16)+3*b*n*x^2*cos(a+b*ln(c*x^n))^2*sin(a+b*ln(c*x^n))/(9*b^2*n^2+4)
```

Rubi [A]

time = 0.03, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4576, 4574}

$$\frac{2x^2\cos^3(a+b\log(cx^n))}{9b^2n^2+4}+\frac{3bnx^2\sin(a+b\log(cx^n))\cos^2(a+b\log(cx^n))}{9b^2n^2+4}+\frac{12b^2n^2x^2\cos(a+b\log(cx^n))}{9b^4n^4+40b^2n^2+16}+\frac{6b^3n^3x^2\sin(a+b\log(cx^n))}{9b^4n^4+40b^2n^2+16}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (12*b^2*n^2*x^2*Cos[a + b*Log[c*x^n]])/(16 + 40*b^2*n^2 + 9*b^4*n^4) + (2*x
^2*Cos[a + b*Log[c*x^n]]^3)/(4 + 9*b^2*n^2) + (6*b^3*n^3*x^2*Sin[a + b*Log[
c*x^n]])/(16 + 40*b^2*n^2 + 9*b^4*n^4) + (3*b*n*x^2*Cos[a + b*Log[c*x^n]]^2
*Sin[a + b*Log[c*x^n]])/(4 + 9*b^2*n^2)
```

Rule 4574

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Rule 4576

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2
*n^2*p^2 + (m + 1)^2)), Int[(e*x)^m*Cos[d*(a + b*Log[c*x^n])]^(p - 2), x],
x] + Simp[b*d*n*p*(e*x)^(m + 1)*Sin[d*(a + b*Log[c*x^n])]*(Cos[d*(a + b*Log
[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x]) /; FreeQ[{a, b, c
, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int x\cos^3(a+b\log(cx^n))\,dx = \frac{2x^2\cos^3(a+b\log(cx^n))}{4+9b^2n^2}+\frac{3bnx^2\cos^2(a+b\log(cx^n))\sin(a+b\log(cx^n))}{4+9b^2n^2}$$
$$= \frac{12b^2n^2x^2\cos(a+b\log(cx^n))}{16+40b^2n^2+9b^4n^4}+\frac{2x^2\cos^3(a+b\log(cx^n))}{4+9b^2n^2}+\frac{6b^3n^3x^2\sin(a-}{16+40b^2n^2}$$

Mathematica [A]
time = 0.55, size = 123, normalized size = 0.78

$$\frac{x^2(6(4+9b^2n^2)\cos(a+b\log(cx^n))+2(4+b^2n^2)\cos(3(a+b\log(cx^n)))+6bn(4+5b^2n^2+(4+b^2n^2)\cos(2(a+b\log(cx^n))))\sin(a+b\log(cx^n)))}{4(16+40b^2n^2+9b^4n^4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (x^2*(6*(4 + 9*b^2*n^2)*Cos[a + b*Log[c*x^n]] + 2*(4 + b^2*n^2)*Cos[3*(a +
b*Log[c*x^n])] + 6*b*n*(4 + 5*b^2*n^2 + (4 + b^2*n^2)*Cos[2*(a + b*Log[c*x^
n])])*Sin[a + b*Log[c*x^n]]))/(4*(16 + 40*b^2*n^2 + 9*b^4*n^4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int x\left(\cos^3(a+b\ln(c\,x^n))\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(x*cos(a+b*ln(c*x^n))^3,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. 2(158) = 316.
time = 0.32, size = 1015, normalized size = 6.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*(b^3*cos(3*b*log(c))*sin(6*b*log(c)) - b^3*cos(6*b*log(c))*sin(3*b*
log(c)) + b^3*sin(3*b*log(c)))*n^3 + 2*(b^2*cos(6*b*log(c))*cos(3*b*log(c))
 + b^2*sin(6*b*log(c))*sin(3*b*log(c)) + b^2*cos(3*b*log(c)))*n^2 + 12*(b*c
os(3*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(3*b*log(c)) + b*sin(
3*b*log(c)))*n + 8*cos(6*b*log(c))*cos(3*b*log(c)) + 8*sin(6*b*log(c))*sin(
```

```
3*b*log(c)) + 8*cos(3*b*log(c)))*x^2*cos(3*b*log(x^n) + 3*a) + 3*(9*(b^3*co
s(3*b*log(c))*sin(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(3*b*log(c)) + b^3*c
os(2*b*log(c))*sin(3*b*log(c)) - b^3*cos(3*b*log(c))*sin(2*b*log(c)))*n^3 +
 18*(b^2*cos(4*b*log(c))*cos(3*b*log(c)) + b^2*cos(3*b*log(c))*cos(2*b*log(
c)) + b^2*sin(4*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c))*sin(2*b*log
(c)))*n^2 + 4*(b*cos(3*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(3*
b*log(c)) + b*cos(2*b*log(c))*sin(3*b*log(c)) - b*cos(3*b*log(c))*sin(2*b*l
og(c)))*n + 8*cos(4*b*log(c))*cos(3*b*log(c)) + 8*cos(3*b*log(c))*cos(2*b*l
og(c)) + 8*sin(4*b*log(c))*sin(3*b*log(c)) + 8*sin(3*b*log(c))*sin(2*b*log(
c)))*x^2*cos(b*log(x^n) + a) + (3*(b^3*cos(6*b*log(c))*cos(3*b*log(c)) + b^
3*sin(6*b*log(c))*sin(3*b*log(c)) + b^3*cos(3*b*log(c)))*n^3 - 2*(b^2*cos(3
*b*log(c))*sin(6*b*log(c)) - b^2*cos(6*b*log(c))*sin(3*b*log(c)) + b^2*sin(
3*b*log(c)))*n^2 + 12*(b*cos(6*b*log(c))*cos(3*b*log(c)) + b*sin(6*b*log(c)
)*sin(3*b*log(c)) + b*cos(3*b*log(c)))*n - 8*cos(3*b*log(c))*sin(6*b*log(c)
) + 8*cos(6*b*log(c))*sin(3*b*log(c)) - 8*sin(3*b*log(c)))*x^2*sin(3*b*log(
x^n) + 3*a) + 3*(9*(b^3*cos(4*b*log(c))*cos(3*b*log(c)) + b^3*cos(3*b*log(c
))*cos(2*b*log(c)) + b^3*sin(4*b*log(c))*sin(3*b*log(c)) + b^3*sin(3*b*log(
c))*sin(2*b*log(c)))*n^3 - 18*(b^2*cos(3*b*log(c))*sin(4*b*log(c)) - b^2*co
s(4*b*log(c))*sin(3*b*log(c)) + b^2*cos(2*b*log(c))*sin(3*b*log(c)) - b^2*c
os(3*b*log(c))*sin(2*b*log(c)))*n^2 + 4*(b*cos(4*b*log(c))*cos(3*b*log(c))
+ b*cos(3*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(3*b*log(c)) + b
*sin(3*b*log(c))*sin(2*b*log(c)))*n - 8*cos(3*b*log(c))*sin(4*b*log(c)) + 8
*cos(4*b*log(c))*sin(3*b*log(c)) - 8*cos(2*b*log(c))*sin(3*b*log(c)) + 8*co
s(3*b*log(c))*sin(2*b*log(c)))*x^2*sin(b*log(x^n) + a))/(9*(b^4*cos(3*b*log
(c))^2 + b^4*sin(3*b*log(c))^2)*n^4 + 40*(b^2*cos(3*b*log(c))^2 + b^2*sin(3
*b*log(c))^2)*n^2 + 16*cos(3*b*log(c))^2 + 16*sin(3*b*log(c))^2)
```

Fricas [A]
time = 2.22, size = 129, normalized size = 0.82

$$\frac{12\,b^2n^2x^2\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+2\,(b^2n^2+4)x^2\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^3+3\,\left(2\,b^3n^3x^2+(b^3n^3+4\,bn)x^2\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{9\,b^4n^4+40\,b^2n^2+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] (12*b^2*n^2*x^2*cos(b*n*log(x) + b*log(c) + a) + 2*(b^2*n^2 + 4)*x^2*cos(b*
n*log(x) + b*log(c) + a)^3 + 3*(2*b^3*n^3*x^2 + (b^3*n^3 + 4*b*n)*x^2*cos(b
*n*log(x) + b*log(c) + a)^2)*sin(b*n*log(x) + b*log(c) + a))/(9*b^4*n^4 + 4
0*b^2*n^2 + 16)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int x\cos^{3}\left(a-\frac{2i\log\left(cx^{n}\right)}{n}\right)dx & \text{for}\: b=-\frac{2i}{n} \\ \int x\cos^{3}\left(a-\frac{2i\log\left(cx^{n}\right)}{3n}\right)dx & \text{for}\: b=-\frac{2i}{3n} \\ \int x\cos^{3}\left(a+\frac{2i\log\left(cx^{n}\right)}{3n}\right)dx & \text{for}\: b=\frac{2i}{3n} \\ \int x\cos^{3}\left(a+\frac{2i\log\left(cx^{n}\right)}{n}\right)dx & \text{for}\: b=\frac{2i}{n} \\ \frac{6b^{3}n^{3}x^{2}\sin^{3}\left(a+b\log\left(cx^{n}\right)\right)}{9b^{4}n^{4}+40b^{2}n^{2}+16}+\frac{9b^{3}n^{3}x^{2}\sin\left(a+b\log\left(cx^{n}\right)\right)\cos^{2}\left(a+b\log\left(cx^{n}\right)\right)}{9b^{4}n^{4}+40b^{2}n^{2}+16}+\frac{12b^{2}n^{2}x^{2}\sin^{2}\left(a+b\log\left(cx^{n}\right)\right)\cos\left(a+b\log\left(cx^{n}\right)\right)}{9b^{4}n^{4}+40b^{2}n^{2}+16}+\frac{14b^{2}n^{2}x^{2}\cos^{3}\left(a+b\log\left(cx^{n}\right)\right)}{9b^{4}n^{4}+40b^{2}n^{2}+16}+\frac{12bnx^{2}\sin\left(a+b\log\left(cx^{n}\right)\right)\cos^{2}\left(a+b\log\left(cx^{n}\right)\right)}{9b^{4}n^{4}+40b^{2}n^{2}+16}+\frac{8x^{2}\cos^{3}\left(a+b\log\left(cx^{n}\right)\right)}{9b^{4}n^{4}+40b^{2}n^{2}+16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*ln(c*x**n))**3,x)
```

```
[Out] Piecewise((Integral(x*cos(a - 2*I*log(c*x**n)/n)**3, x), Eq(b, -2*I/n)), (I
ntegral(x*cos(a - 2*I*log(c*x**n)/(3*n))**3, x), Eq(b, -2*I/(3*n))), (Integ
ral(x*cos(a + 2*I*log(c*x**n)/(3*n))**3, x), Eq(b, 2*I/(3*n))), (Integral(x
*cos(a + 2*I*log(c*x**n)/n)**3, x), Eq(b, 2*I/n)), (6*b**3*n**3*x**2*sin(a
+ b*log(c*x**n))**3/(9*b**4*n**4 + 40*b**2*n**2 + 16) + 9*b**3*n**3*x**2*si
n(a + b*log(c*x**n))*cos(a + b*log(c*x**n))**2/(9*b**4*n**4 + 40*b**2*n**2
+ 16) + 12*b**2*n**2*x**2*sin(a + b*log(c*x**n))**2*cos(a + b*log(c*x**n))/
(9*b**4*n**4 + 40*b**2*n**2 + 16) + 14*b**2*n**2*x**2*cos(a + b*log(c*x**n)
)**3/(9*b**4*n**4 + 40*b**2*n**2 + 16) + 12*b*n*x**2*sin(a + b*log(c*x**n))
*cos(a + b*log(c*x**n))**2/(9*b**4*n**4 + 40*b**2*n**2 + 16) + 8*x**2*cos(a
 + b*log(c*x**n))**3/(9*b**4*n**4 + 40*b**2*n**2 + 16), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 18069 vs. $2(158) = 316$.
time = 1.17, size = 18069, normalized size = 114.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] -1/4*(27*b^3*n^3*x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) -
1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(ab
s(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a) + 27*b^3*n^3*x^2*e^(-1
/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log
(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c))
)^2*tan(3/2*a)^2*tan(1/2*a) + 3*b^3*n^3*x^2*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b
*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)
))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)*tan(1/2*a)^2
 + 3*b^3*n^3*x^2*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/2
*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x
)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)*tan(1/2*a)^2 + 27*b^3*n^3*x^2*e^(1/2*p
i*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs
(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*ta
n(3/2*a)^2*tan(1/2*a)^2 + 27*b^3*n^3*x^2*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n
 - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))
^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(3/2*a)^2*tan(1/2*a)^2 +
 3*b^3*n^3*x^2*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi
*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))*tan(1/2*b*n*log(abs(x)) +
1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 + 3*b^3*n^3*x^2*e^(-3/2*pi*b
*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)
) + 3/2*b*log(abs(c)))*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3
```

/2*a)^2*tan(1/2*a)^2 - b^2*n^2*x^2*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 - 27*b^2*n^2*x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 - 27*b^2*n^2*x^2*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 - b^2*n^2*x^2*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 + 3*b^3*n^3*x^2*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a) + 3*b^3*n^3*x^2*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a) - 27*b^3*n^3*x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(3/2*a)^2 - 27*b^3*n^3*x^2*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(3/2*a)^2 + 3*b^3*n^3*x^2*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2 + 3*b^3*n^3*x^2*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2 + 27*b^3*n^3*x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a) + 27*b^3*n^3*x^2*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a) - 27*b^3*n^3*x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a) - 27*b^3*n^3*x^2*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a) + 27*b^3*n^3*x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a) + 27*b^3*n^3*x^2*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a) + 27*b^3*n^3*x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a)^2 + 27*b^3*n^3*x^2*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a)^2 - 3*b^3*n^3*x^2*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan

```
(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))*tan(1...
```

Mupad [B]
time = 2.95, size = 122, normalized size = 0.77

$$\frac{x^2\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,1\mathrm{i}}}\,3\mathrm{i}}{8\,b\,n+16\mathrm{i}}+\frac{3\,x^2\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,1\mathrm{i}}}{16+b\,n\,8\mathrm{i}}+\frac{x^2\,\mathrm{e}^{-a\,3\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,3\mathrm{i}}}\,1\mathrm{i}}{24\,b\,n+16\mathrm{i}}+\frac{x^2\,\mathrm{e}^{a\,3\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,3\mathrm{i}}}{16+b\,n\,24\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*log(c*x^n))^3,x)
```

```
[Out] (x^2*exp(-a*1i)/(c*x^n)^(b*1i)*3i)/(8*b*n + 16i) + (3*x^2*exp(a*1i)*(c*x^n)
^(b*1i))/(b*n*8i + 16) + (x^2*exp(-a*3i)/(c*x^n)^(b*3i)*1i)/(24*b*n + 16i)
+ (x^2*exp(a*3i)*(c*x^n)^(b*3i))/(b*n*24i + 16)
```

3.98 $\int \cos^3(a+b\log(cx^n))\,dx$

Optimal. Leaf size=149

$$\frac{6b^2n^2x\cos(a+b\log(cx^n))}{1+10b^2n^2+9b^4n^4}+\frac{x\cos^3(a+b\log(cx^n))}{1+9b^2n^2}+\frac{6b^3n^3x\sin(a+b\log(cx^n))}{1+10b^2n^2+9b^4n^4}+\frac{3bnx\cos^2(a+b\log(cx^n))}{1+9b^2}$$

```
[Out] 6*b^2*n^2*x*cos(a+b*ln(c*x^n))/(9*b^4*n^4+10*b^2*n^2+1)+x*cos(a+b*ln(c*x^n)
)^3/(9*b^2*n^2+1)+6*b^3*n^3*x*sin(a+b*ln(c*x^n))/(9*b^4*n^4+10*b^2*n^2+1)+3
*b*n*x*cos(a+b*ln(c*x^n))^2*sin(a+b*ln(c*x^n))/(9*b^2*n^2+1)
```

Rubi [A]
time = 0.03, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4566, 4564}

$$\frac{x\cos^3(a+b\log(cx^n))}{9b^2n^2+1}+\frac{3bnx\sin(a+b\log(cx^n))\cos^2(a+b\log(cx^n))}{9b^2n^2+1}+\frac{6b^2n^2x\cos(a+b\log(cx^n))}{9b^4n^4+10b^2n^2+1}+\frac{6b^3n^3x\sin(a+b\log(cx^n))}{9b^4n^4+10b^2n^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (6*b^2*n^2*x*Cos[a + b*Log[c*x^n]])/(1 + 10*b^2*n^2 + 9*b^4*n^4) + (x*Cos[a
 + b*Log[c*x^n]]^3)/(1 + 9*b^2*n^2) + (6*b^3*n^3*x*Sin[a + b*Log[c*x^n]])/(
1 + 10*b^2*n^2 + 9*b^4*n^4) + (3*b*n*x*Cos[a + b*Log[c*x^n]]^2*Sin[a + b*Lo
g[c*x^n]])/(1 + 9*b^2*n^2)
```

Rule 4564

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*(
Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] + Simp[b*d*n*x*(Sin[d*(a +
 b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

Rule 4566

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Sim
p[x*(Cos[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 + 1)), x] + (Dist[b^2*d^2
*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + 1)), Int[Cos[d*(a + b*Log[c*x^n])]^(p -
2), x], x] + Simp[b*d*n*p*x*Cos[d*(a + b*Log[c*x^n])]^(p - 1)*(Sin[d*(a + b
*Log[c*x^n])]/(b^2*d^2*n^2*p^2 + 1)), x]) /; FreeQ[{a, b, c, d, n}, x] && I
GtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + 1, 0]
```

Rubi steps

$$\int \cos^3(a+b\log(cx^n))\,dx = \frac{x\cos^3(a+b\log(cx^n))}{1+9b^2n^2} + \frac{3bnx\cos^2(a+b\log(cx^n))\sin(a+b\log(cx^n))}{1+9b^2n^2} + \frac{(6$$

$$= \frac{6b^2n^2x\cos(a+b\log(cx^n))}{1+10b^2n^2+9b^4n^4} + \frac{x\cos^3(a+b\log(cx^n))}{1+9b^2n^2} + \frac{6b^3n^3x\sin(a+b\log(}{1+10b^2n^2+9b^4}$$

Mathematica [A]
time = 0.46, size = 117, normalized size = 0.79

$$\frac{x(3(1+9b^2n^2)\cos(a+b\log(cx^n))+(1+b^2n^2)\cos(3(a+b\log(cx^n)))+6bn(1+5b^2n^2+(1+b^2n^2)\cos(2(a+b\log(cx^n))))\sin(a+b\log(cx^n)))}{4+40b^2n^2+36b^4n^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (x*(3*(1 + 9*b^2*n^2)*Cos[a + b*Log[c*x^n]] + (1 + b^2*n^2)*Cos[3*(a + b*Lo
g[c*x^n])] + 6*b*n*(1 + 5*b^2*n^2 + (1 + b^2*n^2)*Cos[2*(a + b*Log[c*x^n])]
)*Sin[a + b*Log[c*x^n]]))/(4 + 40*b^2*n^2 + 36*b^4*n^4)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \cos^3(a+b\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(cos(a+b*ln(c*x^n))^3,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 989 vs. $2(149) = 298$.
time = 0.32, size = 989, normalized size = 6.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*(b^3*cos(3*b*log(c))*sin(6*b*log(c)) - b^3*cos(6*b*log(c))*sin(3*b*
log(c)) + b^3*sin(3*b*log(c)))*n^3 + (b^2*cos(6*b*log(c))*cos(3*b*log(c)) +
 b^2*sin(6*b*log(c))*sin(3*b*log(c)) + b^2*cos(3*b*log(c)))*n^2 + 3*(b*cos(
3*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(3*b*log(c)) + b*sin(3*b
*log(c)))*n + cos(6*b*log(c))*cos(3*b*log(c)) + sin(6*b*log(c))*sin(3*b*log
(c)) + cos(3*b*log(c)))*x*cos(3*b*log(x^n) + 3*a) + 3*(9*(b^3*cos(3*b*log(c
```

```
))*sin(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(3*b*log(c)) + b^3*cos(2*b*log(
c))*sin(3*b*log(c)) - b^3*cos(3*b*log(c))*sin(2*b*log(c)))*n^3 + 9*(b^2*cos
(4*b*log(c))*cos(3*b*log(c)) + b^2*cos(3*b*log(c))*cos(2*b*log(c)) + b^2*si
n(4*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c))*sin(2*b*log(c)))*n^2 +
(b*cos(3*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(3*b*log(c)) + b*
cos(2*b*log(c))*sin(3*b*log(c)) - b*cos(3*b*log(c))*sin(2*b*log(c)))*n + co
s(4*b*log(c))*cos(3*b*log(c)) + cos(3*b*log(c))*cos(2*b*log(c)) + sin(4*b*l
og(c))*sin(3*b*log(c)) + sin(3*b*log(c))*sin(2*b*log(c)))*x*cos(b*log(x^n)
+ a) + (3*(b^3*cos(6*b*log(c))*cos(3*b*log(c)) + b^3*sin(6*b*log(c))*sin(3*
b*log(c)) + b^3*cos(3*b*log(c)))*n^3 - (b^2*cos(3*b*log(c))*sin(6*b*log(c))
 - b^2*cos(6*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c)))*n^2 + 3*(b*co
s(6*b*log(c))*cos(3*b*log(c)) + b*sin(6*b*log(c))*sin(3*b*log(c)) + b*cos(3
*b*log(c)))*n - cos(3*b*log(c))*sin(6*b*log(c)) + cos(6*b*log(c))*sin(3*b*l
og(c)) - sin(3*b*log(c)))*x*sin(3*b*log(x^n) + 3*a) + 3*(9*(b^3*cos(4*b*log
(c))*cos(3*b*log(c)) + b^3*cos(3*b*log(c))*cos(2*b*log(c)) + b^3*sin(4*b*lo
g(c))*sin(3*b*log(c)) + b^3*sin(3*b*log(c))*sin(2*b*log(c)))*n^3 - 9*(b^2*c
os(3*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(3*b*log(c)) + b^2*
cos(2*b*log(c))*sin(3*b*log(c)) - b^2*cos(3*b*log(c))*sin(2*b*log(c)))*n^2
+ (b*cos(4*b*log(c))*cos(3*b*log(c)) + b*cos(3*b*log(c))*cos(2*b*log(c)) +
b*sin(4*b*log(c))*sin(3*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*log(c)))*n -
cos(3*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c))*sin(3*b*log(c)) - cos(2*b
*log(c))*sin(3*b*log(c)) + cos(3*b*log(c))*sin(2*b*log(c)))*x*sin(b*log(x^n
) + a))/(9*(b^4*cos(3*b*log(c))^2 + b^4*sin(3*b*log(c))^2)*n^4 + 10*(b^2*co
s(3*b*log(c))^2 + b^2*sin(3*b*log(c))^2)*n^2 + cos(3*b*log(c))^2 + sin(3*b*
log(c))^2)
```

Fricas [A]
time = 2.84, size = 119, normalized size = 0.80

$$\frac{6\,b^2n^2x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+(b^2n^2+1)x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^3+3\left(2\,b^3n^3x+(b^3n^3+bn)x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{9\,b^4n^4+10\,b^2n^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] (6*b^2*n^2*x*cos(b*n*log(x) + b*log(c) + a) + (b^2*n^2 + 1)*x*cos(b*n*log(x
) + b*log(c) + a)^3 + 3*(2*b^3*n^3*x + (b^3*n^3 + b*n)*x*cos(b*n*log(x) + b
*log(c) + a)^2)*sin(b*n*log(x) + b*log(c) + a))/(9*b^4*n^4 + 10*b^2*n^2 + 1
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \cos^{3}{\left(a - \frac{i \log{\left(c x^{n} \right)}}{n} \right)}\, dx & \text{for}\: b = - \frac{i}{n} \\ \int \cos^{3}{\left(a - \frac{i \log{\left(c x^{n} \right)}}{3 n} \right)}\, dx & \text{for}\: b = - \frac{i}{3 n} \\ \int \cos^{3}{\left(a + \frac{i \log{\left(c x^{n} \right)}}{3 n} \right)}\, dx & \text{for}\: b = \frac{i}{3 n} \\ \int \cos^{3}{\left(a + \frac{i \log{\left(c x^{n} \right)}}{n} \right)}\, dx & \text{for}\: b = \frac{i}{n} \\ \frac{6 b^{3} n^{3} x \sin^{3}{\left(a + b \log{\left(c x^{n} \right)} \right)}}{9 b^{4} n^{4} + 10 b^{2} n^{2} + 1} + \frac{9 b^{3} n^{3} x \sin{\left(a + b \log{\left(c x^{n} \right)} \right)} \cos^{2}{\left(a + b \log{\left(c x^{n} \right)} \right)}}{9 b^{4} n^{4} + 10 b^{2} n^{2} + 1} + \frac{6 b^{2} n^{2} x \sin^{2}{\left(a + b \log{\left(c x^{n} \right)} \right)} \cos{\left(a + b \log{\left(c x^{n} \right)} \right)}}{9 b^{4} n^{4} + 10 b^{2} n^{2} + 1} + \frac{7 b^{2} n^{2} x \cos^{3}{\left(a + b \log{\left(c x^{n} \right)} \right)}}{9 b^{4} n^{4} + 10 b^{2} n^{2} + 1} + \frac{3 b n x \sin{\left(a + b \log{\left(c x^{n} \right)} \right)} \cos^{2}{\left(a + b \log{\left(c x^{n} \right)} \right)}}{9 b^{4} n^{4} + 10 b^{2} n^{2} + 1} + \frac{x \cos^{3}{\left(a + b \log{\left(c x^{n} \right)} \right)}}{9 b^{4} n^{4} + 10 b^{2} n^{2} + 1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*ln(c*x**n))**3,x)
```

```
[Out] Piecewise((Integral(cos(a - I*log(c*x**n)/n)**3, x), Eq(b, -I/n)), (Integra
l(cos(a - I*log(c*x**n)/(3*n))**3, x), Eq(b, -I/(3*n))), (Integral(cos(a +
I*log(c*x**n)/(3*n))**3, x), Eq(b, I/(3*n))), (Integral(cos(a + I*log(c*x**
n)/n)**3, x), Eq(b, I/n)), (6*b**3*n**3*x*sin(a + b*log(c*x**n))**3/(9*b**4
*n**4 + 10*b**2*n**2 + 1) + 9*b**3*n**3*x*sin(a + b*log(c*x**n))*cos(a + b*
log(c*x**n))**2/(9*b**4*n**4 + 10*b**2*n**2 + 1) + 6*b**2*n**2*x*sin(a + b*
log(c*x**n))**2*cos(a + b*log(c*x**n))/(9*b**4*n**4 + 10*b**2*n**2 + 1) + 7
*b**2*n**2*x*cos(a + b*log(c*x**n))**3/(9*b**4*n**4 + 10*b**2*n**2 + 1) + 3
*b*n*x*sin(a + b*log(c*x**n))*cos(a + b*log(c*x**n))**2/(9*b**4*n**4 + 10*b
**2*n**2 + 1) + x*cos(a + b*log(c*x**n))**3/(9*b**4*n**4 + 10*b**2*n**2 + 1
), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 17458 vs. $2(149) = 298$.
time = 0.83, size = 17458, normalized size = 117.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] -1/8*(54*b^3*n^3*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/
2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(
x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a) + 54*b^3*n^3*x*e^(-1/2*p
i*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs
(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*
tan(3/2*a)^2*tan(1/2*a) + 6*b^3*n^3*x*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3
/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*t
an(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)*tan(1/2*a)^2 + 6*b
^3*n^3*x*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/2*pi*b)*t
an(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2
*b*log(abs(c)))^2*tan(3/2*a)*tan(1/2*a)^2 + 54*b^3*n^3*x*e^(1/2*pi*b*n*sgn(
x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2
*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(3/2*a)^2
*tan(1/2*a)^2 + 54*b^3*n^3*x*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*
sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b
*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(3/2*a)^2*tan(1/2*a)^2 + 6*b^3*n^3*x
*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*
n*log(abs(x)) + 3/2*b*log(abs(c)))*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(
c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 + 6*b^3*n^3*x*e^(-3/2*pi*b*n*sgn(x) + 3/2*
pi*b*n - 3/2*pi*b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(ab
s(c)))*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*
```

a)^2 - b^2*n^2*x*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*
pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)
) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 - 27*b^2*n^2*x*e^(1/2*pi
*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(
x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*t
an(3/2*a)^2*tan(1/2*a)^2 - 27*b^2*n^2*x*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n
- 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^
2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2
- b^2*n^2*x*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/2*pi*b
)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) +
1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 + 6*b^3*n^3*x*e^(3/2*pi*b*n*
sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) +
3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/
2*a) + 6*b^3*n^3*x*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3
/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs
(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a) - 54*b^3*n^3*x*e^(1/2*pi*b*n*sgn(x)
- 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*
log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(3/2*a)^2 -
54*b^3*n^3*x*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*
b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) +
1/2*b*log(abs(c)))*tan(3/2*a)^2 + 6*b^3*n^3*x*e^(3/2*pi*b*n*sgn(x) - 3/2*p
i*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs
(c)))*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2 + 6*b^3*n
^3*x*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/2*pi*b)*tan(3
/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))*tan(1/2*b*n*log(abs(x)) + 1/2*b*log
(abs(c)))^2*tan(3/2*a)^2 + 54*b^3*n^3*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n +
1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2
*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a) + 54*b^3*n^3*x*e
^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n
*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs
(c)))^2*tan(1/2*a) - 54*b^3*n^3*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*p
i*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(3
/2*a)^2*tan(1/2*a) - 54*b^3*n^3*x*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*
pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(
3/2*a)^2*tan(1/2*a) + 54*b^3*n^3*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*
pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(
3/2*a)^2*tan(1/2*a) + 54*b^3*n^3*x*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2
*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan
(3/2*a)^2*tan(1/2*a) + 54*b^3*n^3*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2
*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan
(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a)^2 + 54*b^3*n^3*x*e^(-1
/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log
(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c))
)*tan(1/2*a)^2 - 6*b^3*n^3*x*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*s
gn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))*tan(1/2*b*n*

```
log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2...
```

Mupad [B]
time = 2.82, size = 114, normalized size = 0.77

$$\frac{x\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{(c\,x^n)^{b\,1\mathrm{i}}}\,3\mathrm{i}}{8\,b\,n+8\mathrm{i}}+\frac{3\,x\,\mathrm{e}^{a\,1\mathrm{i}}\,(c\,x^n)^{b\,1\mathrm{i}}}{8+b\,n\,8\mathrm{i}}+\frac{x\,\mathrm{e}^{-a\,3\mathrm{i}}\,\frac{1}{(c\,x^n)^{b\,3\mathrm{i}}}\,1\mathrm{i}}{24\,b\,n+8\mathrm{i}}+\frac{x\,\mathrm{e}^{a\,3\mathrm{i}}\,(c\,x^n)^{b\,3\mathrm{i}}}{8+b\,n\,24\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*log(c*x^n))^3,x)
```

```
[Out] (x*exp(-a*1i)/(c*x^n)^(b*1i)*3i)/(8*b*n + 8i) + (3*x*exp(a*1i)*(c*x^n)^(b*1
i))/(b*n*8i + 8) + (x*exp(-a*3i)/(c*x^n)^(b*3i)*1i)/(24*b*n + 8i) + (x*exp(
a*3i)*(c*x^n)^(b*3i))/(b*n*24i + 8)
```

3.99 $\int \frac{\cos^3(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=42

$$\frac{\sin(a+b\log(cx^n))}{bn}-\frac{\sin^3(a+b\log(cx^n))}{3bn}$$

```
[Out] sin(a+b*ln(c*x^n))/b/n-1/3*sin(a+b*ln(c*x^n))^3/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.059$, Rules used = $\{2713\}$

$$\frac{\sin(a+b\log(cx^n))}{bn}-\frac{\sin^3(a+b\log(cx^n))}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] Sin[a + b*Log[c*x^n]]/(b*n) - Sin[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \cos^3(a+bx)\,dx,x,\log(cx^n)\right)}{n}\\ &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(a+b\log(cx^n))\right)}{bn}\\ &= \frac{\sin(a+b\log(cx^n))}{bn}-\frac{\sin^3(a+b\log(cx^n))}{3bn}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 42, normalized size = 1.00

$$\frac{\sin(a+b\log(cx^n))}{bn}-\frac{\sin^3(a+b\log(cx^n))}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] Sin[a + b*Log[c*x^n]]/(b*n) - Sin[a + b*Log[c*x^n]]^3/(3*b*n)
```

Maple [A]
time = 0.06, size = 35, normalized size = 0.83

method	result	size
derivativedivides	$\frac{\left(2+\cos^{2}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)\sin\left(a+b\ln\left(c\,x^{n}\right)\right)}{3nb}$	35
default	$\frac{\left(2+\cos^{2}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)\sin\left(a+b\ln\left(c\,x^{n}\right)\right)}{3nb}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/n/b*(2+cos(a+b*ln(c*x^n))^2)*sin(a+b*ln(c*x^n))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(40) = 80.
time = 0.30, size = 232, normalized size = 5.52

$$\frac{\left(\cos\left(3\,b\log\left(c\right)\right)\sin\left(6\,b\log\left(c\right)\right)-\cos\left(6\,b\log\left(c\right)\right)\sin\left(3\,b\log\left(c\right)\right)+\sin\left(3\,b\log\left(c\right)\right)\right)\cos\left(3\,b\log\left(x^{n}\right)+3\,a\right)+9\left(\cos\left(3\,b\log\left(c\right)\right)\sin\left(4\,b\log\left(c\right)\right)-\cos\left(4\,b\log\left(c\right)\right)\sin\left(3\,b\log\left(c\right)\right)+\cos\left(2\,b\log\left(c\right)\right)\sin\left(3\,b\log\left(c\right)\right)-\cos\left(3\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)\right)\cos\left(b\log\left(x^{n}\right)+a\right)+\left(\cos\left(6\,b\log\left(c\right)\right)\cos\left(3\,b\log\left(c\right)\right)+\sin\left(6\,b\log\left(c\right)\right)\sin\left(3\,b\log\left(c\right)\right)+\cos\left(3\,b\log\left(c\right)\right)\right)\sin\left(3\,b\log\left(x^{n}\right)+3\,a\right)+9\left(\cos\left(4\,b\log\left(c\right)\right)\cos\left(3\,b\log\left(c\right)\right)+\cos\left(3\,b\log\left(c\right)\right)\cos\left(2\,b\log\left(c\right)\right)+\sin\left(4\,b\log\left(c\right)\right)\sin\left(3\,b\log\left(c\right)\right)+\sin\left(3\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)\right)\sin\left(b\log\left(x^{n}\right)+a\right)}{24\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] 1/24*((cos(3*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c))*sin(3*b*log(c)) +
sin(3*b*log(c)))*cos(3*b*log(x^n) + 3*a) + 9*(cos(3*b*log(c))*sin(4*b*log(c
)) - cos(4*b*log(c))*sin(3*b*log(c)) + cos(2*b*log(c))*sin(3*b*log(c)) - co
s(3*b*log(c))*sin(2*b*log(c)))*cos(b*log(x^n) + a) + (cos(6*b*log(c))*cos(3
*b*log(c)) + sin(6*b*log(c))*sin(3*b*log(c)) + cos(3*b*log(c)))*sin(3*b*log
(x^n) + 3*a) + 9*(cos(4*b*log(c))*cos(3*b*log(c)) + cos(3*b*log(c))*cos(2*b
*log(c)) + sin(4*b*log(c))*sin(3*b*log(c)) + sin(3*b*log(c))*sin(2*b*log(c)
))*sin(b*log(x^n) + a))/(b*n)
```

Fricas [A]
time = 7.34, size = 36, normalized size = 0.86

$$\frac{\left(\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{3\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(b*n*log(x) + b*log(c) + a)^2 + 2)*sin(b*n*log(x) + b*log(c) + a)/(
b*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(32) = 64$.
time = 1.76, size = 71, normalized size = 1.69

$$\begin{cases} \log(x)\cos^3(a) & \text{for } b = 0 \wedge (b = 0 \vee n = 0) \\ \log(x)\cos^3(a + b\log(c)) & \text{for } n = 0 \\ \frac{2\sin^3(a+b\log(cx^n))}{3bn} + \frac{\sin(a+b\log(cx^n))\cos^2(a+b\log(cx^n))}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*ln(c*x**n))**3/x,x)
```

```
[Out] Piecewise((log(x)*cos(a)**3, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*cos
(a + b*log(c))**3, Eq(n, 0)), (2*sin(a + b*log(c*x**n))**3/(3*b*n) + sin(a
+ b*log(c*x**n))*cos(a + b*log(c*x**n))**2/(b*n), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] integrate(cos(b*log(c*x^n) + a)^3/x, x)
```

Mupad [B]
time = 2.35, size = 37, normalized size = 0.88

$$\frac{3\,\sin\left(a+b\,\ln\left(c\,x^n\right)\right)-\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{3\,b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*log(c*x^n))^3/x,x)
```

```
[Out] (3*sin(a + b*log(c*x^n)) - sin(a + b*log(c*x^n))^3)/(3*b*n)
```

3.100 $\int \frac{\cos^3(a+b\log(cx^n))}{x^2} dx$

Optimal. Leaf size=158

$$-\frac{6b^2n^2\cos(a+b\log(cx^n))}{(1+10b^2n^2+9b^4n^4)x}-\frac{\cos^3(a+b\log(cx^n))}{(1+9b^2n^2)x}+\frac{6b^3n^3\sin(a+b\log(cx^n))}{(1+10b^2n^2+9b^4n^4)x}+\frac{3bn\cos^2(a+b\log(cx^n))\sin(a+b\log(cx^n))}{(1+9b^2n^2)x}$$

```
[Out] -6*b^2*n^2*cos(a+b*ln(c*x^n))/(9*b^4*n^4+10*b^2*n^2+1)/x-cos(a+b*ln(c*x^n))
^3/(9*b^2*n^2+1)/x+6*b^3*n^3*sin(a+b*ln(c*x^n))/(9*b^4*n^4+10*b^2*n^2+1)/x+
3*b*n*cos(a+b*ln(c*x^n))^2*sin(a+b*ln(c*x^n))/(9*b^2*n^2+1)/x
```

Rubi [A]
time = 0.03, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4576, 4574}

$$-\frac{\cos^3(a+b\log(cx^n))}{x(9b^2n^2+1)}+\frac{3bn\sin(a+b\log(cx^n))\cos^2(a+b\log(cx^n))}{x(9b^2n^2+1)}-\frac{6b^2n^2\cos(a+b\log(cx^n))}{x(9b^4n^4+10b^2n^2+1)}+\frac{6b^3n^3\sin(a+b\log(cx^n))}{x(9b^4n^4+10b^2n^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*Log[c*x^n]]^3/x^2,x]
```

```
[Out] (-6*b^2*n^2*Cos[a + b*Log[c*x^n]])/((1 + 10*b^2*n^2 + 9*b^4*n^4)*x) - Cos[a
 + b*Log[c*x^n]]^3/((1 + 9*b^2*n^2)*x) + (6*b^3*n^3*Sin[a + b*Log[c*x^n]])/
((1 + 10*b^2*n^2 + 9*b^4*n^4)*x) + (3*b*n*Cos[a + b*Log[c*x^n]]^2*Sin[a + b
*Log[c*x^n]])/((1 + 9*b^2*n^2)*x)
```

Rule 4574

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Rule 4576

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2
*n^2*p^2 + (m + 1)^2)), Int[(e*x)^m*Cos[d*(a + b*Log[c*x^n])]^(p - 2), x],
x] + Simp[b*d*n*p*(e*x)^(m + 1)*Sin[d*(a + b*Log[c*x^n])]*(Cos[d*(a + b*Log
[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x]) /; FreeQ[{a, b, c
, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+b\log(cx^n))}{x^2}\,dx = -\frac{\cos^3(a+b\log(cx^n))}{(1+9b^2n^2)\,x} + \frac{3bn\cos^2(a+b\log(cx^n))\sin(a+b\log(cx^n))}{(1+9b^2n^2)\,x} + \frac{(6b^2
$$
$$= -\frac{6b^2n^2\cos(a+b\log(cx^n))}{(1+10b^2n^2+9b^4n^4)\,x} - \frac{\cos^3(a+b\log(cx^n))}{(1+9b^2n^2)\,x} + \frac{6b^3n^3\sin(a+b\log(cx^n
$$

Mathematica [A]
time = 0.50, size = 122, normalized size = 0.77

$$-\frac{3(1+9b^2n^2)\cos(a+b\log(cx^n)) + (1+b^2n^2)\cos(3(a+b\log(cx^n))) - 6bn(1+5b^2n^2+(1+b^2n^2)\cos(2(a+b\log(cx^n))))\sin(a+b\log(cx^n))}{4(1+10b^2n^2+9b^4n^4)\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*Log[c*x^n]]^3/x^2,x]
```

```
[Out] -1/4*(3*(1 + 9*b^2*n^2)*Cos[a + b*Log[c*x^n]] + (1 + b^2*n^2)*Cos[3*(a + b*
Log[c*x^n])] - 6*b*n*(1 + 5*b^2*n^2 + (1 + b^2*n^2)*Cos[2*(a + b*Log[c*x^n]
)])*Sin[a + b*Log[c*x^n]])/((1 + 10*b^2*n^2 + 9*b^4*n^4)*x)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\cos^3(a+b\ln(c\,x^n))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*ln(c*x^n))^3/x^2,x)
```

```
[Out] int(cos(a+b*ln(c*x^n))^3/x^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 994 vs. $2(158) = 316$.
time = 0.33, size = 994, normalized size = 6.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/8*((3*(b^3*cos(3*b*log(c))*sin(6*b*log(c)) - b^3*cos(6*b*log(c))*sin(3*b*
log(c)) + b^3*sin(3*b*log(c)))*n^3 - (b^2*cos(6*b*log(c))*cos(3*b*log(c)) +
 b^2*sin(6*b*log(c))*sin(3*b*log(c)) + b^2*cos(3*b*log(c)))*n^2 + 3*(b*cos(
3*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(3*b*log(c)) + b*sin(3*b
*log(c)))*n - cos(6*b*log(c))*cos(3*b*log(c)) - sin(6*b*log(c))*sin(3*b*log
```

```
(c)) - cos(3*b*log(c)))*cos(3*b*log(x^n) + 3*a) + 3*(9*(b^3*cos(3*b*log(c))
*sin(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(3*b*log(c)) + b^3*cos(2*b*log(c)
)*sin(3*b*log(c)) - b^3*cos(3*b*log(c))*sin(2*b*log(c)))*n^3 - 9*(b^2*cos(4
*b*log(c))*cos(3*b*log(c)) + b^2*cos(3*b*log(c))*cos(2*b*log(c)) + b^2*sin(
4*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c))*sin(2*b*log(c)))*n^2 + (b
*cos(3*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(3*b*log(c)) + b*co
s(2*b*log(c))*sin(3*b*log(c)) - b*cos(3*b*log(c))*sin(2*b*log(c)))*n - cos(
4*b*log(c))*cos(3*b*log(c)) - cos(3*b*log(c))*cos(2*b*log(c)) - sin(4*b*log
(c))*sin(3*b*log(c)) - sin(3*b*log(c))*sin(2*b*log(c)))*cos(b*log(x^n) + a)
 + (3*(b^3*cos(6*b*log(c))*cos(3*b*log(c)) + b^3*sin(6*b*log(c))*sin(3*b*lo
g(c)) + b^3*cos(3*b*log(c)))*n^3 + (b^2*cos(3*b*log(c))*sin(6*b*log(c)) - b
^2*cos(6*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c)))*n^2 + 3*(b*cos(6*
b*log(c))*cos(3*b*log(c)) + b*sin(6*b*log(c))*sin(3*b*log(c)) + b*cos(3*b*l
og(c)))*n + cos(3*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c))*sin(3*b*log(c
)) + sin(3*b*log(c)))*sin(3*b*log(x^n) + 3*a) + 3*(9*(b^3*cos(4*b*log(c))*c
os(3*b*log(c)) + b^3*cos(3*b*log(c))*cos(2*b*log(c)) + b^3*sin(4*b*log(c))*
sin(3*b*log(c)) + b^3*sin(3*b*log(c))*sin(2*b*log(c)))*n^3 + 9*(b^2*cos(3*b
*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(3*b*log(c)) + b^2*cos(2*
b*log(c))*sin(3*b*log(c)) - b^2*cos(3*b*log(c))*sin(2*b*log(c)))*n^2 + (b*c
os(4*b*log(c))*cos(3*b*log(c)) + b*cos(3*b*log(c))*cos(2*b*log(c)) + b*sin(
4*b*log(c))*sin(3*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*log(c)))*n + cos(3*
b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(3*b*log(c)) + cos(2*b*log(c
))*sin(3*b*log(c)) - cos(3*b*log(c))*sin(2*b*log(c)))*sin(b*log(x^n) + a))/
((9*(b^4*cos(3*b*log(c))^2 + b^4*sin(3*b*log(c))^2)*n^4 + 10*(b^2*cos(3*b*l
og(c))^2 + b^2*sin(3*b*log(c))^2)*n^2 + cos(3*b*log(c))^2 + sin(3*b*log(c))
^2)*x)
```

Fricas [A]
time = 2.95, size = 119, normalized size = 0.75

$$-\frac{6\,b^{2}n^{2}\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\left(b^{2}n^{2}+1\right)\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}-3\,\left(2\,b^{3}n^{3}+\left(b^{3}n^{3}+bn\right)\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{\left(9\,b^{4}n^{4}+10\,b^{2}n^{2}+1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^3/x^2,x, algorithm="fricas")
```

```
[Out] -(6*b^2*n^2*cos(b*n*log(x) + b*log(c) + a) + (b^2*n^2 + 1)*cos(b*n*log(x) +
 b*log(c) + a)^3 - 3*(2*b^3*n^3 + (b^3*n^3 + b*n)*cos(b*n*log(x) + b*log(c)
 + a)^2)*sin(b*n*log(x) + b*log(c) + a))/((9*b^4*n^4 + 10*b^2*n^2 + 1)*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 34.68, size = 774, normalized size = 4.90

$$\begin{cases} \frac{3i\sin\left(a-\frac{i\log\left(cx^{n}\right)}{n}\right)}{8x}+\frac{3i\sin\left(3a-\frac{3i\log\left(cx^{n}\right)}{n}\right)}{32x}+\frac{\cos\left(3a-\frac{3i\log\left(cx^{n}\right)}{n}\right)}{32x}+\frac{3i\log\left(cx^{n}\right)\sin\left(a-\frac{i\log\left(cx^{n}\right)}{n}\right)}{8nx}+\frac{3\log\left(cx^{n}\right)\cos\left(a-\frac{i\log\left(cx^{n}\right)}{n}\right)}{8nx} & \text{for}\: b=-\frac{i}{n} \\ -\frac{9i\sin\left(a-\frac{i\log\left(cx^{n}\right)}{3n}\right)}{32x}+\frac{i\sin\left(3a-\frac{i\log\left(cx^{n}\right)}{n}\right)}{8x}-\frac{27\cos\left(a-\frac{i\log\left(cx^{n}\right)}{3n}\right)}{32x}+\frac{i\log\left(cx^{n}\right)\sin\left(3a-\frac{i\log\left(cx^{n}\right)}{n}\right)}{8nx}+\frac{\log\left(cx^{n}\right)\cos\left(3a-\frac{i\log\left(cx^{n}\right)}{n}\right)}{8nx} & \text{for}\: b=-\frac{i}{3n} \\ \frac{9i\sin\left(a+\frac{i\log\left(cx^{n}\right)}{3n}\right)}{32x}-\frac{i\sin\left(3a+\frac{i\log\left(cx^{n}\right)}{n}\right)}{8x}-\frac{27\cos\left(a+\frac{i\log\left(cx^{n}\right)}{3n}\right)}{32x}-\frac{i\log\left(cx^{n}\right)\sin\left(3a+\frac{i\log\left(cx^{n}\right)}{n}\right)}{8nx}+\frac{\log\left(cx^{n}\right)\cos\left(3a+\frac{i\log\left(cx^{n}\right)}{n}\right)}{8nx} & \text{for}\: b=\frac{i}{3n} \\ -\frac{3i\sin\left(3a+\frac{3i\log\left(cx^{n}\right)}{n}\right)}{32x}-\frac{3\cos\left(a+\frac{i\log\left(cx^{n}\right)}{n}\right)}{8x}+\frac{\cos\left(3a+\frac{3i\log\left(cx^{n}\right)}{n}\right)}{32x}-\frac{3i\log\left(cx^{n}\right)\sin\left(a+\frac{i\log\left(cx^{n}\right)}{n}\right)}{8nx}+\frac{3\log\left(cx^{n}\right)\cos\left(a+\frac{i\log\left(cx^{n}\right)}{n}\right)}{8nx} & \text{for}\: b=\frac{i}{n} \\ \frac{6b^{3}n^{3}\sin^{3}\left(a+b\log\left(cx^{n}\right)\right)}{9b^{4}n^{4}x+10b^{2}n^{2}x+x}+\frac{9b^{3}n^{3}\sin\left(a+b\log\left(cx^{n}\right)\right)\cos^{2}\left(a+b\log\left(cx^{n}\right)\right)}{9b^{4}n^{4}x+10b^{2}n^{2}x+x}-\frac{6b^{2}n^{2}\sin^{2}\left(a+b\log\left(cx^{n}\right)\right)\cos\left(a+b\log\left(cx^{n}\right)\right)}{9b^{4}n^{4}x+10b^{2}n^{2}x+x}-\frac{7b^{2}n^{2}\cos^{3}\left(a+b\log\left(cx^{n}\right)\right)}{9b^{4}n^{4}x+10b^{2}n^{2}x+x}+\frac{3bn\sin\left(a+b\log\left(cx^{n}\right)\right)\cos^{2}\left(a+b\log\left(cx^{n}\right)\right)}{9b^{4}n^{4}x+10b^{2}n^{2}x+x}-\frac{\cos^{3}\left(a+b\log\left(cx^{n}\right)\right)}{9b^{4}n^{4}x+10b^{2}n^{2}x+x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*ln(c*x**n))**3/x**2,x)
```

```
[Out] Piecewise((3*I*sin(a - I*log(c*x**n)/n)/(8*x) + 3*I*sin(3*a - 3*I*log(c*x**
n)/n)/(32*x) + cos(3*a - 3*I*log(c*x**n)/n)/(32*x) + 3*I*log(c*x**n)*sin(a
- I*log(c*x**n)/n)/(8*n*x) + 3*log(c*x**n)*cos(a - I*log(c*x**n)/n)/(8*n*x)
, Eq(b, -I/n)), (-9*I*sin(a - I*log(c*x**n)/(3*n))/(32*x) + I*sin(3*a - I*l
og(c*x**n)/n)/(8*x) - 27*cos(a - I*log(c*x**n)/(3*n))/(32*x) + I*log(c*x**n
)*sin(3*a - I*log(c*x**n)/n)/(8*n*x) + log(c*x**n)*cos(3*a - I*log(c*x**n)/
n)/(8*n*x), Eq(b, -I/(3*n))), (9*I*sin(a + I*log(c*x**n)/(3*n))/(32*x) - I*
sin(3*a + I*log(c*x**n)/n)/(8*x) - 27*cos(a + I*log(c*x**n)/(3*n))/(32*x) -
 I*log(c*x**n)*sin(3*a + I*log(c*x**n)/n)/(8*n*x) + log(c*x**n)*cos(3*a + I
*log(c*x**n)/n)/(8*n*x), Eq(b, I/(3*n))), (-3*I*sin(3*a + 3*I*log(c*x**n)/n
)/(32*x) - 3*cos(a + I*log(c*x**n)/n)/(8*x) + cos(3*a + 3*I*log(c*x**n)/n)/
(32*x) - 3*I*log(c*x**n)*sin(a + I*log(c*x**n)/n)/(8*n*x) + 3*log(c*x**n)*c
os(a + I*log(c*x**n)/n)/(8*n*x), Eq(b, I/n)), (6*b**3*n**3*sin(a + b*log(c*
x**n))**3/(9*b**4*n**4*x + 10*b**2*n**2*x + x) + 9*b**3*n**3*sin(a + b*log(
c*x**n))*cos(a + b*log(c*x**n))**2/(9*b**4*n**4*x + 10*b**2*n**2*x + x) - 6
*b**2*n**2*sin(a + b*log(c*x**n))**2*cos(a + b*log(c*x**n))/(9*b**4*n**4*x
+ 10*b**2*n**2*x + x) - 7*b**2*n**2*cos(a + b*log(c*x**n))**3/(9*b**4*n**4*
x + 10*b**2*n**2*x + x) + 3*b*n*sin(a + b*log(c*x**n))*cos(a + b*log(c*x**n
))**2/(9*b**4*n**4*x + 10*b**2*n**2*x + x) - cos(a + b*log(c*x**n))**3/(9*b
**4*n**4*x + 10*b**2*n**2*x + x), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*log(c*x^n) + a)^3/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*log(c*x^n))^3/x^2,x)
```

```
[Out] int(cos(a + b*log(c*x^n))^3/x^2, x)
```

3.101 $\int \cos^4(a + b\log(cx^n))\,dx$

Optimal. Leaf size=191

$$\frac{24b^4n^4x}{1+20b^2n^2+64b^4n^4}+\frac{12b^2n^2x\cos^2(a+b\log(cx^n))}{1+20b^2n^2+64b^4n^4}+\frac{x\cos^4(a+b\log(cx^n))}{1+16b^2n^2}+\frac{24b^3n^3x\cos(a+b\log(cx^n))}{1+20b^2n^2+6}$$

```
[Out] 24*b^4*n^4*x/(64*b^4*n^4+20*b^2*n^2+1)+12*b^2*n^2*x*cos(a+b*ln(c*x^n))^2/(6
4*b^4*n^4+20*b^2*n^2+1)+x*cos(a+b*ln(c*x^n))^4/(16*b^2*n^2+1)+24*b^3*n^3*x*
cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))/(64*b^4*n^4+20*b^2*n^2+1)+4*b*n*x*cos
(a+b*ln(c*x^n))^3*sin(a+b*ln(c*x^n))/(16*b^2*n^2+1)
```

Rubi [A]
time = 0.03, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4566, 8}

$$\frac{x\cos^4(a+b\log(cx^n))}{16b^2n^2+1}+\frac{4bnx\sin(a+b\log(cx^n))\cos^3(a+b\log(cx^n))}{16b^2n^2+1}+\frac{12b^2n^2x\cos^2(a+b\log(cx^n))}{64b^4n^4+20b^2n^2+1}+\frac{24b^3n^3x\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{64b^4n^4+20b^2n^2+1}+\frac{24b^4n^4x}{64b^4n^4+20b^2n^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (24*b^4*n^4*x)/(1 + 20*b^2*n^2 + 64*b^4*n^4) + (12*b^2*n^2*x*Cos[a + b*Log[
c*x^n]]^2)/(1 + 20*b^2*n^2 + 64*b^4*n^4) + (x*Cos[a + b*Log[c*x^n]]^4)/(1 +
 16*b^2*n^2) + (24*b^3*n^3*x*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/(
1 + 20*b^2*n^2 + 64*b^4*n^4) + (4*b*n*x*Cos[a + b*Log[c*x^n]]^3*Sin[a + b*L
og[c*x^n]])/(1 + 16*b^2*n^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4566

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Sim
p[x*(Cos[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 + 1)), x] + (Dist[b^2*d^2
*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + 1)), Int[Cos[d*(a + b*Log[c*x^n])]^(p -
2), x], x] + Simp[b*d*n*p*x*Cos[d*(a + b*Log[c*x^n])]^(p - 1)*(Sin[d*(a + b
*Log[c*x^n])]/(b^2*d^2*n^2*p^2 + 1)), x]) /; FreeQ[{a, b, c, d, n}, x] && I
GtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + 1, 0]
```

Rubi steps

$$\int \cos^4(a+b\log(cx^n))\,dx = \frac{x\cos^4(a+b\log(cx^n))}{1+16b^2n^2} + \frac{4bnx\cos^3(a+b\log(cx^n))\sin(a+b\log(cx^n))}{1+16b^2n^2} + \frac{(12\cdots}{\cdots}$$
$$= \frac{12b^2n^2x\cos^2(a+b\log(cx^n))}{1+20b^2n^2+64b^4n^4} + \frac{x\cos^4(a+b\log(cx^n))}{1+16b^2n^2} + \frac{24b^3n^3x\cos(a+b\log\cdots}{1+20\cdots}$$
$$= \frac{24b^4n^4x}{1+20b^2n^2+64b^4n^4} + \frac{12b^2n^2x\cos^2(a+b\log(cx^n))}{1+20b^2n^2+64b^4n^4} + \frac{x\cos^4(a+b\log(cx^n))}{1+16b^2n^2}$$

Mathematica [A]
time = 0.46, size = 167, normalized size = 0.87

$$\frac{x(3+60b^2n^2+192b^4n^4+(4+64b^2n^2)\cos(2(a+b\log(cx^n)))+(1+4b^2n^2)\cos(4(a+b\log(cx^n)))+8bn\sin(2(a+b\log(cx^n)))+128b^3n^3\sin(2(a+b\log(cx^n)))+4bn\sin(4(a+b\log(cx^n)))+16b^3n^3\sin(4(a+b\log(cx^n))))}{8(1+20b^2n^2+64b^4n^4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x*(3 + 60*b^2*n^2 + 192*b^4*n^4 + (4 + 64*b^2*n^2)*Cos[2*(a + b*Log[c*x^n]
)] + (1 + 4*b^2*n^2)*Cos[4*(a + b*Log[c*x^n])] + 8*b*n*Sin[2*(a + b*Log[c*x
^n])] + 128*b^3*n^3*Sin[2*(a + b*Log[c*x^n])] + 4*b*n*Sin[4*(a + b*Log[c*x^
n])] + 16*b^3*n^3*Sin[4*(a + b*Log[c*x^n])]))/(8*(1 + 20*b^2*n^2 + 64*b^4*n
^4))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \cos^4(a+b\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*ln(c*x^n))^4,x)
```

```
[Out] int(cos(a+b*ln(c*x^n))^4,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1078 vs. $2(191) = 382$.
time = 0.32, size = 1078, normalized size = 5.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

```
[Out] 1/16*((16*(b^3*cos(4*b*log(c))*sin(8*b*log(c)) - b^3*cos(8*b*log(c))*sin(4*
b*log(c)) + b^3*sin(4*b*log(c)))*n^3 + 4*(b^2*cos(8*b*log(c))*cos(4*b*log(c
```

```
)) + b^2*sin(8*b*log(c))*sin(4*b*log(c)) + b^2*cos(4*b*log(c)))*n^2 + 4*(b*
cos(4*b*log(c))*sin(8*b*log(c)) - b*cos(8*b*log(c))*sin(4*b*log(c)) + b*sin
(4*b*log(c)))*n + cos(8*b*log(c))*cos(4*b*log(c)) + sin(8*b*log(c))*sin(4*b
*log(c)) + cos(4*b*log(c)))*x*cos(4*b*log(x^n) + 4*a) + 4*(32*(b^3*cos(4*b*
log(c))*sin(6*b*log(c)) - b^3*cos(6*b*log(c))*sin(4*b*log(c)) + b^3*cos(2*b
*log(c))*sin(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(2*b*log(c)))*n^3 + 16*(b
^2*cos(6*b*log(c))*cos(4*b*log(c)) + b^2*cos(4*b*log(c))*cos(2*b*log(c)) +
b^2*sin(6*b*log(c))*sin(4*b*log(c)) + b^2*sin(4*b*log(c))*sin(2*b*log(c)))*
n^2 + 2*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b*log(
c)) + b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c))
)*n + cos(6*b*log(c))*cos(4*b*log(c)) + cos(4*b*log(c))*cos(2*b*log(c)) + s
in(6*b*log(c))*sin(4*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*x*cos(2*b
*log(x^n) + 2*a) + (16*(b^3*cos(8*b*log(c))*cos(4*b*log(c)) + b^3*sin(8*b*l
og(c))*sin(4*b*log(c)) + b^3*cos(4*b*log(c)))*n^3 - 4*(b^2*cos(4*b*log(c))*
sin(8*b*log(c)) - b^2*cos(8*b*log(c))*sin(4*b*log(c)) + b^2*sin(4*b*log(c))
)*n^2 + 4*(b*cos(8*b*log(c))*cos(4*b*log(c)) + b*sin(8*b*log(c))*sin(4*b*lo
g(c)) + b*cos(4*b*log(c)))*n - cos(4*b*log(c))*sin(8*b*log(c)) + cos(8*b*lo
g(c))*sin(4*b*log(c)) - sin(4*b*log(c)))*x*sin(4*b*log(x^n) + 4*a) + 4*(32*
(b^3*cos(6*b*log(c))*cos(4*b*log(c)) + b^3*cos(4*b*log(c))*cos(2*b*log(c))
+ b^3*sin(6*b*log(c))*sin(4*b*log(c)) + b^3*sin(4*b*log(c))*sin(2*b*log(c))
)*n^3 - 16*(b^2*cos(4*b*log(c))*sin(6*b*log(c)) - b^2*cos(6*b*log(c))*sin(4
*b*log(c)) + b^2*cos(2*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(
2*b*log(c)))*n^2 + 2*(b*cos(6*b*log(c))*cos(4*b*log(c)) + b*cos(4*b*log(c))
*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(4*b*log(c)) + b*sin(4*b*log(c))*si
n(2*b*log(c)))*n - cos(4*b*log(c))*sin(6*b*log(c)) + cos(6*b*log(c))*sin(4*
b*log(c)) - cos(2*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c))*sin(2*b*log(c
)))*x*sin(2*b*log(x^n) + 2*a) + 6*(64*(b^4*cos(4*b*log(c))^2 + b^4*sin(4*b*
log(c))^2)*n^4 + 20*(b^2*cos(4*b*log(c))^2 + b^2*sin(4*b*log(c))^2)*n^2 + c
os(4*b*log(c))^2 + sin(4*b*log(c))^2)*x)/(64*(b^4*cos(4*b*log(c))^2 + b^4*s
in(4*b*log(c))^2)*n^4 + 20*(b^2*cos(4*b*log(c))^2 + b^2*sin(4*b*log(c))^2)*
n^2 + cos(4*b*log(c))^2 + sin(4*b*log(c))^2)
```

Fricas [**A**]
time = 2.58, size = 144, normalized size = 0.75

$$\frac{24\,b^{4}n^{4}x+12\,b^{2}n^{2}x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+\left(4\,b^{2}n^{2}+1\right)x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}+4\,\left(6\,b^{3}n^{3}x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\left(4\,b^{3}n^{3}+bn\right)x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{64\,b^{4}n^{4}+20\,b^{2}n^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] (24*b^4*n^4*x + 12*b^2*n^2*x*cos(b*n*log(x) + b*log(c) + a)^2 + (4*b^2*n^2
+ 1)*x*cos(b*n*log(x) + b*log(c) + a)^4 + 4*(6*b^3*n^3*x*cos(b*n*log(x) + b
*log(c) + a) + (4*b^3*n^3 + b*n)*x*cos(b*n*log(x) + b*log(c) + a)^3)*sin(b*
n*log(x) + b*log(c) + a))/(64*b^4*n^4 + 20*b^2*n^2 + 1)
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \cos^4\left(a - \frac{i\log(cx^n)}{2n}\right) dx & \text{for } b = -\frac{i}{2n} \\ \int \cos^4\left(a - \frac{i\log(cx^n)}{4n}\right) dx & \text{for } b = -\frac{i}{4n} \\ \int \cos^4\left(a + \frac{i\log(cx^n)}{4n}\right) dx & \text{for } b = \frac{i}{4n} \\ \int \cos^4\left(a + \frac{i\log(cx^n)}{2n}\right) dx & \text{for } b = \frac{i}{2n} \\ \frac{24b^4n^4x\sin^4(a+b\log(cx^n))}{64b^4n^4+20b^2n^2+1} + \frac{48b^4n^4x\sin^2(a+b\log(cx^n))\cos^2(a+b\log(cx^n))}{64b^4n^4+20b^2n^2+1} + \frac{24b^4n^4x\cos^4(a+b\log(cx^n))}{64b^4n^4+20b^2n^2+1} + \frac{24b^3n^3x\sin^3(a+b\log(cx^n))\cos(a+b\log(cx^n))}{64b^4n^4+20b^2n^2+1} + \frac{40b^3n^3x\sin(a+b\log(cx^n))\cos^3(a+b\log(cx^n))}{64b^4n^4+20b^2n^2+1} + \frac{12b^2n^2x\sin^2(a+b\log(cx^n))\cos^2(a+b\log(cx^n))}{64b^4n^4+20b^2n^2+1} + \frac{16b^2n^2x\cos^4(a+b\log(cx^n))}{64b^4n^4+20b^2n^2+1} + \frac{4bnx\sin(a+b\log(cx^n))\cos^3(a+b\log(cx^n))}{64b^4n^4+20b^2n^2+1} + \frac{x\cos^4(a+b\log(cx^n))}{64b^4n^4+20b^2n^2+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*ln(c*x**n))**4,x)
```

```
[Out] Piecewise((Integral(cos(a - I*log(c*x**n)/(2*n))**4, x), Eq(b, -I/(2*n))),
(Integral(cos(a - I*log(c*x**n)/(4*n))**4, x), Eq(b, -I/(4*n))), (Integral(
cos(a + I*log(c*x**n)/(4*n))**4, x), Eq(b, I/(4*n))), (Integral(cos(a + I*l
og(c*x**n)/(2*n))**4, x), Eq(b, I/(2*n))), (24*b**4*n**4*x*sin(a + b*log(c*
x**n))**4/(64*b**4*n**4 + 20*b**2*n**2 + 1) + 48*b**4*n**4*x*sin(a + b*log(
c*x**n))**2*cos(a + b*log(c*x**n))**2/(64*b**4*n**4 + 20*b**2*n**2 + 1) + 2
4*b**4*n**4*x*cos(a + b*log(c*x**n))**4/(64*b**4*n**4 + 20*b**2*n**2 + 1) +
 24*b**3*n**3*x*sin(a + b*log(c*x**n))**3*cos(a + b*log(c*x**n))/(64*b**4*n
**4 + 20*b**2*n**2 + 1) + 40*b**3*n**3*x*sin(a + b*log(c*x**n))*cos(a + b*l
og(c*x**n))**3/(64*b**4*n**4 + 20*b**2*n**2 + 1) + 12*b**2*n**2*x*sin(a + b
*log(c*x**n))**2*cos(a + b*log(c*x**n))**2/(64*b**4*n**4 + 20*b**2*n**2 + 1
) + 16*b**2*n**2*x*cos(a + b*log(c*x**n))**4/(64*b**4*n**4 + 20*b**2*n**2 +
 1) + 4*b*n*x*sin(a + b*log(c*x**n))*cos(a + b*log(c*x**n))**3/(64*b**4*n**
4 + 20*b**2*n**2 + 1) + x*cos(a + b*log(c*x**n))**4/(64*b**4*n**4 + 20*b**2
*n**2 + 1), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 16422 vs. $2(191) = 382$.
time = 0.80, size = 16422, normalized size = 85.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] 3/8*x - 1/16*(256*b^3*n^3*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)
*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs
(c)))^2*tan(2*a)^2*tan(a) + 256*b^3*n^3*x*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b
*sgn(c) + pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(
x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a) + 32*b^3*n^3*x*e^(2*pi*b*n*sgn(x)
- 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)
))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)*tan(a)^2 + 32*b^3*n^3*
x*e^(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c) + 2*pi*b)*tan(2*b*n*log(ab
s(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)*
tan(a)^2 + 256*b^3*n^3*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*ta
n(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)
```

))*tan(2*a)^2*tan(a)^2 + 256*b^3*n^3*x*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sg
n(c) + pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x))
 + b*log(abs(c)))*tan(2*a)^2*tan(a)^2 + 32*b^3*n^3*x*e^(2*pi*b*n*sgn(x) - 2
*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))*
tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a)^2 + 32*b^3*n^3*x*e
^(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c) + 2*pi*b)*tan(2*b*n*log(abs(x
)) + 2*b*log(abs(c)))*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan
(a)^2 - 4*b^2*n^2*x*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)
*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs
(c)))^2*tan(2*a)^2*tan(a)^2 - 64*b^2*n^2*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b
*sgn(c) - pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(
x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a)^2 - 64*b^2*n^2*x*e^(-pi*b*n*sgn(x)
 + pi*b*n - pi*b*sgn(c) + pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*
tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a)^2 - 4*b^2*n^2*x*e^
(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c) + 2*pi*b)*tan(2*b*n*log(abs(x)
) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*ta
n(a)^2 + 32*b^3*n^3*x*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*
b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(a
bs(c)))^2*tan(2*a) + 32*b^3*n^3*x*e^(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*s
gn(c) + 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(
x)) + b*log(abs(c)))^2*tan(2*a) - 256*b^3*n^3*x*e^(pi*b*n*sgn(x) - pi*b*n +
 pi*b*sgn(c) - pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log
(abs(x)) + b*log(abs(c)))*tan(2*a)^2 - 256*b^3*n^3*x*e^(-pi*b*n*sgn(x) + pi
*b*n - pi*b*sgn(c) + pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b
*n*log(abs(x)) + b*log(abs(c)))*tan(2*a)^2 + 32*b^3*n^3*x*e^(2*pi*b*n*sgn(x
) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(
c)))*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2 + 32*b^3*n^3*x*e^(-2
*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c) + 2*pi*b)*tan(2*b*n*log(abs(x)) +
 2*b*log(abs(c)))*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2 + 256*b
^3*n^3*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(2*b*n*log(abs(
x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 25
6*b^3*n^3*x*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(2*b*n*log(
abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)
 - 256*b^3*n^3*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(2*b*n*l
og(abs(x)) + 2*b*log(abs(c)))^2*tan(2*a)^2*tan(a) - 256*b^3*n^3*x*e^(-pi*b*
n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs
(c)))^2*tan(2*a)^2*tan(a) + 256*b^3*n^3*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*
sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a) + 2
56*b^3*n^3*x*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(a
bs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a) + 256*b^3*n^3*x*e^(pi*b*n*sgn(x
) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2
*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a)^2 + 256*b^3*n^3*x*e^(-pi*b*n*s
gn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)
))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a)^2 - 32*b^3*n^3*x*e^(2*pi*b
*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*

```
log(abs(c)))*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 - 32*b^3*n^3*x
*e^(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c) + 2*pi*b)*tan(2*b*n*log(abs
(x)) + 2*b*log(abs(c)))*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 + 3
2*b^3*n^3*x*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b
*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(2*a)*tan(a)^2 + 32*b^3*n^3*x*e^(-2*
pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c) + 2*pi*b)*tan(2*b*n*log(abs(x)) +
2*b*log(abs(c)))^2*tan(2*a)*tan(a)^2 - 32*b^3*n^3*x*e^(2*pi*b*n*sgn(x) - 2*
pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan
(2*a)*tan(a)^2 - 32*b^3*n^3*x*e^(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c
) + 2*pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)*tan(a)^2 + 32*b
^3*n^3*x*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b...
```

Mupad [B]
time = 2.82, size = 116, normalized size = 0.61

$$\frac{3\,x}{8}+\frac{x\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,2\mathrm{i}}}\,1\mathrm{i}}{8\,b\,n+4\mathrm{i}}+\frac{x\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}}{4+b\,n\,8\mathrm{i}}+\frac{x\,\mathrm{e}^{-a\,4\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,4\mathrm{i}}}\,1\mathrm{i}}{64\,b\,n+16\mathrm{i}}+\frac{x\,\mathrm{e}^{a\,4\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,4\mathrm{i}}}{16+b\,n\,64\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*log(c*x^n))^4,x)
```

```
[Out] (3*x)/8 + (x*exp(-a*2i)/(c*x^n)^(b*2i)*1i)/(8*b*n + 4i) + (x*exp(a*2i)*(c*x
^n)^(b*2i))/(b*n*8i + 4) + (x*exp(-a*4i)/(c*x^n)^(b*4i)*1i)/(64*b*n + 16i)
+ (x*exp(a*4i)*(c*x^n)^(b*4i))/(b*n*64i + 16)
```

3.102 $\int \frac{\cos^4(a+b\log(cx^n))}{x} \, dx$

Optimal. Leaf size=73

$$\frac{3\log(x)}{8}+\frac{3\cos\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\left(cx^n\right)\right)}{8bn}+\frac{\cos^3\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\left(cx^n\right)\right)}{4bn}$$

```
[Out] 3/8*ln(x)+3/8*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))/b/n+1/4*cos(a+b*ln(c*x^
n))^3*sin(a+b*ln(c*x^n))/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = $\{2715, 8\}$

$$\frac{\sin\left(a+b\log\left(cx^n\right)\right)\cos^3\left(a+b\log\left(cx^n\right)\right)}{4bn}+\frac{3\sin\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{8bn}+\frac{3\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] (3*Log[x])/8 + (3*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/(8*b*n) + (C
os[a + b*Log[c*x^n]]^3*Sin[a + b*Log[c*x^n]])/(4*b*n)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^4\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \cos^4(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}\\
&= \frac{\cos^3\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\left(cx^n\right)\right)}{4bn}+\frac{3\text{Subst}\left(\int \cos^2(a+bx)\,dx, x, \log\left(\right.\right.}{4n}\\
&= \frac{3\cos\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\left(cx^n\right)\right)}{8bn}+\frac{\cos^3\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\right.}{4bn}\\
&= \frac{3\log(x)}{8}+\frac{3\cos\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\left(cx^n\right)\right)}{8bn}+\frac{\cos^3\left(a+b\log\left(cx^n\right)\right)}{4bn}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 51, normalized size = 0.70

$$\frac{12(a+b\log(cx^n))+8\sin(2(a+b\log(cx^n)))+\sin(4(a+b\log(cx^n)))}{32bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] (12*(a + b*Log[c*x^n]) + 8*Sin[2*(a + b*Log[c*x^n])] + Sin[4*(a + b*Log[c*x
^n])])/(32*b*n)
```

Maple [A]
time = 0.06, size = 61, normalized size = 0.84

method	result	size
derivativedivides	$\frac{\frac{\left(\cos^3(a+b\ln(c\,x^n))+\frac{3\cos(a+b\ln(c\,x^n))}{2}\right)\sin(a+b\ln(c\,x^n))}{4}+\frac{3b\ln(c\,x^n)}{8}+\frac{3a}{8}}{nb}$	61
default	$\frac{\frac{\left(\cos^3(a+b\ln(c\,x^n))+\frac{3\cos(a+b\ln(c\,x^n))}{2}\right)\sin(a+b\ln(c\,x^n))}{4}+\frac{3b\ln(c\,x^n)}{8}+\frac{3a}{8}}{nb}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*ln(c*x^n))^4/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(1/4*(cos(a+b*ln(c*x^n))^3+3/2*cos(a+b*ln(c*x^n)))*sin(a+b*ln(c*x^n))
+3/8*b*ln(c*x^n)+3/8*a)
```

Maxima [A]
time = 0.30, size = 93, normalized size = 1.27

$$\frac{12\,bn\log(x)+\cos(4\,b\log(x^n)+4\,a)\sin(4\,b\log(c))+8\,\cos(2\,b\log(x^n)+2\,a)\sin(2\,b\log(c))+\cos(4\,b\log(c))\sin(4\,b\log(x^n)+4\,a)+8\,\cos(2\,b\log(c))\sin(2\,b\log(x^n)+2\,a)}{32\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

```
[Out] 1/32*(12*b*n*log(x) + cos(4*b*log(x^n) + 4*a)*sin(4*b*log(c)) + 8*cos(2*b*l
og(x^n) + 2*a)*sin(2*b*log(c)) + cos(4*b*log(c))*sin(4*b*log(x^n) + 4*a) +
8*cos(2*b*log(c))*sin(2*b*log(x^n) + 2*a))/(b*n)
```

Fricas [A]
time = 2.36, size = 59, normalized size = 0.81

$$\frac{3\,bn\log(x)+\left(2\,\cos(bn\log(x)+b\log(c)+a)^3+3\,\cos(bn\log(x)+b\log(c)+a)\right)\sin(bn\log(x)+b\log(c)+a)}{8\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

```
[Out] 1/8*(3*b*n*log(x) + (2*cos(b*n*log(x) + b*log(c) + a)^3 + 3*cos(b*n*log(x)
+ b*log(c) + a))*sin(b*n*log(x) + b*log(c) + a))/(b*n)
```

Sympy [A]
time = 10.52, size = 100, normalized size = 1.37

$$\frac{\begin{cases} \log(x)\cos(2a) & \text{for } b=0 \wedge (b=0 \vee n=0) \\ \log(x)\cos(2a+2b\log(c)) & \text{for } n=0 \\ \frac{\sin(2a+2b\log(cx^n))}{2bn} & \text{otherwise} \end{cases}}{2} + \frac{\begin{cases} \log(x)\cos(4a) & \text{for } b=0 \wedge (b=0 \vee n=0) \\ \log(x)\cos(4a+4b\log(c)) & \text{for } n=0 \\ \frac{\sin(4a+4b\log(cx^n))}{4bn} & \text{otherwise} \end{cases}}{8} + \frac{3\log(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Piecewise((log(x)*cos(2*a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*cos(
2*a + 2*b*log(c)), Eq(n, 0)), (sin(2*a + 2*b*log(c*x**n))/(2*b*n), True))/2
 + Piecewise((log(x)*cos(4*a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*c
os(4*a + 4*b*log(c)), Eq(n, 0)), (sin(4*a + 4*b*log(c*x**n))/(4*b*n), True)
)/8 + 3*log(x)/8
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] integrate(cos(b*log(c*x^n) + a)^4/x, x)
```

Mupad [B]
time = 2.55, size = 50, normalized size = 0.68

$$\frac{3\,\ln\left(x^n\right)}{8\,n}+\frac{\frac{\sin\left(2\,a+2\,b\,\ln\left(c\,x^n\right)\right)}{4}+\frac{\sin\left(4\,a+4\,b\,\ln\left(c\,x^n\right)\right)}{32}}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*log(c*x^n))^4/x,x)
```

```
[Out] (3*log(x^n))/(8*n) + (sin(2*a + 2*b*log(c*x^n))/4 + sin(4*a + 4*b*log(c*x^n
))/32)/(b*n)
```

3.103 $\int \cos(\log(6+3x))\,dx$

Optimal. Leaf size=29

$$\frac{1}{2}(2+x)\cos(\log(3(2+x)))+\frac{1}{2}(2+x)\sin(\log(3(2+x)))$$

```
[Out] 1/2*(2+x)*cos(ln(6+3*x))+1/2*(2+x)*sin(ln(6+3*x))
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4564}

$$\frac{1}{2}(x+2)\sin(\log(3(x+2)))+\frac{1}{2}(x+2)\cos(\log(3(x+2)))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Log[6 + 3*x]],x]
```

```
[Out] ((2 + x)*Cos[Log[3*(2 + x)]])/2 + ((2 + x)*Sin[Log[3*(2 + x)]])/2
```

Rule 4564

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*(
Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] + Simp[b*d*n*x*(Sin[d*(a +
 b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(\log(6+3x))\,dx &= \frac{1}{3}\text{Subst}\left(\int \cos(\log(x))\,dx, x, 6+3x\right)\\ &= \frac{1}{2}(2+x)\cos(\log(3(2+x)))+\frac{1}{2}(2+x)\sin(\log(3(2+x)))\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.76

$$\frac{1}{2}(2+x)(\cos(\log(3(2+x)))+\sin(\log(3(2+x))))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Log[6 + 3*x]],x]
```

```
[Out] ((2 + x)*(Cos[Log[3*(2 + x)]] + Sin[Log[3*(2 + x)]]))/2
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 34, normalized size = 1.17

method	result	size
risch	$\left(\frac{1}{4}-\frac{i}{4}\right)(2+x)(6+3x)^{i}+\left(\frac{1}{4}+\frac{i}{4}\right)(2+x)(6+3x)^{-i}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(ln(6+3*x)),x,method=_RETURNVERBOSE)
[Out] (1/4-1/4*I)*(2+x)*(6+3*x)^I+(1/4+1/4*I)*(2+x)/((6+3*x)^I)
```

Maxima [A]
time = 0.27, size = 20, normalized size = 0.69

$$\frac{1}{2}(x+2)(\cos(\log(3x+6))+\sin(\log(3x+6)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(6+3*x)),x, algorithm="maxima")
[Out] 1/2*(x + 2)*(cos(log(3*x + 6)) + sin(log(3*x + 6)))
```

Fricas [A]
time = 1.96, size = 25, normalized size = 0.86

$$\frac{1}{2}(x+2)\cos(\log(3x+6))+\frac{1}{2}(x+2)\sin(\log(3x+6))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(6+3*x)),x, algorithm="fricas")
[Out] 1/2*(x + 2)*cos(log(3*x + 6)) + 1/2*(x + 2)*sin(log(3*x + 6))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\cos(\log(3x+6))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(ln(6+3*x)),x)
[Out] Integral(cos(log(3*x + 6)), x)
```

Giac [A]
time = 0.41, size = 25, normalized size = 0.86

$$\frac{1}{2}\left(x+2\right)\cos\left(\log\left(3\,x+6\right)\right)+\frac{1}{2}\left(x+2\right)\sin\left(\log\left(3\,x+6\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(6+3*x)),x, algorithm="giac")
```

```
[Out] 1/2*(x + 2)*cos(log(3*x + 6)) + 1/2*(x + 2)*sin(log(3*x + 6))
```

Mupad [B]
time = 2.17, size = 21, normalized size = 0.72

$$\frac{\sqrt{2}\,\sin\left(\frac{\pi}{4}+\ln\left(3\,x+6\right)\right)\,\left(3\,x+6\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(log(3*x + 6)),x)
```

```
[Out] (2^(1/2)*sin(pi/4 + log(3*x + 6))*(3*x + 6))/6
```

3.104 $\int x^m \cos\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)dx$

Optimal. Leaf size=101

$$\frac{e^{\frac{a(1+m)}{\sqrt{-\frac{(1+m)^2}{n^2}}\,n}}x^{1+m}(cx^n)^{\frac{1+m}{n}}}{4(1+m)}+\frac{1}{2}e^{\frac{a\sqrt{-\frac{(1+m)^2}{n^2}}\,n}{1+m}}x^{1+m}(cx^n)^{-\frac{1+m}{n}}\log(x)$$

[Out] 1/4*exp(a*(1+m)/n/(-(1+m)^2/n^2)^(1/2))*x^(1+m)*(c*x^n)^((1+m)/n)/(1+m)+1/2*exp(a*n*(-(1+m)^2/n^2)^(1/2)/(1+m))*x^(1+m)*ln(x)/((c*x^n)^((1+m)/n))

Rubi [A]
time = 0.11, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}=0.071$, Rules used = {4582, 4578}

$$\frac{x^{m+1}e^{\frac{a(m+1)}{n\sqrt{-\frac{(m+1)^2}{n^2}}}}(cx^n)^{\frac{m+1}{n}}}{4(m+1)}+\frac{1}{2}x^{m+1}\log(x)e^{\frac{an\sqrt{-\frac{(m+1)^2}{n^2}}}{m+1}}(cx^n)^{-\frac{m+1}{n}}$$

Antiderivative was successfully verified.

[In] Int[x^m*Cos[a + Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n]],x]

[Out] (E^((a*(1 + m))/(Sqrt[-((1 + m)^2/n^2)]*n))*x^(1 + m)*(c*x^n)^((1 + m)/n))/(4*(1 + m)) + (E^((a*Sqrt[-((1 + m)^2/n^2)]*n)/(1 + m))*x^(1 + m)*Log[x])/(2*(c*x^n)^((1 + m)/n))

Rule 4578

```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[1/2^p, Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^2*(p/(m + 1)))/x^((m +
 1)/p) + x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x], x], x] /; FreeQ[{a,
b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m + 1)^2, 0]
```

Rule 4582

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Cos[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^m \cos\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)dx = \frac{\left(x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{n}}\cos\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\right.\right.}{n}$$

$$= \frac{\left(x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int\left(\frac{e^{\frac{a\sqrt{-\frac{(1+m)^2}{n^2}}n}{1+m}}}{x}+e^{\frac{a(1+m)}{\sqrt{-\frac{(1+m)^2}{n^2}}n}}x\right.\right.}{2n}$$

$$= \frac{e^{\frac{a(1+m)}{\sqrt{-\frac{(1+m)^2}{n^2}}n}}x^{1+m}(cx^n)^{\frac{1+m}{n}}}{4(1+m)}+\frac{1}{2}e^{\frac{a\sqrt{-\frac{(1+m)^2}{n^2}}n}{1+m}}x^{1+m}(cx^n)^{-\frac{1+m}{n}}\text{l}$$

Mathematica [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int x^m \cos\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*Cos[a + Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n]],x]
[Out] Integrate[x^m*Cos[a + Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n]], x]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^m \cos\left(a+\ln(c\,x^n)\sqrt{-\frac{(1+m)^2}{n^2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a+ln(c*x^n)*(-(1+m)^2/n^2)^(1/2)),x)
[Out] int(x^m*cos(a+ln(c*x^n)*(-(1+m)^2/n^2)^(1/2)),x)
```

Maxima [A]
time = 0.31, size = 82, normalized size = 0.81

$$\frac{c^{\frac{2m}{n}+\frac{2}{n}}x\cos(a)\,e^{\left(m\log(x)+\frac{m\log(x^n)}{n}+\frac{\log(x^n)}{n}\right)}+2\,(m\cos(a)+\cos(a))\log(x)}{4\left(c^{\frac{m}{n}+\frac{1}{n}}m+c^{\frac{m}{n}+\frac{1}{n}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+log(c*x^n)*(-(1+m)^2/n^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*(c^(2*m/n + 2/n)*x*cos(a)*e^(m*log(x) + m*log(x^n)/n + log(x^n)/n) + 2*
(m*cos(a) + cos(a))*log(x))/(c^(m/n + 1/n)*m + c^(m/n + 1/n))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.44, size = 60, normalized size = 0.59

$$\frac{\left(x^2x^{2\,m}+2\,(m+1)e^{\left(\frac{2\,(i\,an-(m+1)\log(c))}{n}\right)}\log\left(x\right)\right)e^{\left(-\frac{i\,an-(m+1)\log(c)}{n}\right)}}{4\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+log(c*x^n)*(-(1+m)^2/n^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(x^2*x^(2*m) + 2*(m + 1)*e^(2*(I*a*n - (m + 1)*log(c))/n)*log(x))*e^(-(
I*a*n - (m + 1)*log(c))/n)/(m + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\cos\left(a+\sqrt{-\frac{m^2}{n^2}-\frac{2m}{n^2}-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cos(a+ln(c*x**n)*(-(1+m)**2/n**2)**(1/2)),x)
```

```
[Out] Integral(x**m*cos(a + sqrt(-m**2/n**2 - 2*m/n**2 - 1/n**2)*log(c*x**n)), x)
```

Giac [C] Result contains complex when optimal does not.
time = 1.04, size = 267, normalized size = 2.64

$$\frac{mn^2xx^me^{\left(i\,a-\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}+mn^2xx^me^{\left(-i\,a+\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}+n^2xx^me^{\left(i\,a-\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}+nxx^m|mn+n|e^{\left(i\,a-\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}+n^2xx^me^{\left(-i\,a+\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}-nxx^m|mn+n|e^{\left(-i\,a+\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}}{2\left(m^2n^2+2\,mn^2-(mn+n)^2+n^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+log(c*x^n)*(-(1+m)^2/n^2)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*(m*n^2*x*x^m*e^(I*a - (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2
) + m*n^2*x*x^m*e^(-I*a + (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2
) + n^2*x*x^m*e^(I*a - (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) +
 n*x*x^m*abs(m*n + n)*e^(I*a - (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)
)/n^2) + n^2*x*x^m*e^(-I*a + (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/
n^2) - n*x*x^m*abs(m*n + n)*e^(-I*a + (n*abs(m*n + n)*log(x) + abs(m*n + n)
*log(c))/n^2))/(m^2*n^2 + 2*m*n^2 - (m*n + n)^2 + n^2)
```

Mupad [B]
time = 3.78, size = 131, normalized size = 1.30

$$\frac{x\,x^m\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,1\mathrm{i}}}}{2\,m+2-n\,\sqrt{-\frac{{\left(m+1\right)}^2}{n^2}}\,2\mathrm{i}}+\frac{x\,x^m\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,1\mathrm{i}}}{2\,m+2+n\,\sqrt{-\frac{{\left(m+1\right)}^2}{n^2}}\,2\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a + log(c*x^n)*(-(m + 1)^2/n^2)^(1/2)),x)
```

```
[Out] (x*x^m*exp(-a*1i)/(c*x^n)^((- (2*m)/n^2 - 1/n^2 - m^2/n^2)^(1/2)*1i))/(2*m
- n*(-(m + 1)^2/n^2)^(1/2)*2i + 2) + (x*x^m*exp(a*1i)*(c*x^n)^((- (2*m)/n^2
 - 1/n^2 - m^2/n^2)^(1/2)*1i))/(2*m + n*(-(m + 1)^2/n^2)^(1/2)*2i + 2)
```

3.105 $\int \cos\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=62

$$\frac{1}{4}e^{-a\sqrt{-\frac{1}{n^2}}n}x(cx^n)^{\frac{1}{n}}+\frac{1}{2}e^{a\sqrt{-\frac{1}{n^2}}n}x(cx^n)^{-1/n}\log(x)$$

```
[Out] 1/4*x*(c*x^n)^(1/n)/exp(a*n*(-1/n^2)^(1/2))+1/2*exp(a*n*(-1/n^2)^(1/2))*x*l
n(x)/((c*x^n)^(1/n))
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {4572, 4578}

$$\frac{1}{4}xe^{-a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{1}{n}}+\frac{1}{2}xe^{a\sqrt{-\frac{1}{n^2}}n}\log(x)\left(cx^n\right)^{-1/n}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + Sqrt[-n^(-2)]*Log[c*x^n]],x]
```

```
[Out] (x*(c*x^n)^n^(-1))/(4*E^(a*Sqrt[-n^(-2)]*n)) + (E^(a*Sqrt[-n^(-2)]*n)*x*Log
[x])/(2*(c*x^n)^n^(-1))
```

Rule 4572

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Cos[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4578

```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[1/2^p, Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^2*(p/(m + 1)))/x^((m +
 1)/p) + x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x], x], x] /; FreeQ[{a,
b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int \cos\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\cos\left(a+\sqrt{-\frac{1}{n^2}}\log(x)\right)dx,x,cx^n\right)}{n}$$
$$= \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int\left(\frac{e^{a\sqrt{-\frac{1}{n^2}}n}}{x}+e^{-a\sqrt{-\frac{1}{n^2}}n}x^{-1+\frac{2}{n}}\right)dx,x,cx^n\right)}{2n}$$
$$= \frac{1}{4}e^{-a\sqrt{-\frac{1}{n^2}}n}x(cx^n)^{\frac{1}{n}}+\frac{1}{2}e^{a\sqrt{-\frac{1}{n^2}}n}x(cx^n)^{-1/n}\log(x)$$

Mathematica [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \cos\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[a + Sqrt[-n^(-2)]*Log[c*x^n]],x]
[Out] Integrate[Cos[a + Sqrt[-n^(-2)]*Log[c*x^n]], x]
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \cos\left(a+\ln\left(c\,x^n\right)\sqrt{-\frac{1}{n^2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+ln(c*x^n)*(-1/n^2)^(1/2)),x)
[Out] int(cos(a+ln(c*x^n)*(-1/n^2)^(1/2)),x)
```

Maxima [A]
time = 0.30, size = 29, normalized size = 0.47

$$\frac{c^{\frac{2}{n}}x^2\cos(a)+2\cos(a)\log(x)}{4\,c^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+log(c*x^n)*(-1/n^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*(c^(2/n)*x^2*cos(a) + 2*cos(a)*log(x))/c^(1/n)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.43, size = 40, normalized size = 0.65

$$\frac{1}{4}\left(x^{2}+2\,e^{\left(\frac{2\,(i\,an-\log(c))}{n}\right)}\log\left(x\right)\right)e^{\left(-\frac{i\,an-\log(c)}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+log(c*x^n)*(-1/n^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(x^2 + 2*e^(2*(I*a*n - log(c))/n)*log(x))*e^(-(I*a*n - log(c))/n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\cos\left(a+\sqrt{-\frac{1}{n^{2}}}\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+ln(c*x**n)*(-1/n**2)**(1/2)),x)
```

```
[Out] Integral(cos(a + sqrt(-1/n**2)*log(c*x**n)), x)
```

Giac [A]
time = 0.48, size = 1, normalized size = 0.02

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+log(c*x^n)*(-1/n^2)^(1/2)),x, algorithm="giac")
```

```
[Out] +Infinity
```

Mupad [B]
time = 2.78, size = 83, normalized size = 1.34

$$\frac{x\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}}\,1\mathrm{i}}{2\,n\,\sqrt{-\frac{1}{n^2}}+2\mathrm{i}}-\frac{x\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}\,1\mathrm{i}}{2\,n\,\sqrt{-\frac{1}{n^2}}-2\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + log(c*x^n)*(-1/n^2)^(1/2)),x)
```

```
[Out] (x*exp(-a*1i)/(c*x^n)^((-1/n^2)^(1/2)*1i)*1i)/(2*n*(-1/n^2)^(1/2) + 2i) - (
x*exp(a*1i)*(c*x^n)^((-1/n^2)^(1/2)*1i)*1i)/(2*n*(-1/n^2)^(1/2) - 2i)
```

3.106 $\int x^m \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=117

$$\frac{x^{1+m}}{2(1+m)}+\frac{e^{-\frac{2a\sqrt{-\frac{(1+m)^2}{n^2}}\,n}{1+m}}x^{1+m}(cx^n)^{\frac{1+m}{n}}}{8(1+m)}+\frac{1}{4}e^{\frac{2a\sqrt{-\frac{(1+m)^2}{n^2}}\,n}{1+m}}x^{1+m}(cx^n)^{-\frac{1+m}{n}}\log(x)$$

```
[Out] 1/2*x^(1+m)/(1+m)+1/8*x^(1+m)*(c*x^n)^((1+m)/n)/exp(2*a*n*(-(1+m)^2/n^2)^(1
/2)/(1+m))/(1+m)+1/4*exp(2*a*n*(-(1+m)^2/n^2)^(1/2)/(1+m))*x^(1+m)*ln(x)/((
c*x^n)^((1+m)/n))
```

Rubi [A]
time = 0.09, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {4582, 4578}

$$\frac{x^{m+1}e^{-\frac{2an\sqrt{-\frac{(m+1)^2}{n^2}}}{m+1}}(cx^n)^{\frac{m+1}{n}}}{8(m+1)}+\frac{1}{4}x^{m+1}\log(x)e^{\frac{2an\sqrt{-\frac{(m+1)^2}{n^2}}}{m+1}}(cx^n)^{-\frac{m+1}{n}}+\frac{x^{m+1}}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cos[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2]^2,x]
```

```
[Out] x^(1 + m)/(2*(1 + m)) + (x^(1 + m)*(c*x^n)^((1 + m)/n))/(8*E^((2*a*Sqrt[-((
1 + m)^2/n^2)]*n)/(1 + m))*(1 + m)) + (E^((2*a*Sqrt[-((1 + m)^2/n^2)]*n)/(1
 + m))*x^(1 + m)*Log[x])/(4*(c*x^n)^((1 + m)/n))
```

Rule 4578

```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[1/2^p, Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^2*(p/(m + 1)))/x^((m +
 1)/p) + x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x], x], x] /; FreeQ[{a,
b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m + 1)^2, 0]
```

Rule 4582

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Cos[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^m \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)dx = \frac{\left(x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{n}}\cos^2\left(a+\frac{1}{2}\sqrt{-\frac{(1}{}}\right.\right.}{n}$$

$$= \frac{\left(x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int\left(\frac{e^{\frac{2a\sqrt{-\frac{(1+m)^2}{n^2}}\,n}{1+m}}}{x}+2x^{-1+\frac{1+m}{n}}\right.\right.}{4n}$$

$$= \frac{x^{1+m}}{2(1+m)}+\frac{e^{-\frac{2a\sqrt{-\frac{(1+m)^2}{n^2}}\,n}{1+m}}x^{1+m}(cx^n)^{\frac{1+m}{n}}}{8(1+m)}+\frac{1}{4}e^{\frac{2a\sqrt{-\frac{(1+m}{n^2}}}{1+m}}$$

Mathematica [F]
time = 0.39, size = 0, normalized size = 0.00

$$\int x^m \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*Cos[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2]^2,x]
[Out] Integrate[x^m*Cos[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2]^2, x]
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int x^m\left(\cos^2\left(a+\frac{\ln\left(c\,x^n\right)\sqrt{-\frac{(1+m)^2}{n^2}}}{2}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a+1/2*ln(c*x^n)*(-(1+m)^2/n^2)^(1/2))^2,x)
[Out] int(x^m*cos(a+1/2*ln(c*x^n)*(-(1+m)^2/n^2)^(1/2))^2,x)
```

Maxima [A]
time = 0.33, size = 172, normalized size = 1.47

$$\frac{4\left(\cos(2a)^2+\sin(2a)^2\right)c^{\frac{m}{n}+\frac{1}{n}}xx^m+c^{\frac{2m}{n}+\frac{2}{n}}x\cos(2a)e^{\left(m\log(x)+\frac{m\log(x^n)}{n}+\frac{\log(x^n)}{n}\right)}+2\left(\cos(2a)^3+\cos(2a)\sin(2a)^2+\left(\cos(2a)^3+\cos(2a)\sin(2a)^2\right)m\right)\log(x)}{8\left(\left(\cos(2a)^2+\sin(2a)^2\right)c^{\frac{m}{n}+\frac{1}{n}}m+\left(\cos(2a)^2+\sin(2a)^2\right)c^{\frac{m}{n}+\frac{1}{n}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+1/2*log(c*x^n)*(-(1+m)^2/n^2)^(1/2))^2,x, algorithm="ma
xima")
```

```
[Out] 1/8*(4*(cos(2*a)^2 + sin(2*a)^2)*c^(m/n + 1/n)*x*x^m + c^(2*m/n + 2/n)*x*co
s(2*a)*e^(m*log(x) + m*log(x^n)/n + log(x^n)/n) + 2*(cos(2*a)^3 + cos(2*a)*
sin(2*a)^2 + (cos(2*a)^3 + cos(2*a)*sin(2*a)^2)*m)*log(x))/((cos(2*a)^2 + s
in(2*a)^2)*c^(m/n + 1/n)*m + (cos(2*a)^2 + sin(2*a)^2)*c^(m/n + 1/n))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.20, size = 107, normalized size = 0.91

$$\frac{\left(2\,(m+1)e^{\left(-\frac{2\,((m+1)n\log(x)-2i\,an+(m+1)\log(c))}{n}\right)}\log(x)+4\,e^{\left(-\frac{(m+1)n\log(x)-2i\,an+(m+1)\log(c)}{n}\right)}+1\right)e^{\left(\frac{2\,((m+1)n\log(x)-2i\,an+(m+1)\log(c))}{n}+\frac{2i\,an-(m+1)\log(c)}{n}\right)}}{8\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+1/2*log(c*x^n)*(-(1+m)^2/n^2)^(1/2))^2,x, algorithm="fr
icas")
```

```
[Out] 1/8*(2*(m + 1)*e^(-2*((m + 1)*n*log(x) - 2*I*a*n + (m + 1)*log(c))/n)*log(x
) + 4*e^(-((m + 1)*n*log(x) - 2*I*a*n + (m + 1)*log(c))/n) + 1)*e^(2*((m +
1)*n*log(x) - 2*I*a*n + (m + 1)*log(c))/n + (2*I*a*n - (m + 1)*log(c))/n)/(
m + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\cos^2\left(a+\frac{\sqrt{-\frac{m^2}{n^2}-\frac{2m}{n^2}-\frac{1}{n^2}}\,\log(cx^n)}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cos(a+1/2*ln(c*x**n)*(-(1+m)**2/n**2)**(1/2))**2,x)
```

```
[Out] Integral(x**m*cos(a + sqrt(-m**2/n**2 - 2*m/n**2 - 1/n**2)*log(c*x**n)/2)**
2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 3.40, size = 498, normalized size = 4.26

$$\frac{m^2n^2xx^me^{\left(2ia-\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}+m^2n^2xx^me^{\left(-2ia+\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}+2m^2n^2xx^m+2mn^2xx^me^{\left(2ia-\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}+mnxx^m|mn+n|e^{\left(2ia-\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}+2mn^2xx^me^{\left(-2ia+\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}-mnxx^m|mn+n|e^{\left(-2ia+\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}+4mn^2xx^m+n^2xx^me^{\left(2ia-\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}+nxx^m|mn+n|e^{\left(2ia-\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}+n^2xx^me^{\left(-2ia+\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}-nxx^m|mn+n|e^{\left(-2ia+\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}-2\,(mn+n)^2xx^m+2\,n^2xx^m}{4\left(m^3n^2+3\,m^2n^2-(mn+n)^2m+3\,mn^2-(mn+n)^2+n^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+1/2*log(c*x^n)*(-(1+m)^2/n^2)^(1/2))^2,x, algorithm="gi
ac")
```

```
[Out] 1/4*(m^2*n^2*x*x^m*e^(2*I*a - (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))
/n^2) + m^2*n^2*x*x^m*e^(-2*I*a + (n*abs(m*n + n)*log(x) + abs(m*n + n)*log
(c))/n^2) + 2*m^2*n^2*x*x^m + 2*m*n^2*x*x^m*e^(2*I*a - (n*abs(m*n + n)*log(
x) + abs(m*n + n)*log(c))/n^2) + m*n*x*x^m*abs(m*n + n)*e^(2*I*a - (n*abs(m
*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 2*m*n^2*x*x^m*e^(-2*I*a + (n*a
bs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - m*n*x*x^m*abs(m*n + n)*e^(
-2*I*a + (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 4*m*n^2*x*x^m
 + n^2*x*x^m*e^(2*I*a - (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)
+ n*x*x^m*abs(m*n + n)*e^(2*I*a - (n*abs(m*n + n)*log(x) + abs(m*n + n)*log
(c))/n^2) + n^2*x*x^m*e^(-2*I*a + (n*abs(m*n + n)*log(x) + abs(m*n + n)*log
(c))/n^2) - n*x*x^m*abs(m*n + n)*e^(-2*I*a + (n*abs(m*n + n)*log(x) + abs(m
*n + n)*log(c))/n^2) - 2*(m*n + n)^2*x*x^m + 2*n^2*x*x^m)/(m^3*n^2 + 3*m^2*
n^2 - (m*n + n)^2*m + 3*m*n^2 - (m*n + n)^2 + n^2)
```

Mupad [B]
time = 3.71, size = 143, normalized size = 1.22

$$\frac{x\,x^m}{2\,m+2}+\frac{x\,x^m\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,1\mathrm{i}}}}{4\,m+4-n\,\sqrt{-\frac{{\left(m+1\right)}^2}{n^2}}\,4\mathrm{i}}+\frac{x\,x^m\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,1\mathrm{i}}}{4\,m+4+n\,\sqrt{-\frac{{\left(m+1\right)}^2}{n^2}}\,4\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a + (log(c*x^n)*(-(m + 1)^2/n^2)^(1/2))/2)^2,x)
```

```
[Out] (x*x^m)/(2*m + 2) + (x*x^m*exp(-a*2i)/(c*x^n)^((- (2*m)/n^2 - 1/n^2 - m^2/n
^2)^(1/2)*1i))/(4*m - n*(-(m + 1)^2/n^2)^(1/2)*4i + 4) + (x*x^m*exp(a*2i)*(
c*x^n)^((- (2*m)/n^2 - 1/n^2 - m^2/n^2)^(1/2)*1i))/(4*m + n*(-(m + 1)^2/n^2
)^(1/2)*4i + 4)
```

3.107 $\int \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right) dx$

Optimal. Leaf size=68

$$\frac{x}{2}+\frac{1}{8}e^{-2a\sqrt{-\frac{1}{n^2}}\,n}x(cx^n)^{\frac{1}{n}}+\frac{1}{4}e^{2a\sqrt{-\frac{1}{n^2}}\,n}x(cx^n)^{-1/n}\log(x)$$

```
[Out] 1/2*x+1/8*x*(c*x^n)^(1/n)/exp(2*a*n*(-1/n^2)^(1/2))+1/4*exp(2*a*n*(-1/n^2)^
(1/2))*x*ln(x)/((c*x^n)^(1/n))
```

Rubi [A]
time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4572, 4578}

$$\frac{1}{8}xe^{-2a\sqrt{-\frac{1}{n^2}}\,n}(cx^n)^{\frac{1}{n}}+\frac{1}{4}xe^{2a\sqrt{-\frac{1}{n^2}}\,n}\log(x)\left(cx^n\right)^{-1/n}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + (Sqrt[-n^(-2)]*Log[c*x^n])/2]^2,x]
```

```
[Out] x/2 + (x*(c*x^n)^n^(-1))/(8*E^(2*a*Sqrt[-n^(-2)]*n)) + (E^(2*a*Sqrt[-n^(-2)
]*n)*x*Log[x])/(4*(c*x^n)^n^(-1))
```

Rule 4572

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Cos[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4578

```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[1/2^p, Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^2*(p/(m + 1)))/x^((m +
 1)/p) + x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x], x], x] /; FreeQ[{a,
b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\cos^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log(x)\right)dx, x, c\right.}{n}$$
$$= \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int\left(\frac{e^{2a\sqrt{-\frac{1}{n^2}}n}}{x}+2x^{-1+\frac{1}{n}}+e^{-2a\sqrt{-\frac{1}{n^2}}n}x^{-1+\frac{2}{n}}\right.\right.}{4n}$$
$$= \frac{x}{2}+\frac{1}{8}e^{-2a\sqrt{-\frac{1}{n^2}}n}x(cx^n)^{\frac{1}{n}}+\frac{1}{4}e^{2a\sqrt{-\frac{1}{n^2}}n}x(cx^n)^{-1/n}\log(x)$$

Mathematica [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[a + (Sqrt[-n^(-2)]*Log[c*x^n])/2]^2,x]
[Out] Integrate[Cos[a + (Sqrt[-n^(-2)]*Log[c*x^n])/2]^2, x]
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \cos^2\left(a+\frac{\ln\left(c\,x^n\right)\sqrt{-\frac{1}{n^2}}}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+1/2*ln(c*x^n)*(-1/n^2)^(1/2))^2,x)
[Out] int(cos(a+1/2*ln(c*x^n)*(-1/n^2)^(1/2))^2,x)
```

Maxima [A]
time = 0.31, size = 41, normalized size = 0.60

$$\frac{c^{\frac{2}{n}}x^2\cos\left(2\,a\right)+4\,c^{\left(\frac{1}{n}\right)}x+2\,\cos\left(2\,a\right)\log\left(x\right)}{8\,c^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+1/2*log(c*x^n)*(-1/n^2)^(1/2))^2,x, algorithm="maxima")
[Out] 1/8*(c^(2/n)*x^2*cos(2*a) + 4*c^(1/n)*x + 2*cos(2*a)*log(x))/c^(1/n)
```

Fricas [C] Result contains complex when optimal does not.
time = 3.29, size = 57, normalized size = 0.84

$$\frac{1}{8}\left(x^2+4\,xe^{\left(\frac{2i\,an-\log(c)}{n}\right)}+2\,e^{\left(\frac{2\,(2i\,an-\log(c))}{n}\right)}\log\left(x\right)\right)e^{\left(-\frac{2i\,an-\log(c)}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+1/2*log(c*x^n)*(-1/n^2)^(1/2))^2,x, algorithm="fricas")
[Out] 1/8*(x^2 + 4*x*e^((2*I*a*n - log(c))/n) + 2*e^(2*(2*I*a*n - log(c))/n)*log(
x))*e^(-(2*I*a*n - log(c))/n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\cos^2\left(a+\frac{\sqrt{-\frac{1}{n^2}}\,\log\left(cx^n\right)}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+1/2*ln(c*x**n)*(-1/n**2)**(1/2))**2,x)
[Out] Integral(cos(a + sqrt(-1/n**2)*log(c*x**n)/2)**2, x)
```

Giac [A]
time = 0.68, size = 1, normalized size = 0.01

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+1/2*log(c*x^n)*(-1/n^2)^(1/2))^2,x, algorithm="giac")
[Out] +Infinity
```

Mupad [B]
time = 2.71, size = 86, normalized size = 1.26

$$\frac{x}{2}+\frac{x\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}}\,1\mathrm{i}}{4\,n\,\sqrt{-\frac{1}{n^2}}+4\mathrm{i}}-\frac{x\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}\,1\mathrm{i}}{4\,n\,\sqrt{-\frac{1}{n^2}}-4\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + (log(c*x^n)*(-1/n^2)^(1/2))/2)^2,x)
[Out] x/2 + (x*exp(-a*2i)/(c*x^n)^((-1/n^2)^(1/2)*1i)*1i)/(4*n*(-1/n^2)^(1/2) + 4
i) - (x*exp(a*2i)*(c*x^n)^((-1/n^2)^(1/2)*1i)*1i)/(4*n*(-1/n^2)^(1/2) - 4i)
```

3.108 $\int x^m \cos^3\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=226

$$\frac{8x^{1+m}\cos\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)}{5(1+m)}-\frac{4x^{1+m}\cos^3\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)}{5(1+m)}+\frac{4\sqrt{-\frac{(1+m)^2}{n^2}}}{}$$

```
[Out] 8/5*x^(1+m)*cos(a+1/2*ln(c*x^n)*(-(1+m)^2/n^2)^(1/2))/(1+m)-4/5*x^(1+m)*cos
(a+1/2*ln(c*x^n)*(-(1+m)^2/n^2)^(1/2))^3/(1+m)+4/5*n*x^(1+m)*sin(a+1/2*ln(c
*x^n)*(-(1+m)^2/n^2)^(1/2))*(-(1+m)^2/n^2)^(1/2)/(1+m)^2-6/5*n*x^(1+m)*cos(
a+1/2*ln(c*x^n)*(-(1+m)^2/n^2)^(1/2))^2*sin(a+1/2*ln(c*x^n)*(-(1+m)^2/n^2)^
(1/2))*(-(1+m)^2/n^2)^(1/2)/(1+m)^2
```

Rubi [A]
time = 0.06, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {4576, 4574}

$$\frac{4n\sqrt{-\frac{(m+1)^2}{n^2}}\,x^{m+1}\sin\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)}{5(m+1)^2}-\frac{4x^{m+1}\cos^3\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)}{5(m+1)}+\frac{8x^{m+1}\cos\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)}{5(m+1)}-\frac{6n\sqrt{-\frac{(m+1)^2}{n^2}}\,x^{m+1}\sin\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)\cos^2\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)}{5(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cos[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2]^3,x]
```

```
[Out] (8*x^(1 + m)*Cos[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2])/(5*(1 + m)) -
(4*x^(1 + m)*Cos[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2]^3)/(5*(1 + m))
+ (4*Sqrt[-((1 + m)^2/n^2)]*n*x^(1 + m)*Sin[a + (Sqrt[-((1 + m)^2/n^2)]*Log
[c*x^n])/2])/(5*(1 + m)^2) - (6*Sqrt[-((1 + m)^2/n^2)]*n*x^(1 + m)*Cos[a +
(Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2]^2*Sin[a + (Sqrt[-((1 + m)^2/n^2)]*Lo
g[c*x^n])/2])/(5*(1 + m)^2)
```

Rule 4574

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Rule 4576

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2
```

```
*n^2*p^2 + (m + 1)^2)), Int[(e*x)^m*Cos[d*(a + b*Log[c*x^n])]^(p - 2), x],
x] + Simp[b*d*n*p*(e*x)^(m + 1)*Sin[d*(a + b*Log[c*x^n])]*(Cos[d*(a + b*Log
[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x]) /; FreeQ[{a, b, c
, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int x^m \cos^3\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)dx = -\frac{4x^{1+m}\cos^3\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)}{5(1+m)} - \ldots$$

$$= \frac{8x^{1+m}\cos\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)}{5(1+m)} - \ldots$$

Mathematica [A]
time = 1.56, size = 158, normalized size = 0.70

$$\frac{x^{1+m}\left(10(1+m)\cos\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)-2(1+m)\cos\left(3a+\frac{3}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)+\sqrt{-\frac{(1+m)^2}{n^2}}\,n\left(5\sin\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)-3\sin\left(3a+\frac{3}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\right)\right)}{10(1+m)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cos[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2]^3,x]
```

```
[Out] (x^(1 + m)*(10*(1 + m)*Cos[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2] - 2*(
1 + m)*Cos[3*a + (3*Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2] + Sqrt[-((1 + m)^
2/n^2)]*n*(5*Sin[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2] - 3*Sin[3*a + (
3*Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2])))/(10*(1 + m)^2)
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int x^m\left(\cos^3\left(a+\frac{\ln\left(c\,x^n\right)\sqrt{-\frac{(1+m)^2}{n^2}}}{2}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a+1/2*ln(c*x^n)*(-(1+m)^2/n^2)^(1/2))^3,x)
```

```
[Out] int(x^m*cos(a+1/2*ln(c*x^n)*(-(1+m)^2/n^2)^(1/2))^3,x)
```

Maxima [A]
time = 0.36, size = 195, normalized size = 0.86

$$\frac{\left(c^{\frac{3m}{n}+\frac{3}{n}}x\cos\left(3a\right)e^{\left(m\log\left(x\right)+\frac{3m\log\left(x^{n}\right)}{n}+\frac{3\log\left(x^{n}\right)}{n}\right)}+5c^{\frac{2m}{n}+\frac{2}{n}}x\cos\left(a\right)e^{\left(m\log\left(x\right)+\frac{2m\log\left(x^{n}\right)}{n}+\frac{2\log\left(x^{n}\right)}{n}\right)}+15c^{\frac{m}{n}+\frac{1}{n}}x\cos\left(a\right)e^{\left(m\log\left(x\right)+\frac{m\log\left(x^{n}\right)}{n}+\frac{\log\left(x^{n}\right)}{n}\right)}-5xx^{m}\cos\left(3a\right)\right)e^{\left(-\frac{3m\log\left(x^{n}\right)}{2n}-\frac{3\log\left(x^{n}\right)}{2n}\right)}}{20\left(c^{\frac{3m}{2n}+\frac{3}{2n}}m+c^{\frac{3m}{2n}+\frac{3}{2n}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+1/2*log(c*x^n)*(-(1+m)^2/n^2)^(1/2))^3,x, algorithm="ma
xima")
```

```
[Out] 1/20*(c^(3*m/n + 3/n)*x*cos(3*a)*e^(m*log(x) + 3*m*log(x^n)/n + 3*log(x^n)/
n) + 5*c^(2*m/n + 2/n)*x*cos(a)*e^(m*log(x) + 2*m*log(x^n)/n + 2*log(x^n)/n
) + 15*c^(m/n + 1/n)*x*cos(a)*e^(m*log(x) + m*log(x^n)/n + log(x^n)/n) - 5*
x*x^m*cos(3*a))*e^(-3/2*m*log(x^n)/n - 3/2*log(x^n)/n)/(c^(3/2*m/n + 3/2/n)
*m + c^(3/2*m/n + 3/2/n))
```

Fricas [C] Result contains complex when optimal does not.
time = 2.04, size = 128, normalized size = 0.57

$$\frac{\left(5e^{\left(-\frac{(m+1)n\log\left(x\right)-2ian+(m+1)\log\left(c\right)}{n}\right)}+15e^{\left(-\frac{2\left((m+1)n\log\left(x\right)-2ian+(m+1)\log\left(c\right)\right)}{n}\right)}-5e^{\left(-\frac{3\left((m+1)n\log\left(x\right)-2ian+(m+1)\log\left(c\right)\right)}{n}\right)}+1\right)e^{\left(\frac{5\left((m+1)n\log\left(x\right)-2ian+(m+1)\log\left(c\right)\right)}{2n}+\frac{2ian-(m+1)\log\left(c\right)}{n}\right)}}{20\left(m+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+1/2*log(c*x^n)*(-(1+m)^2/n^2)^(1/2))^3,x, algorithm="fr
icas")
```

```
[Out] 1/20*(5*e^(-((m + 1)*n*log(x) - 2*I*a*n + (m + 1)*log(c))/n) + 15*e^(-2*((m
 + 1)*n*log(x) - 2*I*a*n + (m + 1)*log(c))/n) - 5*e^(-3*((m + 1)*n*log(x) -
 2*I*a*n + (m + 1)*log(c))/n) + 1)*e^(5/2*((m + 1)*n*log(x) - 2*I*a*n + (m
+ 1)*log(c))/n + (2*I*a*n - (m + 1)*log(c))/n)/(m + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m}\cos^{3}\left(a+\frac{\sqrt{-\frac{m^{2}}{n^{2}}-\frac{2m}{n^{2}}-\frac{1}{n^{2}}}\log\left(cx^{n}\right)}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cos(a+1/2*ln(c*x**n)*(-(1+m)**2/n**2)**(1/2))**3,x)
```

```
[Out] Integral(x**m*cos(a + sqrt(-m**2/n**2 - 2*m/n**2 - 1/n**2)*log(c*x**n)/2)**
3, x)
```

Giac [C] Result contains complex when optimal does not.
time = 8.02, size = 1870, normalized size = 8.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+1/2*log(c*x^n)*(-(1+m)^2/n^2)^(1/2))^3,x, algorithm="gi
ac")
```

```
[Out] 1/4*(8*m^3*n^4*x*x^m*e^(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*l
og(c))/n^2) + 24*m^3*n^4*x*x^m*e^(I*a - 1/2*(n*abs(m*n + n)*log(x) + abs(m*
n + n)*log(c))/n^2) + 24*m^3*n^4*x*x^m*e^(-I*a + 1/2*(n*abs(m*n + n)*log(x)
 + abs(m*n + n)*log(c))/n^2) + 8*m^3*n^4*x*x^m*e^(-3*I*a + 3/2*(n*abs(m*n +
 n)*log(x) + abs(m*n + n)*log(c))/n^2) + 24*m^2*n^4*x*x^m*e^(3*I*a - 3/2*(n
*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 12*m^2*n^3*x*x^m*abs(m*n
 + n)*e^(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 7
2*m^2*n^4*x*x^m*e^(I*a - 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/
n^2) + 12*m^2*n^3*x*x^m*abs(m*n + n)*e^(I*a - 1/2*(n*abs(m*n + n)*log(x) +
abs(m*n + n)*log(c))/n^2) + 72*m^2*n^4*x*x^m*e^(-I*a + 1/2*(n*abs(m*n + n)*
log(x) + abs(m*n + n)*log(c))/n^2) - 12*m^2*n^3*x*x^m*abs(m*n + n)*e^(-I*a
+ 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 24*m^2*n^4*x*x^m
*e^(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - 12*m^
2*n^3*x*x^m*abs(m*n + n)*e^(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + abs(m*n +
 n)*log(c))/n^2) - 2*(m*n + n)^2*m*n^2*x*x^m*e^(3*I*a - 3/2*(n*abs(m*n + n)
*log(x) + abs(m*n + n)*log(c))/n^2) + 24*m*n^4*x*x^m*e^(3*I*a - 3/2*(n*abs(
m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 24*m*n^3*x*x^m*abs(m*n + n)*e
^(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - 54*(m*n
+ n)^2*m*n^2*x*x^m*e^(I*a - 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c
))/n^2) + 72*m*n^4*x*x^m*e^(I*a - 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)
*log(c))/n^2) + 24*m*n^3*x*x^m*abs(m*n + n)*e^(I*a - 1/2*(n*abs(m*n + n)*lo
g(x) + abs(m*n + n)*log(c))/n^2) - 54*(m*n + n)^2*m*n^2*x*x^m*e^(-I*a + 1/2
*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 72*m*n^4*x*x^m*e^(-I*
a + 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - 24*m*n^3*x*x^m
*abs(m*n + n)*e^(-I*a + 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n
^2) - 2*(m*n + n)^2*m*n^2*x*x^m*e^(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + ab
s(m*n + n)*log(c))/n^2) + 24*m*n^4*x*x^m*e^(-3*I*a + 3/2*(n*abs(m*n + n)*lo
g(x) + abs(m*n + n)*log(c))/n^2) - 24*m*n^3*x*x^m*abs(m*n + n)*e^(-3*I*a +
3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - 2*(m*n + n)^2*n^2*
x*x^m*e^(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 8
*n^4*x*x^m*e^(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2
) - 3*(m*n + n)^2*n*x*x^m*abs(m*n + n)*e^(3*I*a - 3/2*(n*abs(m*n + n)*log(x
) + abs(m*n + n)*log(c))/n^2) + 12*n^3*x*x^m*abs(m*n + n)*e^(3*I*a - 3/2*(n
*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - 54*(m*n + n)^2*n^2*x*x^m
*e^(I*a - 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 24*n^4*x
*x^m*e^(I*a - 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - 27*(
```

```
m*n + n)^2*n*x*x^m*abs(m*n + n)*e^(I*a - 1/2*(n*abs(m*n + n)*log(x) + abs(m
*n + n)*log(c))/n^2) + 12*n^3*x*x^m*abs(m*n + n)*e^(I*a - 1/2*(n*abs(m*n +
n)*log(x) + abs(m*n + n)*log(c))/n^2) - 54*(m*n + n)^2*n^2*x*x^m*e^(-I*a +
1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 24*n^4*x*x^m*e^(-I
*a + 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 27*(m*n + n)^
2*n*x*x^m*abs(m*n + n)*e^(-I*a + 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*
log(c))/n^2) - 12*n^3*x*x^m*abs(m*n + n)*e^(-I*a + 1/2*(n*abs(m*n + n)*log(
x) + abs(m*n + n)*log(c))/n^2) - 2*(m*n + n)^2*n^2*x*x^m*e^(-3*I*a + 3/2*(n
*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 8*n^4*x*x^m*e^(-3*I*a +
3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 3*(m*n + n)^2*n*x*
x^m*abs(m*n + n)*e^(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(
c))/n^2) - 12*n^3*x*x^m*abs(m*n + n)*e^(-3*I*a + 3/2*(n*abs(m*n + n)*log(x)
 + abs(m*n + n)*log(c))/n^2))/(16*m^4*n^4 + 64*m^3*n^4 - 40*(m*n + n)^2*m^2
*n^2 + 96*m^2*n^4 - 80*(m*n + n)^2*m*n^2 + 64*m*n^4 + 9*(m*n + n)^4 - 40*(m
*n + n)^2*n^2 + 16*n^4)
```

Mupad [B]
time = 4.71, size = 277, normalized size = 1.23

$$\frac{x\,x^m\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\frac{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,1\mathrm{i}}{2}}}\,\left(2\,m+2+n\,\sqrt{-\frac{{\left(m+1\right)}^2}{n^2}}\,1\mathrm{i}\right)}{4\,{\left(m+1\right)}^2}+\frac{x\,x^m\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{\frac{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,1\mathrm{i}}{2}}\,\left(2\,m+2-n\,\sqrt{-\frac{{\left(m+1\right)}^2}{n^2}}\,1\mathrm{i}\right)}{4\,{\left(m+1\right)}^2}-\frac{x\,x^m\,\mathrm{e}^{-a\,3\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\frac{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,3\mathrm{i}}{2}}}\,\left(2\,m+2+n\,\sqrt{-\frac{{\left(m+1\right)}^2}{n^2}}\,3\mathrm{i}\right)}{20\,{\left(m+1\right)}^2}-\frac{x\,x^m\,\mathrm{e}^{a\,3\mathrm{i}}\,{\left(c\,x^n\right)}^{\frac{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,3\mathrm{i}}{2}}\,\left(2\,m+2-n\,\sqrt{-\frac{{\left(m+1\right)}^2}{n^2}}\,3\mathrm{i}\right)}{20\,{\left(m+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a + (log(c*x^n)*(-(m + 1)^2/n^2)^(1/2))/2)^3,x)
```

```
[Out] (x*x^m*exp(-a*1i)/(c*x^n)^(((- (2*m)/n^2 - 1/n^2 - m^2/n^2)^(1/2)*1i)/2)*(2
*m + n*(-(m + 1)^2/n^2)^(1/2)*1i + 2))/(4*(m + 1)^2) + (x*x^m*exp(a*1i)*(c*
x^n)^(((- (2*m)/n^2 - 1/n^2 - m^2/n^2)^(1/2)*1i)/2)*(2*m - n*(-(m + 1)^2/n^
2)^(1/2)*1i + 2))/(4*(m + 1)^2) - (x*x^m*exp(-a*3i)/(c*x^n)^(((- (2*m)/n^2
- 1/n^2 - m^2/n^2)^(1/2)*3i)/2)*(2*m + n*(-(m + 1)^2/n^2)^(1/2)*3i + 2))/(2
0*(m + 1)^2) - (x*x^m*exp(a*3i)*(c*x^n)^(((- (2*m)/n^2 - 1/n^2 - m^2/n^2)^(
1/2)*3i)/2)*(2*m - n*(-(m + 1)^2/n^2)^(1/2)*3i + 2))/(20*(m + 1)^2)
```

3.109 $\int \cos^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)\,dx$

Optimal. Leaf size=128

$$\frac{9}{16}e^{a\sqrt{-\frac{1}{n^2}}\,n}x(cx^n)^{-\frac{1}{3}/n}+\frac{9}{32}e^{-a\sqrt{-\frac{1}{n^2}}\,n}x(cx^n)^{\frac{1}{3}/n}+\frac{1}{16}e^{-3a\sqrt{-\frac{1}{n^2}}\,n}x(cx^n)^{\frac{1}{n}}+\frac{1}{8}e^{3a\sqrt{-\frac{1}{n^2}}\,n}x(cx^n)^{-1/n}\log(x)$$

```
[Out] 9/16*exp(a*n*(-1/n^2)^(1/2))*x/((c*x^n)^(1/3/n))+9/32*x*(c*x^n)^(1/3/n)/exp
(a*n*(-1/n^2)^(1/2))+1/16*x*(c*x^n)^(1/n)/exp(3*a*n*(-1/n^2)^(1/2))+1/8*exp
(3*a*n*(-1/n^2)^(1/2))*x*ln(x)/((c*x^n)^(1/n))
```

Rubi [A]
time = 0.07, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {4572, 4578}

$$\frac{9}{16}xe^{a\sqrt{-\frac{1}{n^2}}\,n}(cx^n)^{-\frac{1}{3}/n}+\frac{9}{32}xe^{-a\sqrt{-\frac{1}{n^2}}\,n}(cx^n)^{\frac{1}{3}/n}+\frac{1}{16}xe^{-3a\sqrt{-\frac{1}{n^2}}\,n}(cx^n)^{\frac{1}{n}}+\frac{1}{8}xe^{3a\sqrt{-\frac{1}{n^2}}\,n}\log(x)\,(cx^n)^{-1/n}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + (Sqrt[-n^(-2)]*Log[c*x^n])/3]^3,x]
```

```
[Out] (9*E^(a*Sqrt[-n^(-2)]*n)*x)/(16*(c*x^n)^(1/(3*n))) + (9*x*(c*x^n)^(1/(3*n))
)/(32*E^(a*Sqrt[-n^(-2)]*n)) + (x*(c*x^n)^n^(-1))/(16*E^(3*a*Sqrt[-n^(-2)]*
n)) + (E^(3*a*Sqrt[-n^(-2)]*n)*x*Log[x])/(8*(c*x^n)^n^(-1))
```

Rule 4572

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Cos[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4578

```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[1/2^p, Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^2*(p/(m + 1)))/x^((m +
 1)/p) + x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x], x], x] /; FreeQ[{a,
b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int \cos^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\cos^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log(x)\right)dx, x, c\right.}{n}$$

$$= \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int\left(\frac{e^{3a\sqrt{-\frac{1}{n^2}}n}}{x}+3e^{a\sqrt{-\frac{1}{n^2}}n}x^{-1+\frac{2}{3n}}+3e^{-a\sqrt{-}}\right.\right.}{8n}$$

$$= \frac{9}{16}e^{a\sqrt{-\frac{1}{n^2}}n}x(cx^n)^{-\frac{1}{3}/n}+\frac{9}{32}e^{-a\sqrt{-\frac{1}{n^2}}n}x(cx^n)^{\frac{1}{3}/n}+\frac{1}{16}e^{-3a\sqrt{-\frac{1}{n^2}}n}$$

Mathematica [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \cos^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[a + (Sqrt[-n^(-2)]*Log[c*x^n])/3]^3,x]
[Out] Integrate[Cos[a + (Sqrt[-n^(-2)]*Log[c*x^n])/3]^3, x]
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \cos^3\left(a+\frac{\ln(c\,x^n)\sqrt{-\frac{1}{n^2}}}{3}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+1/3*ln(c*x^n)*(-1/n^2)^(1/2))^3,x)
[Out] int(cos(a+1/3*ln(c*x^n)*(-1/n^2)^(1/2))^3,x)
```

Maxima [A]
time = 0.30, size = 106, normalized size = 0.83

$$\frac{9\,c^{\frac{5}{3\,n}}x(x^n)^{\frac{2}{3\,n}}\cos(a)+4\,c^{\frac{1}{3\,n}}(x^n)^{\frac{1}{3\,n}}\cos(3\,a)\log(x)+18\,c^{\left(\frac{1}{n}\right)}x\cos(a)+2\,c^{\frac{7}{3\,n}}\cos(3\,a)\,e^{\left(\frac{\log(x^n)}{3\,n}+2\,\log(x)\right)}}{32\,c^{\frac{4}{3\,n}}(x^n)^{\frac{1}{3\,n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+1/3*log(c*x^n)*(-1/n^2)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] 1/32*(9*c^(5/3/n)*x*(x^n)^(2/3/n)*cos(a) + 4*c^(1/3/n)*(x^n)^(1/3/n)*cos(3*
a)*log(x) + 18*c^(1/n)*x*cos(a) + 2*c^(7/3/n)*cos(3*a)*e^(1/3*log(x^n)/n +
2*log(x)))/(c^(4/3/n)*(x^n)^(1/3/n))
```

Fricas [C] Result contains complex when optimal does not.
time = 2.00, size = 84, normalized size = 0.66

$$\frac{1}{32}\left(9\,x^{\frac{4}{3}}e^{\left(\frac{2\,(3i\,an-\log(c))}{3\,n}\right)}+2\,x^2+12\,e^{\left(\frac{2\,(3i\,an-\log(c))}{n}\right)}\log\left(x^{\frac{1}{3}}\right)+18\,x^{\frac{2}{3}}e^{\left(\frac{4\,(3i\,an-\log(c))}{3\,n}\right)}\right)e^{\left(-\frac{3i\,an-\log(c)}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+1/3*log(c*x^n)*(-1/n^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(9*x^(4/3)*e^(2/3*(3*I*a*n - log(c))/n) + 2*x^2 + 12*e^(2*(3*I*a*n - l
og(c))/n)*log(x^(1/3)) + 18*x^(2/3)*e^(4/3*(3*I*a*n - log(c))/n))*e^(-(3*I*
a*n - log(c))/n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\cos^3\left(a+\frac{\sqrt{-\frac{1}{n^2}}\,\log\left(cx^n\right)}{3}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+1/3*ln(c*x**n)*(-1/n**2)**(1/2))**3,x)
```

```
[Out] Integral(cos(a + sqrt(-1/n**2)*log(c*x**n)/3)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+1/3*log(c*x^n)*(-1/n^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (9*sa
geVARn^4*sageVARx*exp((-3*i)*sageVARa)*exp((sageVARn*abs(sageVARn)*ln(sageV
ARx)+abs(sageVARn)*ln(sageVARc))/sageVARn^2)+27*sageVARn^4*sageVARx*exp((-i
)*sageVARa)*exp(
```

Mupad [B]
time = 3.01, size = 158, normalized size = 1.23

$$x\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\frac{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}{3}}}\,\left(\frac{27}{64}+\frac{n\,\sqrt{-\frac{1}{n^2}}\,9\mathrm{i}}{64}\right)-x\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{\frac{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}{3}}\,\left(-\frac{27}{64}+\frac{n\,\sqrt{-\frac{1}{n^2}}\,9\mathrm{i}}{64}\right)+\frac{x\,\mathrm{e}^{-a\,3\mathrm{i}}\,\frac{1}{\left(c\,x^n\right)^{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}}\,1\mathrm{i}}{8\,n\,\sqrt{-\frac{1}{n^2}}+8\mathrm{i}}-\frac{x\,\mathrm{e}^{a\,3\mathrm{i}}\,{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}\,1\mathrm{i}}{8\,n\,\sqrt{-\frac{1}{n^2}}-8\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + (log(c*x^n)*(-1/n^2)^(1/2))/3)^3,x)
```

```
[Out] x*exp(-a*1i)/(c*x^n)^(((-1/n^2)^(1/2)*1i)/3)*((n*(-1/n^2)^(1/2)*9i)/64 + 27
/64) - x*exp(a*1i)*(c*x^n)^(((-1/n^2)^(1/2)*1i)/3)*((n*(-1/n^2)^(1/2)*9i)/6
4 - 27/64) + (x*exp(-a*3i)/(c*x^n)^((-1/n^2)^(1/2)*1i)*1i)/(8*n*(-1/n^2)^(1
/2) + 8i) - (x*exp(a*3i)*(c*x^n)^((-1/n^2)^(1/2)*1i)*1i)/(8*n*(-1/n^2)^(1/2
) - 8i)
```

3.110 $\int \sqrt{\cos(a+b\log(cx^n))}\, dx$

Optimal. Leaf size=110

$$\frac{2x\sqrt{\cos(a+b\log(cx^n))}\,{}_2F_1\left(-\frac{1}{2},-\frac{2i+bn}{4bn};\frac{1}{4}\left(3-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{(2-ibn)\sqrt{1+e^{2ia}\left(cx^n\right)^{2ib}}}$$

[Out] 2*x*hypergeom([-1/2, 1/4*(-2*I-b*n)/b/n],[3/4-1/2*I/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))*cos(a+b*ln(c*x^n))^(1/2)/(2-I*b*n)/(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(1/2)

Rubi [A]
time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4572, 4580, 371}

$$\frac{2x\,{}_2F_1\left(-\frac{1}{2},-\frac{bn+2i}{4bn};\frac{1}{4}\left(3-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)\sqrt{\cos(a+b\log(cx^n))}}{(2-ibn)\sqrt{1+e^{2ia}\left(cx^n\right)^{2ib}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cos[a + b*Log[c*x^n]]],x]

[Out] (2*x*Sqrt[Cos[a + b*Log[c*x^n]]]*Hypergeometric2F1[-1/2, -1/4*(2*I + b*n)/(b*n), (3 - (2*I)/(b*n))/4, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 - I*b*n)* Sqrt[1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)])

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4572

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Cos[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4580

```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Dist[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
```

```
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}\,dx &= \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\sqrt{\cos(a+b\log(x))}\,dx, x, cx^n\right)}{n}\\ &= \frac{\left(x(cx^n)^{\frac{ib}{2}-\frac{1}{n}}\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}\right)\text{Subst}\left(\int x^{-1-\frac{ib}{2}+\frac{1}{n}}\sqrt{1+e^{2ia}x^{2ib}}\,dx,\right.}{n\sqrt{1+e^{2ia}\left(cx^n\right)^{2ib}}}\\ &= \frac{2x\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}\,{}_2F_1\left(-\frac{1}{2},-\frac{2i+bn}{4bn};\frac{1}{4}\left(3-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{(2-ibn)\sqrt{1+e^{2ia}\left(cx^n\right)^{2ib}}}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 361 vs. $2(110) = 220$.
time = 5.54, size = 361, normalized size = 3.28

$$\frac{2i\sqrt{2}\,be^{-ia}nx(cx^n)^{-ib}\left((2i+bn)\left(1+e^{2ia}(cx^n)^{2ib}\right)+\sqrt{1+e^{2ia}\left(cx^n\right)^{2ib}}\left(-2i-bn+e^{2ia}(-2i+bn)x^{-2ibn}(cx^n)^{2ib}\right){}_2F_1\left(\frac{1}{2},-\frac{2i+bn}{4bn};\frac{3}{4}-\frac{i}{2bn};-e^{2ia}(cx^n)^{2ib}\right)\right)}{(4+b^2n^2)\sqrt{e^{-ia}\left(cx^n\right)^{-ib}+e^{ia}\left(cx^n\right)^{ib}}\left(-2i-bn+e^{2ia}(-2i+bn)x^{-2ibn}\left(cx^n\right)^{2ib}\right)}-\frac{2x\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}\cos\left(a-bn\log(x)+b\log\left(cx^n\right)\right)}{-2\cos\left(a-bn\log(x)+b\log\left(cx^n\right)\right)+bn\sin\left(a-bn\log(x)+b\log\left(cx^n\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[a + b*Log[c*x^n]]],x]
```

```
[Out] ((2*I)*Sqrt[2]*b*n*x*((2*I + b*n)*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)) + Sqr
t[1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)]*(-2*I - b*n + (E^((2*I)*a)*(-2*I + b*n
)*(c*x^n)^((2*I)*b))/x^((2*I)*b*n))*Hypergeometric2F1[1/2, -1/4*(2*I + b*n)
/(b*n), 3/4 - (I/2)/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))]))/(E^(I*a)*(4
+ b^2*n^2)*(c*x^n)^(I*b)*Sqrt[1/(E^(I*a)*(c*x^n)^(I*b)) + E^(I*a)*(c*x^n)^(
I*b)]*(-2*I - b*n + (E^((2*I)*a)*(-2*I + b*n)*(c*x^n)^((2*I)*b))/x^((2*I)*b
*n))) - (2*x*Sqrt[Cos[a + b*Log[c*x^n]]]*Cos[a - b*n*Log[x] + b*Log[c*x^n]]
)/(-2*Cos[a - b*n*Log[x] + b*Log[c*x^n]] + b*n*Sin[a - b*n*Log[x] + b*Log[c
*x^n]])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \sqrt{\cos}\left(a+b\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*ln(c*x^n))^(1/2),x)
```

```
[Out] int(cos(a+b*ln(c*x^n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(cos(b*log(c*x^n) + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\cos \left(a + b \log \left(c x^{n}\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*ln(c*x**n))**(1/2),x)
[Out] Integral(sqrt(cos(a + b*log(c*x**n))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(cos(b*log(c*x^n) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos \left(a+b\,\ln \left(c\,x^n\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*log(c*x^n))^(1/2),x)
[Out] int(cos(a + b*log(c*x^n))^(1/2), x)
```

3.111 $\int \frac{\sqrt{\cos(a+b\log(cx^n))}}{x}\,dx$

Optimal. Leaf size=24

$$\frac{2E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{bn}$$

```
[Out] 2*(cos(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cos(1/2*a+1/2*b*ln(c*x^n))*EllipticE
(sin(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{2719\}$

$$\frac{2E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] (2*EllipticE[(a + b*Log[c*x^n])/2, 2])/(b*n)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cos(a+b\log(cx^n))}}{x}\,dx &= \frac{\text{Subst}\left(\int \sqrt{\cos(a+bx)}\,dx, x, \log(cx^n)\right)}{n}\\ &= \frac{2E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{bn}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 24, normalized size = 1.00

$$\frac{2E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] (2*EllipticE[(a + b*Log[c*x^n])/2, 2])/(b*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(60) = 120$.
time = 0.22, size = 181, normalized size = 7.54

method	result
derivativedivides	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(a+b\ln(cx^n))}{2}}\,\sqrt{-2\left(\cos\right.}}{n\sqrt{-2\left(\sin^4\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+\sin^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)}\,\sin\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\sqrt{}}$
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(a+b\ln(cx^n))}{2}}\,\sqrt{-2\left(\cos\right.}}{n\sqrt{-2\left(\sin^4\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+\sin^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)}\,\sin\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/n*((2*cos(1/2*a+1/2*b*ln(c*x^n))^2-1)*sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)
*(sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cos(1/2*a+1/2*b*ln(c*x^n))^2+1)^(
1/2)*EllipticE(cos(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/(-2*sin(1/2*a+1/2*b*ln(c
*x^n))^4+sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/sin(1/2*a+1/2*b*ln(c*x^n))/(2*
cos(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(b*log(c*x^n) + a))/x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.55, size = 84, normalized size = 3.50

$$\frac{i\sqrt{2}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bn\log(x)+b\log(c)+a)+i\sin(bn\log(x)+b\log(c)+a)))-i\sqrt{2}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bn\log(x)+b\log(c)+a)-i\sin(bn\log(x)+b\log(c)+a)))}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] (I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*n*log(x)
 + b*log(c) + a) + I*sin(b*n*log(x) + b*log(c) + a))) - I*sqrt(2)*weierstra
```

```
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*n*log(x) + b*log(c) + a) - I
*sin(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos\left(a+b\log\left(cx^{n}\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*ln(c*x**n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(cos(a + b*log(c*x**n)))/x, x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(b*log(c*x^n) + a))/x, x)
```

Mupad [B]

time = 2.37, size = 23, normalized size = 0.96

$$\frac{2\,\mathrm{E}\left(\frac{a}{2}+\frac{b\,\ln\left(c\,x^n\right)}{2}\middle|2\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*log(c*x^n))^(1/2)/x,x)
```

```
[Out] (2*ellipticE(a/2 + (b*log(c*x^n))/2, 2))/(b*n)
```

3.112 $\int \cos^{\frac{3}{2}}(a+b\log(cx^n))\,dx$

Optimal. Leaf size=109

$$\frac{2x\cos^{\frac{3}{2}}(a+b\log(cx^n))\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{2i}{bn}\right);\frac{1}{4}\left(1-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{(2-3ibn)\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

```
[Out] 2*x*cos(a+b*ln(c*x^n))^(3/2)*hypergeom([-3/2, -3/4-1/2*I/b/n],[1/4-1/2*I/b/
n],-exp(2*I*a)*(c*x^n)^(2*I*b))/(2-3*I*b*n)/(1+exp(2*I*a)*(c*x^n)^(2*I*b))^
(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4572, 4580, 371}

$$\frac{2x\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{2i}{bn}\right);\frac{1}{4}\left(1-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)\cos^{\frac{3}{2}}(a+b\log(cx^n))}{(2-3ibn)\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*Log[c*x^n]]^(3/2),x]
```

```
[Out] (2*x*Cos[a + b*Log[c*x^n]]^(3/2)*Hypergeometric2F1[-3/2, (-3 - (2*I)/(b*n))
/4, (1 - (2*I)/(b*n))/4, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 - (3*I)*b*n
)*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4572

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Cos[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4580

```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Dist[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
```

```
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\,dx &= \frac{\left(x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\cos^{\frac{3}{2}}(a+b\log(x))\,dx,x,cx^n\right)}{n}\\ &= \frac{\left(x\left(cx^n\right)^{\frac{3ib}{2}-\frac{1}{n}}\cos^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\right)\operatorname{Subst}\left(\int x^{-1-\frac{3ib}{2}+\frac{1}{n}}\left(1+e^{2ia}x^{2ib}\right)^{3/2}\,dx,x,cx^n\right)}{n\left(1+e^{2ia}\left(cx^n\right)^{2ib}\right)^{3/2}}\\ &= \frac{2x\cos^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{2i}{bn}\right);\frac{1}{4}\left(1-\frac{2i}{bn}\right);-e^{2ia}\left(cx^n\right)^{2ib}\right)}{(2-3ibn)\left(1+e^{2ia}\left(cx^n\right)^{2ib}\right)^{3/2}}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 220 vs. $2(109) = 218$.
time = 1.04, size = 220, normalized size = 2.02

$$-\frac{6i\sqrt{2}\,b^2\sqrt{1+e^{2i(a+b\log(cx^n))}}\,n^2x\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}-\frac{i}{2bn};\frac{5}{4}-\frac{i}{2bn};-e^{2i(a+b\log(cx^n))}\right)}{\sqrt{e^{-i(a+b\log(cx^n))}\left(1+e^{2i(a+b\log(cx^n))}\right)}\,(-2i+bn)(-2i+3bn)(2i+3bn)}+\frac{2x\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}\left(2\cos\left(a+b\log\left(cx^n\right)\right)+3bn\sin\left(a+b\log\left(cx^n\right)\right)\right)}{4+9b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*Log[c*x^n]]^(3/2),x]
[Out] ((-6*I)*Sqrt[2]*b^2*Sqrt[1 + E^((2*I)*(a + b*Log[c*x^n]))]*n^2*x*Hypergeome
tric2F1[1/2, 1/4 - (I/2)/(b*n), 5/4 - (I/2)/(b*n), -E^((2*I)*(a + b*Log[c*x
^n]))])/(Sqrt[(1 + E^((2*I)*(a + b*Log[c*x^n])))/E^(I*(a + b*Log[c*x^n]))]*
(-2*I + b*n)*(-2*I + 3*b*n)*(2*I + 3*b*n)) + (2*x*Sqrt[Cos[a + b*Log[c*x^n]
]]*(2*Cos[a + b*Log[c*x^n]] + 3*b*n*Sin[a + b*Log[c*x^n]]))/(4 + 9*b^2*n^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \cos^{\frac{3}{2}}\left(a+b\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*ln(c*x^n))^(3/2),x)
[Out] int(cos(a+b*ln(c*x^n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*log(c*x^n) + a)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^{\frac{3}{2}}\left(a + b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Integral(cos(a + b*log(c*x**n))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*log(c*x^n) + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*log(c*x^n))^(3/2),x)
```

```
[Out] int(cos(a + b*log(c*x^n))^(3/2), x)
```

3.113 $\int \frac{\cos^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=63

$$\frac{2F\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{3bn}+\frac{2\sqrt{\cos(a+b\log(cx^n))}\sin(a+b\log(cx^n))}{3bn}$$

```
[Out] 2/3*(cos(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cos(1/2*a+1/2*b*ln(c*x^n))*Ellipti
cF(sin(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n+2/3*sin(a+b*ln(c*x^n))*cos(a+b*l
n(c*x^n))^(1/2)/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2715, 2720}

$$\frac{2F\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{3bn}+\frac{2\sin(a+b\log(cx^n))\sqrt{\cos(a+b\log(cx^n))}}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (2*EllipticF[(a + b*Log[c*x^n])/2, 2])/(3*b*n) + (2*Sqrt[Cos[a + b*Log[c*x^
n]]]*Sin[a + b*Log[c*x^n]])/(3*b*n)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\text{Subst}\left(\int \cos^{\frac{3}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2\sqrt{\cos(a+b\log(cx^n))}\,\sin(a+b\log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx, x,\right.}{3n}$$

$$= \frac{2F\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{3bn} + \frac{2\sqrt{\cos(a+b\log(cx^n))}\,\sin(a+b\log(cx^n))}{3bn}$$

Mathematica [A]
time = 0.12, size = 54, normalized size = 0.86

$$\frac{2\left(F\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)+\sqrt{\cos(a+b\log(cx^n))}\,\sin(a+b\log(cx^n))\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (2*(EllipticF[(a + b*Log[c*x^n])/2, 2] + Sqrt[Cos[a + b*Log[c*x^n]]]*Sin[a
+ b*Log[c*x^n]]))/(3*b*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(93) = 186$.
time = 0.25, size = 247, normalized size = 3.92

method	result
derivativedivides	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\left(4\cos\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\left(\sin^4\left(\frac{a}{2}+\frac{b\ln(cx}{2}\right.\right.\right.}{3n\sqrt{-2\left(\sin^4\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+}}$
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\left(4\cos\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\left(\sin^4\left(\frac{a}{2}+\frac{b\ln(cx}{2}\right.\right.\right.}{3n\sqrt{-2\left(\sin^4\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/n*((2*cos(1/2*a+1/2*b*ln(c*x^n))^2-1)*sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1
/2)*(4*cos(1/2*a+1/2*b*ln(c*x^n))*sin(1/2*a+1/2*b*ln(c*x^n))^4-2*sin(1/2*a+
```

```
1/2*b*ln(c*x^n))^2*cos(1/2*a+1/2*b*ln(c*x^n))+(sin(1/2*a+1/2*b*ln(c*x^n))^2
)^(1/2)*(2*sin(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)*EllipticF(cos(1/2*a+1/2*b*
ln(c*x^n)),2^(1/2)))/(-2*sin(1/2*a+1/2*b*ln(c*x^n))^4+sin(1/2*a+1/2*b*ln(c*
x^n))^2)^(1/2)/sin(1/2*a+1/2*b*ln(c*x^n))/(2*cos(1/2*a+1/2*b*ln(c*x^n))^2-1
)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(cos(b*log(c*x^n) + a)^(3/2)/x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.40, size = 107, normalized size = 1.70

$$\frac{2\sqrt{\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}\,\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-i\sqrt{2}\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+i\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)+i\sqrt{2}\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-i\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)}{3\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(cos(b*n*log(x) + b*log(c) + a))*sin(b*n*log(x) + b*log(c) + a)
- I*sqrt(2)*weierstrassPInverse(-4, 0, cos(b*n*log(x) + b*log(c) + a) + I*s
in(b*n*log(x) + b*log(c) + a)) + I*sqrt(2)*weierstrassPInverse(-4, 0, cos(b
*n*log(x) + b*log(c) + a) - I*sin(b*n*log(x) + b*log(c) + a)))/(b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*ln(c*x**n))**(3/2)/x,x)
```

```
[Out] Integral(cos(a + b*log(c*x**n))**(3/2)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(cos(b*log(c*x^n) + a)^(3/2)/x, x)
```

Mupad [B]
time = 2.30, size = 56, normalized size = 0.89

$$\frac{2\,\mathrm{F}\left(\frac{a}{2}+\frac{b\,\ln(c\,x^n)}{2}\middle|2\right)}{3\,b\,n}+\frac{2\,\sqrt{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}\,\,\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}{3\,b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*log(c*x^n))^(3/2)/x,x)
```

```
[Out] (2*ellipticF(a/2 + (b*log(c*x^n))/2, 2))/(3*b*n) + (2*cos(a + b*log(c*x^n))
^(1/2)*sin(a + b*log(c*x^n)))/(3*b*n)
```

3.114 $\int \cos^{\frac{5}{2}}(a+b\log(cx^n))\,dx$

Optimal. Leaf size=110

$$\frac{2x\cos^{\frac{5}{2}}(a+b\log(cx^n))\,{}_2F_1\left(-\frac{5}{2},\frac{1}{4}\left(-5-\frac{2i}{bn}\right);-\frac{2i+bn}{4bn};-e^{2ia}(cx^n)^{2ib}\right)}{(2-5ibn)\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}}$$

```
[Out] 2*x*cos(a+b*ln(c*x^n))^(5/2)*hypergeom([-5/2, -5/4-1/2*I/b/n],[1/4*(-2*I-b*
n)/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))/(2-5*I*b*n)/(1+exp(2*I*a)*(c*x^n)^(2*I
*b))^(5/2)
```

Rubi [A]
time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4572, 4580, 371}

$$\frac{2x\,{}_2F_1\left(-\frac{5}{2},\frac{1}{4}\left(-5-\frac{2i}{bn}\right);-\frac{bn+2i}{4bn};-e^{2ia}(cx^n)^{2ib}\right)\cos^{\frac{5}{2}}(a+b\log(cx^n))}{(2-5ibn)\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*Log[c*x^n]]^(5/2),x]
```

```
[Out] (2*x*Cos[a + b*Log[c*x^n]]^(5/2)*Hypergeometric2F1[-5/2, (-5 - (2*I)/(b*n))
/4, -1/4*(2*I + b*n)/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 - (5*I)*
b*n)*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^(5/2))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4572

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Cos[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4580

```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Dist[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
```

```
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \cos^{\frac{5}{2}}(a+b\log(cx^n))\,dx &= \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\cos^{\frac{5}{2}}(a+b\log(x))\,dx,x,cx^n\right)}{n} \\
&= \frac{\left(x(cx^n)^{\frac{5ib}{2}-\frac{1}{n}}\cos^{\frac{5}{2}}(a+b\log(cx^n))\right)\operatorname{Subst}\left(\int x^{-1-\frac{5ib}{2}+\frac{1}{n}}\left(1+e^{2ia}x^{2ib}\right)^{5/2}\,dx,x,\right.}{n\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}} \\
&= \frac{2x\cos^{\frac{5}{2}}(a+b\log(cx^n))\,{}_2F_1\left(-\frac{5}{2},\frac{1}{4}\left(-5-\frac{2i}{bn}\right);-\frac{2i+bn}{4bn};-e^{2ia}(cx^n)^{2ib}\right)}{(2-5ibn)\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}}
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 681 vs. $2(110) = 220$.
time = 7.23, size = 681, normalized size = 6.19

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*Log[c*x^n]]^(5/2),x]
```

```
[Out] ((30*I)*Sqrt[2]*b^3*n^3*x^(1 - I*b*n)*((2*I + b*n)*(1 + E^((2*I)*(a + b*(-(
n*Log[x]) + Log[c*x^n])))*x^((2*I)*b*n)) + (-2*I - b*n + E^((2*I)*(a + b*(-
(n*Log[x]) + Log[c*x^n])))*(-2*I + b*n))*Sqrt[1 + E^((2*I)*(a + b*(-(n*Log[
x]) + Log[c*x^n])))*x^((2*I)*b*n)]*Hypergeometric2F1[1/2, -1/4*(2*I + b*n)/
(b*n), 3/4 - (I/2)/(b*n), -(E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n])))*x^
((2*I)*b*n))]))/(E^(I*(a + b*(-(n*Log[x]) + Log[c*x^n])))*(-2*I + 5*b*n)*(2
*I + 5*b*n)*(4 + b^2*n^2)*(-2*I - b*n + E^((2*I)*(a + b*(-(n*Log[x]) + Log[
c*x^n])))*(-2*I + b*n))*Sqrt[(1 + E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n]
)))*x^((2*I)*b*n))/(E^(I*(a + b*(-(n*Log[x]) + Log[c*x^n])))*x^(I*b*n))]) +
 Sqrt[Cos[a + b*n*Log[x] + b*(-(n*Log[x]) + Log[c*x^n])]]*((-2*x*(2*Cos[a +
 b*(-(n*Log[x]) + Log[c*x^n])] + 15*b^2*n^2*Cos[a + b*(-(n*Log[x]) + Log[c*
x^n])] - b*n*Sin[a + b*(-(n*Log[x]) + Log[c*x^n])]))/((-2*I + 5*b*n)*(2*I +
 5*b*n)*(-2*Cos[a + b*(-(n*Log[x]) + Log[c*x^n])] + b*n*Sin[a + b*(-(n*Log[
x]) + Log[c*x^n])])) + (x*Sin[2*b*n*Log[x]]*(5*b*n*Cos[2*(a + b*(-(n*Log[x]
) + Log[c*x^n]))] - 2*Sin[2*(a + b*(-(n*Log[x]) + Log[c*x^n]))]))/((-2*I +
5*b*n)*(2*I + 5*b*n)) + (x*Cos[2*b*n*Log[x]]*(2*Cos[2*(a + b*(-(n*Log[x]) +
 Log[c*x^n]))] + 5*b*n*Sin[2*(a + b*(-(n*Log[x]) + Log[c*x^n]))]))/((-2*I +
 5*b*n)*(2*I + 5*b*n)))
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \cos^{\frac{5}{2}}\left(a+b\ln\left(c\,x^{n}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*ln(c*x^n))^(5/2),x)
[Out] int(cos(a+b*ln(c*x^n))^(5/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
[Out] integrate(cos(b*log(c*x^n) + a)^(5/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*ln(c*x**n))**(5/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*log(c*x^n) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,\ln\left(c\,x^n\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*log(c*x^n))^(5/2),x)
```

```
[Out] int(cos(a + b*log(c*x^n))^(5/2), x)
```

3.115 $\int \frac{\cos^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=63

$$\frac{6E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{5bn}+\frac{2\cos^{\frac{3}{2}}(a+b\log(cx^n))\sin(a+b\log(cx^n))}{5bn}$$

```
[Out] 6/5*(cos(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cos(1/2*a+1/2*b*ln(c*x^n))*Ellipti
cE(sin(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n+2/5*cos(a+b*ln(c*x^n))^(3/2)*sin
(a+b*ln(c*x^n))/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2715, 2719}

$$\frac{6E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{5bn}+\frac{2\sin(a+b\log(cx^n))\cos^{\frac{3}{2}}(a+b\log(cx^n))}{5bn}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (6*EllipticE[(a + b*Log[c*x^n])/2, 2])/(5*b*n) + (2*Cos[a + b*Log[c*x^n]]^(
3/2)*Sin[a + b*Log[c*x^n]])/(5*b*n)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \cos^{\frac{5}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\
&= \frac{2\cos^{\frac{3}{2}}(a+b\log(cx^n))\sin(a+b\log(cx^n))}{5bn}+\frac{3\text{Subst}\left(\int \sqrt{\cos(a+bx)}\,dx, x\right.}{5n}\\
&= \frac{6E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{5bn}+\frac{2\cos^{\frac{3}{2}}(a+b\log(cx^n))\sin(a+b\log(cx^n))}{5bn}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 58, normalized size = 0.92

$$\frac{6E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)+\sqrt{\cos(a+b\log(cx^n))}\sin(2(a+b\log(cx^n)))}{5bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (6*EllipticE[(a + b*Log[c*x^n])/2, 2] + Sqrt[Cos[a + b*Log[c*x^n]]]*Sin[2*(
a + b*Log[c*x^n])])/(5*b*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(93) = 186$.
time = 0.23, size = 280, normalized size = 4.44

method	result
derivativedivides	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\left(-8\cos\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\left(\sin^6\left(\frac{a}{2}+\frac{b\ln(c}{2}\right.\right.\right.}{5n\sqrt{-2\left(\sin^4\left(\right.\right.}}$
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\left(-8\cos\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\left(\sin^6\left(\frac{a}{2}+\frac{b\ln(c}{2}\right.\right.\right.}{5n\sqrt{-2\left(\sin^4\left(\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*ln(c*x^n))^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/n*((2*cos(1/2*a+1/2*b*ln(c*x^n))^2-1)*sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1
/2)*(-8*cos(1/2*a+1/2*b*ln(c*x^n))*sin(1/2*a+1/2*b*ln(c*x^n))^6+8*cos(1/2*a
+1/2*b*ln(c*x^n))*sin(1/2*a+1/2*b*ln(c*x^n))^4-2*sin(1/2*a+1/2*b*ln(c*x^n))
^2*cos(1/2*a+1/2*b*ln(c*x^n))-3*(sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(2*sin
(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)*EllipticE(cos(1/2*a+1/2*b*ln(c*x^n)),2^(
1/2)))/(-2*sin(1/2*a+1/2*b*ln(c*x^n))^4+sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)
/sin(1/2*a+1/2*b*ln(c*x^n))/(2*cos(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(cos(b*log(c*x^n) + a)^(5/2)/x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.78, size = 113, normalized size = 1.79

$$\frac{2\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{\frac{3}{2}}\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+3i\sqrt{2}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+i\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)))-3i\sqrt{2}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-i\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)))}{5\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] 1/5*(2*cos(b*n*log(x) + b*log(c) + a)^(3/2)*sin(b*n*log(x) + b*log(c) + a)
+ 3*I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*n*log
(x) + b*log(c) + a) + I*sin(b*n*log(x) + b*log(c) + a))) - 3*I*sqrt(2)*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*n*log(x) + b*log(c) + a
) - I*sin(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*ln(c*x**n))**(5/2)/x,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate(cos(b*log(c*x^n) + a)^(5/2)/x, x)
```

Mupad [B]
time = 2.38, size = 65, normalized size = 1.03

$$-\frac{2\cos\left(a+b\,\ln\left(c\,x^n\right)\right)^{7/2}\,\sin\left(a+b\,\ln\left(c\,x^n\right)\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(a+b\,\ln\left(c\,x^n\right)\right)^2\right)}{7\,b\,n\,\sqrt{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*log(c*x^n))^(5/2)/x,x)
```

```
[Out] -(2*cos(a + b*log(c*x^n))^(7/2)*sin(a + b*log(c*x^n))*hypergeom([1/2, 7/4],
 11/4, cos(a + b*log(c*x^n))^2))/(7*b*n*(sin(a + b*log(c*x^n))^2)^(1/2))
```

3.116 $\int \frac{1}{\sqrt{\cos (a+b \log (c x^n))}} \, dx$

Optimal. Leaf size=109

$$\frac{2x\sqrt{1+e^{2ia}\left(cx^n\right)^{2ib}}\ {}_2F_1\left(\frac{1}{2},\frac{1}{4}\left(1-\frac{2i}{bn}\right);\frac{1}{4}\left(5-\frac{2i}{bn}\right);-e^{2ia}\left(cx^n\right)^{2ib}\right)}{(2+ibn)\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] 2*x*hypergeom([1/2, 1/4-1/2*I/b/n],[5/4-1/2*I/b/n],-exp(2*I*a)*(c*x^n)^(2*I
*b))*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(1/2)/(2+I*b*n)/cos(a+b*ln(c*x^n))^(1/2
)
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4572, 4580, 371}

$$\frac{2x\sqrt{1+e^{2ia}\left(cx^n\right)^{2ib}}\ {}_2F_1\left(\frac{1}{2},\frac{1}{4}\left(1-\frac{2i}{bn}\right);\frac{1}{4}\left(5-\frac{2i}{bn}\right);-e^{2ia}\left(cx^n\right)^{2ib}\right)}{(2+ibn)\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Cos[a + b*Log[c*x^n]]],x]
```

```
[Out] (2*x*Sqrt[1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Hypergeometric2F1[1/2, (1 - (2
*I)/(b*n))/4, (5 - (2*I)/(b*n))/4, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 +
 I*b*n)*Sqrt[Cos[a + b*Log[c*x^n]]])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4572

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Cos[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4580

```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Dist[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
```

```
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}}\,dx &= \frac{\left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\sqrt{\cos(a+b\log(x))}}\,dx,x,cx^n\right)}{n}\\ &= \frac{\left(x\left(cx^n\right)^{-\frac{ib}{2}-\frac{1}{n}}\sqrt{1+e^{2ia}\left(cx^n\right)^{2ib}}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{ib}{2}+\frac{1}{n}}}{\sqrt{1+e^{2ia}x^{2ib}}}\,dx,x,cx^n\right)}{n\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}}\\ &= \frac{2x\sqrt{1+e^{2ia}\left(cx^n\right)^{2ib}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}\left(1-\frac{2i}{bn}\right);\frac{1}{4}\left(5-\frac{2i}{bn}\right);-e^{2ia}\left(cx^n\right)^{2ib}\right)}{(2+ibn)\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}}\end{aligned}$$

Mathematica [A]
time = 0.46, size = 134, normalized size = 1.23

$$-\frac{2i\sqrt{2}\sqrt{1+e^{2i(a+b\log(cx^n))}}\,x\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}-\frac{i}{2bn};\frac{5}{4}-\frac{i}{2bn};-e^{2i(a+b\log(cx^n))}\right)}{\sqrt{e^{-i(a+b\log(cx^n))}\left(1+e^{2i(a+b\log(cx^n))}\right)}\,(-2i+bn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Cos[a + b*Log[c*x^n]]],x]
[Out] ((-2*I)*Sqrt[2]*Sqrt[1 + E^((2*I)*(a + b*Log[c*x^n]))]*x*Hypergeometric2F1[
1/2, 1/4 - (I/2)/(b*n), 5/4 - (I/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))])/
(Sqrt[(1 + E^((2*I)*(a + b*Log[c*x^n])))/E^(I*(a + b*Log[c*x^n]))]*(-2*I +
b*n))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cos\left(a+b\ln\left(c\,x^{n}\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a+b*ln(c*x^n))^(1/2),x)
[Out] int(1/cos(a+b*ln(c*x^n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(cos(b*log(c*x^n) + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cos\left(a + b\log\left(cx^{n}\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(1/sqrt(cos(a + b*log(c*x**n))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(cos(b*log(c*x^n) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos\left(a + b\,\ln\left(c\,x^{n}\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*log(c*x^n))^(1/2),x)
```

```
[Out] int(1/cos(a + b*log(c*x^n))^(1/2), x)
```

3.117 $\int \frac{1}{x\sqrt{\cos(a+b\log(cx^n))}}\,dx$

Optimal. Leaf size=24

$$\frac{2F\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{bn}$$

```
[Out] 2*(cos(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cos(1/2*a+1/2*b*ln(c*x^n))*EllipticF
(sin(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2720}

$$\frac{2F\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Cos[a + b*Log[c*x^n]]]),x]
```

```
[Out] (2*EllipticF[(a + b*Log[c*x^n])/2, 2])/(b*n)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{\cos(a+b\log(cx^n))}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx, x, \log(cx^n)\right)}{n}\\ &= \frac{2F\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{bn}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 24, normalized size = 1.00

$$\frac{2F\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[Cos[a + b*Log[c*x^n]]]),x]
[Out] (2*EllipticF[(a + b*Log[c*x^n])/2, 2])/(b*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.06, size = 26, normalized size = 1.08

method	result	size
derivativedivides	$\frac{2\,\mathrm{am}^{-1}\left(\frac{a}{2}+\frac{b\ln\left(c\,x^{n}\right)}{2}\middle|\sqrt{2}\right)}{nb}$	26
default	$\frac{2\,\mathrm{am}^{-1}\left(\frac{a}{2}+\frac{b\ln\left(c\,x^{n}\right)}{2}\middle|\sqrt{2}\right)}{nb}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/cos(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/n/b*InverseJacobiAM(1/2*a+1/2*b*ln(c*x^n),2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(x*sqrt(cos(b*log(c*x^n) + a))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.75, size = 78, normalized size = 3.25

$$\frac{-i\sqrt{2}\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+i\,\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)+i\sqrt{2}\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-i\,\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
[Out] (-I*sqrt(2)*weierstrassPInverse(-4, 0, cos(b*n*log(x) + b*log(c) + a) + I*s
in(b*n*log(x) + b*log(c) + a)) + I*sqrt(2)*weierstrassPInverse(-4, 0, cos(b
*n*log(x) + b*log(c) + a) - I*sin(b*n*log(x) + b*log(c) + a)))/(b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\cos\left(a+b\log\left(cx^{n}\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(a+b*ln(c*x**n))**(1/2),x)
[Out] Integral(1/(x*sqrt(cos(a + b*log(c*x**n)))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
[Out] integrate(1/(x*sqrt(cos(b*log(c*x^n) + a))), x)
```

Mupad [B]
time = 2.37, size = 23, normalized size = 0.96

$$\frac{2\,\mathrm{F}\left(\frac{a}{2}+\frac{b\,\ln\left(c\,x^n\right)}{2}\middle|2\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cos(a + b*log(c*x^n))^(1/2)),x)
[Out] (2*ellipticF(a/2 + (b*log(c*x^n))/2, 2))/(b*n)
```

3.118 $\int \frac{1}{\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=109

$$\frac{2x\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}{}_2F_1\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i}{bn}\right);\frac{1}{4}\left(7-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{(2+3ibn)\cos^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

```
[Out] 2*x*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(3/2)*hypergeom([3/2, 3/4-1/2*I/b/n],[7/
4-1/2*I/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))/(2+3*I*b*n)/cos(a+b*ln(c*x^n))^(3
/2)
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4572, 4580, 371}

$$\frac{2x\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}{}_2F_1\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i}{bn}\right);\frac{1}{4}\left(7-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{(2+3ibn)\cos^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*Log[c*x^n]]^(-3/2),x]
```

```
[Out] (2*x*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2)*Hypergeometric2F1[3/2, (3 -
(2*I)/(b*n))/4, (7 - (2*I)/(b*n))/4, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2
 + (3*I)*b*n)*Cos[a + b*Log[c*x^n]]^(3/2))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4572

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Cos[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4580

```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Dist[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
```

```
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx = \frac{\left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\cos^{\frac{3}{2}}(a+b\log(x))}\,dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x\left(cx^n\right)^{-\frac{3ib}{2}-\frac{1}{n}}\left(1+e^{2ia}\left(cx^n\right)^{2ib}\right)^{3/2}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{3ib}{2}+\frac{1}{n}}}{\left(1+e^{2ia}x^{2ib}\right)^{3/2}}\,dx, x, cx^n\right)}{n\cos^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

$$= \frac{2x\left(1+e^{2ia}\left(cx^n\right)^{2ib}\right)^{3/2}{}_2F_1\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i}{bn}\right);\frac{1}{4}\left(7-\frac{2i}{bn}\right);-e^{2ia}\left(cx^n\right)^{2ib}\right)}{(2+3ibn)\cos^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 847 vs. 2(109) = 218.
time = 7.41, size = 847, normalized size = 7.77

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*Log[c*x^n]]^(-3/2),x]
```

```
[Out] (-4*Sqrt[2]*x^(1 - I*b*n)*((2*I + b*n)*(1 + E^((2*I)*(a + b*(-(n*Log[x]) +
Log[c*x^n])))*x^((2*I)*b*n)) + (-2*I - b*n + E^((2*I)*(a + b*(-(n*Log[x]) +
 Log[c*x^n])))*(-2*I + b*n))*Sqrt[1 + E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*
x^n])))*x^((2*I)*b*n)]*Hypergeometric2F1[1/2, -1/4*(2*I + b*n)/(b*n), 3/4 -
 (I/2)/(b*n), -(E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n])))*x^((2*I)*b*n))
]))/(b*E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n])))*n*(4 + b^2*n^2)*Sqrt[(1
 + E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n])))*x^((2*I)*b*n))/(E^(I*(a + b
*(-(n*Log[x]) + Log[c*x^n])))*x^(I*b*n))]*(-2*Cos[a + b*(-(n*Log[x]) + Log[
c*x^n])] + b*n*Sin[a + b*(-(n*Log[x]) + Log[c*x^n])])) - (Sqrt[2]*b*n*x^(1
- I*b*n)*((2*I + b*n)*(1 + E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n])))*x^(
(2*I)*b*n)) + (-2*I - b*n + E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n])))*(-
2*I + b*n))*Sqrt[1 + E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n])))*x^((2*I)*
b*n)]*Hypergeometric2F1[1/2, -1/4*(2*I + b*n)/(b*n), 3/4 - (I/2)/(b*n), -(E
^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n])))*x^((2*I)*b*n))]))/(E^((2*I)*(a
+ b*(-(n*Log[x]) + Log[c*x^n])))*(4 + b^2*n^2)*Sqrt[(1 + E^((2*I)*(a + b*(-
(n*Log[x]) + Log[c*x^n])))*x^((2*I)*b*n))/(E^(I*(a + b*(-(n*Log[x]) + Log[c
*x^n])))*x^(I*b*n))]*(-2*Cos[a + b*(-(n*Log[x]) + Log[c*x^n])] + b*n*Sin[a
+ b*(-(n*Log[x]) + Log[c*x^n])])) + Sqrt[Cos[a + b*n*Log[x] + b*(-(n*Log[x]
```

```
) + Log[c*x^n])]]*((2*x*Sec[a + b*(-(n*Log[x]) + Log[c*x^n])]*Sec[a + b*n*L
og[x] + b*(-(n*Log[x]) + Log[c*x^n])]*Sin[b*n*Log[x]])/(b*n) + (2*x*Sec[a +
 b*(-(n*Log[x]) + Log[c*x^n])])/(-2*Cos[a + b*(-(n*Log[x]) + Log[c*x^n])] +
 b*n*Sin[a + b*(-(n*Log[x]) + Log[c*x^n])]))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos \left(a+b\ln \left(c\,x^{n}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a+b*ln(c*x^n))^(3/2),x)
```

```
[Out] int(1/cos(a+b*ln(c*x^n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*log(c*x^n) + a)^(-3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos^{\frac{3}{2}}\left(a+b\log \left(cx^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Integral(cos(a + b*log(c*x**n))**(-3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*log(c*x^n))^(3/2),x)
```

```
[Out] int(1/cos(a + b*log(c*x^n))^(3/2), x)
```

3.119 $\int \frac{1}{x\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=59

$$-\frac{2E\left(\frac{1}{2}(a+b\log\left(cx^n\right))\middle|2\right)}{bn}+\frac{2\sin\left(a+b\log\left(cx^n\right)\right)}{bn\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] -2*(cos(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cos(1/2*a+1/2*b*ln(c*x^n))*Elliptic
E(sin(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n+2*sin(a+b*ln(c*x^n))/b/n/cos(a+b*
ln(c*x^n))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2716, 2719}

$$\frac{2\sin\left(a+b\log\left(cx^n\right)\right)}{bn\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}}-\frac{2E\left(\frac{1}{2}(a+b\log\left(cx^n\right))\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Cos[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (-2*EllipticE[(a + b*Log[c*x^n])/2, 2])/(b*n) + (2*Sin[a + b*Log[c*x^n]])/(
b*n*Sqrt[Cos[a + b*Log[c*x^n]]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{x\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2\sin(a+b\log(cx^n))}{bn\sqrt{\cos(a+b\log(cx^n))}} - \frac{\text{Subst}\left(\int \sqrt{\cos(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{bn} + \frac{2\sin(a+b\log(cx^n))}{bn\sqrt{\cos(a+b\log(cx^n))}}$$

Mathematica [A]
time = 0.17, size = 54, normalized size = 0.92

$$\frac{2\left(-E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right) + \frac{\sin(a+b\log(cx^n))}{\sqrt{\cos(a+b\log(cx^n))}}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Cos[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (2*(-EllipticE[(a + b*Log[c*x^n])/2, 2] + Sin[a + b*Log[c*x^n]]/Sqrt[Cos[a
+ b*Log[c*x^n]]]))/(b*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(93) = 186$.
time = 0.24, size = 250, normalized size = 4.24

method	result
derivativedivides	$-\frac{2\left(-2\cos\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+\sin^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)}\left(\sin^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right.}{n\sqrt{-2\left(\sin^4\left(\frac{a}{2}+\frac{b\ln(cx\right.}{2}\right.}}$
default	$-\frac{2\left(-2\cos\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+\sin^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)}\left(\sin^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right.}{n\sqrt{-2\left(\sin^4\left(\frac{a}{2}+\frac{b\ln(cx\right.}{2}\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/cos(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/n*(-2*cos(1/2*a+1/2*b*ln(c*x^n))*(-2*sin(1/2*a+1/2*b*ln(c*x^n))^4+sin(1/
2*a+1/2*b*ln(c*x^n))^2)^(1/2)*sin(1/2*a+1/2*b*ln(c*x^n))^2+(sin(1/2*a+1/2*b
*ln(c*x^n))^2)^(1/2)*(2*sin(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)*(-2*sin(1/2*a
+1/2*b*ln(c*x^n))^4+sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*EllipticE(cos(1/2*a
+1/2*b*ln(c*x^n)),2^(1/2)))/(-2*sin(1/2*a+1/2*b*ln(c*x^n))^4+sin(1/2*a+1/2*
b*ln(c*x^n))^2)^(1/2)/sin(1/2*a+1/2*b*ln(c*x^n))/(2*cos(1/2*a+1/2*b*ln(c*x^
n))^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*cos(b*log(c*x^n) + a)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.80, size = 150, normalized size = 2.54

$$\frac{-i\sqrt{2}\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+i\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)))+i\sqrt{2}\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-i\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)))+2\sqrt{\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{bn\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*cos(b*n*log(x) + b*log(c) + a)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(b*n*log(x) + b*log(c) + a) + I*sin(b*n*log(x) + b*log
(c) + a))) + I*sqrt(2)*cos(b*n*log(x) + b*log(c) + a)*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(b*n*log(x) + b*log(c) + a) - I*sin(b*n*log
(x) + b*log(c) + a))) + 2*sqrt(cos(b*n*log(x) + b*log(c) + a))*sin(b*n*log(
x) + b*log(c) + a))/(b*n*cos(b*n*log(x) + b*log(c) + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\cos^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Integral(1/(x*cos(a + b*log(c*x**n))**(3/2)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 2.67, size = 65, normalized size = 1.10

$$\frac{2\,\sin\left(a+b\,\ln\left(c\,x^n\right)\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(a+b\,\ln\left(c\,x^n\right)\right)^2\right)}{b\,n\,\sqrt{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}\,\sqrt{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cos(a + b*log(c*x^n))^(3/2)),x)
```

```
[Out] (2*sin(a + b*log(c*x^n))*hypergeom([-1/4, 1/2], 3/4, cos(a + b*log(c*x^n))^
2))/(b*n*cos(a + b*log(c*x^n))^(1/2)*(sin(a + b*log(c*x^n))^2)^(1/2))
```

3.120 $\int \frac{1}{\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=109

$$\frac{2x\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}{}_2F_1\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i}{bn}\right);\frac{1}{4}\left(9-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{(2+5ibn)\cos^{\frac{5}{2}}(a+b\log(cx^n))}$$

```
[Out] 2*x*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(5/2)*hypergeom([5/2, 5/4-1/2*I/b/n],[9/
4-1/2*I/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))/(2+5*I*b*n)/cos(a+b*ln(c*x^n))^(5
/2)
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4572, 4580, 371}

$$\frac{2x\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}{}_2F_1\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i}{bn}\right);\frac{1}{4}\left(9-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{(2+5ibn)\cos^{\frac{5}{2}}(a+b\log(cx^n))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*Log[c*x^n]]^(-5/2),x]
```

```
[Out] (2*x*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^(5/2)*Hypergeometric2F1[5/2, (5 -
(2*I)/(b*n))/4, (9 - (2*I)/(b*n))/4, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2
 + (5*I)*b*n)*Cos[a + b*Log[c*x^n]]^(5/2))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4572

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Cos[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4580

```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Dist[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
```

```
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\cos^{\frac{5}{2}}(a+b\log(x))}\,dx, x, cx^n\right)}{n}$$
$$= \frac{\left(x(cx^n)^{-\frac{5ib}{2}-\frac{1}{n}}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{5ib}{2}+\frac{1}{n}}}{\left(1+e^{2ia}x^{2ib}\right)^{5/2}}\,dx, x, cx^n\right)}{n\cos^{\frac{5}{2}}(a+b\log(cx^n))}$$
$$= \frac{2x\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}{}_2F_1\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i}{bn}\right);\frac{1}{4}\left(9-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{(2+5ibn)\cos^{\frac{5}{2}}(a+b\log(cx^n))}$$

Mathematica [A]
time = 2.56, size = 188, normalized size = 1.72

$$\frac{2x\left(\frac{(2-ibn)\sqrt{2+2e^{2ia}(cx^n)^{2ib}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}-\frac{i}{2bn};\frac{5}{4}-\frac{i}{2bn};-e^{2i(a+b\log(cx^n))}\right)}{\sqrt{e^{-ia}(cx^n)^{-ib}+e^{ia}(cx^n)^{ib}}}+\frac{-2\cos(a+b\log(cx^n))+bn\sin(a+b\log(cx^n))}{\cos^{\frac{3}{2}}(a+b\log(cx^n))}\right)}{3b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*Log[c*x^n]]^(-5/2),x]
```

```
[Out] (2*x*(((2 - I*b*n)*Sqrt[2 + 2*E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Hypergeometric
2F1[1/2, 1/4 - (I/2)/(b*n), 5/4 - (I/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n])
)])/Sqrt[1/(E^(I*a)*(c*x^n)^(I*b)) + E^(I*a)*(c*x^n)^(I*b)] + (-2*Cos[a + b
*Log[c*x^n]] + b*n*Sin[a + b*Log[c*x^n]])/Cos[a + b*Log[c*x^n]]^(3/2)))/(3*
b^2*n^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos\left(a+b\ln\left(c\,x^n\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a+b*ln(c*x^n))^(5/2),x)
```

```
[Out] int(1/cos(a+b*ln(c*x^n))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*log(c*x^n) + a)^(-5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*log(c*x^n))^(5/2),x)
```

```
[Out] int(1/cos(a + b*log(c*x^n))^(5/2), x)
```

3.121 $\int \frac{1}{x\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=63

$$\frac{2F\left(\frac{1}{2}(a+b\log\left(cx^n\right))\middle|\,2\right)}{3bn}+\frac{2\sin\left(a+b\log\left(cx^n\right)\right)}{3bn\cos^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

```
[Out] 2/3*(cos(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cos(1/2*a+1/2*b*ln(c*x^n))*Ellipti
cF(sin(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n+2/3*sin(a+b*ln(c*x^n))/b/n/cos(a
+b*ln(c*x^n))^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2716, 2720}

$$\frac{2F\left(\frac{1}{2}(a+b\log\left(cx^n\right))\middle|\,2\right)}{3bn}+\frac{2\sin\left(a+b\log\left(cx^n\right)\right)}{3bn\cos^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Cos[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (2*EllipticF[(a + b*Log[c*x^n])/2, 2])/(3*b*n) + (2*Sin[a + b*Log[c*x^n]])/
(3*b*n*Cos[a + b*Log[c*x^n]]^(3/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{x\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\cos^{\frac{5}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2\sin(a+b\log(cx^n))}{3bn\cos^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx, x, \log(cx^n)\right)}{3n}$$

$$= \frac{2F\left(\frac{1}{2}(a+b\log(cx^n))\middle|\,2\right)}{3bn} + \frac{2\sin(a+b\log(cx^n))}{3bn\cos^{\frac{3}{2}}(a+b\log(cx^n))}$$

Mathematica [A]
time = 0.19, size = 54, normalized size = 0.86

$$\frac{2\left(F\left(\frac{1}{2}(a+b\log(cx^n))\middle|\,2\right) + \frac{\sin(a+b\log(cx^n))}{\cos^{\frac{3}{2}}(a+b\log(cx^n))}\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Cos[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (2*(EllipticF[(a + b*Log[c*x^n])/2, 2] + Sin[a + b*Log[c*x^n]]/Cos[a + b*Lo
g[c*x^n]]^(3/2)))/(3*b*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(93) = 186$.
time = 0.26, size = 291, normalized size = 4.62

method	result
derivativedivides	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(a+b\ln(c\,x^n))}{2}}\,\sqrt{2\left(\sin^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)-1}\;\mathrm{EllipticF}\left(\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right),\sqrt{2}\right)(\mathrm{si}\right.}{}$
default	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(a+b\ln(c\,x^n))}{2}}\,\sqrt{2\left(\sin^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)-1}\;\mathrm{EllipticF}\left(\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right),\sqrt{2}\right)(\mathrm{si}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/cos(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/n*(-2*(sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(2*sin(1/2*a+1/2*b*ln(c*x^n
))^2-1)^(1/2)*EllipticF(cos(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))*sin(1/2*a+1/2*b
*ln(c*x^n))^2-2*sin(1/2*a+1/2*b*ln(c*x^n))^2*cos(1/2*a+1/2*b*ln(c*x^n))+(si
n(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(2*sin(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)*
EllipticF(cos(1/2*a+1/2*b*ln(c*x^n)),2^(1/2)))*((2*cos(1/2*a+1/2*b*ln(c*x^n
))^2-1)*sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/(-2*sin(1/2*a+1/2*b*ln(c*x^n))^
4+sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/(2*cos(1/2*a+1/2*b*ln(c*x^n))^2-1)^(3
/2)/sin(1/2*a+1/2*b*ln(c*x^n))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*cos(b*log(c*x^n) + a)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.42, size = 149, normalized size = 2.37

$$\frac{-i\sqrt{2}\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+i\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)+i\sqrt{2}\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-i\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)+2\sqrt{\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{3\,bn\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*cos(b*n*log(x) + b*log(c) + a)^2*weierstrassPInverse(-4, 0,
 cos(b*n*log(x) + b*log(c) + a) + I*sin(b*n*log(x) + b*log(c) + a)) + I*sqr
t(2)*cos(b*n*log(x) + b*log(c) + a)^2*weierstrassPInverse(-4, 0, cos(b*n*lo
g(x) + b*log(c) + a) - I*sin(b*n*log(x) + b*log(c) + a)) + 2*sqrt(cos(b*n*l
og(x) + b*log(c) + a))*sin(b*n*log(x) + b*log(c) + a))/(b*n*cos(b*n*log(x)
+ b*log(c) + a)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 2.71, size = 65, normalized size = 1.03

$$\frac{2\,\sin\left(a+b\,\ln\left(c\,x^n\right)\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(a+b\,\ln\left(c\,x^n\right)\right)^2\right)}{3\,b\,n\cos\left(a+b\,\ln\left(c\,x^n\right)\right)^{3/2}\,\sqrt{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cos(a + b*log(c*x^n))^(5/2)),x)
```

```
[Out] (2*sin(a + b*log(c*x^n))*hypergeom([-3/4, 1/2], 1/4, cos(a + b*log(c*x^n))^
2))/(3*b*n*cos(a + b*log(c*x^n))^(3/2)*(sin(a + b*log(c*x^n))^2)^(1/2))
```

3.122 $\int \frac{1}{\cos^{\frac{3}{2}}(a-2i\log(cx))}\,dx$

Optimal. Leaf size=48

$$-\frac{e^{-2ia}(1+c^4e^{2ia}x^4)}{2c^4x^3\cos^{\frac{3}{2}}(a-2i\log(cx))}$$

```
[Out] 1/2*(-1-c^4*exp(2*I*a)*x^4)/c^4/exp(2*I*a)/x^3/cos(a-2*I*ln(c*x))^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4572, 4570, 267}

$$-\frac{e^{-2ia}(1+e^{2ia}c^4x^4)}{2c^4x^3\cos^{\frac{3}{2}}(a-2i\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a - (2*I)*Log[c*x]]^(-3/2),x]
```

```
[Out] -1/2*(1 + c^4*E^((2*I)*a)*x^4)/(c^4*E^((2*I)*a)*x^3*Cos[a - (2*I)*Log[c*x]]
^(3/2))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4570

```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :> Dist[Cos[d*(a + b
*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), Int[(1 + E^(2*I*
a*d)*x^(2*I*b*d))^p/x^(I*b*d*p), x], x] /; FreeQ[{a, b, d, p}, x] &&  !Inte
gerQ[p]
```

Rule 4572

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Cos[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(a-2i\log(cx))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\cos^{\frac{3}{2}}(a-2i\log(x))}\,dx, x, cx\right)}{c}$$
$$= \frac{(1+c^4e^{2ia}x^4)^{3/2}\,\text{Subst}\left(\int \frac{x^3}{(1+e^{2ia}x^4)^{3/2}}\,dx, x, cx\right)}{c^4x^3\cos^{\frac{3}{2}}(a-2i\log(cx))}$$
$$= -\frac{e^{-2ia}(1+c^4e^{2ia}x^4)}{2c^4x^3\cos^{\frac{3}{2}}(a-2i\log(cx))}$$

Mathematica [A]
time = 0.12, size = 82, normalized size = 1.71

$$-\frac{x(\cos(a)-i\sin(a))\sqrt{\frac{2(1+c^4x^4)\cos(a)+2i(-1+c^4x^4)\sin(a)}{c^2x^2}}}{(1+c^4x^4)\cos(a)+i(-1+c^4x^4)\sin(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a - (2*I)*Log[c*x]]^(-3/2),x]
[Out] -((x*(Cos[a] - I*Sin[a])*Sqrt[(2*(1 + c^4*x^4)*Cos[a] + (2*I)*(-1 + c^4*x^4
)*Sin[a])/(c^2*x^2)])/((1 + c^4*x^4)*Cos[a] + I*(-1 + c^4*x^4)*Sin[a]))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos\left(a-2i\ln(cx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a-2*I*ln(c*x))^(3/2),x)
[Out] int(1/cos(a-2*I*ln(c*x))^(3/2),x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(36) = 72$.
time = 0.53, size = 187, normalized size = 3.90

$$-\frac{\left(\left(\sqrt{2}\cos\left(\frac{3}{2}a\right)+i\sqrt{2}\sin\left(\frac{3}{2}a\right)\right)c^4x^4+\sqrt{2}\cos\left(\frac{1}{2}a\right)-i\sqrt{2}\sin\left(\frac{1}{2}a\right)\right)\cos\left(\frac{3}{2}\arctan\left(c^4x^4\sin(2a),c^4x^4\cos(2a)+1\right)\right)+\left(\left(-i\sqrt{2}\cos\left(\frac{3}{2}a\right)+\sqrt{2}\sin\left(\frac{3}{2}a\right)\right)c^4x^4-i\sqrt{2}\cos\left(\frac{1}{2}a\right)-\sqrt{2}\sin\left(\frac{1}{2}a\right)\right)\sin\left(\frac{3}{2}\arctan\left(c^4x^4\sin(2a),c^4x^4\cos(2a)+1\right)\right)}{\left(\left(\cos(2a)^2+\sin(2a)^2\right)c^8x^8+2c^4x^4\cos(2a)+1\right)^{\frac{3}{4}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(a-2*I*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] -((((sqrt(2)*cos(3/2*a) + I*sqrt(2)*sin(3/2*a))*c^4*x^4 + sqrt(2)*cos(1/2*a)
 - I*sqrt(2)*sin(1/2*a))*cos(3/2*arctan2(c^4*x^4*sin(2*a), c^4*x^4*cos(2*a)
 + 1)) + ((-I*sqrt(2)*cos(3/2*a) + sqrt(2)*sin(3/2*a))*c^4*x^4 - I*sqrt(2)*
cos(1/2*a) - sqrt(2)*sin(1/2*a))*sin(3/2*arctan2(c^4*x^4*sin(2*a), c^4*x^4*
cos(2*a) + 1)))/(((cos(2*a)^2 + sin(2*a)^2)*c^8*x^8 + 2*c^4*x^4*cos(2*a) +
1)^(3/4)*c)
```

Fricas [A]
time = 1.92, size = 39, normalized size = 0.81

$$-\frac{2\sqrt{\frac{1}{2}}\,\sqrt{c^4x^4+e^{(-2i\,a)}}\,e^{\left(-\frac{3}{2}i\,a\right)}}{c^5x^4+ce^{(-2i\,a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(a-2*I*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(1/2)*sqrt(c^4*x^4 + e^(-2*I*a))*e^(-3/2*I*a)/(c^5*x^4 + c*e^(-2*I*a
))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos^{\frac{3}{2}}\left(a-2i\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(a-2*I*ln(c*x))**(3/2),x)
```

```
[Out] Integral(cos(a - 2*I*log(c*x))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(a-2*I*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(a - 2*I*log(c*x))^(-3/2), x)
```

Mupad [B]
time = 2.79, size = 48, normalized size = 1.00

$$-\frac{2\,x\,\sqrt{\frac{\mathrm{e}^{-a\,1\mathrm{i}}}{2\,c^2\,x^2}+\frac{c^2\,x^2\,\mathrm{e}^{a\,1\mathrm{i}}}{2}}}{\mathrm{e}^{a\,2\mathrm{i}}\,c^4\,x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a - log(c*x)*2i)^(3/2),x)
```

```
[Out] -(2*x*(exp(-a*1i)/(2*c^2*x^2) + (c^2*x^2*exp(a*1i))/2)^(1/2))/(c^4*x^4*exp(
a*2i) + 1)
```

3.123 $\int x^m \cos^4(a+b\log(cx^n))\,dx$

Optimal. Leaf size=266

$$\frac{24b^4n^4x^{1+m}}{(1+m)\left((1+m)^2+4b^2n^2\right)\left((1+m)^2+16b^2n^2\right)}+\frac{12b^2(1+m)n^2x^{1+m}\cos^2(a+b\log(cx^n))}{\left((1+m)^2+4b^2n^2\right)\left((1+m)^2+16b^2n^2\right)}+\frac{(1+m)x^{1+m}}{(1+}$$

```
[Out] 24*b^4*n^4*x^(1+m)/(1+m)/((1+m)^2+4*b^2*n^2)/((1+m)^2+16*b^2*n^2)+12*b^2*(1
+m)*n^2*x^(1+m)*cos(a+b*ln(c*x^n))^2/((1+m)^2+4*b^2*n^2)/((1+m)^2+16*b^2*n^
2)+(1+m)*x^(1+m)*cos(a+b*ln(c*x^n))^4/((1+m)^2+16*b^2*n^2)+24*b^3*n^3*x^(1+
m)*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))/((1+m)^2+4*b^2*n^2)/((1+m)^2+16*b^
2*n^2)+4*b*n*x^(1+m)*cos(a+b*ln(c*x^n))^3*sin(a+b*ln(c*x^n))/((1+m)^2+16*b^
2*n^2)
```

Rubi [A]
time = 0.09, antiderivative size = 260, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4576, 30}

$$\frac{(m+1)x^{m+1}\cos^4(a+b\log(cx^n))}{16b^2n^2+(m+1)^2}+\frac{4bnx^{m+1}\sin(a+b\log(cx^n))\cos^3(a+b\log(cx^n))}{16b^2n^2+(m+1)^2}+\frac{12b^2(m+1)n^2x^{m+1}\cos^2(a+b\log(cx^n))}{64b^4n^4+20b^2(m+1)^2n^2+(m+1)^4}+\frac{24b^3n^3x^{m+1}\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{64b^4n^4+20b^2(m+1)^2n^2+(m+1)^4}+\frac{24b^4n^4x^{m+1}}{(m+1)\left(4b^2n^2+(m+1)^2\right)\left(16b^2n^2+(m+1)^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cos[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (24*b^4*n^4*x^(1 + m))/((1 + m)*((1 + m)^2 + 4*b^2*n^2)*((1 + m)^2 + 16*b^2
*n^2)) + (12*b^2*(1 + m)*n^2*x^(1 + m)*Cos[a + b*Log[c*x^n]]^2)/((1 + m)^4
+ 20*b^2*(1 + m)^2*n^2 + 64*b^4*n^4) + ((1 + m)*x^(1 + m)*Cos[a + b*Log[c*x
^n]]^4)/((1 + m)^2 + 16*b^2*n^2) + (24*b^3*n^3*x^(1 + m)*Cos[a + b*Log[c*x^
n]]*Sin[a + b*Log[c*x^n]])/((1 + m)^4 + 20*b^2*(1 + m)^2*n^2 + 64*b^4*n^4)
+ (4*b*n*x^(1 + m)*Cos[a + b*Log[c*x^n]]^3*Sin[a + b*Log[c*x^n]])/((1 + m)^
2 + 16*b^2*n^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4576

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2
*n^2*p^2 + (m + 1)^2)), Int[(e*x)^m*Cos[d*(a + b*Log[c*x^n])]^(p - 2), x],
x] + Simp[b*d*n*p*(e*x)^(m + 1)*Sin[d*(a + b*Log[c*x^n])]*(Cos[d*(a + b*Log
[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x]) /; FreeQ[{a, b, c
, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^m \cos^4(a+b\log(cx^n))\,dx &= \frac{(1+m)x^{1+m}\cos^4(a+b\log(cx^n))}{(1+m)^2+16b^2n^2} + \frac{4bnx^{1+m}\cos^3(a+b\log(cx^n))\sin(a+\cdots}{(1+m)^2+16b^2n^2} \\
&= \frac{12b^2(1+m)n^2x^{1+m}\cos^2(a+b\log(cx^n))}{(1+m)^4+20b^2(1+m)^2n^2+64b^4n^4} + \frac{(1+m)x^{1+m}\cos^4(a+b\log(cx\cdots}{(1+m)^2+16b^2n^2} \\
&= \frac{24b^4n^4x^{1+m}}{(1+m)\left((1+m)^4+20b^2(1+m)^2n^2+64b^4n^4\right)} + \frac{12b^2(1+m)n^2x^{1+m}\cos^2\cdots}{(1+m)^4+20b^2(1+m\cdots}
\end{aligned}
$$

Mathematica [A]
time = 4.26, size = 312, normalized size = 1.17

$$
\frac{1}{8}x^{1+m}\left(\frac{3}{1+m}-\frac{4\sin(2bn\log(x))\left(-2bn\cos\left(2(a-bn\log(x)+b\log\left(cx^n\right))\right)+(1+m)\sin\left(2(a-bn\log(x)+b\log\left(cx^n\right))\right)\right)}{1+2m+m^2+4b^2n^2}+\frac{4\cos(2bn\log(x))\left((1+m)\cos\left(2(a-bn\log(x)+b\log\left(cx^n\right))\right)+2bn\sin\left(2(a-bn\log(x)+b\log\left(cx^n\right))\right)\right)}{1+2m+m^2+4b^2n^2}-\frac{\sin(4bn\log(x))\left(-4bn\cos\left(4(a-bn\log(x)+b\log\left(cx^n\right))\right)+(1+m)\sin\left(4(a-bn\log(x)+b\log\left(cx^n\right))\right)\right)}{1+2m+m^2+16b^2n^2}+\frac{\cos(4bn\log(x))\left((1+m)\cos\left(4(a-bn\log(x)+b\log\left(cx^n\right))\right)+4bn\sin\left(4(a-bn\log(x)+b\log\left(cx^n\right))\right)\right)}{1+2m+m^2+16b^2n^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cos[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x^(1 + m)*(3/(1 + m) - (4*Sin[2*b*n*Log[x]]*(-2*b*n*Cos[2*(a - b*n*Log[x]
+ b*Log[c*x^n])] + (1 + m)*Sin[2*(a - b*n*Log[x] + b*Log[c*x^n])]))/(1 + 2*
m + m^2 + 4*b^2*n^2) + (4*Cos[2*b*n*Log[x]]*((1 + m)*Cos[2*(a - b*n*Log[x]
+ b*Log[c*x^n])] + 2*b*n*Sin[2*(a - b*n*Log[x] + b*Log[c*x^n])]))/(1 + 2*m
+ m^2 + 4*b^2*n^2) - (Sin[4*b*n*Log[x]]*(-4*b*n*Cos[4*(a - b*n*Log[x] + b*L
og[c*x^n])] + (1 + m)*Sin[4*(a - b*n*Log[x] + b*Log[c*x^n])]))/(1 + 2*m + m
^2 + 16*b^2*n^2) + (Cos[4*b*n*Log[x]]*((1 + m)*Cos[4*(a - b*n*Log[x] + b*Lo
g[c*x^n])] + 4*b*n*Sin[4*(a - b*n*Log[x] + b*Log[c*x^n])]))/(1 + 2*m + m^2
+ 16*b^2*n^2)))/8
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$
\int x^m\left(\cos^4\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a+b*ln(c*x^n))^4,x)
```

```
[Out] int(x^m*cos(a+b*ln(c*x^n))^4,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3537 vs. 2(266) = 532.
time = 0.47, size = 3537, normalized size = 13.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

```
[Out] 1/16*(((cos(8*b*log(c))*cos(4*b*log(c)) + sin(8*b*log(c))*sin(4*b*log(c)) +
 cos(4*b*log(c)))*m^4 + 4*(cos(8*b*log(c))*cos(4*b*log(c)) + sin(8*b*log(c)
)*sin(4*b*log(c)) + cos(4*b*log(c)))*m^3 + 16*(b^3*cos(4*b*log(c))*sin(8*b*
log(c)) - b^3*cos(8*b*log(c))*sin(4*b*log(c)) + b^3*sin(4*b*log(c)) + (b^3*
cos(4*b*log(c))*sin(8*b*log(c)) - b^3*cos(8*b*log(c))*sin(4*b*log(c)) + b^3
*sin(4*b*log(c)))*m)*n^3 + 6*(cos(8*b*log(c))*cos(4*b*log(c)) + sin(8*b*log
(c))*sin(4*b*log(c)) + cos(4*b*log(c)))*m^2 + 4*(b^2*cos(8*b*log(c))*cos(4*
b*log(c)) + b^2*sin(8*b*log(c))*sin(4*b*log(c)) + (b^2*cos(8*b*log(c))*cos(
4*b*log(c)) + b^2*sin(8*b*log(c))*sin(4*b*log(c)) + b^2*cos(4*b*log(c)))*m^
2 + b^2*cos(4*b*log(c)) + 2*(b^2*cos(8*b*log(c))*cos(4*b*log(c)) + b^2*sin(
8*b*log(c))*sin(4*b*log(c)) + b^2*cos(4*b*log(c)))*m)*n^2 + 4*(cos(8*b*log(
c))*cos(4*b*log(c)) + sin(8*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c)))*m
+ 4*((b*cos(4*b*log(c))*sin(8*b*log(c)) - b*cos(8*b*log(c))*sin(4*b*log(c))
 + b*sin(4*b*log(c)))*m^3 + 3*(b*cos(4*b*log(c))*sin(8*b*log(c)) - b*cos(8*
b*log(c))*sin(4*b*log(c)) + b*sin(4*b*log(c)))*m^2 + b*cos(4*b*log(c))*sin(
8*b*log(c)) - b*cos(8*b*log(c))*sin(4*b*log(c)) + 3*(b*cos(4*b*log(c))*sin(
8*b*log(c)) - b*cos(8*b*log(c))*sin(4*b*log(c)) + b*sin(4*b*log(c)))*m + b*
sin(4*b*log(c)))*n + cos(8*b*log(c))*cos(4*b*log(c)) + sin(8*b*log(c))*sin(
4*b*log(c)) + cos(4*b*log(c)))*x*x^m*cos(4*b*log(x^n) + 4*a) + 4*((cos(6*b*
log(c))*cos(4*b*log(c)) + cos(4*b*log(c))*cos(2*b*log(c)) + sin(6*b*log(c))
*sin(4*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*m^4 + 4*(cos(6*b*log(c)
)*cos(4*b*log(c)) + cos(4*b*log(c))*cos(2*b*log(c)) + sin(6*b*log(c))*sin(4
*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*m^3 + 32*(b^3*cos(4*b*log(c))
*sin(6*b*log(c)) - b^3*cos(6*b*log(c))*sin(4*b*log(c)) + b^3*cos(2*b*log(c)
)*sin(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(2*b*log(c)) + (b^3*cos(4*b*log(
c))*sin(6*b*log(c)) - b^3*cos(6*b*log(c))*sin(4*b*log(c)) + b^3*cos(2*b*log
(c))*sin(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(2*b*log(c)))*m)*n^3 + 6*(cos
(6*b*log(c))*cos(4*b*log(c)) + cos(4*b*log(c))*cos(2*b*log(c)) + sin(6*b*lo
g(c))*sin(4*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*m^2 + 16*(b^2*cos(
6*b*log(c))*cos(4*b*log(c)) + b^2*cos(4*b*log(c))*cos(2*b*log(c)) + b^2*sin
(6*b*log(c))*sin(4*b*log(c)) + b^2*sin(4*b*log(c))*sin(2*b*log(c)) + (b^2*c
os(6*b*log(c))*cos(4*b*log(c)) + b^2*cos(4*b*log(c))*cos(2*b*log(c)) + b^2*
sin(6*b*log(c))*sin(4*b*log(c)) + b^2*sin(4*b*log(c))*sin(2*b*log(c)))*m^2
+ 2*(b^2*cos(6*b*log(c))*cos(4*b*log(c)) + b^2*cos(4*b*log(c))*cos(2*b*log(
c)) + b^2*sin(6*b*log(c))*sin(4*b*log(c)) + b^2*sin(4*b*log(c))*sin(2*b*log
(c)))*m)*n^2 + 4*(cos(6*b*log(c))*cos(4*b*log(c)) + cos(4*b*log(c))*cos(2*b
*log(c)) + sin(6*b*log(c))*sin(4*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)
))*m + 2*((b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b*lo
g(c)) + b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c
)))*m^3 + 3*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b*
log(c)) + b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log
```

```
(c)))*m^2 + b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b*l
og(c)) + b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(
c)) + 3*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b*log(
c)) + b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c))
)*m)*n + cos(6*b*log(c))*cos(4*b*log(c)) + cos(4*b*log(c))*cos(2*b*log(c))
 + sin(6*b*log(c))*sin(4*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*x*x^m*
cos(2*b*log(x^n) + 2*a) - ((cos(4*b*log(c))*sin(8*b*log(c)) - cos(8*b*log(c
))*sin(4*b*log(c)) + sin(4*b*log(c)))*m^4 + 4*(cos(4*b*log(c))*sin(8*b*log(
c)) - cos(8*b*log(c))*sin(4*b*log(c)) + sin(4*b*log(c)))*m^3 - 16*(b^3*cos(
8*b*log(c))*cos(4*b*log(c)) + b^3*sin(8*b*log(c))*sin(4*b*log(c)) + b^3*cos
(4*b*log(c)) + (b^3*cos(8*b*log(c))*cos(4*b*log(c)) + b^3*sin(8*b*log(c))*s
in(4*b*log(c)) + b^3*cos(4*b*log(c)))*m)*n^3 + 6*(cos(4*b*log(c))*sin(8*b*l
og(c)) - cos(8*b*log(c))*sin(4*b*log(c)) + sin(4*b*log(c)))*m^2 + 4*(b^2*co
s(4*b*log(c))*sin(8*b*log(c)) - b^2*cos(8*b*log(c))*sin(4*b*log(c)) + (b^2*
cos(4*b*log(c))*sin(8*b*log(c)) - b^2*cos(8*b*log(c))*sin(4*b*log(c)) + b^2
*sin(4*b*log(c)))*m^2 + b^2*sin(4*b*log(c)) + 2*(b^2*cos(4*b*log(c))*sin(8*
b*log(c)) - b^2*cos(8*b*log(c))*sin(4*b*log(c)) + b^2*sin(4*b*log(c)))*m)*n
^2 + 4*(cos(4*b*log(c))*sin(8*b*log(c)) - cos(8*b*log(c))*sin(4*b*log(c)) +
 sin(4*b*log(c)))*m - 4*((b*cos(8*b*log(c))*cos(4*b*log(c)) + b*sin(8*b*log
(c))*sin(4*b*log(c)) + b*cos(4*b*log(c)))*m^3 + 3*(b*cos(8*b*log(c))*cos(4*
b*log(c)) + b*sin(8*b*log(c))*sin(4*b*log(c)) + b*cos(4*b*log(c)))*m^2 + b*
cos(8*b*log(c))*cos(4*b*log(c)) + b*sin(8*b*log(c))*sin(4*b*log(c)) + 3*(b*
cos(8*b*log(c))*cos(4*b*log(c)) + b*sin(8*b*log(c))*sin(4*b*log(c)) + b*cos
(4*b*log(c)))*m + b*cos(4*b*log(c)))*n + cos(4*b*log(c))*sin(8*b*log(c)) -
cos(8*b*log(c))*sin(4*b*log(c)) + sin(4*b*log(c)))*x*x^m*sin(4*b*log(x^n) +
 4*a) - 4*((cos(4*b*log(c))*sin(6*b*log(c)) - c...
```

Fricas [A]
time = 2.27, size = 273, normalized size = 1.03

$$\frac{4\,{\left(6\,{\left(b^{3} m + b^{3}\right)} n^{3} x \cos\left(b n \log\left(x\right) + b \log\left(c\right) + a\right) + {\left(4\,{\left(b^{3} m + b^{3}\right)} n^{3} + {\left(b m^{3} + 3\,b m^{2} + 3\,b m + b\right)} n\right)} x \cos\left(b n \log\left(x\right) + b \log\left(c\right) + a\right)^{3}\right)} x^{m} \sin\left(b n \log\left(x\right) + b \log\left(c\right) + a\right) + {\left(24\,b^{4} n^{4} x + 12\,{\left(b^{2} m^{2} + 2\,b^{2} m + b^{2}\right)} n^{2} x \cos\left(b n \log\left(x\right) + b \log\left(c\right) + a\right)^{2} + {\left(m^{4} + 4\,m^{3} + 4\,{\left(b^{2} m^{2} + 2\,b^{2} m + b^{2}\right)} n^{2} + 6\,m^{2} + 4\,m + 1\right)} x \cos\left(b n \log\left(x\right) + b \log\left(c\right) + a\right)^{4}\right)} x^{m}}{m^{5} + 64\,{\left(b^{4} m + b^{4}\right)} n^{4} + 5\,m^{4} + 10\,m^{3} + 20\,{\left(b^{2} m^{3} + 3\,b^{2} m^{2} + 3\,b^{2} m + b^{2}\right)} n^{2} + 10\,m^{2} + 5\,m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] (4*(6*(b^3*m + b^3)*n^3*x*cos(b*n*log(x) + b*log(c) + a) + (4*(b^3*m + b^3)
*n^3 + (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cos(b*n*log(x) + b*log(c) + a)^3)
*x^m*sin(b*n*log(x) + b*log(c) + a) + (24*b^4*n^4*x + 12*(b^2*m^2 + 2*b^2*m
 + b^2)*n^2*x*cos(b*n*log(x) + b*log(c) + a)^2 + (m^4 + 4*m^3 + 4*(b^2*m^2
+ 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cos(b*n*log(x) + b*log(c) + a)^4)
*x^m)/(m^5 + 64*(b^4*m + b^4)*n^4 + 5*m^4 + 10*m^3 + 20*(b^2*m^3 + 3*b^2*m^
2 + 3*b^2*m + b^2)*n^2 + 10*m^2 + 5*m + 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cos(a+b*ln(c*x**n))**4,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 225232 vs. $2(266) = 532$.
time = 5.25, size = 225232, normalized size = 846.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] -1/16*(384*b^4*n^4*x*abs(x)^m*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*ta
n(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(
2*a)^2*tan(a)^2 - 256*b^3*m*n^3*x*abs(x)^m*e^(pi*b*n*sgn(x) - pi*b*n + pi*b
*sgn(c) - pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(
x)) + b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(2*a)^2*tan(a)
 - 256*b^3*m*n^3*x*abs(x)^m*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)
*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs
(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(2*a)^2*tan(a) - 32*b^3*m*n^3*
x*abs(x)^m*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b*
n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*t
an(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(2*a)*tan(a)^2 - 32*b^3*m*n^3*x*abs(x)^
m*e^(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c) + 2*pi*b)*tan(2*b*n*log(ab
s(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4*p
i*m*sgn(x) - 1/4*pi*m)^2*tan(2*a)*tan(a)^2 + 32*b^3*m*n^3*x*abs(x)^m*e^(2*p
i*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b*n*log(abs(x)) + 2
*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x
) - 1/4*pi*m)*tan(2*a)^2*tan(a)^2 + 256*b^3*m*n^3*x*abs(x)^m*e^(pi*b*n*sgn(
x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^
2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)*ta
n(2*a)^2*tan(a)^2 - 256*b^3*m*n^3*x*abs(x)^m*e^(-pi*b*n*sgn(x) + pi*b*n - p
i*b*sgn(c) + pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(a
bs(x)) + b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)*tan(2*a)^2*tan(a)
^2 - 32*b^3*m*n^3*x*abs(x)^m*e^(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c)
 + 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) +
 b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)*tan(2*a)^2*tan(a)^2 - 256
*b^3*m*n^3*x*abs(x)^m*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(2
*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))*
tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(2*a)^2*tan(a)^2 - 256*b^3*m*n^3*x*abs
(x)^m*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(2*b*n*log(abs(x)
) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(1/4*pi*m*sg
n(x) - 1/4*pi*m)^2*tan(2*a)^2*tan(a)^2 - 32*b^3*m*n^3*x*abs(x)^m*e^(2*pi*b*
```

```
n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*l
og(abs(c)))*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/
4*pi*m)^2*tan(2*a)^2*tan(a)^2 - 32*b^3*m*n^3*x*abs(x)^m*e^(-2*pi*b*n*sgn(x)
 + 2*pi*b*n - 2*pi*b*sgn(c) + 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c
)))*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^
2*tan(2*a)^2*tan(a)^2 + 4*b^2*m^2*n^2*x*abs(x)^m*e^(2*pi*b*n*sgn(x) - 2*pi*
b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*ta
n(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(
2*a)^2*tan(a)^2 + 64*b^2*m^2*n^2*x*abs(x)^m*e^(pi*b*n*sgn(x) - pi*b*n + pi*
b*sgn(c) - pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs
(x)) + b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(2*a)^2*tan(a)
^2 + 64*b^2*m^2*n^2*x*abs(x)^m*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + p
i*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log
(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*m
^2*n^2*x*abs(x)^m*e^(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2*pi*b*sgn(c) + 2*pi*b)*
tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(
c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(2*a)^2*tan(a)^2 + 384*b^4*n^4*
x*abs(x)^m*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) +
 b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(2*a)^2 - 256*b^3*n^
3*x*abs(x)^m*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(2*b*n*log(
abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4
*pi*m*sgn(x) - 1/4*pi*m)^2*tan(2*a)^2*tan(a) - 256*b^3*n^3*x*abs(x)^m*e^(-p
i*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log
(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/
4*pi*m)^2*tan(2*a)^2*tan(a) + 384*b^4*n^4*x*abs(x)^m*tan(2*b*n*log(abs(x))
+ 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4*pi*m*sg
n(x) - 1/4*pi*m)^2*tan(a)^2 - 32*b^3*n^3*x*abs(x)^m*e^(2*pi*b*n*sgn(x) - 2*
pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2
*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*t
an(2*a)*tan(a)^2 - 32*b^3*n^3*x*abs(x)^m*e^(-2*pi*b*n*sgn(x) + 2*pi*b*n - 2
*pi*b*sgn(c) + 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*l
og(abs(x)) + b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(2*a)*ta
n(a)^2 - 384*b^4*n^4*x*abs(x)^m*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*
tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a)^2 + 32*b^3*n^3*x*a
bs(x)^m*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b*n*l
og(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs...
```

Mupad [B]
time = 3.59, size = 152, normalized size = 0.57

$$\frac{3\,x\,x^m}{8\,m+8}+\frac{x\,x^m\,{\mathrm{e}}^{a\,2{}\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2{}\mathrm{i}}}{4\,m+4+b\,n\,8{}\mathrm{i}}+\frac{x\,x^m\,{\mathrm{e}}^{-a\,2{}\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,2{}\mathrm{i}}}\,1{}\mathrm{i}}{m\,4{}\mathrm{i}+8\,b\,n+4{}\mathrm{i}}+\frac{x\,x^m\,{\mathrm{e}}^{a\,4{}\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,4{}\mathrm{i}}}{16\,m+16+b\,n\,64{}\mathrm{i}}+\frac{x\,x^m\,{\mathrm{e}}^{-a\,4{}\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,4{}\mathrm{i}}}\,1{}\mathrm{i}}{m\,16{}\mathrm{i}+64\,b\,n+16{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a + b*log(c*x^n))^4,x)
```

```
[Out] (3*x*x^m)/(8*m + 8) + (x*x^m*exp(a*2i)*(c*x^n)^(b*2i))/(4*m + b*n*8i + 4) +
 (x*x^m*exp(-a*2i)/(c*x^n)^(b*2i)*1i)/(m*4i + 8*b*n + 4i) + (x*x^m*exp(a*4i
)*(c*x^n)^(b*4i))/(16*m + b*n*64i + 16) + (x*x^m*exp(-a*4i)/(c*x^n)^(b*4i)*
1i)/(m*16i + 64*b*n + 16i)
```

3.124 $\int x^m \cos^3(a + b\log(cx^n))\, dx$

Optimal. Leaf size=201

$$\frac{6b^2(1+m)n^2x^{1+m}\cos(a+b\log(cx^n))}{((1+m)^2+b^2n^2)((1+m)^2+9b^2n^2)}+\frac{(1+m)x^{1+m}\cos^3(a+b\log(cx^n))}{(1+m)^2+9b^2n^2}+\frac{6b^3n^3x^{1+m}\sin(a+b\log(cx}{((1+m)^2+b^2n^2)((1+m)^2+}$$

```
[Out] 6*b^2*(1+m)*n^2*x^(1+m)*cos(a+b*ln(c*x^n))/((1+m)^2+b^2*n^2)/((1+m)^2+9*b^2
*n^2)+(1+m)*x^(1+m)*cos(a+b*ln(c*x^n))^3/((1+m)^2+9*b^2*n^2)+6*b^3*n^3*x^(1
+m)*sin(a+b*ln(c*x^n))/((1+m)^2+b^2*n^2)/((1+m)^2+9*b^2*n^2)+3*b*n*x^(1+m)*
cos(a+b*ln(c*x^n))^2*sin(a+b*ln(c*x^n))/((1+m)^2+9*b^2*n^2)
```

Rubi [A]

time = 0.05, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4576, 4574}

$$\frac{(m+1)x^{m+1}\cos^3(a+b\log(cx^n))}{9b^2n^2+(m+1)^2}+\frac{6b^2(m+1)n^2x^{m+1}\cos(a+b\log(cx^n))}{(b^2n^2+(m+1)^2)(9b^2n^2+(m+1)^2)}+\frac{3bnx^{m+1}\sin(a+b\log(cx^n))\cos^2(a+b\log(cx^n))}{9b^2n^2+(m+1)^2}+\frac{6b^3n^3x^{m+1}\sin(a+b\log(cx^n))}{(b^2n^2+(m+1)^2)(9b^2n^2+(m+1)^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cos[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (6*b^2*(1 + m)*n^2*x^(1 + m)*Cos[a + b*Log[c*x^n]])/(((1 + m)^2 + b^2*n^2)*
((1 + m)^2 + 9*b^2*n^2)) + ((1 + m)*x^(1 + m)*Cos[a + b*Log[c*x^n]]^3)/((1
+ m)^2 + 9*b^2*n^2) + (6*b^3*n^3*x^(1 + m)*Sin[a + b*Log[c*x^n]])/(((1 + m)
^2 + b^2*n^2)*((1 + m)^2 + 9*b^2*n^2)) + (3*b*n*x^(1 + m)*Cos[a + b*Log[c*x
^n]]^2*Sin[a + b*Log[c*x^n]])/((1 + m)^2 + 9*b^2*n^2)
```

Rule 4574

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Rule 4576

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2
*n^2*p^2 + (m + 1)^2)), Int[(e*x)^m*Cos[d*(a + b*Log[c*x^n])]^(p - 2), x],
x] + Simp[b*d*n*p*(e*x)^(m + 1)*Sin[d*(a + b*Log[c*x^n])]*(Cos[d*(a + b*Log
[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x]) /; FreeQ[{a, b, c
, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^m \cos^3(a+b\log(cx^n))\,dx &= \frac{(1+m)x^{1+m}\cos^3(a+b\log(cx^n))}{(1+m)^2+9b^2n^2}+\frac{3bnx^{1+m}\cos^2(a+b\log(cx^n))\sin(a}{(1+m)^2+9b^2n^2} \\
&= \frac{6b^2(1+m)n^2x^{1+m}\cos(a+b\log(cx^n))}{((1+m)^2+b^2n^2)((1+m)^2+9b^2n^2)}+\frac{(1+m)x^{1+m}\cos^3(a+b\log(cx^{}}{(1+m)^2+9b^2n^2}
\end{aligned}
$$

Mathematica [A]
time = 2.05, size = 292, normalized size = 1.45

$$
\frac{1}{4}x^{1+m}\left(-\frac{3\sin(bn\log(x))\left(-bn\cos(a-bn\log(x)+b\log(cx^n))+(1+m)\sin(a-bn\log(x)+b\log(cx^n))\right)}{1+2m+m^2+b^2n^2}+\frac{3\cos(bn\log(x))\left((1+m)\cos(a-bn\log(x)+b\log(cx^n))+bn\sin(a-bn\log(x)+b\log(cx^n))\right)}{1+2m+m^2+b^2n^2}-\frac{\sin(3bn\log(x))\left(-3bn\cos(3(a-bn\log(x)+b\log(cx^n)))+(1+m)\sin(3(a-bn\log(x)+b\log(cx^n)))\right)}{1+2m+m^2+9b^2n^2}+\frac{\cos(3bn\log(x))\left((1+m)\cos(3(a-bn\log(x)+b\log(cx^n)))+3bn\sin(3(a-bn\log(x)+b\log(cx^n)))\right)}{1+2m+m^2+9b^2n^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cos[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (x^(1 + m)*((-3*Sin[b*n*Log[x]]*(-(b*n*Cos[a - b*n*Log[x] + b*Log[c*x^n]])
+ (1 + m)*Sin[a - b*n*Log[x] + b*Log[c*x^n]]))/(1 + 2*m + m^2 + b^2*n^2) +
(3*Cos[b*n*Log[x]]*((1 + m)*Cos[a - b*n*Log[x] + b*Log[c*x^n]] + b*n*Sin[a
- b*n*Log[x] + b*Log[c*x^n]]))/(1 + 2*m + m^2 + b^2*n^2) - (Sin[3*b*n*Log[x
]]*(-3*b*n*Cos[3*(a - b*n*Log[x] + b*Log[c*x^n])] + (1 + m)*Sin[3*(a - b*n*
Log[x] + b*Log[c*x^n])]))/(1 + 2*m + m^2 + 9*b^2*n^2) + (Cos[3*b*n*Log[x]]*
((1 + m)*Cos[3*(a - b*n*Log[x] + b*Log[c*x^n])] + 3*b*n*Sin[3*(a - b*n*Log[
x] + b*Log[c*x^n])]))/(1 + 2*m + m^2 + 9*b^2*n^2)))/4
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$
\int x^m\left(\cos^3(a+b\ln(c\,x^n))\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(x^m*cos(a+b*ln(c*x^n))^3,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2352 vs. 2(201) = 402.
time = 0.40, size = 2352, normalized size = 11.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(((cos(6*b*log(c))*cos(3*b*log(c)) + sin(6*b*log(c))*sin(3*b*log(c)) +
cos(3*b*log(c)))*m^3 + 3*(b^3*cos(3*b*log(c))*sin(6*b*log(c)) - b^3*cos(6*b
*log(c))*sin(3*b*log(c)) + b^3*sin(3*b*log(c)))*n^3 + 3*(cos(6*b*log(c))*co
s(3*b*log(c)) + sin(6*b*log(c))*sin(3*b*log(c)) + cos(3*b*log(c)))*m^2 + (b
^2*cos(6*b*log(c))*cos(3*b*log(c)) + b^2*sin(6*b*log(c))*sin(3*b*log(c)) +
b^2*cos(3*b*log(c)) + (b^2*cos(6*b*log(c))*cos(3*b*log(c)) + b^2*sin(6*b*lo
g(c))*sin(3*b*log(c)) + b^2*cos(3*b*log(c)))*m)*n^2 + 3*(cos(6*b*log(c))*co
s(3*b*log(c)) + sin(6*b*log(c))*sin(3*b*log(c)) + cos(3*b*log(c)))*m + 3*((
b*cos(3*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(3*b*log(c)) + b*s
in(3*b*log(c)))*m^2 + b*cos(3*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))
*sin(3*b*log(c)) + 2*(b*cos(3*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))
*sin(3*b*log(c)) + b*sin(3*b*log(c)))*m + b*sin(3*b*log(c)))*n + cos(6*b*lo
g(c))*cos(3*b*log(c)) + sin(6*b*log(c))*sin(3*b*log(c)) + cos(3*b*log(c)))*
x*x^m*cos(3*b*log(x^n) + 3*a) + 3*((cos(4*b*log(c))*cos(3*b*log(c)) + cos(3
*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(3*b*log(c)) + sin(3*b*log(
c))*sin(2*b*log(c)))*m^3 + 9*(b^3*cos(3*b*log(c))*sin(4*b*log(c)) - b^3*cos
(4*b*log(c))*sin(3*b*log(c)) + b^3*cos(2*b*log(c))*sin(3*b*log(c)) - b^3*co
s(3*b*log(c))*sin(2*b*log(c)))*n^3 + 3*(cos(4*b*log(c))*cos(3*b*log(c)) + c
os(3*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(3*b*log(c)) + sin(3*b*
log(c))*sin(2*b*log(c)))*m^2 + 9*(b^2*cos(4*b*log(c))*cos(3*b*log(c)) + b^2
*cos(3*b*log(c))*cos(2*b*log(c)) + b^2*sin(4*b*log(c))*sin(3*b*log(c)) + b^
2*sin(3*b*log(c))*sin(2*b*log(c)) + (b^2*cos(4*b*log(c))*cos(3*b*log(c)) +
b^2*cos(3*b*log(c))*cos(2*b*log(c)) + b^2*sin(4*b*log(c))*sin(3*b*log(c)) +
 b^2*sin(3*b*log(c))*sin(2*b*log(c)))*m)*n^2 + 3*(cos(4*b*log(c))*cos(3*b*l
og(c)) + cos(3*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(3*b*log(c))
+ sin(3*b*log(c))*sin(2*b*log(c)))*m + ((b*cos(3*b*log(c))*sin(4*b*log(c))
- b*cos(4*b*log(c))*sin(3*b*log(c)) + b*cos(2*b*log(c))*sin(3*b*log(c)) - b
*cos(3*b*log(c))*sin(2*b*log(c)))*m^2 + b*cos(3*b*log(c))*sin(4*b*log(c)) -
 b*cos(4*b*log(c))*sin(3*b*log(c)) + b*cos(2*b*log(c))*sin(3*b*log(c)) - b*
cos(3*b*log(c))*sin(2*b*log(c)) + 2*(b*cos(3*b*log(c))*sin(4*b*log(c)) - b*
cos(4*b*log(c))*sin(3*b*log(c)) + b*cos(2*b*log(c))*sin(3*b*log(c)) - b*cos
(3*b*log(c))*sin(2*b*log(c)))*m)*n + cos(4*b*log(c))*cos(3*b*log(c)) + cos(
3*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(3*b*log(c)) + sin(3*b*log
(c))*sin(2*b*log(c)))*x*x^m*cos(b*log(x^n) + a) - ((cos(3*b*log(c))*sin(6*b
*log(c)) - cos(6*b*log(c))*sin(3*b*log(c)) + sin(3*b*log(c)))*m^3 - 3*(b^3*
cos(6*b*log(c))*cos(3*b*log(c)) + b^3*sin(6*b*log(c))*sin(3*b*log(c)) + b^3
*cos(3*b*log(c)))*n^3 + 3*(cos(3*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c)
)*sin(3*b*log(c)) + sin(3*b*log(c)))*m^2 + (b^2*cos(3*b*log(c))*sin(6*b*log
(c)) - b^2*cos(6*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c)) + (b^2*cos
(3*b*log(c))*sin(6*b*log(c)) - b^2*cos(6*b*log(c))*sin(3*b*log(c)) + b^2*si
n(3*b*log(c)))*m)*n^2 + 3*(cos(3*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c)
)*sin(3*b*log(c)) + sin(3*b*log(c)))*m - 3*((b*cos(6*b*log(c))*cos(3*b*log(
c)) + b*sin(6*b*log(c))*sin(3*b*log(c)) + b*cos(3*b*log(c)))*m^2 + b*cos(6*
b*log(c))*cos(3*b*log(c)) + b*sin(6*b*log(c))*sin(3*b*log(c)) + 2*(b*cos(6*
b*log(c))*cos(3*b*log(c)) + b*sin(6*b*log(c))*sin(3*b*log(c)) + b*cos(3*b*l
```

```
og(c)))*m + b*cos(3*b*log(c)))*n + cos(3*b*log(c))*sin(6*b*log(c)) - cos(6*
b*log(c))*sin(3*b*log(c)) + sin(3*b*log(c)))*x*x^m*sin(3*b*log(x^n) + 3*a)
- 3*((cos(3*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(3*b*log(c)) + c
os(2*b*log(c))*sin(3*b*log(c)) - cos(3*b*log(c))*sin(2*b*log(c)))*m^3 - 9*(
b^3*cos(4*b*log(c))*cos(3*b*log(c)) + b^3*cos(3*b*log(c))*cos(2*b*log(c)) +
 b^3*sin(4*b*log(c))*sin(3*b*log(c)) + b^3*sin(3*b*log(c))*sin(2*b*log(c)))
*n^3 + 3*(cos(3*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(3*b*log(c))
 + cos(2*b*log(c))*sin(3*b*log(c)) - cos(3*b*log(c))*sin(2*b*log(c)))*m^2 +
 9*(b^2*cos(3*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(3*b*log(c
)) + b^2*cos(2*b*log(c))*sin(3*b*log(c)) - b^2*cos(3*b*log(c))*sin(2*b*log(
c)) + (b^2*cos(3*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(3*b*lo
g(c)) + b^2*cos(2*b*log(c))*sin(3*b*log(c)) - b^2*cos(3*b*log(c))*sin(2*b*l
og(c)))*m)*n^2 + 3*(cos(3*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(3
*b*log(c)) + cos(2*b*log(c))*sin(3*b*log(c)) - cos(3*b*log(c))*sin(2*b*log(
c)))*m - ((b*cos(4*b*log(c))*cos(3*b*log(c)) + b*cos(3*b*log(c))*cos(2*b*lo
g(c)) + b*sin(4*b*log(c))*sin(3*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*log(c
)))*m^2 + b*cos(4*b*log(c))*cos(3*b*log(c)) + b*cos(3*b*log(c))*cos(2*b*log
(c)) + b*sin(4*b*log(c))*sin(3*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*log(c)
) + 2*(b*cos(4*b*log(c))*cos(3*b*log(c)) + b*cos(3*b*log(c))*cos(2*b*log(c)
) + b*sin(4*b*log(c))*sin(3*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*log(c)))*
m)*n + cos(3*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(3*b*log(c)) +
cos(2*b*log(c))*sin(3*b*log(c)) - cos(3*b*log(c...
```

Fricas [A]
time = 1.86, size = 190, normalized size = 0.95

$$\frac{3\left(2\,b^3n^3x+(b^3n^3+(bm^2+2\,bm+b)n)x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2\right)x^m\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\left(6\,(b^2m+b^2)n^2x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+(m^3+(b^2m+b^2)n^2+3\,m^2+3\,m+1)x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^3\right)x^m}{9\,b^4n^4+m^4+4\,m^3+10\,(b^2m^2+2\,b^2m+b^2)n^2+6\,m^2+4\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] (3*(2*b^3*n^3*x + (b^3*n^3 + (b*m^2 + 2*b*m + b)*n)*x*cos(b*n*log(x) + b*lo
g(c) + a)^2)*x^m*sin(b*n*log(x) + b*log(c) + a) + (6*(b^2*m + b^2)*n^2*x*co
s(b*n*log(x) + b*log(c) + a) + (m^3 + (b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*
x*cos(b*n*log(x) + b*log(c) + a)^3)*x^m)/(9*b^4*n^4 + m^4 + 4*m^3 + 10*(b^2
*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cos(a+b*ln(c*x**n))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 159584 vs. 2(201) = 402.
time = 3.97, size = 159584, normalized size = 793.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] 1/8*(54*b^3*n^3*x*abs(x)^m*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn
(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*
log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(3/
2*a)^2*tan(1/2*a) + 54*b^3*n^3*x*abs(x)^m*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*
n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c))
)^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/
4*pi*m)^2*tan(3/2*a)^2*tan(1/2*a) + 6*b^3*n^3*x*abs(x)^m*e^(3/2*pi*b*n*sgn(
x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2
*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi
*m*sgn(x) - 1/4*pi*m)^2*tan(3/2*a)*tan(1/2*a)^2 + 6*b^3*n^3*x*abs(x)^m*e^(-
3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*lo
g(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)
))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(3/2*a)*tan(1/2*a)^2 - 6*b^3*n^3*
x*abs(x)^m*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*
tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/
2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)*tan(3/2*a)^2*tan(1/2*a)^
2 - 54*b^3*n^3*x*abs(x)^m*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(
c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*l
og(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)*tan(3/2*a
)^2*tan(1/2*a)^2 + 54*b^3*n^3*x*abs(x)^m*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n
 - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))
^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4
*pi*m)*tan(3/2*a)^2*tan(1/2*a)^2 + 6*b^3*n^3*x*abs(x)^m*e^(-3/2*pi*b*n*sgn(
x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2
*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi
*m*sgn(x) - 1/4*pi*m)*tan(3/2*a)^2*tan(1/2*a)^2 + 54*b^3*n^3*x*abs(x)^m*e^(
1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*lo
g(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)
))*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(3/2*a)^2*tan(1/2*a)^2 + 54*b^3*n^3
*x*abs(x)^m*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b
)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) +
1/2*b*log(abs(c)))*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(3/2*a)^2*tan(1/2*a
)^2 + 6*b^3*n^3*x*abs(x)^m*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn
(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))*tan(1/2*b*n*lo
g(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(3/2*
a)^2*tan(1/2*a)^2 + 6*b^3*n^3*x*abs(x)^m*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n
```

```
 - 3/2*pi*b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))
*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*p
i*m)^2*tan(3/2*a)^2*tan(1/2*a)^2 - b^2*m*n^2*x*abs(x)^m*e^(3/2*pi*b*n*sgn(x
) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*
b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi*
m*sgn(x) - 1/4*pi*m)^2*tan(3/2*a)^2*tan(1/2*a)^2 - 27*b^2*m*n^2*x*abs(x)^m*
e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n
*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs
(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(3/2*a)^2*tan(1/2*a)^2 - 27*b^
2*m*n^2*x*abs(x)^m*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1
/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs
(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(3/2*a)^2*
tan(1/2*a)^2 - b^2*m*n^2*x*abs(x)^m*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/
2*pi*b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*ta
n(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m
)^2*tan(3/2*a)^2*tan(1/2*a)^2 - b^2*n^2*x*abs(x)^m*e^(3/2*pi*b*n*sgn(x) - 3
/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log
(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn
(x) - 1/4*pi*m)^2*tan(3/2*a)^2*tan(1/2*a)^2 - 27*b^2*n^2*x*abs(x)^m*e^(1/2*
pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(ab
s(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2
*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(3/2*a)^2*tan(1/2*a)^2 - 27*b^2*n^2*x
*abs(x)^m*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*
tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/
2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(3/2*a)^2*tan(1/2*a
)^2 - b^2*n^2*x*abs(x)^m*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(
c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*l
og(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(3/2
*a)^2*tan(1/2*a)^2 + 6*b^3*n^3*x*abs(x)^m*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n
 + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))
^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))...
```

Mupad [B]

time = 3.53, size = 140, normalized size = 0.70

$$\frac{3\,x\,x^m\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,1\mathrm{i}}}{8\,m+8+b\,n\,8\mathrm{i}}+\frac{x\,x^m\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,1\mathrm{i}}}\,3\mathrm{i}}{m\,8\mathrm{i}+8\,b\,n+8\mathrm{i}}+\frac{x\,x^m\,\mathrm{e}^{a\,3\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,3\mathrm{i}}}{8\,m+8+b\,n\,24\mathrm{i}}+\frac{x\,x^m\,\mathrm{e}^{-a\,3\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,3\mathrm{i}}}\,1\mathrm{i}}{m\,8\mathrm{i}+24\,b\,n+8\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a + b*log(c*x^n))^3,x)
```

```
[Out] (3*x*x^m*exp(a*1i)*(c*x^n)^(b*1i))/(8*m + b*n*8i + 8) + (x*x^m*exp(-a*1i)/(
c*x^n)^(b*1i)*3i)/(m*8i + 8*b*n + 8i) + (x*x^m*exp(a*3i)*(c*x^n)^(b*3i))/(8
*m + b*n*24i + 8) + (x*x^m*exp(-a*3i)/(c*x^n)^(b*3i)*1i)/(m*8i + 24*b*n + 8
i)
```

3.125 $\int x^m \cos^2(a + b\log(cx^n))\, dx$

Optimal. Leaf size=120

$$\frac{2b^2n^2x^{1+m}}{(1+m)\left((1+m)^2+4b^2n^2\right)}+\frac{(1+m)x^{1+m}\cos^2(a+b\log(cx^n))}{(1+m)^2+4b^2n^2}+\frac{2bnx^{1+m}\cos(a+b\log(cx^n))\sin(a+b\log}{(1+m)^2+4b^2n^2}$$

```
[Out] 2*b^2*n^2*x^(1+m)/(1+m)/((1+m)^2+4*b^2*n^2)+(1+m)*x^(1+m)*cos(a+b*ln(c*x^n)
)^2/((1+m)^2+4*b^2*n^2)+2*b*n*x^(1+m)*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))
/((1+m)^2+4*b^2*n^2)
```

Rubi [A]
time = 0.02, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4576, 30}

$$\frac{(m+1)x^{m+1}\cos^2(a+b\log(cx^n))}{4b^2n^2+(m+1)^2}+\frac{2bnx^{m+1}\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+(m+1)^2}+\frac{2b^2n^2x^{m+1}}{(m+1)\left(4b^2n^2+(m+1)^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cos[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (2*b^2*n^2*x^(1 + m))/((1 + m)*((1 + m)^2 + 4*b^2*n^2)) + ((1 + m)*x^(1 + m
)*Cos[a + b*Log[c*x^n]]^2)/((1 + m)^2 + 4*b^2*n^2) + (2*b*n*x^(1 + m)*Cos[a
 + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/((1 + m)^2 + 4*b^2*n^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4576

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2
*n^2*p^2 + (m + 1)^2)), Int[(e*x)^m*Cos[d*(a + b*Log[c*x^n])]^(p - 2), x],
x] + Simp[b*d*n*p*(e*x)^(m + 1)*Sin[d*(a + b*Log[c*x^n])]*(Cos[d*(a + b*Log
[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x]) /; FreeQ[{a, b, c
, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Rubi steps

$$\begin{aligned}\int x^m\cos^2(a+b\log(cx^n))\,dx &= \frac{(1+m)x^{1+m}\cos^2(a+b\log(cx^n))}{(1+m)^2+4b^2n^2}+\frac{2bnx^{1+m}\cos(a+b\log(cx^n))\sin(a+}{(1+m)^2+4b^2n^2}\\ &= \frac{2b^2n^2x^{1+m}}{(1+m)\left((1+m)^2+4b^2n^2\right)}+\frac{(1+m)x^{1+m}\cos^2(a+b\log(cx^n))}{(1+m)^2+4b^2n^2}+\frac{2bnx^{1+}}{}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.38, size = 91, normalized size = 0.76

$$\frac{x^{1+m}(1+2m+m^2+4b^2n^2+(1+m)^2\cos(2(a+b\log(cx^n)))+2b(1+m)n\sin(2(a+b\log(cx^n))))}{2(1+m)(1+m-2ibn)(1+m+2ibn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cos[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x^(1 + m)*(1 + 2*m + m^2 + 4*b^2*n^2 + (1 + m)^2*Cos[2*(a + b*Log[c*x^n])]
 + 2*b*(1 + m)*n*Sin[2*(a + b*Log[c*x^n])]))/(2*(1 + m)*(1 + m - (2*I)*b*n)
*(1 + m + (2*I)*b*n))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int x^m\left(\cos^2\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(x^m*cos(a+b*ln(c*x^n))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 646 vs. $2(120) = 240$.
time = 0.32, size = 646, normalized size = 5.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(((cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)) +
cos(2*b*log(c)))*m^2 + 2*(cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))
*sin(2*b*log(c)) + cos(2*b*log(c)))*m + 2*(b*cos(2*b*log(c))*sin(4*b*log(c)
) - b*cos(4*b*log(c))*sin(2*b*log(c)) + (b*cos(2*b*log(c))*sin(4*b*log(c))
- b*cos(4*b*log(c))*sin(2*b*log(c)) + b*sin(2*b*log(c)))*m + b*sin(2*b*log(
c)))*n + cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c))
+ cos(2*b*log(c)))*x*x^m*cos(2*b*log(x^n) + 2*a) - ((cos(2*b*log(c))*sin(4*
b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)) + sin(2*b*log(c)))*m^2 + 2*(cos
(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)) + sin(2*b*lo
g(c)))*m - 2*(b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(2*b
*log(c)) + (b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*l
og(c)) + b*cos(2*b*log(c)))*m + b*cos(2*b*log(c)))*n + cos(2*b*log(c))*sin(
4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)) + sin(2*b*log(c)))*x*x^m*sin(
```

```
2*b*log(x^n) + 2*a) + 2*((cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*m^2 + 4*(b
^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + 2*(cos(2*b*log(c))^2 +
sin(2*b*log(c))^2)*m + cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*x*x^m)/((cos(
2*b*log(c))^2 + sin(2*b*log(c))^2)*m^3 + 3*(cos(2*b*log(c))^2 + sin(2*b*log
(c))^2)*m^2 + 4*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2 + (b^2*cos(2
*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*m)*n^2 + 3*(cos(2*b*log(c))^2 + sin(2
*b*log(c))^2)*m + cos(2*b*log(c))^2 + sin(2*b*log(c))^2)
```

Fricas [A]
time = 2.76, size = 105, normalized size = 0.88

$$\frac{2\,(bm+b)nxx^{m}\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\left(2\,b^{2}n^{2}x+(m^{2}+2\,m+1)x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}\right)x^{m}}{m^{3}+4\,(b^{2}m+b^{2})n^{2}+3\,m^{2}+3\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] (2*(b*m + b)*n*x*x^m*cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(x) + b*log(
c) + a) + (2*b^2*n^2*x + (m^2 + 2*m + 1)*x*cos(b*n*log(x) + b*log(c) + a)^2
)*x^m)/(m^3 + 4*(b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \log\left(x\right)\cos^{2}\left(a\right) & \text{for}\: b=0\wedge m=-1 \\ \int x^{m}\cos^{2}\left(-a+\frac{im\log\left(cx^{n}\right)}{2n}+\frac{i\log\left(cx^{n}\right)}{2n}\right)\,dx & \text{for}\: b=\frac{i(-m-1)}{2n} \\ \int x^{m}\cos^{2}\left(a+\frac{im\log\left(cx^{n}\right)}{2n}+\frac{i\log\left(cx^{n}\right)}{2n}\right)\,dx & \text{for}\: b=\frac{i(m+1)}{2n} \\ \frac{\begin{cases} \log\left(x\right)\cos\left(2a\right) & \text{for}\: b=0\wedge\left(b=0\vee n=0\right) \\ \log\left(x\right)\cos\left(2a+2b\log\left(c\right)\right) & \text{for}\: n=0 \\ \frac{\sin\left(2a+2b\log\left(cx^{n}\right)\right)}{2bn} & \text{otherwise} \end{cases}}{2}+\frac{\log\left(x\right)}{2} & \text{for}\: m=-1 \\ \frac{2b^{2}n^{2}xx^{m}\sin^{2}\left(a+b\log\left(cx^{n}\right)\right)}{4b^{2}mn^{2}+4b^{2}n^{2}+m^{3}+3m^{2}+3m+1}+\frac{2b^{2}n^{2}xx^{m}\cos^{2}\left(a+b\log\left(cx^{n}\right)\right)}{4b^{2}mn^{2}+4b^{2}n^{2}+m^{3}+3m^{2}+3m+1}+\frac{2bmnxx^{m}\sin\left(a+b\log\left(cx^{n}\right)\right)\cos\left(a+b\log\left(cx^{n}\right)\right)}{4b^{2}mn^{2}+4b^{2}n^{2}+m^{3}+3m^{2}+3m+1}+\frac{2bnxx^{m}\sin\left(a+b\log\left(cx^{n}\right)\right)\cos\left(a+b\log\left(cx^{n}\right)\right)}{4b^{2}mn^{2}+4b^{2}n^{2}+m^{3}+3m^{2}+3m+1}+\frac{m^{2}xx^{m}\cos^{2}\left(a+b\log\left(cx^{n}\right)\right)}{4b^{2}mn^{2}+4b^{2}n^{2}+m^{3}+3m^{2}+3m+1}+\frac{2mxx^{m}\cos^{2}\left(a+b\log\left(cx^{n}\right)\right)}{4b^{2}mn^{2}+4b^{2}n^{2}+m^{3}+3m^{2}+3m+1}+\frac{xx^{m}\cos^{2}\left(a+b\log\left(cx^{n}\right)\right)}{4b^{2}mn^{2}+4b^{2}n^{2}+m^{3}+3m^{2}+3m+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cos(a+b*ln(c*x**n))**2,x)
```

```
[Out] Piecewise((log(x)*cos(a)**2, Eq(b, 0) & Eq(m, -1)), (Integral(x**m*cos(-a +
 I*m*log(c*x**n)/(2*n) + I*log(c*x**n)/(2*n))**2, x), Eq(b, I*(-m - 1)/(2*n
))), (Integral(x**m*cos(a + I*m*log(c*x**n)/(2*n) + I*log(c*x**n)/(2*n))**2
, x), Eq(b, I*(m + 1)/(2*n))), (Piecewise((log(x)*cos(2*a), Eq(b, 0) & (Eq(
b, 0) | Eq(n, 0))), (log(x)*cos(2*a + 2*b*log(c)), Eq(n, 0)), (sin(2*a + 2*
b*log(c*x**n))/(2*b*n), True))/2 + log(x)/2, Eq(m, -1)), (2*b**2*n**2*x*x**
m*sin(a + b*log(c*x**n))**2/(4*b**2*m*n**2 + 4*b**2*n**2 + m**3 + 3*m**2 +
3*m + 1) + 2*b**2*n**2*x*x**m*cos(a + b*log(c*x**n))**2/(4*b**2*m*n**2 + 4*
b**2*n**2 + m**3 + 3*m**2 + 3*m + 1) + 2*b*m*n*x*x**m*sin(a + b*log(c*x**n)
)*cos(a + b*log(c*x**n))/(4*b**2*m*n**2 + 4*b**2*n**2 + m**3 + 3*m**2 + 3*m
 + 1) + 2*b*n*x*x**m*sin(a + b*log(c*x**n))*cos(a + b*log(c*x**n))/(4*b**2*
m*n**2 + 4*b**2*n**2 + m**3 + 3*m**2 + 3*m + 1) + m**2*x*x**m*cos(a + b*log
(c*x**n))**2/(4*b**2*m*n**2 + 4*b**2*n**2 + m**3 + 3*m**2 + 3*m + 1) + 2*m*
x*x**m*cos(a + b*log(c*x**n))**2/(4*b**2*m*n**2 + 4*b**2*n**2 + m**3 + 3*m*
```

```
*2 + 3*m + 1) + x*x**m*cos(a + b*log(c*x**n))**2/(4*b**2*m*n**2 + 4*b**2*n*
*2 + m**3 + 3*m**2 + 3*m + 1), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 8742 vs. $2(120) = 240$.
time = 0.71, size = 8742, normalized size = 72.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] -1/4*(8*b^2*n^2*x*abs(x)^m*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4*p
i*m*sgn(x) - 1/4*pi*m)^2*tan(a)^2 - 4*b*m*n*x*abs(x)^m*e^(pi*b*n*sgn(x) - p
i*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4*
pi*m*sgn(x) - 1/4*pi*m)^2*tan(a) - 4*b*m*n*x*abs(x)^m*e^(-pi*b*n*sgn(x) + p
i*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4*
pi*m*sgn(x) - 1/4*pi*m)^2*tan(a) + 4*b*m*n*x*abs(x)^m*e^(pi*b*n*sgn(x) - pi
*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4*p
i*m*sgn(x) - 1/4*pi*m)*tan(a)^2 - 4*b*m*n*x*abs(x)^m*e^(-pi*b*n*sgn(x) + pi
*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4*p
i*m*sgn(x) - 1/4*pi*m)*tan(a)^2 - 4*b*m*n*x*abs(x)^m*e^(pi*b*n*sgn(x) - pi*
b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(1/4*pi*m
*sgn(x) - 1/4*pi*m)^2*tan(a)^2 - 4*b*m*n*x*abs(x)^m*e^(-pi*b*n*sgn(x) + pi*
b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(1/4*pi*m
*sgn(x) - 1/4*pi*m)^2*tan(a)^2 + m^2*x*abs(x)^m*e^(pi*b*n*sgn(x) - pi*b*n +
 pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4*pi*m*sg
n(x) - 1/4*pi*m)^2*tan(a)^2 + m^2*x*abs(x)^m*e^(-pi*b*n*sgn(x) + pi*b*n - p
i*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4*pi*m*sgn(
x) - 1/4*pi*m)^2*tan(a)^2 + 8*b^2*n^2*x*abs(x)^m*tan(b*n*log(abs(x)) + b*lo
g(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2 - 4*b*n*x*abs(x)^m*e^(pi*b*n
*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))
^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(a) - 4*b*n*x*abs(x)^m*e^(-pi*b*n*s
gn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2
*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(a) - 8*b^2*n^2*x*abs(x)^m*tan(b*n*lo
g(abs(x)) + b*log(abs(c)))^2*tan(a)^2 + 4*b*n*x*abs(x)^m*e^(pi*b*n*sgn(x) -
 pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/
4*pi*m*sgn(x) - 1/4*pi*m)*tan(a)^2 - 4*b*n*x*abs(x)^m*e^(-pi*b*n*sgn(x) + p
i*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4*
pi*m*sgn(x) - 1/4*pi*m)*tan(a)^2 + 8*b^2*n^2*x*abs(x)^m*tan(1/4*pi*m*sgn(x)
 - 1/4*pi*m)^2*tan(a)^2 - 4*b*n*x*abs(x)^m*e^(pi*b*n*sgn(x) - pi*b*n + pi*b
*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(1/4*pi*m*sgn(x) -
1/4*pi*m)^2*tan(a)^2 - 4*b*n*x*abs(x)^m*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*s
gn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(1/4*pi*m*sgn(x) - 1/
4*pi*m)^2*tan(a)^2 + 2*m^2*x*abs(x)^m*tan(b*n*log(abs(x)) + b*log(abs(c)))^
```

```
2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(a)^2 + 2*m*x*abs(x)^m*e^(pi*b*n*sgn
(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*t
an(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(a)^2 + 2*m*x*abs(x)^m*e^(-pi*b*n*sgn(x
) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan
(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(a)^2 - 4*b*m*n*x*abs(x)^m*e^(pi*b*n*sgn(
x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*ta
n(1/4*pi*m*sgn(x) - 1/4*pi*m) + 4*b*m*n*x*abs(x)^m*e^(-pi*b*n*sgn(x) + pi*b
*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4*pi*
m*sgn(x) - 1/4*pi*m) + 4*b*m*n*x*abs(x)^m*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*
sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(1/4*pi*m*sgn(x) - 1
/4*pi*m)^2 + 4*b*m*n*x*abs(x)^m*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) +
pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^
2 - m^2*x*abs(x)^m*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*
log(abs(x)) + b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2 - m^2*x*ab
s(x)^m*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x))
 + b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2 + 4*b*m*n*x*abs(x)^m*
e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log
(abs(c)))^2*tan(a) + 4*b*m*n*x*abs(x)^m*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*s
gn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) - 16*b*m*n*x*ab
s(x)^m*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x))
+ b*log(abs(c)))*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)*tan(a) + 16*b*m*n*x*abs(x)
^m*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b
*log(abs(c)))*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)*tan(a) + 4*m^2*x*abs(x)^m*e^(
pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(ab
s(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)*tan(a) - 4*m^2*x*abs(x)^m*e^(-pi*b
*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)
))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)*tan(a) + 4*b*m*n*x*abs(x)^m*e^(pi*b*n*
sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan
(a) + 4*b*m*n*x*abs(x)^m*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*t
an(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(a) - 4*m^2*x*abs(x)^m*e^(pi*b*n*sgn(x)
 - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(1/
4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(a) - 4*m^2*x*abs(x)^m*e^(-pi*b*n*sgn(x) + p
i*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(1/4*pi
*m*sgn(x) - 1/4*pi*m)^2*tan(a) + 4*b*m*n*x*abs(...
```

Mupad [B]
time = 2.79, size = 82, normalized size = 0.68

$$\frac{x\,x^m}{2\,m+2}+\frac{x\,x^m\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}}{4\,m+4+b\,n\,8\mathrm{i}}+\frac{x\,x^m\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,2\mathrm{i}}}\,1\mathrm{i}}{m\,4\mathrm{i}+8\,b\,n+4\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a + b*log(c*x^n))^2,x)
```

```
[Out] (x*x^m)/(2*m + 2) + (x*x^m*exp(a*2i)*(c*x^n)^(b*2i))/(4*m + b*n*8i + 4) + (
x*x^m*exp(-a*2i)/(c*x^n)^(b*2i)*1i)/(m*4i + 8*b*n + 4i)
```

3.126 $\int x^m \cos(a + b\log(cx^n))\,dx$

Optimal. Leaf size=70

$$\frac{(1+m)x^{1+m}\cos(a+b\log(cx^n))}{(1+m)^2+b^2n^2}+\frac{bnx^{1+m}\sin(a+b\log(cx^n))}{(1+m)^2+b^2n^2}$$

```
[Out] (1+m)*x^(1+m)*cos(a+b*ln(c*x^n))/((1+m)^2+b^2*n^2)+b*n*x^(1+m)*sin(a+b*ln(c
*x^n))/((1+m)^2+b^2*n^2)
```

Rubi [A]
time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {4574}

$$\frac{bnx^{m+1}\sin(a+b\log(cx^n))}{b^2n^2+(m+1)^2}+\frac{(m+1)x^{m+1}\cos(a+b\log(cx^n))}{b^2n^2+(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cos[a + b*Log[c*x^n]],x]
```

```
[Out] ((1 + m)*x^(1 + m)*Cos[a + b*Log[c*x^n]])/((1 + m)^2 + b^2*n^2) + (b*n*x^(1
 + m)*Sin[a + b*Log[c*x^n]])/((1 + m)^2 + b^2*n^2)
```

Rule 4574

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Rubi steps

$$\int x^m\cos(a+b\log(cx^n))\,dx=\frac{(1+m)x^{1+m}\cos(a+b\log(cx^n))}{(1+m)^2+b^2n^2}+\frac{bnx^{1+m}\sin(a+b\log(cx^n))}{(1+m)^2+b^2n^2}$$

Mathematica [A]
time = 0.15, size = 53, normalized size = 0.76

$$\frac{x^{1+m}((1+m)\cos(a+b\log(cx^n))+bn\sin(a+b\log(cx^n)))}{1+2m+m^2+b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cos[a + b*Log[c*x^n]],x]
```

```
[Out] (x^(1 + m)*((1 + m)*Cos[a + b*Log[c*x^n]] + b*n*Sin[a + b*Log[c*x^n]]))/(1
+ 2*m + m^2 + b^2*n^2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^{m} \cos \left(a+b \ln \left(c x^{n}\right)\right) d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a+b*ln(c*x^n)),x)
```

```
[Out] int(x^m*cos(a+b*ln(c*x^n)),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(70) = 140$.
time = 0.30, size = 313, normalized size = 4.47

$$\frac{\left((\cos (2 b \log (c)) \cos (b \log (c))+\sin (2 b \log (c)) \sin (b \log (c))+\cos (b \log (c))) m+(b \cos (b \log (c)) \sin (2 b \log (c))-b \cos (2 b \log (c)) \sin (b \log (c))+b \sin (b \log (c))) n+\cos (2 b \log (c)) \cos (b \log (c))+\sin (2 b \log (c)) \sin (b \log (c))+\cos (b \log (c))\right) x x^{m} \cos \left(b \log \left(x^{n}\right)+a\right)-\left((\cos (b \log (c)) \sin (2 b \log (c))-\cos (2 b \log (c)) \sin (b \log (c))+\sin (b \log (c))) m-(b \cos (2 b \log (c)) \cos (b \log (c))+b \sin (2 b \log (c)) \sin (b \log (c))+b \cos (b \log (c))) n+\cos (b \log (c)) \sin (2 b \log (c))-\cos (2 b \log (c)) \sin (b \log (c))+\sin (b \log (c))\right) x x^{m} \sin \left(b \log \left(x^{n}\right)+a\right)}{2\left(\left(\cos (b \log (c))^{2}+\sin (b \log (c))^{2}\right) m^{2}+\left(b^{2} \cos (b \log (c))^{2}+b^{2} \sin (b \log (c))^{2}\right) n^{2}+2\left(\cos (b \log (c))^{2}+\sin (b \log (c))^{2}\right) m+\cos (b \log (c))^{2}+\sin (b \log (c))^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] 1/2*(((cos(2*b*log(c))*cos(b*log(c)) + sin(2*b*log(c))*sin(b*log(c)) + cos(
b*log(c)))*m + (b*cos(b*log(c))*sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(b*l
og(c)) + b*sin(b*log(c)))*n + cos(2*b*log(c))*cos(b*log(c)) + sin(2*b*log(c
))*sin(b*log(c)) + cos(b*log(c)))*x*x^m*cos(b*log(x^n) + a) - ((cos(b*log(c
))*sin(2*b*log(c)) - cos(2*b*log(c))*sin(b*log(c)) + sin(b*log(c)))*m - (b*
cos(2*b*log(c))*cos(b*log(c)) + b*sin(2*b*log(c))*sin(b*log(c)) + b*cos(b*l
og(c)))*n + cos(b*log(c))*sin(2*b*log(c)) - cos(2*b*log(c))*sin(b*log(c)) +
 sin(b*log(c)))*x*x^m*sin(b*log(x^n) + a))/((cos(b*log(c))^2 + sin(b*log(c)
)^2)*m^2 + (b^2*cos(b*log(c))^2 + b^2*sin(b*log(c))^2)*n^2 + 2*(cos(b*log(c
))^2 + sin(b*log(c))^2)*m + cos(b*log(c))^2 + sin(b*log(c))^2)
```

Fricas [A]
time = 2.13, size = 58, normalized size = 0.83

$$\frac{b n x x^{m} \sin \left(b n \log (x)+b \log (c)+a\right)+(m+1) x x^{m} \cos \left(b n \log (x)+b \log (c)+a\right)}{b^{2} n^{2}+m^{2}+2 m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] (b*n*x*x^m*sin(b*n*log(x) + b*log(c) + a) + (m + 1)*x*x^m*cos(b*n*log(x) +
b*log(c) + a))/(b^2*n^2 + m^2 + 2*m + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \log(x)\cos(a) & \text{for } b = 0 \wedge m = -1 \\ \int x^m \cos\left(-a + \frac{im\log(cx^n)}{n} + \frac{i\log(cx^n)}{n}\right) dx & \text{for } b = \frac{i(-m-1)}{n} \\ \int x^m \cos\left(a + \frac{im\log(cx^n)}{n} + \frac{i\log(cx^n)}{n}\right) dx & \text{for } b = \frac{i(m+1)}{n} \\ \frac{bnxx^m\sin(a+b\log(cx^n))}{b^2n^2+m^2+2m+1} + \frac{mxx^m\cos(a+b\log(cx^n))}{b^2n^2+m^2+2m+1} + \frac{xx^m\cos(a+b\log(cx^n))}{b^2n^2+m^2+2m+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cos(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((log(x)*cos(a), Eq(b, 0) & Eq(m, -1)), (Integral(x**m*cos(-a + I*
m*log(c*x**n)/n + I*log(c*x**n)/n), x), Eq(b, I*(-m - 1)/n)), (Integral(x**
m*cos(a + I*m*log(c*x**n)/n + I*log(c*x**n)/n), x), Eq(b, I*(m + 1)/n)), (b
*n*x*x**m*sin(a + b*log(c*x**n))/(b**2*n**2 + m**2 + 2*m + 1) + m*x*x**m*co
s(a + b*log(c*x**n))/(b**2*n**2 + m**2 + 2*m + 1) + x*x**m*cos(a + b*log(c*
x**n))/(b**2*n**2 + m**2 + 2*m + 1), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 5162 vs. $2(70) = 140$.
time = 0.55, size = 5162, normalized size = 73.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*n*x*abs(x)^m*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) -
 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn(
x) - 1/4*pi*m)^2*tan(1/2*a) + 2*b*n*x*abs(x)^m*e^(-1/2*pi*b*n*sgn(x) + 1/2*
pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(ab
s(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(1/2*a) - 2*b*n*x*abs(x)^m*e^
(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*l
og(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)*tan(1/2*a
)^2 + 2*b*n*x*abs(x)^m*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c)
 + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sg
n(x) - 1/4*pi*m)*tan(1/2*a)^2 + 2*b*n*x*abs(x)^m*e^(1/2*pi*b*n*sgn(x) - 1/2
*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(a
bs(c)))*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(1/2*a)^2 + 2*b*n*x*abs(x)^m*e
^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n
*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(1/2
*a)^2 - m*x*abs(x)^m*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) -
1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x
```

```
) - 1/4*pi*m)^2*tan(1/2*a)^2 - m*x*abs(x)^m*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*
b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c
)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(1/2*a)^2 - x*abs(x)^m*e^(1/2*pi
*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(
x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(1/2*a)^2 -
 x*abs(x)^m*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b
)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*
pi*m)^2*tan(1/2*a)^2 + 2*b*n*x*abs(x)^m*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n +
 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2
*tan(1/4*pi*m*sgn(x) - 1/4*pi*m) - 2*b*n*x*abs(x)^m*e^(-1/2*pi*b*n*sgn(x) +
 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*l
og(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m) - 2*b*n*x*abs(x)^m*e^(1/2*pi*
b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x
)) + 1/2*b*log(abs(c)))*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2 - 2*b*n*x*abs(x)^
m*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*
b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2 + m*
x*abs(x)^m*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*
tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi
*m)^2 + m*x*abs(x)^m*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) +
 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn(
x) - 1/4*pi*m)^2 - 2*b*n*x*abs(x)^m*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2
*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan
(1/2*a) - 2*b*n*x*abs(x)^m*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sg
n(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)
 + 8*b*n*x*abs(x)^m*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/
2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/4*pi*m*sgn(x) -
1/4*pi*m)*tan(1/2*a) - 8*b*n*x*abs(x)^m*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n
- 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*
tan(1/4*pi*m*sgn(x) - 1/4*pi*m)*tan(1/2*a) - 4*m*x*abs(x)^m*e^(1/2*pi*b*n*s
gn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) +
1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)*tan(1/2*a) + 4*m*x*abs
(x)^m*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(
1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)*
tan(1/2*a) - 2*b*n*x*abs(x)^m*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*
sgn(c) - 1/2*pi*b)*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(1/2*a) - 2*b*n*x*a
bs(x)^m*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*ta
n(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(1/2*a) + 4*m*x*abs(x)^m*e^(1/2*pi*b*n*s
gn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) +
1/2*b*log(abs(c)))*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(1/2*a) + 4*m*x*abs
(x)^m*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(
1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*
tan(1/2*a) - 2*b*n*x*abs(x)^m*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*
sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a)^
2 - 2*b*n*x*abs(x)^m*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) +
 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a)^2 + m*x*
```

```
abs(x)^m*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*ta
n(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 + m*x*abs(x)^m*e^
(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*
log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 + 2*b*n*x*abs(x)^m*e^(1/2*p
i*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/4*pi*m*sgn(x)
 - 1/4*pi*m)*tan(1/2*a)^2 - 2*b*n*x*abs(x)^m*e^...
```

Mupad [B]
time = 2.67, size = 70, normalized size = 1.00

$$\frac{x\,x^m\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,1\mathrm{i}}}{2\,m+2+b\,n\,2\mathrm{i}}+\frac{x\,x^m\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,1\mathrm{i}}}\,1\mathrm{i}}{m\,2\mathrm{i}+2\,b\,n+2\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a + b*log(c*x^n)),x)
```

```
[Out] (x*x^m*exp(a*1i)*(c*x^n)^(b*1i))/(2*m + b*n*2i + 2) + (x*x^m*exp(-a*1i)/(c*
x^n)^(b*1i)*1i)/(m*2i + 2*b*n + 2i)
```

3.127 $\int x^m \cos^{\frac{3}{2}}(a+b\log(cx^n))\,dx$

Optimal. Leaf size=130

$$\frac{2x^{1+m}\cos^{\frac{3}{2}}(a+b\log(cx^n))\,{}_2F_1\left(-\frac{3}{2},-\frac{2i+2im+3bn}{4bn};-\frac{2i+2im-bn}{4bn};-e^{2ia}(cx^n)^{2ib}\right)}{(2+2m-3ibn)\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

```
[Out] 2*x^(1+m)*cos(a+b*ln(c*x^n))^(3/2)*hypergeom([-3/2, 1/4*(-2*I-2*I*m-3*b*n)/
b/n],[1/4*(-2*I-2*I*m+b*n)/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))/(2+2*m-3*I*b*n
)/(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4582, 4580, 371}

$$\frac{2x^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(-\frac{2i(m+1)}{bn}-3\right);-\frac{2im-bn+2i}{4bn};-e^{2ia}(cx^n)^{2ib}\right)\cos^{\frac{3}{2}}(a+b\log(cx^n))}{(-3ibn+2m+2)\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cos[a + b*Log[c*x^n]]^(3/2),x]
```

```
[Out] (2*x^(1 + m)*Cos[a + b*Log[c*x^n]]^(3/2)*Hypergeometric2F1[-3/2, (-3 - ((2*
I)*(1 + m))/(b*n))/4, -1/4*(2*I + (2*I)*m - b*n)/(b*n), -(E^((2*I)*a)*(c*x^
n)^((2*I)*b))])/((2 + 2*m - (3*I)*b*n)*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^
(3/2))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4580

```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Dist[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 4582

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Cos[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^m \cos^{\frac{3}{2}}(a+b\log(cx^n))\,dx = \frac{\left(x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\mathrm{Subst}\left(\int x^{-1+\frac{1+m}{n}}\cos^{\frac{3}{2}}(a+b\log(x))\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x^{1+m}(cx^n)^{\frac{3ib}{2}-\frac{1+m}{n}}\cos^{\frac{3}{2}}(a+b\log(cx^n))\right)\mathrm{Subst}\left(\int x^{-1-\frac{3ib}{2}+\frac{1+m}{n}}\left(1+e^{2ia}\right.\right.}{n\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

$$= \frac{2x^{1+m}\cos^{\frac{3}{2}}(a+b\log(cx^n))\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{2i(1+m)}{bn}\right);-\frac{2i+2im-bn}{4bn};-e^{2i\alpha}\right.}{(2+2m-3ibn)\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

Mathematica [A]
time = 2.44, size = 238, normalized size = 1.83

$$\frac{2\left(\frac{3b^2n^2x^{1+m}\sqrt{2+2e^{2ia}(cx^n)^{2ib}}\,{}_2F_1\left(\frac{1}{2},\frac{-2i-2im+bn}{4bn};-\frac{2i+2im-5bn}{4bn};-e^{2i(a+b\log(cx^n))}\right)}{(2+2m+ibn)\sqrt{e^{-ia}(cx^n)^{-ib}+e^{ia}(cx^n)^{ib}}}+x^{1+m}\sqrt{\cos(a+b\log(cx^n))}\,(2(1+m)\cos(a+b\log(cx^n))+3bn\sin(a+b\log(cx^n)))\right)}{4+8m+4m^2+9b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cos[a + b*Log[c*x^n]]^(3/2),x]
```

```
[Out] (2*((3*b^2*n^2*x^(1 + m)*Sqrt[2 + 2*E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Hypergeo
metric2F1[1/2, (-2*I - (2*I)*m + b*n)/(4*b*n), -1/4*(2*I + (2*I)*m - 5*b*n)
/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))])/((2 + 2*m + I*b*n)*Sqrt[1/(E^(I*a)*
(c*x^n)^(I*b)) + E^(I*a)*(c*x^n)^(I*b)]) + x^(1 + m)*Sqrt[Cos[a + b*Log[c*x
^n]]]*(2*(1 + m)*Cos[a + b*Log[c*x^n]] + 3*b*n*Sin[a + b*Log[c*x^n]])))/(4
+ 8*m + 4*m^2 + 9*b^2*n^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^m\left(\cos^{\frac{3}{2}}(a+b\ln(c\,x^n))\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a+b*ln(c*x^n))^(3/2),x)
```

```
[Out] int(x^m*cos(a+b*ln(c*x^n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
[Out] integrate(x^m*cos(b*log(c*x^n) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cos(a+b*ln(c*x**n))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6436 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
[Out] integrate(x^m*cos(b*log(c*x^n) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \cos\left(a+b\ln\left(c\,x^n\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a + b*log(c*x^n))^(3/2),x)
[Out] int(x^m*cos(a + b*log(c*x^n))^(3/2), x)
```

3.128 $\int x^m \sqrt{\cos(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=129

$$\frac{2x^{1+m}\sqrt{\cos(a+b\log(cx^n))}\,{}_2F_1\left(-\frac{1}{2},-\frac{2i+2im+bn}{4bn};-\frac{2i+2im-3bn}{4bn};-e^{2ia}(cx^n)^{2ib}\right)}{(2+2m-ibn)\sqrt{1+e^{2ia}(cx^n)^{2ib}}}$$

```
[Out] 2*x^(1+m)*hypergeom([-1/2, 1/4*(-2*I-2*I*m-b*n)/b/n],[1/4*(-2*I-2*I*m+3*b*n
)/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))*cos(a+b*ln(c*x^n))^(1/2)/(2+2*m-I*b*n)/
(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 126, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4582, 4580, 371}

$$\frac{2x^{m+1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4}\left(-\frac{2i(m+1)}{bn}-1\right);-\frac{2im-3bn+2i}{4bn};-e^{2ia}(cx^n)^{2ib}\right)\sqrt{\cos(a+b\log(cx^n))}}{(-ibn+2m+2)\sqrt{1+e^{2ia}(cx^n)^{2ib}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sqrt[Cos[a + b*Log[c*x^n]]],x]
```

```
[Out] (2*x^(1 + m)*Sqrt[Cos[a + b*Log[c*x^n]]]*Hypergeometric2F1[-1/2, (-1 - ((2*
I)*(1 + m))/(b*n))/4, -1/4*(2*I + (2*I)*m - 3*b*n)/(b*n), -(E^((2*I)*a)*(c*
x^n)^((2*I)*b))])/((2 + 2*m - I*b*n)*Sqrt[1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)
])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4580

```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Dist[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
eQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rule 4582

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
```

```
((m + 1)/n - 1)*Cos[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^m \sqrt{\cos(a+b\log(cx^n))}\,dx &= \frac{\left(x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1+m}{n}}\sqrt{\cos(a+b\log(x))}\,dx, x, cx^n\right)}{n}\\ &= \frac{\left(x^{1+m}(cx^n)^{\frac{ib}{2}-\frac{1+m}{n}}\sqrt{\cos(a+b\log(cx^n))}\right)\operatorname{Subst}\left(\int x^{-1-\frac{ib}{2}+\frac{1+m}{n}}\sqrt{1+e^{2}\cdots}\right.}{n\sqrt{1+e^{2ia}(cx^n)^{2ib}}}\\ &= \frac{2x^{1+m}\sqrt{\cos(a+b\log(cx^n))}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4}\left(-1-\frac{2i(1+m)}{bn}\right);-\frac{2i+2im-3bn}{4bn};-e\cdots\right.}{(2+2m-ibn)\sqrt{1+e^{2ia}(cx^n)^{2ib}}}\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 436 vs. $2(129) = 258$.
time = 6.19, size = 436, normalized size = 3.38

$$-\frac{2be^{ia}nx^{1+m}(cx^n)^{ib}\sqrt{2+2e^{2ia}(cx^n)^{2ib}}\left((2i+2im+bn)x^{2ibn}\,{}_2F_1\left(\frac{1}{2},-\frac{2i+2im-3bn}{4bn};-\frac{2i+2im-7bn}{4bn};-e^{2ia}(cx^n)^{2ib}\right)+(-2i-2im+3bn)\,{}_2F_1\left(\frac{1}{2},-\frac{2i+2im+bn}{4bn};-\frac{2i+2im-3bn}{4bn};-e^{2ia}(cx^n)^{2ib}\right)\right)}{(2+2m-ibn)(2+2m+3ibn)\sqrt{e^{-ia}(cx^n)^{-ib}+e^{ia}(cx^n)^{ib}}\left((2+2m-ibn)x^{2ibn}+e^{2ia}(2+2m+ibn)(cx^n)^{2ib}\right)}+\frac{2x^{1+m}\sqrt{\cos(a+b\log(cx^n))}\cos(a-bn\log(x)+b\log(cx^n))}{2(1+m)\cos(a-bn\log(x)+b\log(cx^n))-bn\sin(a-bn\log(x)+b\log(cx^n))}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sqrt[Cos[a + b*Log[c*x^n]]],x]
```

```
[Out] (-2*b*E^(I*a)*n*x^(1 + m)*(c*x^n)^(I*b)*Sqrt[2 + 2*E^((2*I)*a)*(c*x^n)^((2*
I)*b)]*((2*I + (2*I)*m + b*n)*x^((2*I)*b*n)*Hypergeometric2F1[1/2, -1/4*(2*
I + (2*I)*m - 3*b*n)/(b*n), -1/4*(2*I + (2*I)*m - 7*b*n)/(b*n), -(E^((2*I)*
a)*(c*x^n)^((2*I)*b))] + (-2*I - (2*I)*m + 3*b*n)*Hypergeometric2F1[1/2, -1
/4*(2*I + (2*I)*m + b*n)/(b*n), -1/4*(2*I + (2*I)*m - 3*b*n)/(b*n), -(E^((2
*I)*a)*(c*x^n)^((2*I)*b))]))/((2 + 2*m - I*b*n)*(2 + 2*m + (3*I)*b*n)*Sqrt[
1/(E^(I*a)*(c*x^n)^(I*b)) + E^(I*a)*(c*x^n)^(I*b)]*((2 + 2*m - I*b*n)*x^((2
*I)*b*n) + E^((2*I)*a)*(2 + 2*m + I*b*n)*(c*x^n)^((2*I)*b))) + (2*x^(1 + m)
*Sqrt[Cos[a + b*Log[c*x^n]]]*Cos[a - b*n*Log[x] + b*Log[c*x^n]])/(2*(1 + m)
*Cos[a - b*n*Log[x] + b*Log[c*x^n]] - b*n*Sin[a - b*n*Log[x] + b*Log[c*x^n]
])
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^m\left(\sqrt{\cos}\,(a+b\ln(c\,x^n))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a+b*ln(c*x^n))^(1/2),x)
```

```
[Out] int(x^m*cos(a+b*ln(c*x^n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*sqrt(cos(b*log(c*x^n) + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sqrt{\cos\left(a + b\log\left(cx^n\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cos(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(cos(a + b*log(c*x**n))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*sqrt(cos(b*log(c*x^n) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \sqrt{\cos \left(a + b \ln \left(c x^n\right)\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a + b*log(c*x^n))^(1/2),x)
[Out] int(x^m*cos(a + b*log(c*x^n))^(1/2), x)
```

3.129 $\int \frac{x^m}{\sqrt{\cos(a+b\log(cx^n))}} \, dx$

Optimal. Leaf size=130

$$\frac{2x^{1+m}\sqrt{1+e^{2ia}(cx^n)^{2ib}}\ {}_2F_1\left(\frac{1}{2},-\frac{2i+2im-bn}{4bn};-\frac{2i+2im-5bn}{4bn};-e^{2ia}(cx^n)^{2ib}\right)}{(2+2m+ibn)\sqrt{\cos(a+b\log(cx^n))}}$$

```
[Out] 2*x^(1+m)*hypergeom([1/2, 1/4*(-2*I-2*I*m+b*n)/b/n],[1/4*(-2*I-2*I*m+5*b*n)
/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(1/2)/(2+
2*m+I*b*n)/cos(a+b*ln(c*x^n))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4582, 4580, 371}

$$\frac{2x^{m+1}\sqrt{1+e^{2ia}(cx^n)^{2ib}}\ {}_2F_1\left(\frac{1}{2},-\frac{2im-bn+2i}{4bn};-\frac{2im-5bn+2i}{4bn};-e^{2ia}(cx^n)^{2ib}\right)}{(ibn+2m+2)\sqrt{\cos(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[Cos[a + b*Log[c*x^n]]],x]
```

```
[Out] (2*x^(1 + m)*Sqrt[1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Hypergeometric2F1[1/2,
 -1/4*(2*I + (2*I)*m - b*n)/(b*n), -1/4*(2*I + (2*I)*m - 5*b*n)/(b*n), -(E^
((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 + 2*m + I*b*n)*Sqrt[Cos[a + b*Log[c*x^n]
]])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4580

```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Dist[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
eQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rule 4582

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Cos[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^m}{\sqrt{\cos(a+b\log(cx^n))}}\,dx = \frac{\left(x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}}{\sqrt{\cos(a+b\log(x))}}\,dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x^{1+m}(cx^n)^{-\frac{ib}{2}-\frac{1+m}{n}}\sqrt{1+e^{2ia}(cx^n)^{2ib}}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{ib}{2}+\frac{1+m}{n}}}{\sqrt{1+e^{2ia}x^{2ib}}}\,dx, x, cx^n\right)}{n\sqrt{\cos(a+b\log(cx^n))}}$$

$$= \frac{2x^{1+m}\sqrt{1+e^{2ia}(cx^n)^{2ib}}\,{}_2F_1\left(\frac{1}{2}, -\frac{2i+2im-bn}{4bn}; -\frac{2i+2im-5bn}{4bn}; -e^{2ia}(cx^n)^{2ib}\right)}{(2+2m+ibn)\sqrt{\cos(a+b\log(cx^n))}}$$

Mathematica [A]
time = 0.75, size = 153, normalized size = 1.18

$$\frac{2\sqrt{2}\sqrt{1+e^{2i(a+b\log(cx^n))}}\,x^{1+m}\,{}_2F_1\left(\frac{1}{2}, \frac{-2i-2im+bn}{4bn}; -\frac{2i+2im-5bn}{4bn}; -e^{2i(a+b\log(cx^n))}\right)}{\sqrt{e^{-i(a+b\log(cx^n))}\left(1+e^{2i(a+b\log(cx^n))}\right)}\,(2+2m+ibn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[Cos[a + b*Log[c*x^n]]],x]
```

```
[Out] (2*Sqrt[2]*Sqrt[1 + E^((2*I)*(a + b*Log[c*x^n]))]*x^(1 + m)*Hypergeometric2
F1[1/2, (-2*I - (2*I)*m + b*n)/(4*b*n), -1/4*(2*I + (2*I)*m - 5*b*n)/(b*n),
 -E^((2*I)*(a + b*Log[c*x^n]))])/(Sqrt[(1 + E^((2*I)*(a + b*Log[c*x^n])))/E
^(I*(a + b*Log[c*x^n]))]*(2 + 2*m + I*b*n))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{\cos(a+b\ln(c\,x^n))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/cos(a+b*ln(c*x^n))^(1/2),x)
```

```
[Out] int(x^m/cos(a+b*ln(c*x^n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/cos(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
[Out] integrate(x^m/sqrt(cos(b*log(c*x^n) + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/cos(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{\cos\left(a + b\log\left(cx^n\right)\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/cos(a+b*ln(c*x**n))**(1/2),x)
[Out] Integral(x**m/sqrt(cos(a + b*log(c*x**n))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/cos(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
[Out] integrate(x^m/sqrt(cos(b*log(c*x^n) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{\sqrt{\cos\left(a + b\,\ln\left(c\,x^n\right)\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/cos(a + b*log(c*x^n))^(1/2),x)
[Out] int(x^m/cos(a + b*log(c*x^n))^(1/2), x)
```

3.130 $\int \frac{x^m}{\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=130

$$\frac{2x^{1+m}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},-\frac{2i+2im-3bn}{4bn};-\frac{2i+2im-7bn}{4bn};-e^{2ia}(cx^n)^{2ib}\right)}{(2+2m+3ibn)\cos^{\frac{3}{2}}(a+b\log(cx^n))}$$

```
[Out] 2*x^(1+m)*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(3/2)*hypergeom([3/2, 1/4*(-2*I-2*
I*m+3*b*n)/b/n],[1/4*(-2*I-2*I*m+7*b*n)/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))/(
2+2*m+3*I*b*n)/cos(a+b*ln(c*x^n))^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4582, 4580, 371}

$$\frac{2x^{m+1}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i(m+1)}{bn}\right);-\frac{2im-7bn+2i}{4bn};-e^{2ia}(cx^n)^{2ib}\right)}{(3ibn+2m+2)\cos^{\frac{3}{2}}(a+b\log(cx^n))}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Cos[a + b*Log[c*x^n]]^(3/2),x]
```

```
[Out] (2*x^(1 + m)*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2)*Hypergeometric2F1[3/
2, (3 - ((2*I)*(1 + m))/(b*n))/4, -1/4*(2*I + (2*I)*m - 7*b*n)/(b*n), -(E^(
(2*I)*a)*(c*x^n)^((2*I)*b))])/((2 + 2*m + (3*I)*b*n)*Cos[a + b*Log[c*x^n]]^
(3/2))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4580

```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Dist[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 4582

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
```

```
((m + 1)/n - 1)*Cos[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^m}{\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx &= \frac{\left(x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\operatorname{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}}{\cos^{\frac{3}{2}}(a+b\log(x))}\,dx, x, cx^n\right)}{n} \\
&= \frac{\left(x^{1+m}(cx^n)^{-\frac{3ib}{2}-\frac{1+m}{n}}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\right)\operatorname{Subst}\left(\int \frac{x^{-1+\frac{3ib}{2}+\frac{1+m}{n}}}{\left(1+e^{2ia}x^{2ib}\right)^{3/2}}\,dx, x, cx^n\right.}{n\cos^{\frac{3}{2}}(a+b\log(cx^n))} \\
&= \frac{2x^{1+m}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i(1+m)}{bn}\right);-\frac{2i+2im-7bn}{4bn};-e^{2ia}(cx^n)^{2i}\right.}{(2+2m+3ibn)\cos^{\frac{3}{2}}(a+b\log(cx^n))}
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 487 vs. 2(130) = 260.
time = 5.70, size = 487, normalized size = 3.75

$$
-\frac{x^{1+m-ibn}\left(\left(4+8m+4m^2+b^2n^2\right)x^{2ibn}\sqrt{2+2e^{2ia}(cx^n)^{2ib}}\sqrt{\cos(a+b\log(cx^n))}\,{}_2F_1\left(\frac{1}{2},-\frac{2i+2im-3bn}{4bn};-\frac{2i+2im-7bn}{4bn};-e^{2ia}(cx^n)^{2ib}\right)+(-2i-2im+3bn)\left((-2i-2im+bn)\sqrt{2+2e^{2ia}(cx^n)^{2ib}}\sqrt{\cos(a+b\log(cx^n))}\,{}_2F_1\left(\frac{1}{2},-\frac{2i+2im+bn}{4bn};-\frac{2i+2im-3bn}{4bn};-e^{2ia}(cx^n)^{2ib}\right)-2x^{ibn}\sqrt{e^{-ia}(cx^n)^{-ib}+e^{ia}(cx^n)^{ib}}\,(bn\cos(bn\log(x))-2(1+m)\sin(bn\log(x)))\right)\right)}{bn(-2i-2im+3bn)\sqrt{e^{-ia}(cx^n)^{-ib}+e^{ia}(cx^n)^{ib}}\sqrt{\cos(a+b\log(cx^n))}\,(-2(1+m)\cos(a-bn\log(x)+b\log(cx^n))+bn\sin(a-bn\log(x)+b\log(cx^n)))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Cos[a + b*Log[c*x^n]]^(3/2),x]
```

```
[Out] -((x^(1 + m - I*b*n)*((4 + 8*m + 4*m^2 + b^2*n^2)*x^((2*I)*b*n)*Sqrt[2 + 2*
E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[Cos[a + b*Log[c*x^n]]]*Hypergeometric2F
1[1/2, -1/4*(2*I + (2*I)*m - 3*b*n)/(b*n), -1/4*(2*I + (2*I)*m - 7*b*n)/(b*
n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))] + (-2*I - (2*I)*m + 3*b*n)*((-2*I - (
2*I)*m + b*n)*Sqrt[2 + 2*E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[Cos[a + b*Log[
c*x^n]]]*Hypergeometric2F1[1/2, -1/4*(2*I + (2*I)*m + b*n)/(b*n), -1/4*(2*I
 + (2*I)*m - 3*b*n)/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))] - 2*x^(I*b*n)*
Sqrt[1/(E^(I*a)*(c*x^n)^(I*b)) + E^(I*a)*(c*x^n)^(I*b)]*(b*n*Cos[b*n*Log[x]
] - 2*(1 + m)*Sin[b*n*Log[x]]))))/(b*n*(-2*I - (2*I)*m + 3*b*n)*Sqrt[1/(E^(
I*a)*(c*x^n)^(I*b)) + E^(I*a)*(c*x^n)^(I*b)]*Sqrt[Cos[a + b*Log[c*x^n]]]*(-
2*(1 + m)*Cos[a - b*n*Log[x] + b*Log[c*x^n]] + b*n*Sin[a - b*n*Log[x] + b*L
og[c*x^n]])))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$
\int \frac{x^m}{\cos(a+b\ln(c\,x^n))^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/cos(a+b*ln(c*x^n))^(3/2),x)
```

```
[Out] int(x^m/cos(a+b*ln(c*x^n))^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/cos(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/cos(b*log(c*x^n) + a)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/cos(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\cos^{\frac{3}{2}}\left(a + b\log\left(cx^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/cos(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Integral(x**m/cos(a + b*log(c*x**n))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/cos(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/cos(a + b*log(c*x^n))^(3/2),x)
```

```
[Out] int(x^m/cos(a + b*log(c*x^n))^(3/2), x)
```

3.131 $\int \frac{x^m}{\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=130

$$\frac{2x^{1+m}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},-\frac{2i+2im-5bn}{4bn};-\frac{2i+2im-9bn}{4bn};-e^{2ia}(cx^n)^{2ib}\right)}{(2+2m+5ibn)\cos^{\frac{5}{2}}(a+b\log(cx^n))}$$

```
[Out] 2*x^(1+m)*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(5/2)*hypergeom([5/2, 1/4*(-2*I-2*
I*m+5*b*n)/b/n],[1/4*(-2*I-2*I*m+9*b*n)/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))/(
2+2*m+5*I*b*n)/cos(a+b*ln(c*x^n))^(5/2)
```

Rubi [A]
time = 0.07, antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4582, 4580, 371}

$$\frac{2x^{m+1}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i(m+1)}{bn}\right);-\frac{2im-9bn+2i}{4bn};-e^{2ia}(cx^n)^{2ib}\right)}{(5ibn+2m+2)\cos^{\frac{5}{2}}(a+b\log(cx^n))}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Cos[a + b*Log[c*x^n]]^(5/2),x]
```

```
[Out] (2*x^(1 + m)*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^(5/2)*Hypergeometric2F1[5/
2, (5 - ((2*I)*(1 + m))/(b*n))/4, -1/4*(2*I + (2*I)*m - 9*b*n)/(b*n), -(E^(
(2*I)*a)*(c*x^n)^((2*I)*b))])/((2 + 2*m + (5*I)*b*n)*Cos[a + b*Log[c*x^n]]^
(5/2))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4580

```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Dist[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 4582

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
```

```
((m + 1)/n - 1)*Cos[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^m}{\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\left(x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}}{\cos^{\frac{5}{2}}(a+b\log(x))}\,dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x^{1+m}(cx^n)^{-\frac{5ib}{2}-\frac{1+m}{n}}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{5ib}{2}+\frac{1+m}{n}}}{\left(1+e^{2ia}x^{2ib}\right)^{5/2}}\,dx, x, cx^n\right.}{n\cos^{\frac{5}{2}}(a+b\log(cx^n))}$$

$$= \frac{2x^{1+m}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i(1+m)}{bn}\right);-\frac{2i+2im-9bn}{4bn};-e^{2ia}(cx^n)^{2i}\right.}{(2+2m+5ibn)\cos^{\frac{5}{2}}(a+b\log(cx^n))}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 263 vs. 2(130) = 260.
time = 3.46, size = 263, normalized size = 2.02

$$\frac{x^{1+m}\left(-4(1+m)\cos(a+b\log(cx^n))+\frac{(2+2m-ibn)\sqrt{1+e^{2ia}(cx^n)^{2ib}}\,{}_2F_1\left(\frac{1}{2},\frac{-2i-2im+bn}{4bn};-\frac{2i+2im-5bn}{4bn};-e^{2i(a+b\log(cx^n))}\right)\left(e^{-ia}\left((1+e^{2ia})\cos(b\log(cx^n))+i(-1+e^{2ia})\sin(b\log(cx^n))\right)\right)^{3/2}}{\sqrt{e^{-ia}(cx^n)^{-ib}+e^{ia}(cx^n)^{ib}}}+2bn\sin(a+b\log(cx^n))\right)}{3b^2n^2\cos^{\frac{3}{2}}(a+b\log(cx^n))}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Cos[a + b*Log[c*x^n]]^(5/2),x]
```

```
[Out] (x^(1 + m)*(-4*(1 + m)*Cos[a + b*Log[c*x^n]] + ((2 + 2*m - I*b*n)*Sqrt[1 +
E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Hypergeometric2F1[1/2, (-2*I - (2*I)*m + b*n
)/(4*b*n), -1/4*(2*I + (2*I)*m - 5*b*n)/(b*n), -E^((2*I)*(a + b*Log[c*x^n])
)]*(((1 + E^((2*I)*a))*Cos[b*Log[c*x^n]] + I*(-1 + E^((2*I)*a))*Sin[b*Log[c
*x^n]])/E^(I*a))^(3/2))/Sqrt[1/(E^(I*a)*(c*x^n)^(I*b)) + E^(I*a)*(c*x^n)^(I
*b)] + 2*b*n*Sin[a + b*Log[c*x^n]]))/(3*b^2*n^2*Cos[a + b*Log[c*x^n]]^(3/2)
)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\cos(a+b\ln(cx^n))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/cos(a+b*ln(c*x^n))^(5/2),x)
```

```
[Out] int(x^m/cos(a+b*ln(c*x^n))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/cos(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
[Out] integrate(x^m/cos(b*log(c*x^n) + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/cos(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/cos(a+b*ln(c*x**n))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/cos(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/cos(a + b*log(c*x^n))^(5/2),x)
[Out] int(x^m/cos(a + b*log(c*x^n))^(5/2), x)
```

3.132 $\int (ex)^m \cos^p\left(d(a+b\log\left(cx^n\right))\right)\,dx$

Optimal. Leaf size=144

$$\frac{(ex)^{1+m}\left(1+e^{2iad}(cx^n)^{2ibd}\right)^{-p}\cos^p\left(d(a+b\log\left(cx^n\right))\right)\,{}_2F_1\left(-p,-\frac{i+im+bdnp}{2bdn};\frac{1}{2}\left(2-\frac{i(1+m)}{bdn}-p\right);-e^{2iad}(cx^n}{e(1+m-ibdnp)}$$

```
[Out] (e*x)^(1+m)*cos(d*(a+b*ln(c*x^n)))^p*hypergeom([-p, 1/2*(-I-I*m-b*d*n*p)/b/
d/n],[1-1/2*I*(1+m)/b/d/n-1/2*p],-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/e/(1+m-I*
b*d*n*p)/((1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^p)
```

Rubi [A]

time = 0.08, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4582, 4580, 371}

$$\frac{(ex)^{m+1}\left(1+e^{2iad}(cx^n)^{2ibd}\right)^{-p}\,{}_2F_1\left(-p,-\frac{im+bdnp+i}{2bdn};\frac{1}{2}\left(-\frac{i(m+1)}{bdn}-p+2\right);-e^{2iad}(cx^n)^{2ibd}\right)\cos^p\left(d(a+b\log\left(cx^n\right))\right)}{e(-ibdnp+m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Cos[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] ((e*x)^(1 + m)*Cos[d*(a + b*Log[c*x^n])]^p*Hypergeometric2F1[-p, -1/2*(I +
I*m + b*d*n*p)/(b*d*n), (2 - (I*(1 + m))/(b*d*n) - p)/2, -(E^((2*I)*a*d)*(c
*x^n)^((2*I)*b*d))])/(e*(1 + m - I*b*d*n*p)*(1 + E^((2*I)*a*d)*(c*x^n)^((2*
I)*b*d))^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4580

```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Dist[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 4582

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Cos[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
```

```
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int (ex)^m \cos^p\left(d(a+b\log\left(cx^n\right))\right)\,dx = \frac{\left((ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1+m}{n}}\cos^p(d(a+b\log(x)))\,dx, x, cx$$
$$\overline{en}$$
$$= \frac{\left((ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}+ibdp}\left(1+e^{2iad}(cx^n)^{2ibd}\right)^{-p}\cos^p\left(d(a+b\log\left(cx^n\right)\right.\right.}{en}$$
$$= \frac{(ex)^{1+m}\left(1+e^{2iad}(cx^n)^{2ibd}\right)^{-p}\cos^p\left(d(a+b\log\left(cx^n\right))\right)\,{}_2F_1\left(-p,-\frac{i+i}{2}\right.}{e(1+m-ibdnp)}$$

Mathematica [A]
time = 1.86, size = 170, normalized size = 1.18

$$\frac{x(ex)^m\left(e^{-iad}(cx^n)^{-ibd}+e^{iad}(cx^n)^{ibd}\right)^p\left(2+2e^{-2iad}(cx^n)^{-2ibd}\right)^{-p}\,{}_2F_1\left(-p,\frac{i(1+m+ibdnp)}{2bdn};1+\frac{i(1+m)}{2bdn}-\frac{p}{2};-e^{-2iad}(cx^n)^{-2ibd}\right)}{1+m+ibdnp}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^m*Cos[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] (x*(e*x)^m*(1/(E^(I*a*d)*(c*x^n)^(I*b*d)) + E^(I*a*d)*(c*x^n)^(I*b*d))^p*Hy
pergeometric2F1[-p, ((I/2)*(1 + m + I*b*d*n*p))/(b*d*n), 1 + ((I/2)*(1 + m)
)/(b*d*n) - p/2, -(1/(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))])/((1 + m + I*b*d
*n*p)*(2 + 2/(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))^p)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (ex)^m\left(\cos^p\left(d(a+b\ln\left(c\,x^n\right))\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*cos(d*(a+b*ln(c*x^n)))^p,x)
```

```
[Out] int((e*x)^m*cos(d*(a+b*ln(c*x^n)))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cos(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e)^m*cos((b*log(c*x^n) + a)*d)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cos(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

```
[Out] integral((x*e)^m*cos(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*cos(d*(a+b*ln(c*x**n)))**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cos(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*cos((b*log(c*x^n) + a)*d)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(d\left(a+b\,\ln\left(c\,x^n\right)\right)\right)^p\,{\left(e\,x\right)}^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*(a + b*log(c*x^n)))^p*(e*x)^m,x)
```

```
[Out] int(cos(d*(a + b*log(c*x^n)))^p*(e*x)^m, x)
```

3.133 $\int x \cos^p(a+b\log(cx^n))\,dx$

Optimal. Leaf size=114

$$\frac{x^2\left(1+e^{2ia}(cx^n)^{2ib}\right)^{-p}\cos^p(a+b\log(cx^n))\,{}_2F_1\left(\frac{1}{2}\left(-\frac{2i}{bn}-p\right),-p;\frac{1}{2}\left(2-\frac{2i}{bn}-p\right);-e^{2ia}(cx^n)^{2ib}\right)}{2-ibnp}$$

```
[Out] x^2*cos(a+b*ln(c*x^n))^p*hypergeom([-p, -I/b/n-1/2*p],[1-I/b/n-1/2*p],-exp(
2*I*a)*(c*x^n)^(2*I*b))/(2-I*b*n*p)/((1+exp(2*I*a)*(c*x^n)^(2*I*b))^p)
```

Rubi [A]
time = 0.05, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4582, 4580, 371}

$$\frac{x^2\left(1+e^{2ia}(cx^n)^{2ib}\right)^{-p}\,{}_2F_1\left(\frac{1}{2}\left(-p-\frac{2i}{bn}\right),-p;\frac{1}{2}\left(-p-\frac{2i}{bn}+2\right);-e^{2ia}(cx^n)^{2ib}\right)\cos^p(a+b\log(cx^n))}{2-ibnp}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*Log[c*x^n]]^p,x]
```

```
[Out] (x^2*Cos[a + b*Log[c*x^n]]^p*Hypergeometric2F1[((-2*I)/(b*n) - p)/2, -p, (2
 - (2*I)/(b*n) - p)/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 - I*b*n*p)*(1
 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4580

```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Dist[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
eQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rule 4582

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Cos[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x\cos^p(a+b\log(cx^n))\,dx = \frac{\left(x^2(cx^n)^{-2/n}\right)\mathrm{Subst}\left(\int x^{-1+\frac{2}{n}}\cos^p(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(x^2(cx^n)^{-\frac{2}{n}+ibp}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{-p}\cos^p(a+b\log(cx^n))\right)\mathrm{Subst}\left(\int x^{-1+\frac{2}{n}}\right.}{n}$$
$$= \frac{x^2\left(1+e^{2ia}(cx^n)^{2ib}\right)^{-p}\cos^p(a+b\log(cx^n))\,{}_2F_1\left(\frac{1}{2}\left(-\frac{2i}{bn}-p\right),-p;\frac{1}{2}\left(2-\frac{2i}{bn}\right.\right.}{2-ibnp}$$

Mathematica [A]
time = 0.97, size = 141, normalized size = 1.24

$$\frac{ix^2\left(e^{-ia}(cx^n)^{-ib}+e^{ia}(cx^n)^{ib}\right)^p\left(2+2e^{2ia}(cx^n)^{2ib}\right)^{-p}{}_2F_1\left(-\frac{i}{bn}-\frac{p}{2},-p;1-\frac{i}{bn}-\frac{p}{2};-e^{2ia}(cx^n)^{2ib}\right)}{2i+bnp}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*Log[c*x^n]]^p,x]
```

```
[Out] (I*x^2*(1/(E^(I*a)*(c*x^n)^(I*b)) + E^(I*a)*(c*x^n)^(I*b))^p*Hypergeometric
2F1[(-I)/(b*n) - p/2, -p, 1 - I/(b*n) - p/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b
))])/((2*I + b*n*p)*(2 + 2*E^((2*I)*a)*(c*x^n)^((2*I)*b))^p)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x\left(\cos^p(a+b\ln(c\,x^n))\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a+b*ln(c*x^n))^p,x)
```

```
[Out] int(x*cos(a+b*ln(c*x^n))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*log(c*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*log(c*x^n) + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*log(c*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral(x*cos(b*log(c*x^n) + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos^{p}\left(a+b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*ln(c*x**n))**p,x)
```

```
[Out] Integral(x*cos(a + b*log(c*x**n))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*log(c*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*log(c*x^n) + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\cos\left(a+b\,\ln\left(c\,x^{n}\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*log(c*x^n))^p,x)
```

```
[Out] int(x*cos(a + b*log(c*x^n))^p, x)
```

3.134 $\int \cos^p(a+b\log(cx^n))\,dx$

Optimal. Leaf size=112

$$\frac{x\left(1+e^{2ia}(cx^n)^{2ib}\right)^{-p}\cos^p(a+b\log(cx^n))\,{}_2F_1\left(-p,-\frac{i+bnp}{2bn};\frac{1}{2}\left(2-\frac{i}{bn}-p\right);-e^{2ia}(cx^n)^{2ib}\right)}{1-ibnp}$$

```
[Out] x*cos(a+b*ln(c*x^n))^p*hypergeom([-p, 1/2*(-I-b*n*p)/b/n],[1-1/2*I/b/n-1/2*
p],-exp(2*I*a)*(c*x^n)^(2*I*b))/(1-I*b*n*p)/((1+exp(2*I*a)*(c*x^n)^(2*I*b))
^p)
```

Rubi [A]
time = 0.05, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4572, 4580, 371}

$$\frac{x\left(1+e^{2ia}(cx^n)^{2ib}\right)^{-p}\,{}_2F_1\left(-p,-\frac{bnp+i}{2bn};\frac{1}{2}\left(-p-\frac{i}{bn}+2\right);-e^{2ia}(cx^n)^{2ib}\right)\cos^p(a+b\log(cx^n))}{1-ibnp}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*Log[c*x^n]]^p,x]
```

```
[Out] (x*Cos[a + b*Log[c*x^n]]^p*Hypergeometric2F1[-p, -1/2*(I + b*n*p)/(b*n), (2
 - I/(b*n) - p)/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((1 - I*b*n*p)*(1 + E
^((2*I)*a)*(c*x^n)^((2*I)*b))^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4572

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Cos[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4580

```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Dist[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
), Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fre
eQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cos^p\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\cos^p(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$=\frac{\left(x(cx^n)^{-\frac{1}{n}+ibp}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{-p}\cos^p\left(a+b\log\left(cx^n\right)\right)\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}-ibp}\left(\right.\right.}{n}$$
$$=\frac{x\left(1+e^{2ia}(cx^n)^{2ib}\right)^{-p}\cos^p\left(a+b\log\left(cx^n\right)\right)\,{}_2F_1\left(-p,-\frac{i+bnp}{2bn};\frac{1}{2}\left(2-\frac{i}{bn}-p\right);-e\right.}{1-ibnp}$$

Mathematica [A]
time = 0.74, size = 143, normalized size = 1.28

$$\frac{ix\left(e^{-ia}(cx^n)^{-ib}+e^{ia}(cx^n)^{ib}\right)^p\left(2+2e^{2ia}(cx^n)^{2ib}\right)^{-p}\,{}_2F_1\left(-p,-\frac{i+bnp}{2bn};1-\frac{i}{2bn}-\frac{p}{2};-e^{2ia}(cx^n)^{2ib}\right)}{i+bnp}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*Log[c*x^n]]^p,x]
[Out] (I*x*(1/(E^(I*a)*(c*x^n)^(I*b)) + E^(I*a)*(c*x^n)^(I*b))^p*Hypergeometric2F
1[-p, -1/2*(I + b*n*p)/(b*n), 1 - (I/2)/(b*n) - p/2, -(E^((2*I)*a)*(c*x^n)^
((2*I)*b))])/((I + b*n*p)*(2 + 2*E^((2*I)*a)*(c*x^n)^((2*I)*b))^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \cos^p\left(a+b\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*ln(c*x^n))^p,x)
[Out] int(cos(a+b*ln(c*x^n))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate(cos(b*log(c*x^n) + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral(cos(b*log(c*x^n) + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^{p}\left(a + b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*ln(c*x**n))**p,x)
```

```
[Out] Integral(cos(a + b*log(c*x**n))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate(cos(b*log(c*x^n) + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*log(c*x^n))^p,x)
```

```
[Out] int(cos(a + b*log(c*x^n))^p, x)
```

3.135 $\int x^3 \tan(a + i\log(x))\,dx$

Optimal. Leaf size=47

$$-ie^{2ia}x^2 + \frac{ix^4}{4} + ie^{4ia}\log\left(e^{2ia} + x^2\right)$$

```
[Out] -I*exp(2*I*a)*x^2+1/4*I*x^4+I*exp(4*I*a)*ln(exp(2*I*a)+x^2)
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {4591, 456, 457, 78}

$$-ie^{2ia}x^2 + ie^{4ia}\log\left(x^2 + e^{2ia}\right) + \frac{ix^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Tan[a + I*Log[x]],x]
```

```
[Out] (-I)*E^((2*I)*a)*x^2 + (I/4)*x^4 + I*E^((4*I)*a)*Log[E^((2*I)*a) + x^2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 456

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int x^3 \tan(a + i\log(x))\,dx = \int x^3 \tan(a + i\log(x))\,dx$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 132 vs. $2(47) = 94$.
time = 0.04, size = 132, normalized size = 2.81

$$\frac{ix^4}{4} - ix^2\cos(2a) + \text{ArcTan}\left(\frac{(1+x^2)\cos(a)}{\sin(a) - x^2\sin(a)}\right)\cos(4a) + \frac{1}{2}i\cos(4a)\log\left(1 + x^4 + 2x^2\cos(2a)\right) + x^2\sin(2a) + i\text{ArcTan}\left(\frac{(1+x^2)\cos(a)}{\sin(a) - x^2\sin(a)}\right)\sin(4a) - \frac{1}{2}\log\left(1 + x^4 + 2x^2\cos(2a)\right)\sin(4a)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Tan[a + I*Log[x]],x]
```

```
[Out] (I/4)*x^4 - I*x^2*Cos[2*a] + ArcTan[((1 + x^2)*Cos[a])/(Sin[a] - x^2*Sin[a]
)]*Cos[4*a] + (I/2)*Cos[4*a]*Log[1 + x^4 + 2*x^2*Cos[2*a]] + x^2*Sin[2*a] +
 I*ArcTan[((1 + x^2)*Cos[a])/(Sin[a] - x^2*Sin[a])]*Sin[4*a] - (Log[1 + x^4
 + 2*x^2*Cos[2*a]]*Sin[4*a])/2
```

Maple [**A**]
time = 0.06, size = 37, normalized size = 0.79

method	result	size
risch	$-i e^{2ia} x^2 + \frac{ix^4}{4} + i e^{4ia} \ln\left(e^{2ia} + x^2\right)$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tan(a+I*ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*exp(2*I*a)*x^2+1/4*I*x^4+I*exp(4*I*a)*ln(exp(2*I*a)+x^2)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(30) = 60$.
time = 0.29, size = 88, normalized size = 1.87

$$\frac{1}{4}i\,x^4 + x^2(-i\,\cos(2\,a) + \sin(2\,a)) - (\cos(4\,a) + i\,\sin(4\,a))\arctan\left(\sin(2\,a), x^2 + \cos(2\,a)\right) + \frac{1}{2}(i\,\cos(4\,a) - \sin(4\,a))\log\left(x^4 + 2\,x^2\cos(2\,a) + \cos(2\,a)^2 + \sin(2\,a)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(a+I*log(x)),x, algorithm="maxima")
```

```
[Out] 1/4*I*x^4 + x^2*(-I*cos(2*a) + sin(2*a)) - (cos(4*a) + I*sin(4*a))*arctan2(
sin(2*a), x^2 + cos(2*a)) + 1/2*(I*cos(4*a) - sin(4*a))*log(x^4 + 2*x^2*cos
(2*a) + cos(2*a)^2 + sin(2*a)^2)
```

Fricas [A]
time = 1.73, size = 30, normalized size = 0.64

$$\frac{1}{4}i\,x^4 - i\,x^2e^{(2i\,a)} + i\,e^{(4i\,a)}\log\left(x^2 + e^{(2i\,a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(a+I*log(x)),x, algorithm="fricas")
```

```
[Out] 1/4*I*x^4 - I*x^2*e^(2*I*a) + I*e^(4*I*a)*log(x^2 + e^(2*I*a))
```

Sympy [A]
time = 0.10, size = 37, normalized size = 0.79

$$\frac{ix^4}{4} - ix^2e^{2ia} + ie^{4ia}\log\left(x^2 + e^{2ia}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*tan(a+I*ln(x)),x)
```

```
[Out] I*x**4/4 - I*x**2*exp(2*I*a) + I*exp(4*I*a)*log(x**2 + exp(2*I*a))
```

Giac [A]
time = 0.45, size = 34, normalized size = 0.72

$$\frac{1}{4}i\,x^4 - i\,x^2e^{(2i\,a)} + i\,e^{(4i\,a)}\log\left(i\,x^2 + i\,e^{(2i\,a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(a+I*log(x)),x, algorithm="giac")
```

```
[Out] 1/4*I*x^4 - I*x^2*e^(2*I*a) + I*e^(4*I*a)*log(I*x^2 + I*e^(2*I*a))
```

Mupad [B]
time = 2.22, size = 36, normalized size = 0.77

$$\mathrm{e}^{a\,4\mathrm{i}}\,\ln\left(x^2 + \mathrm{e}^{a\,2\mathrm{i}}\right)\,1\mathrm{i} - x^2\,\mathrm{e}^{a\,2\mathrm{i}}\,1\mathrm{i} + \frac{x^4\,1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tan(a + log(x)*1i),x)
```

```
[Out] exp(a*4i)*log(exp(a*2i) + x^2)*1i - x^2*exp(a*2i)*1i + (x^4*1i)/4
```

3.136 $\int x^2 \tan(a + i\log(x))\,dx$

Optimal. Leaf size=43

$$-2ie^{2ia}x + \frac{ix^3}{3} + 2ie^{3ia}\mathrm{ArcTan}\left(e^{-ia}x\right)$$

```
[Out] -2*I*exp(2*I*a)*x+1/3*I*x^3+2*I*exp(3*I*a)*arctan(x/exp(I*a))
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {4591, 456, 470, 327, 209}

$$2ie^{3ia}\mathrm{ArcTan}\left(e^{-ia}x\right) - 2ie^{2ia}x + \frac{ix^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Tan[a + I*Log[x]],x]
```

```
[Out] (-2*I)*E^((2*I)*a)*x + (I/3)*x^3 + (2*I)*E^((3*I)*a)*ArcTan[x/E^(I*a)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 456

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
```

```
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int x^2 \tan(a+i\log(x))\,dx = \int x^2 \tan(a+i\log(x))\,dx$$

Mathematica [A]
time = 0.02, size = 66, normalized size = 1.53

$$\frac{ix^3}{3} - 2ix\cos(2a) + 2i\mathrm{ArcTan}(x\cos(a) - ix\sin(a))\cos(3a) + 2x\sin(2a) - 2\mathrm{ArcTan}(x\cos(a) - ix\sin(a))\sin(3a)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Tan[a + I*Log[x]],x]
[Out] (I/3)*x^3 - (2*I)*x*Cos[2*a] + (2*I)*ArcTan[x*Cos[a] - I*x*Sin[a]]*Cos[3*a]
 + 2*x*Sin[2*a] - 2*ArcTan[x*Cos[a] - I*x*Sin[a]]*Sin[3*a]
```

Maple [A]
time = 0.05, size = 33, normalized size = 0.77

method	result	size
risch	$\frac{ix^3}{3} - 2i\mathrm{e}^{2ia}x + 2i\arctan\left(x\,\mathrm{e}^{-ia}\right)\mathrm{e}^{3ia}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*tan(a+I*ln(x)),x,method=_RETURNVERBOSE)
[Out] 1/3*I*x^3-2*I*exp(2*I*a)*x+2*I*arctan(x*exp(-I*a))*exp(3*I*a)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(26) = 52.
time = 0.51, size = 149, normalized size = 3.47

$$\frac{1}{3}i\,x^3 - 2\,x(i\,\cos(2\,a) - \sin(2\,a)) - (i\,\cos(3\,a) - \sin(3\,a))\arctan\left(\frac{2\,x\cos(a)}{x^2 + \cos(a)^2 - 2\,x\sin(a) + \sin(a)^2}, \frac{x^2 - \cos(a)^2 - \sin(a)^2}{x^2 + \cos(a)^2 - 2\,x\sin(a) + \sin(a)^2}\right) + \frac{1}{2}(\cos(3\,a) + i\,\sin(3\,a))\log\left(\frac{x^2 + \cos(a)^2 + 2\,x\sin(a) + \sin(a)^2}{x^2 + \cos(a)^2 - 2\,x\sin(a) + \sin(a)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tan(a+I*log(x)),x, algorithm="maxima")
```

```
[Out] 1/3*I*x^3 - 2*x*(I*cos(2*a) - sin(2*a)) - (I*cos(3*a) - sin(3*a))*arctan2(2
*x*cos(a)/(x^2 + cos(a)^2 - 2*x*sin(a) + sin(a)^2), (x^2 - cos(a)^2 - sin(a
)^2)/(x^2 + cos(a)^2 - 2*x*sin(a) + sin(a)^2)) + 1/2*(cos(3*a) + I*sin(3*a)
)*log((x^2 + cos(a)^2 + 2*x*sin(a) + sin(a)^2)/(x^2 + cos(a)^2 - 2*x*sin(a)
 + sin(a)^2))
```

Fricas [A]
time = 1.96, size = 42, normalized size = 0.98

$$\frac{1}{3}i\,x^3 - 2i\,xe^{(2i\,a)} - e^{(3i\,a)}\log\left(x + i\,e^{(i\,a)}\right) + e^{(3i\,a)}\log\left(x - i\,e^{(i\,a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tan(a+I*log(x)),x, algorithm="fricas")
```

```
[Out] 1/3*I*x^3 - 2*I*x*e^(2*I*a) - e^(3*I*a)*log(x + I*e^(I*a)) + e^(3*I*a)*log(
x - I*e^(I*a))
```

Sympy [A]
time = 0.10, size = 61, normalized size = 1.42

$$\frac{ix^3}{3} - 2ixe^{2ia} + \left(\log\left(xe^{2ia} - ie^{3ia}\right) - \log\left(xe^{2ia} + ie^{3ia}\right)\right)e^{3ia}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*tan(a+I*ln(x)),x)
```

```
[Out] I*x**3/3 - 2*I*x*exp(2*I*a) + (log(x*exp(2*I*a) - I*exp(3*I*a)) - log(x*exp
(2*I*a) + I*exp(3*I*a)))*exp(3*I*a)
```

Giac [A]
time = 0.46, size = 26, normalized size = 0.60

$$\frac{1}{3}i\,x^3 + 2i\,\arctan\left(xe^{(-i\,a)}\right)e^{(3i\,a)} - 2i\,xe^{(2i\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tan(a+I*log(x)),x, algorithm="giac")
```

```
[Out] 1/3*I*x^3 + 2*I*arctan(x*e^(-I*a))*e^(3*I*a) - 2*I*x*e^(2*I*a)
```

Mupad [B]
time = 2.21, size = 36, normalized size = 0.84

$$\left(\mathrm{e}^{a\,2\mathrm{i}}\right)^{3/2}\mathrm{atan}\left(\frac{x}{\sqrt{\mathrm{e}^{a\,2\mathrm{i}}}}\right)2\mathrm{i} + \frac{x^3\,1\mathrm{i}}{3} - x\,\mathrm{e}^{a\,2\mathrm{i}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*tan(a + log(x)*1i),x)
```

```
[Out] exp(a*2i)^(3/2)*atan(x/exp(a*2i)^(1/2))*2i + (x^3*1i)/3 - x*exp(a*2i)*2i
```

3.137 $\int x \tan(a + i \log(x)) \, dx$

Optimal. Leaf size=33

$$\frac{ix^2}{2} - ie^{2ia} \log \left(e^{2ia} + x^2\right)$$

[Out] 1/2*I*x^2-I*exp(2*I*a)*ln(exp(2*I*a)+x^2)

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4591, 456, 455, 45}

$$\frac{ix^2}{2} - ie^{2ia} \log \left(x^2 + e^{2ia}\right)$$

Antiderivative was successfully verified.

[In] Int[x*Tan[a + I*Log[x]],x]

[Out] (I/2)*x^2 - I*E^((2*I)*a)*Log[E^((2*I)*a) + x^2]

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 456

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int x\tan(a+i\log(x))\,dx=\int x\tan(a+i\log(x))\,dx$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 114 vs. $2(33)=66$.
time = 0.03, size = 114, normalized size = 3.45

$$\frac{ix^2}{2}-\text{ArcTan}\left(\frac{(1+x^2)\cos(a)}{\sin(a)-x^2\sin(a)}\right)\cos(2a)-\frac{1}{2}i\cos(2a)\log\left(1+x^4+2x^2\cos(2a)\right)-i\text{ArcTan}\left(\frac{(1+x^2)\cos(a)}{\sin(a)-x^2\sin(a)}\right)\sin(2a)+\frac{1}{2}\log\left(1+x^4+2x^2\cos(2a)\right)\sin(2a)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Tan[a + I*Log[x]],x]
```

```
[Out] (I/2)*x^2 - ArcTan[((1 + x^2)*Cos[a])/(Sin[a] - x^2*Sin[a])]*Cos[2*a] - (I/
2)*Cos[2*a]*Log[1 + x^4 + 2*x^2*Cos[2*a]] - I*ArcTan[((1 + x^2)*Cos[a])/(Si
n[a] - x^2*Sin[a])]*Sin[2*a] + (Log[1 + x^4 + 2*x^2*Cos[2*a]]*Sin[2*a])/2
```

Maple [A]
time = 0.04, size = 26, normalized size = 0.79

method	result	size
risch	$\frac{ix^2}{2}-ie^{2ia}\ln\left(e^{2ia}+x^2\right)$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(a+I*ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*x^2-I*exp(2*I*a)*ln(exp(2*I*a)+x^2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(21)=42$.
time = 0.29, size = 70, normalized size = 2.12

$$\frac{1}{2}i\,x^2+(\cos(2\,a)+i\,\sin(2\,a))\arctan\left(\sin(2\,a)\,,x^2+\cos(2\,a)\right)+\frac{1}{2}(-i\,\cos(2\,a)+\sin(2\,a))\log\left(x^4+2\,x^2\cos(2\,a)+\cos(2\,a)^2+\sin(2\,a)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(a+I*log(x)),x, algorithm="maxima")
```

```
[Out] 1/2*I*x^2 + (cos(2*a) + I*sin(2*a))*arctan2(sin(2*a), x^2 + cos(2*a)) + 1/2
*(-I*cos(2*a) + sin(2*a))*log(x^4 + 2*x^2*cos(2*a) + cos(2*a)^2 + sin(2*a)^
2)
```

Fricas [A]
time = 3.56, size = 21, normalized size = 0.64

$$\frac{1}{2} i x^2 - i e^{(2 i a)} \log\left(x^2 + e^{(2 i a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(a+I*log(x)),x, algorithm="fricas")
[Out] 1/2*I*x^2 - I*e^(2*I*a)*log(x^2 + e^(2*I*a))
```

Sympy [A]
time = 0.09, size = 26, normalized size = 0.79

$$\frac{ix^2}{2} - ie^{2ia} \log\left(x^2 + e^{2ia}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(a+I*ln(x)),x)
[Out] I*x**2/2 - I*exp(2*I*a)*log(x**2 + exp(2*I*a))
```

Giac [A]
time = 0.46, size = 25, normalized size = 0.76

$$\frac{1}{2} i x^2 - i e^{(2 i a)} \log\left(-i x^2 - i e^{(2 i a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(a+I*log(x)),x, algorithm="giac")
[Out] 1/2*I*x^2 - I*e^(2*I*a)*log(-I*x^2 - I*e^(2*I*a))
```

Mupad [B]
time = 2.19, size = 25, normalized size = 0.76

$$-\mathrm{e}^{a\,2\mathrm{i}} \ln\left(x^2 + \mathrm{e}^{a\,2\mathrm{i}}\right)\,1\mathrm{i} + \frac{x^2\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(a + log(x)*1i),x)
[Out] (x^2*1i)/2 - exp(a*2i)*log(exp(a*2i) + x^2)*1i
```

3.138 $\int \tan(a + i \log(x))\, dx$

Optimal. Leaf size=27

$$ix - 2ie^{ia}\mathrm{ArcTan}\left(e^{-ia}x\right)$$

```
[Out] I*x-2*I*exp(I*a)*arctan(x/exp(I*a))
```

Rubi [A]

time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {4587, 381, 396, 209}

$$ix - 2ie^{ia}\mathrm{ArcTan}\left(e^{-ia}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + I*Log[x]],x]
```

```
[Out] I*x - (2*I)*E^(I*a)*ArcTan[x/E^(I*a)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 381

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4587

```
Int[Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[((I - I*E^(2
*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d)))^p, x] /; FreeQ[{a, b, d
, p}, x]
```

Rubi steps

$$\int \tan(a + i\log(x))\,dx = \int \tan(a + i\log(x))\,dx$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 1.56

$$ix - 2i\text{ArcTan}(x\cos(a) - ix\sin(a))\cos(a) + 2\text{ArcTan}(x\cos(a) - ix\sin(a))\sin(a)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + I*Log[x]],x]
```

```
[Out] I*x - (2*I)*ArcTan[x*Cos[a] - I*x*Sin[a]]*Cos[a] + 2*ArcTan[x*Cos[a] - I*x*
Sin[a]]*Sin[a]
```

Maple [A]
time = 0.04, size = 22, normalized size = 0.81

method	result	size
risch	$ix - 2i\arctan\left(x\,\mathrm{e}^{-ia}\right)\mathrm{e}^{ia}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a+I*ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] I*x-2*I*arctan(x*exp(-I*a))*exp(I*a)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(17) = 34.
time = 0.52, size = 122, normalized size = 4.52

$$(i\cos(a) - \sin(a))\arctan\left(\frac{2\,x\cos(a)}{x^2 + \cos(a)^2 - 2\,x\sin(a) + \sin(a)^2}, \frac{x^2 - \cos(a)^2 - \sin(a)^2}{x^2 + \cos(a)^2 - 2\,x\sin(a) + \sin(a)^2}\right) - \frac{1}{2}(\cos(a) + i\sin(a))\log\left(\frac{x^2 + \cos(a)^2 + 2\,x\sin(a) + \sin(a)^2}{x^2 + \cos(a)^2 - 2\,x\sin(a) + \sin(a)^2}\right) + i\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x)),x, algorithm="maxima")
```

```
[Out] (I*cos(a) - sin(a))*arctan2(2*x*cos(a)/(x^2 + cos(a)^2 - 2*x*sin(a) + sin(a
)^2), (x^2 - cos(a)^2 - sin(a)^2)/(x^2 + cos(a)^2 - 2*x*sin(a) + sin(a)^2))
 - 1/2*(cos(a) + I*sin(a))*log((x^2 + cos(a)^2 + 2*x*sin(a) + sin(a)^2)/(x^
2 + cos(a)^2 - 2*x*sin(a) + sin(a)^2)) + I*x
```

Fricas [A]
time = 2.48, size = 33, normalized size = 1.22

$$e^{(i\,a)}\log\left(x + i\,e^{(i\,a)}\right) - e^{(i\,a)}\log\left(x - i\,e^{(i\,a)}\right) + i\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x)),x, algorithm="fricas")
```

```
[Out] e^(I*a)*log(x + I*e^(I*a)) - e^(I*a)*log(x - I*e^(I*a)) + I*x
```

Sympy [A]
time = 0.14, size = 27, normalized size = 1.00

$$ix + \left(-\log\left(x - ie^{ia}\right) + \log\left(x + ie^{ia}\right)\right)e^{ia}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*ln(x)),x)
```

```
[Out] I*x + (-log(x - I*exp(I*a)) + log(x + I*exp(I*a)))*exp(I*a)
```

Giac [A]
time = 0.44, size = 30, normalized size = 1.11

$$\frac{2\arctan\left(\frac{i\,x}{\sqrt{-e^{(2i\,a)}}}\right)e^{(2i\,a)}}{\sqrt{-e^{(2i\,a)}}} + i\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x)),x, algorithm="giac")
```

```
[Out] 2*arctan(I*x/sqrt(-e^(2*I*a)))*e^(2*I*a)/sqrt(-e^(2*I*a)) + I*x
```

Mupad [B]
time = 2.17, size = 25, normalized size = 0.93

$$x\,1\mathrm{i} - \sqrt{\mathrm{e}^{a\,2\mathrm{i}}}\,\mathrm{atan}\left(\frac{x}{\sqrt{\mathrm{e}^{a\,2\mathrm{i}}}}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + log(x)*1i),x)
```

```
[Out] x*1i - exp(a*2i)^(1/2)*atan(x/exp(a*2i)^(1/2))*2i
```

3.139 $\int \frac{\tan(a+i\log(x))}{x}\,dx$

Optimal. Leaf size=14

$$i\log(\cos(a+i\log(x)))$$

[Out] I*ln(cos(a+I*ln(x)))

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3556}

$$i\log(\cos(a+i\log(x)))$$

Antiderivative was successfully verified.

[In] Int[Tan[a + I*Log[x]]/x,x]

[Out] I*Log[Cos[a + I*Log[x]]]

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(a+i\log(x))}{x}\,dx &= \text{Subst}\left(\int \tan(a+ix)\,dx, x, \log(x)\right)\\ &= i\log(\cos(a+i\log(x)))\end{aligned}$$

Mathematica [A]
time = 0.03, size = 14, normalized size = 1.00

$$i\log(\cos(a+i\log(x)))$$

Antiderivative was successfully verified.

[In] Integrate[Tan[a + I*Log[x]]/x,x]

[Out] I*Log[Cos[a + I*Log[x]]]

Maple [A]
time = 0.04, size = 17, normalized size = 1.21

method	result	size
derivativedivides	$-\frac{i\ln(1+\tan^2(a+i\ln(x)))}{2}$	17
default	$-\frac{i\ln(1+\tan^2(a+i\ln(x)))}{2}$	17
norman	$-\frac{i\ln(1+\tan^2(a+i\ln(x)))}{2}$	17
risch	$i\ln(x)+2a+i\ln\left(1+\frac{e^{2ia}}{x^2}\right)$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a+I*ln(x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*ln(1+tan(a+I*ln(x))^2)
```

Maxima [A]
time = 0.27, size = 10, normalized size = 0.71

$$-i\log\left(\sec\left(a+i\log\left(x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x))/x,x, algorithm="maxima")
```

```
[Out] -I*log(sec(a + I*log(x)))
```

Fricas [A]
time = 3.24, size = 16, normalized size = 1.14

$$i\log\left(x^2+e^{(2ia)}\right)-i\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x))/x,x, algorithm="fricas")
```

```
[Out] I*log(x^2 + e^(2*I*a)) - I*log(x)
```

Sympy [A]
time = 0.14, size = 17, normalized size = 1.21

$$-i\log\left(x\right)+i\log\left(x^2+e^{2ia}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*ln(x))/x,x)
```

```
[Out] -I*log(x) + I*log(x**2 + exp(2*I*a))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(10) = 20$.
time = 0.43, size = 73, normalized size = 5.21

$$i \log\left(\sqrt{-\frac{1}{8}\left(\frac{\left(|x|^2+1\right)^2}{|x|^2}-\frac{\left(|x|^2-1\right)^2}{|x|^2}\right)\cos\left(\pi \mathrm{sgn}\left(x\right)+2\,a\right)+\frac{\left(|x|^2+1\right)^2}{8\,|x|^2}+\frac{\left(|x|^2-1\right)^2}{8\,|x|^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x))/x,x, algorithm="giac")
```

```
[Out] I*log(sqrt(-1/8*((abs(x)^2 + 1)^2/abs(x)^2 - (abs(x)^2 - 1)^2/abs(x)^2)*cos
(pi*sgn(x) + 2*a) + 1/8*(abs(x)^2 + 1)^2/abs(x)^2 + 1/8*(abs(x)^2 - 1)^2/ab
s(x)^2))
```

Mupad [B]
time = 3.73, size = 16, normalized size = 1.14

$$-\frac{\ln\left(\tan(a+\ln\left(x\right)\,1\mathrm{i})^2+1\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + log(x)*1i)/x,x)
```

```
[Out] -(log(tan(a + log(x)*1i)^2 + 1)*1i)/2
```

3.140 $\int \frac{\tan(a+i\log(x))}{x^2}\,dx$

Optimal. Leaf size=29

$$\frac{i}{x}+2ie^{-ia}\mathrm{ArcTan}\left(e^{-ia}x\right)$$

[Out] I/x+2*I*arctan(x/exp(I*a))/exp(I*a)

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.308$, Rules used = {4591, 456, 464, 209}

$$2ie^{-ia}\mathrm{ArcTan}\left(e^{-ia}x\right)+\frac{i}{x}$$

Antiderivative was successfully verified.

[In] Int[Tan[a + I*Log[x]]/x^2,x]

[Out] I/x + ((2*I)*ArcTan[x/E^(I*a)])/E^(I*a)

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 456

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
```

```
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int \frac{\tan(a+i\log(x))}{x^2}\,dx = \int \frac{\tan(a+i\log(x))}{x^2}\,dx$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 1.52

$$\frac{i}{x} + 2i\text{ArcTan}(x\cos(a) - ix\sin(a))\cos(a) + 2\text{ArcTan}(x\cos(a) - ix\sin(a))\sin(a)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + I*Log[x]]/x^2,x]
```

```
[Out] I/x + (2*I)*ArcTan[x*Cos[a] - I*x*Sin[a]]*Cos[a] + 2*ArcTan[x*Cos[a] - I*x*
Sin[a]]*Sin[a]
```

Maple [A]
time = 0.04, size = 24, normalized size = 0.83

method	result	size
risch	$\frac{i}{x}+2i\arctan\left(x\,\mathrm{e}^{-ia}\right)\mathrm{e}^{-ia}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a+I*ln(x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] I/x+2*I*arctan(x*exp(-I*a))*exp(-I*a)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(19) = 38.
time = 0.52, size = 127, normalized size = 4.38

$$\frac{2\,x(-i\,\cos(a)-\sin(a))\arctan\left(\frac{2\,x\cos(a)}{x^2+\cos(a)^2-2\,x\sin(a)+\sin(a)^2}, \frac{x^2-\cos(a)^2-\sin(a)^2}{x^2+\cos(a)^2-2\,x\sin(a)+\sin(a)^2}\right)+x(\cos(a)-i\,\sin(a))\log\left(\frac{x^2+\cos(a)^2+2\,x\sin(a)+\sin(a)^2}{x^2+\cos(a)^2-2\,x\sin(a)+\sin(a)^2}\right)+2i}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*x*(-I*cos(a) - sin(a))*arctan2(2*x*cos(a)/(x^2 + cos(a)^2 - 2*x*sin(
a) + sin(a)^2), (x^2 - cos(a)^2 - sin(a)^2)/(x^2 + cos(a)^2 - 2*x*sin(a) +
sin(a)^2)) + x*(cos(a) - I*sin(a))*log((x^2 + cos(a)^2 + 2*x*sin(a) + sin(a
)^2)/(x^2 + cos(a)^2 - 2*x*sin(a) + sin(a)^2)) + 2*I)/x
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(19) = 38$.
time = 2.84, size = 39, normalized size = 1.34

$$-\frac{\left(x\log\left(x+i\,e^{(i\,a)}\right)-x\log\left(x-i\,e^{(i\,a)}\right)-i\,e^{(i\,a)}\right)e^{(-i\,a)}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x))/x^2,x, algorithm="fricas")
[Out] -(x*log(x + I*e^(I*a)) - x*log(x - I*e^(I*a)) - I*e^(I*a))*e^(-I*a)/x
```

Sympy [A]
time = 0.18, size = 27, normalized size = 0.93

$$\left(\log\left(x-ie^{ia}\right)-\log\left(x+ie^{ia}\right)\right)e^{-ia}+\frac{i}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*ln(x))/x**2,x)
[Out] (log(x - I*exp(I*a)) - log(x + I*exp(I*a)))*exp(-I*a) + I/x
```

Giac [A]
time = 0.47, size = 28, normalized size = 0.97

$$-\frac{2\arctan\left(\frac{i\,x}{\sqrt{-e^{(2i\,a)}}}\right)}{\sqrt{-e^{(2i\,a)}}}+\frac{i}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x))/x^2,x, algorithm="giac")
[Out] -2*arctan(I*x/sqrt(-e^(2*I*a)))/sqrt(-e^(2*I*a)) + I/x
```

Mupad [B]
time = 2.27, size = 27, normalized size = 0.93

$$\frac{\mathrm{atan}\left(\frac{x}{\sqrt{\mathrm{e}^{a\,2\mathrm{i}}}}\right)\,2\mathrm{i}}{\sqrt{\mathrm{e}^{a\,2\mathrm{i}}}}+\frac{1\mathrm{i}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + log(x)*1i)/x^2,x)
[Out] (atan(x/exp(a*2i)^(1/2))*2i)/exp(a*2i)^(1/2) + 1i/x
```

3.141 $\int \frac{\tan(a+i\log(x))}{x^3} dx$

Optimal. Leaf size=35

$$\frac{i}{2x^2} - ie^{-2ia}\log\left(1+\frac{e^{2ia}}{x^2}\right)$$

```
[Out] 1/2*I/x^2-I*ln(1+exp(2*I*a)/x^2)/exp(2*I*a)
```

Rubi [A]
time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4591, 455, 45}

$$\frac{i}{2x^2} - ie^{-2ia}\log\left(1+\frac{e^{2ia}}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + I*Log[x]]/x^3,x]
```

```
[Out] (I/2)/x^2 - (I*Log[1 + E^((2*I)*a)/x^2])/E^((2*I)*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int \frac{\tan(a+i\log(x))}{x^3}\,dx = \int \frac{\tan(a+i\log(x))}{x^3}\,dx$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 132 vs. $2(35) = 70$.
time = 0.04, size = 132, normalized size = 3.77

$$\frac{i}{2x^2} - \text{ArcTan}\left(\frac{(1+x^2)\cos(a)}{\sin(a) - x^2\sin(a)}\right)\cos(2a) + 2i\cos(2a)\log(x) - \frac{1}{2}i\cos(2a)\log\left(1 + x^4 + 2x^2\cos(2a)\right) + i\text{ArcTan}\left(\frac{(1+x^2)\cos(a)}{\sin(a) - x^2\sin(a)}\right)\sin(2a) + 2\log(x)\sin(2a) - \frac{1}{2}\log\left(1 + x^4 + 2x^2\cos(2a)\right)\sin(2a)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + I*Log[x]]/x^3,x]
```

```
[Out] (I/2)/x^2 - ArcTan[((1 + x^2)*Cos[a])/(Sin[a] - x^2*Sin[a])]*Cos[2*a] + (2*
I)*Cos[2*a]*Log[x] - (I/2)*Cos[2*a]*Log[1 + x^4 + 2*x^2*Cos[2*a]] + I*ArcTa
n[((1 + x^2)*Cos[a])/(Sin[a] - x^2*Sin[a])]*Sin[2*a] + 2*Log[x]*Sin[2*a] -
(Log[1 + x^4 + 2*x^2*Cos[2*a]]*Sin[2*a])/2
```

Maple [**A**]
time = 0.06, size = 36, normalized size = 1.03

method	result	size
risch	$\frac{i}{2x^2} + 2i\mathrm{e}^{-2ia}\ln(x) - i\mathrm{e}^{-2ia}\ln\left(\mathrm{e}^{2ia} + x^2\right)$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a+I*ln(x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/x^2+2*I*exp(-2*I*a)*ln(x)-I*exp(-2*I*a)*ln(exp(2*I*a)+x^2)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(23) = 46$.
time = 0.28, size = 94, normalized size = 2.69

$$-\frac{x^2(i\cos(2a) + \sin(2a))\log\left(x^4 + 2x^2\cos(2a) + \cos(2a)^2 + \sin(2a)^2\right) - 2\left((\cos(2a) - i\sin(2a))\arctan(\sin(2a), x^2 + \cos(2a)) + 2(i\cos(2a) + \sin(2a))\log(x)\right)x^2 - i}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(x^2*(I*cos(2*a) + sin(2*a))*log(x^4 + 2*x^2*cos(2*a) + cos(2*a)^2 + s
in(2*a)^2) - 2*((cos(2*a) - I*sin(2*a))*arctan2(sin(2*a), x^2 + cos(2*a)) +
 2*(I*cos(2*a) + sin(2*a))*log(x))*x^2 - I)/x^2
```

Fricas [**A**]
time = 2.45, size = 37, normalized size = 1.06

$$\frac{\left(-2i\,x^2\log\left(x^2 + e^{(2i\,a)}\right) + 4i\,x^2\log(x) + i\,e^{(2i\,a)}\right)e^{(-2i\,a)}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x))/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(-2*I*x^2*log(x^2 + e^(2*I*a)) + 4*I*x^2*log(x) + I*e^(2*I*a))*e^(-2*I*
a)/x^2
```

Sympy [A]
time = 0.28, size = 39, normalized size = 1.11

$$2ie^{-2ia}\log\left(x\right) - ie^{-2ia}\log\left(x^{2} + e^{2ia}\right) + \frac{i}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*ln(x))/x**3,x)
```

```
[Out] 2*I*exp(-2*I*a)*log(x) - I*exp(-2*I*a)*log(x**2 + exp(2*I*a)) + I/(2*x**2)
```

Giac [A]
time = 0.47, size = 33, normalized size = 0.94

$$-i\,e^{\left(-2i\,a\right)}\log\left(-i\,x^{2} - i\,e^{\left(2i\,a\right)}\right) + 2i\,e^{\left(-2i\,a\right)}\log\left(x\right) + \frac{i}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x))/x^3,x, algorithm="giac")
```

```
[Out] -I*e^(-2*I*a)*log(-I*x^2 - I*e^(2*I*a)) + 2*I*e^(-2*I*a)*log(x) + 1/2*I/x^2
```

Mupad [B]
time = 2.29, size = 35, normalized size = 1.00

$$-\mathrm{e}^{-a\,2\mathrm{i}}\,\ln\left(x^2+\mathrm{e}^{a\,2\mathrm{i}}\right)\,1\mathrm{i}+\mathrm{e}^{-a\,2\mathrm{i}}\,\ln\left(x\right)\,2\mathrm{i}+\frac{1\mathrm{i}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + log(x)*1i)/x^3,x)
```

```
[Out] exp(-a*2i)*log(x)*2i - exp(-a*2i)*log(exp(a*2i) + x^2)*1i + 1i/(2*x^2)
```

3.142 $\int \frac{\tan(a+i\log(x))}{x^4}\,dx$

Optimal. Leaf size=45

$$\frac{i}{3x^3}-\frac{2ie^{-2ia}}{x}-2ie^{-3ia}\mathrm{ArcTan}\left(e^{-ia}x\right)$$

```
[Out] 1/3*I/x^3-2*I/exp(2*I*a)/x-2*I*arctan(x/exp(I*a))/exp(3*I*a)
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.385$, Rules used = {4591, 456, 464, 331, 209}

$$-2ie^{-3ia}\mathrm{ArcTan}\left(e^{-ia}x\right)-\frac{2ie^{-2ia}}{x}+\frac{i}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + I*Log[x]]/x^4,x]
```

```
[Out] (I/3)/x^3 - (2*I)/(E^((2*I)*a)*x) - ((2*I)*ArcTan[x/E^(I*a)])/E^((3*I)*a)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 456

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int \frac{\tan(a+i\log(x))}{x^4}\,dx = \int \frac{\tan(a+i\log(x))}{x^4}\,dx$$

Mathematica [A]
time = 0.03, size = 70, normalized size = 1.56

$$\frac{i}{3x^3} - \frac{2i\cos(2a)}{x} - 2i\mathrm{ArcTan}(x\cos(a) - ix\sin(a))\cos(3a) - \frac{2\sin(2a)}{x} - 2\mathrm{ArcTan}(x\cos(a) - ix\sin(a))\sin(3a)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + I*Log[x]]/x^4,x]
```

```
[Out] (I/3)/x^3 - ((2*I)*Cos[2*a])/x - (2*I)*ArcTan[x*Cos[a] - I*x*Sin[a]]*Cos[3*
a] - (2*Sin[2*a])/x - 2*ArcTan[x*Cos[a] - I*x*Sin[a]]*Sin[3*a]
```

Maple [A]
time = 0.05, size = 35, normalized size = 0.78

method	result	size
risch	$\frac{i}{3x^3} - \frac{2i e^{-2ia}}{x} - 2i\arctan\left(x\,e^{-ia}\right)e^{-3ia}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a+I*ln(x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*I/x^3-2*I*exp(-2*I*a)/x-2*I*arctan(x*exp(-I*a))*exp(-3*I*a)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(28) = 56.
time = 0.54, size = 156, normalized size = 3.47

$$-\frac{6\,x^3(-i\,\cos(3\,a) - \sin(3\,a))\arctan\left(\frac{2\,x\cos(a)}{x^2+\cos(a)^2-2\,x\sin(a)+\sin(a)^2}, \frac{x^2-\cos(a)^2-\sin(a)^2}{x^2+\cos(a)^2-2\,x\sin(a)+\sin(a)^2}\right) + 3\,x^3(\cos(3\,a) - i\,\sin(3\,a))\log\left(\frac{x^2+\cos(a)^2+2\,x\sin(a)+\sin(a)^2}{x^2+\cos(a)^2-2\,x\sin(a)+\sin(a)^2}\right) + 12\,x^2(i\,\cos(2\,a) + \sin(2\,a)) - 2i}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(6*x^3*(-I*cos(3*a) - sin(3*a))*arctan2(2*x*cos(a)/(x^2 + cos(a)^2 - 2
*x*sin(a) + sin(a)^2), (x^2 - cos(a)^2 - sin(a)^2)/(x^2 + cos(a)^2 - 2*x*si
n(a) + sin(a)^2)) + 3*x^3*(cos(3*a) - I*sin(3*a))*log((x^2 + cos(a)^2 + 2*x
*sin(a) + sin(a)^2)/(x^2 + cos(a)^2 - 2*x*sin(a) + sin(a)^2)) + 12*x^2*(I*c
os(2*a) + sin(2*a)) - 2*I)/x^3
```

Fricas [A]
time = 2.79, size = 53, normalized size = 1.18

$$\frac{\left(3\,x^3\log\left(x+i\,e^{(i\,a)}\right)-3\,x^3\log\left(x-i\,e^{(i\,a)}\right)-6i\,x^2e^{(i\,a)}+i\,e^{(3i\,a)}\right)e^{(-3i\,a)}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*x^3*log(x + I*e^(I*a)) - 3*x^3*log(x - I*e^(I*a)) - 6*I*x^2*e^(I*a)
+ I*e^(3*I*a))*e^(-3*I*a)/x^3
```

Sympy [A]
time = 0.23, size = 53, normalized size = 1.18

$$\left(-\log\left(x-ie^{ia}\right)+\log\left(x+ie^{ia}\right)\right)e^{-3ia}+\frac{\left(-6ix^2+ie^{2ia}\right)e^{-2ia}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*ln(x))/x**4,x)
```

```
[Out] (-log(x - I*exp(I*a)) + log(x + I*exp(I*a)))*exp(-3*I*a) + (-6*I*x**2 + I*e
xp(2*I*a))*exp(-2*I*a)/(3*x**3)
```

Giac [A]
time = 0.48, size = 28, normalized size = 0.62

$$-2i\,\arctan\left(xe^{(-i\,a)}\right)e^{(-3i\,a)}-\frac{2i\,e^{(-2i\,a)}}{x}+\frac{i}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x))/x^4,x, algorithm="giac")
```

```
[Out] -2*I*arctan(x*e^(-I*a))*e^(-3*I*a) - 2*I*e^(-2*I*a)/x + 1/3*I/x^3
```

Mupad [B]
time = 2.30, size = 40, normalized size = 0.89

$$-\frac{\mathrm{atan}\left(\frac{x}{\sqrt{\mathrm{e}^{a\,2\mathrm{i}}}}\right)\,2\mathrm{i}}{\left(\mathrm{e}^{a\,2\mathrm{i}}\right)^{3/2}}-\frac{x^2\,\mathrm{e}^{-a\,2\mathrm{i}}\,2\mathrm{i}-\frac{1}{3}\mathrm{i}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + log(x)*1i)/x^4,x)
```

```
[Out] - (atan(x/exp(a*2i)^(1/2))*2i)/exp(a*2i)^(3/2) - (x^2*exp(-a*2i)*2i - 1i/3)
/x^3
```

3.143 $\int x^3 \tan^2(a + i\log(x))\,dx$

Optimal. Leaf size=63

$$2e^{2ia}x^2 - \frac{x^4}{4} - \frac{2e^{6ia}}{e^{2ia} + x^2} - 4e^{4ia}\log\left(e^{2ia} + x^2\right)$$

```
[Out] 2*exp(2*I*a)*x^2-1/4*x^4-2*exp(6*I*a)/(exp(2*I*a)+x^2)-4*exp(4*I*a)*ln(exp(
2*I*a)+x^2)
```

Rubi [A]
time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {4591, 456, 457, 78}

$$2e^{2ia}x^2 - \frac{2e^{6ia}}{x^2 + e^{2ia}} - 4e^{4ia}\log\left(x^2 + e^{2ia}\right) - \frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Tan[a + I*Log[x]]^2,x]
```

```
[Out] 2*E^((2*I)*a)*x^2 - x^4/4 - (2*E^((6*I)*a))/(E^((2*I)*a) + x^2) - 4*E^((4*I
)*a)*Log[E^((2*I)*a) + x^2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 456

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int x^3 \tan^2(a + i\log(x))\, dx = \int x^3 \tan^2(a + i\log(x))\, dx$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 155 vs. 2(63) = 126.
time = 0.21, size = 155, normalized size = 2.46

$$-\frac{x^4}{4} + 2x^2\cos(2a) - 4i\text{ArcTan}\left(\frac{(1+x^2)\cot(a)}{-1+x^2}\right)\cos(4a) - 2\cos(4a)\log\left(1+x^4+2x^2\cos(2a)\right) + 2ix^2\sin(2a) + 4\text{ArcTan}\left(\frac{(1+x^2)\cot(a)}{-1+x^2}\right)\sin(4a) - 2i\log\left(1+x^4+2x^2\cos(2a)\right)\sin(4a) - \frac{2(\cos(5a)+i\sin(5a))}{(1+x^2)\cos(a) - i(-1+x^2)\sin(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Tan[a + I*Log[x]]^2,x]
```

```
[Out] -1/4*x^4 + 2*x^2*Cos[2*a] - (4*I)*ArcTan[((1 + x^2)*Cot[a])/(-1 + x^2)]*Cos
[4*a] - 2*Cos[4*a]*Log[1 + x^4 + 2*x^2*Cos[2*a]] + (2*I)*x^2*Sin[2*a] + 4*A
rcTan[((1 + x^2)*Cot[a])/(-1 + x^2)]*Sin[4*a] - (2*I)*Log[1 + x^4 + 2*x^2*C
os[2*a]]*Sin[4*a] - (2*(Cos[5*a] + I*Sin[5*a]))/((1 + x^2)*Cos[a] - I*(-1 +
 x^2)*Sin[a])
```

Maple [**A**]
time = 0.06, size = 52, normalized size = 0.83

method	result	size
risch	$-\frac{9x^4}{4} + \frac{2x^4}{1+\frac{e^{2ia}}{x^2}} + 4\,e^{2ia}x^2 - 4\,e^{4ia}\ln\left(e^{2ia} + x^2\right)$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tan(a+I*ln(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -9/4*x^4+2*x^4/(1+exp(2*I*a)/x^2)+4*exp(2*I*a)*x^2-4*exp(4*I*a)*ln(exp(2*I*
a)+x^2)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(46) = 92.
time = 0.29, size = 217, normalized size = 3.44

$$-\frac{x^6 - 7x^4(\cos(2a) + i\sin(2a)) - 8\left(2(-i\cos(4a) + \sin(4a))\arctan\left(\sin(2a), x^2 + \cos(2a)\right) + \cos(4a) + i\sin(4a)\right)x^2 - 16\left((-i\cos(2a) + \sin(2a))\cos(4a) + (\cos(2a) + i\sin(2a))\sin(4a)\right)\arctan\left(\sin(2a), x^2 + \cos(2a)\right) + 8\left(x^2(\cos(4a) + i\sin(4a)) + (\cos(2a) + i\sin(2a))\cos(4a) - (-i\cos(2a) + \sin(2a))\sin(4a)\right)\log\left(x^4 + 2x^2\cos(2a) + \cos(2a)^2 + \sin(2a)^2\right) + 8\cos(6a) + 8i\sin(6a)}{4\left(x^2 + \cos(2a) + i\sin(2a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(a+I*log(x))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(x^6 - 7*x^4*(cos(2*a) + I*sin(2*a)) - 8*(2*(-I*cos(4*a) + sin(4*a))*a
rctan2(sin(2*a), x^2 + cos(2*a)) + cos(4*a) + I*sin(4*a))*x^2 - 16*((-I*cos
(2*a) + sin(2*a))*cos(4*a) + (cos(2*a) + I*sin(2*a))*sin(4*a))*arctan2(sin(
2*a), x^2 + cos(2*a)) + 8*(x^2*(cos(4*a) + I*sin(4*a)) + (cos(2*a) + I*sin(
2*a))*cos(4*a) - (-I*cos(2*a) + sin(2*a))*sin(4*a))*log(x^4 + 2*x^2*cos(2*a
) + cos(2*a)^2 + sin(2*a)^2) + 8*cos(6*a) + 8*I*sin(6*a))/(x^2 + cos(2*a) +
 I*sin(2*a))
```

Fricas [A]
time = 3.25, size = 64, normalized size = 1.02

$$-\frac{x^{6}-7\,x^{4}e^{(2i\,a)}-8\,x^{2}e^{(4i\,a)}+16\,\left(x^{2}e^{(4i\,a)}+e^{(6i\,a)}\right)\log\left(x^{2}+e^{(2i\,a)}\right)+8\,e^{(6i\,a)}}{4\,\left(x^{2}+e^{(2i\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(a+I*log(x))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(x^6 - 7*x^4*e^(2*I*a) - 8*x^2*e^(4*I*a) + 16*(x^2*e^(4*I*a) + e^(6*I*
a))*log(x^2 + e^(2*I*a)) + 8*e^(6*I*a))/(x^2 + e^(2*I*a))
```

Sympy [A]
time = 0.25, size = 54, normalized size = 0.86

$$-\frac{x^{4}}{4}+2x^{2}e^{2ia}-4e^{4ia}\log\left(x^{2}+e^{2ia}\right)-\frac{2e^{6ia}}{x^{2}+e^{2ia}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*tan(a+I*ln(x))**2,x)
```

```
[Out] -x**4/4 + 2*x**2*exp(2*I*a) - 4*exp(4*I*a)*log(x**2 + exp(2*I*a)) - 2*exp(6
*I*a)/(x**2 + exp(2*I*a))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(46) = 92$.
time = 0.57, size = 261, normalized size = 4.14

$$-\frac{x^{6}}{4\,\left(x^{2}+\frac{e^{(4i\,a)}}{x^{2}}+2\,e^{(2i\,a)}\right)}+\frac{3\,x^{4}e^{(2i\,a)}}{2\,\left(x^{2}+\frac{e^{(4i\,a)}}{x^{2}}+2\,e^{(2i\,a)}\right)}-\frac{4\,x^{2}e^{(4i\,a)}\log\left(-x^{2}-e^{(2i\,a)}\right)}{x^{2}+\frac{e^{(4i\,a)}}{x^{2}}+2\,e^{(2i\,a)}}+\frac{17\,x^{2}e^{(4i\,a)}}{4\,\left(x^{2}+\frac{e^{(4i\,a)}}{x^{2}}+2\,e^{(2i\,a)}\right)}-\frac{8\,e^{(6i\,a)}\log\left(-x^{2}-e^{(2i\,a)}\right)}{x^{2}+\frac{e^{(4i\,a)}}{x^{2}}+2\,e^{(2i\,a)}}+\frac{e^{(6i\,a)}}{x^{2}+\frac{e^{(4i\,a)}}{x^{2}}+2\,e^{(2i\,a)}}-\frac{4\,e^{(8i\,a)}\log\left(-x^{2}-e^{(2i\,a)}\right)}{\left(x^{2}+\frac{e^{(4i\,a)}}{x^{2}}+2\,e^{(2i\,a)}\right)x^{2}}-\frac{3\,e^{(8i\,a)}}{2\,\left(x^{2}+\frac{e^{(4i\,a)}}{x^{2}}+2\,e^{(2i\,a)}\right)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(a+I*log(x))^2,x, algorithm="giac")
```

```
[Out] -1/4*x^6/(x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a)) + 3/2*x^4*e^(2*I*a)/(x^2 + e^(
4*I*a)/x^2 + 2*e^(2*I*a)) - 4*x^2*e^(4*I*a)*log(-x^2 - e^(2*I*a))/(x^2 + e^
```

```
(4*I*a)/x^2 + 2*e^(2*I*a)) + 17/4*x^2*e^(4*I*a)/(x^2 + e^(4*I*a)/x^2 + 2*e^
(2*I*a)) - 8*e^(6*I*a)*log(-x^2 - e^(2*I*a))/(x^2 + e^(4*I*a)/x^2 + 2*e^(2*
I*a)) + e^(6*I*a)/(x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a)) - 4*e^(8*I*a)*log(-x^
2 - e^(2*I*a))/((x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a))*x^2) - 3/2*e^(8*I*a)/((
x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a))*x^2)
```

Mupad [B]
time = 2.25, size = 51, normalized size = 0.81

$$-\frac{2\,\mathrm{e}^{a\,6\mathrm{i}}}{x^2+\mathrm{e}^{a\,2\mathrm{i}}}-4\,\mathrm{e}^{a\,4\mathrm{i}}\,\ln\left(x^2+\mathrm{e}^{a\,2\mathrm{i}}\right)+2\,x^2\,\mathrm{e}^{a\,2\mathrm{i}}-\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tan(a + log(x)*1i)^2,x)
```

```
[Out] 2*x^2*exp(a*2i) - 4*exp(a*4i)*log(exp(a*2i) + x^2) - (2*exp(a*6i))/(exp(a*2
i) + x^2) - x^4/4
```

3.144 $\int x^2 \tan^2(a + i\log(x))\, dx$

Optimal. Leaf size=62

$$6e^{2ia}x - \frac{x^3}{3} - \frac{2e^{2ia}x^3}{e^{2ia} + x^2} - 6e^{3ia}\text{ArcTan}\left(e^{-ia}x\right)$$

```
[Out] 6*exp(2*I*a)*x-1/3*x^3-2*exp(2*I*a)*x^3/(exp(2*I*a)+x^2)-6*exp(3*I*a)*arcta
n(x/exp(I*a))
```

Rubi [A]
time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4591, 456, 474, 470, 327, 209}

$$-6e^{3ia}\text{ArcTan}\left(e^{-ia}x\right) - \frac{2e^{2ia}x^3}{x^2 + e^{2ia}} + 6e^{2ia}x - \frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Tan[a + I*Log[x]]^2,x]
```

```
[Out] 6*E^((2*I)*a)*x - x^3/3 - (2*E^((2*I)*a)*x^3)/(E^((2*I)*a) + x^2) - 6*E^((3
*I)*a)*ArcTan[x/E^(I*a)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 456

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int x^2\tan^2(a+i\log(x))\,dx=\int x^2\tan^2(a+i\log(x))\,dx$$

Mathematica [A]
time = 0.14, size = 100, normalized size = 1.61

$$-\frac{x^3}{3}+4x\cos(2a)-6\text{ArcTan}(x(\cos(a)-i\sin(a)))\cos(3a)+4ix\sin(2a)+\frac{2x(\cos(3a)+i\sin(3a))}{(1+x^2)\cos(a)-i\left(-1+x^2\right)\sin(a)}-6i\text{ArcTan}(x(\cos(a)-i\sin(a)))\sin(3a)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Tan[a + I*Log[x]]^2,x]
```

```
[Out] -1/3*x^3 + 4*x*Cos[2*a] - 6*ArcTan[x*(Cos[a] - I*Sin[a])]*Cos[3*a] + (4*I)*
x*Sin[2*a] + (2*x*(Cos[3*a] + I*Sin[3*a]))/((1 + x^2)*Cos[a] - I*(-1 + x^2)
*Sin[a]) - (6*I)*ArcTan[x*(Cos[a] - I*Sin[a])]*Sin[3*a]
```

Maple [A]
time = 0.05, size = 48, normalized size = 0.77

method	result	size

risch	$-\frac{7x^3}{3}+\frac{2x^3}{1+\frac{\mathrm{e}^{2ia}}{x^2}}+6\,\mathrm{e}^{2ia}x-6\arctan\left(x\,\mathrm{e}^{-ia}\right)\mathrm{e}^{3ia}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*tan(a+I*ln(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -7/3*x^3+2*x^3/(1+exp(2*I*a)/x^2)+6*exp(2*I*a)*x-6*arctan(x*exp(-I*a))*exp(
3*I*a)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(45) = 90$.
time = 0.52, size = 254, normalized size = 4.10

$$-\frac{2\,x^5-22\,x^3(\cos(2\,a)+i\,\sin(2\,a))-36\,x(\cos(4\,a)+i\,\sin(4\,a))-18\left(x^2(\cos(3\,a)+i\,\sin(3\,a))+(\cos(2\,a)+i\,\sin(2\,a))\cos(3\,a)-(-i\,\cos(2\,a)+\sin(2\,a))\sin(3\,a)\right)\arctan\left(\frac{2\,x\cos(a)}{x^2+\cos(a)^2-2\,x\sin(a)+\sin(a)^2},\frac{x^2-\cos(a)^2-\sin(a)^2}{x^2+\cos(a)^2-2\,x\sin(a)+\sin(a)^2}\right)+9\left(x^2(-i\,\cos(3\,a)+\sin(3\,a))+(-i\,\cos(2\,a)+\sin(2\,a))\cos(3\,a)+(\cos(2\,a)+i\,\sin(2\,a))\sin(3\,a)\right)\log\left(\frac{x^2+\cos(a)^2+2\,x\sin(a)+\sin(a)^2}{x^2+\cos(a)^2-2\,x\sin(a)+\sin(a)^2}\right)}{6\left(x^2+\cos(2\,a)+i\,\sin(2\,a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tan(a+I*log(x))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*x^5 - 22*x^3*(cos(2*a) + I*sin(2*a)) - 36*x*(cos(4*a) + I*sin(4*a))
 - 18*(x^2*(cos(3*a) + I*sin(3*a)) + (cos(2*a) + I*sin(2*a))*cos(3*a) - (-I
*cos(2*a) + sin(2*a))*sin(3*a))*arctan2(2*x*cos(a)/(x^2 + cos(a)^2 - 2*x*si
n(a) + sin(a)^2), (x^2 - cos(a)^2 - sin(a)^2)/(x^2 + cos(a)^2 - 2*x*sin(a)
+ sin(a)^2)) + 9*(x^2*(-I*cos(3*a) + sin(3*a)) + (-I*cos(2*a) + sin(2*a))*c
os(3*a) + (cos(2*a) + I*sin(2*a))*sin(3*a))*log((x^2 + cos(a)^2 + 2*x*sin(a
) + sin(a)^2)/(x^2 + cos(a)^2 - 2*x*sin(a) + sin(a)^2)))/(x^2 + cos(2*a) +
I*sin(2*a))
```

Fricas [A]
time = 3.15, size = 86, normalized size = 1.39

$$-\frac{x^5-11\,x^3e^{(2\,i\,a)}-18\,xe^{(4\,i\,a)}+9\left(i\,x^2e^{(3\,i\,a)}+i\,e^{(5\,i\,a)}\right)\log\left(x+i\,e^{(i\,a)}\right)+9\left(-i\,x^2e^{(3\,i\,a)}-i\,e^{(5\,i\,a)}\right)\log\left(x-i\,e^{(i\,a)}\right)}{3\left(x^2+e^{(2\,i\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tan(a+I*log(x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(x^5 - 11*x^3*e^(2*I*a) - 18*x*e^(4*I*a) + 9*(I*x^2*e^(3*I*a) + I*e^(5
*I*a))*log(x + I*e^(I*a)) + 9*(-I*x^2*e^(3*I*a) - I*e^(5*I*a))*log(x - I*e^
(I*a)))/(x^2 + e^(2*I*a))
```

Sympy [A]
time = 0.27, size = 66, normalized size = 1.06

$$-\frac{x^3}{3}+4xe^{2ia}+\frac{2xe^{4ia}}{x^2+e^{2ia}}-3\left(-i\log\left(x-ie^{ia}\right)+i\log\left(x+ie^{ia}\right)\right)e^{3ia}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*tan(a+I*ln(x))**2,x)
```

```
[Out] -x**3/3 + 4*x*exp(2*I*a) + 2*x*exp(4*I*a)/(x**2 + exp(2*I*a)) - 3*(-I*log(x
 - I*exp(I*a)) + I*log(x + I*exp(I*a)))*exp(3*I*a)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(45) = 90$.
time = 0.57, size = 141, normalized size = 2.27

$$-\frac{x^5}{3\left(x^2+\frac{e^{(4ia)}}{x^2}+2e^{(2ia)}\right)}+\frac{10x^3e^{(2ia)}}{3\left(x^2+\frac{e^{(4ia)}}{x^2}+2e^{(2ia)}\right)}-6\arctan\left(xe^{(-ia)}\right)e^{(3ia)}+\frac{35xe^{(4ia)}}{3\left(x^2+\frac{e^{(4ia)}}{x^2}+2e^{(2ia)}\right)}+\frac{2xe^{(4ia)}}{x^2+e^{(2ia)}}+\frac{8e^{(6ia)}}{\left(x^2+\frac{e^{(4ia)}}{x^2}+2e^{(2ia)}\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tan(a+I*log(x))^2,x, algorithm="giac")
```

```
[Out] -1/3*x^5/(x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a)) + 10/3*x^3*e^(2*I*a)/(x^2 + e^
(4*I*a)/x^2 + 2*e^(2*I*a)) - 6*arctan(x*e^(-I*a))*e^(3*I*a) + 35/3*x*e^(4*I
*a)/(x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a)) + 2*x*e^(4*I*a)/(x^2 + e^(2*I*a)) +
 8*e^(6*I*a)/((x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a))*x)
```

Mupad [B]
time = 2.23, size = 52, normalized size = 0.84

$$-6\left(\mathrm{e}^{a\,2\mathrm{i}}\right)^{3/2}\mathrm{atan}\left(\frac{x}{\sqrt{\mathrm{e}^{a\,2\mathrm{i}}}}\right)-\frac{x^3}{3}+4\,x\,\mathrm{e}^{a\,2\mathrm{i}}+\frac{2\,x\,\mathrm{e}^{a\,4\mathrm{i}}}{x^2+\mathrm{e}^{a\,2\mathrm{i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*tan(a + log(x)*1i)^2,x)
```

```
[Out] 4*x*exp(a*2i) - x^3/3 - 6*exp(a*2i)^(3/2)*atan(x/exp(a*2i)^(1/2)) + (2*x*ex
p(a*4i))/(exp(a*2i) + x^2)
```

3.145 $\int x \tan^2(a + i \log(x))\, dx$

Optimal. Leaf size=51

$$-\frac{x^2}{2} + \frac{2e^{4ia}}{e^{2ia} + x^2} + 2e^{2ia} \log\left(e^{2ia} + x^2\right)$$

[Out] -1/2*x^2+2*exp(4*I*a)/(exp(2*I*a)+x^2)+2*exp(2*I*a)*ln(exp(2*I*a)+x^2)

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {4591, 456, 455, 45}

$$\frac{2e^{4ia}}{x^2 + e^{2ia}} + 2e^{2ia} \log\left(x^2 + e^{2ia}\right) - \frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Int[x*Tan[a + I*Log[x]]^2,x]

[Out] -1/2*x^2 + (2*E^((4*I)*a))/(E^((2*I)*a) + x^2) + 2*E^((2*I)*a)*Log[E^((2*I)*a) + x^2]

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 456

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
```

```
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int x\tan^2(a+i\log(x))\,dx=\int x\tan^2(a+i\log(x))\,dx$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 135 vs. 2(51) = 102.
time = 0.14, size = 135, normalized size = 2.65

$$-\frac{x^2}{2}+2i\text{ArcTan}\left(\frac{(1+x^2)\cot(a)}{-1+x^2}\right)\cos(2a)+\cos(2a)\log\left(1+x^4+2x^2\cos(2a)\right)-2\text{ArcTan}\left(\frac{(1+x^2)\cot(a)}{-1+x^2}\right)\sin(2a)+i\log\left(1+x^4+2x^2\cos(2a)\right)\sin(2a)+\frac{2\cos(3a)+2i\sin(3a)}{(1+x^2)\cos(a)-i(-1+x^2)\sin(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Tan[a + I*Log[x]]^2,x]
```

```
[Out] -1/2*x^2 + (2*I)*ArcTan[((1 + x^2)*Cot[a])/(-1 + x^2)]*Cos[2*a] + Cos[2*a]*
Log[1 + x^4 + 2*x^2*Cos[2*a]] - 2*ArcTan[((1 + x^2)*Cot[a])/(-1 + x^2)]*Sin
[2*a] + I*Log[1 + x^4 + 2*x^2*Cos[2*a]]*Sin[2*a] + (2*Cos[3*a] + (2*I)*Sin[
3*a])/((1 + x^2)*Cos[a] - I*(-1 + x^2)*Sin[a])
```

Maple [A]
time = 0.04, size = 42, normalized size = 0.82

method	result	size
risch	$-\frac{5x^2}{2}+\frac{2x^2}{1+\frac{e^{2ia}}{x^2}}+2\,e^{2ia}\ln\left(e^{2ia}+x^2\right)$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(a+I*ln(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -5/2*x^2+2*x^2/(1+exp(2*I*a)/x^2)+2*exp(2*I*a)*ln(exp(2*I*a)+x^2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(37) = 74.
time = 0.29, size = 185, normalized size = 3.63

$$-\frac{x^4+\left(4\left(-i\cos(2a)+\sin(2a)\right)\arctan\left(\sin(2a),x^2+\cos(2a)\right)+\cos(2a)+i\sin(2a)\right)x^2+4\left(-i\cos(2a)^2+2\cos(2a)\sin(2a)+i\sin(2a)^2\right)\arctan\left(\sin(2a),x^2+\cos(2a)\right)-2\left(x^2(\cos(2a)+i\sin(2a))+\cos(2a)^2+2i\cos(2a)\sin(2a)-\sin(2a)^2\right)\log\left(x^4+2x^2\cos(2a)+\cos(2a)^2+\sin(2a)^2\right)-4\cos(4a)-4i\sin(4a)}{2\left(x^2+\cos(2a)+i\sin(2a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(a+I*log(x))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x^4 + (4*(-I*cos(2*a) + sin(2*a))*arctan2(sin(2*a), x^2 + cos(2*a)) +
 cos(2*a) + I*sin(2*a))*x^2 + 4*(-I*cos(2*a)^2 + 2*cos(2*a)*sin(2*a) + I*si
n(2*a)^2)*arctan2(sin(2*a), x^2 + cos(2*a)) - 2*(x^2*(cos(2*a) + I*sin(2*a)
) + cos(2*a)^2 + 2*I*cos(2*a)*sin(2*a) - sin(2*a)^2)*log(x^4 + 2*x^2*cos(2*
a) + cos(2*a)^2 + sin(2*a)^2) - 4*cos(4*a) - 4*I*sin(4*a))/(x^2 + cos(2*a)
+ I*sin(2*a))
```

Fricas [**A**]
time = 3.55, size = 54, normalized size = 1.06

$$-\frac{x^4 + x^2 e^{(2ia)} - 4\left(x^2 e^{(2ia)} + e^{(4ia)}\right)\log\left(x^2 + e^{(2ia)}\right) - 4e^{(4ia)}}{2\left(x^2 + e^{(2ia)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(a+I*log(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(x^4 + x^2*e^(2*I*a) - 4*(x^2*e^(2*I*a) + e^(4*I*a))*log(x^2 + e^(2*I*
a)) - 4*e^(4*I*a))/(x^2 + e^(2*I*a))
```

Sympy [**A**]
time = 0.15, size = 42, normalized size = 0.82

$$-\frac{x^2}{2} + 2e^{2ia}\log\left(x^2 + e^{2ia}\right) + \frac{2e^{4ia}}{x^2 + e^{2ia}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(a+I*ln(x))**2,x)
```

```
[Out] -x**2/2 + 2*exp(2*I*a)*log(x**2 + exp(2*I*a)) + 2*exp(4*I*a)/(x**2 + exp(2*
I*a))
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(37) = 74.
time = 0.58, size = 221, normalized size = 4.33

$$-\frac{x^4}{2\left(x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}\right)} + \frac{2x^2 e^{(2ia)}\log\left(x^2 + e^{(2ia)}\right)}{x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}} - \frac{5x^2 e^{(2ia)}}{2\left(x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}\right)} + \frac{4e^{(4ia)}\log\left(x^2 + e^{(2ia)}\right)}{x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}} - \frac{3e^{(4ia)}}{2\left(x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}\right)} + \frac{2e^{(6ia)}\log\left(x^2 + e^{(2ia)}\right)}{\left(x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}\right)x^2} + \frac{e^{(6ia)}}{2\left(x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(a+I*log(x))^2,x, algorithm="giac")
```

```
[Out] -1/2*x^4/(x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a)) + 2*x^2*e^(2*I*a)*log(x^2 + e^
(2*I*a))/(x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a)) - 5/2*x^2*e^(2*I*a)/(x^2 + e^(
4*I*a)/x^2 + 2*e^(2*I*a)) + 4*e^(4*I*a)*log(x^2 + e^(2*I*a))/(x^2 + e^(4*I*
a)/x^2 + 2*e^(2*I*a)) - 3/2*e^(4*I*a)/(x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a)) +
 2*e^(6*I*a)*log(x^2 + e^(2*I*a))/((x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a))*x^2)
 + 1/2*e^(6*I*a)/((x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a))*x^2)
```

Mupad [B]
time = 2.21, size = 41, normalized size = 0.80

$$\frac{2\,\mathrm{e}^{a\,4\mathrm{i}}}{x^2+\mathrm{e}^{a\,2\mathrm{i}}}+2\,\mathrm{e}^{a\,2\mathrm{i}}\,\ln\left(x^2+\mathrm{e}^{a\,2\mathrm{i}}\right)-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(a + log(x)*1i)^2,x)
[Out] (2*exp(a*4i))/(exp(a*2i) + x^2) + 2*exp(a*2i)*log(exp(a*2i) + x^2) - x^2/2
```

3.146 $\int \tan^2(a + i\log(x))\,dx$

Optimal. Leaf size=46

$$-x - \frac{2e^{2ia}x}{e^{2ia} + x^2} + 2e^{ia}\mathrm{ArcTan}\left(e^{-ia}x\right)$$

```
[Out] -x-2*exp(2*I*a)*x/(exp(2*I*a)+x^2)+2*exp(I*a)*arctan(x/exp(I*a))
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {4587, 381, 398, 294, 209}

$$2e^{ia}\mathrm{ArcTan}\left(e^{-ia}x\right) - \frac{2e^{2ia}x}{x^2 + e^{2ia}} - x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + I*Log[x]]^2,x]
```

```
[Out] -x - (2*E^((2*I)*a)*x)/(E^((2*I)*a) + x^2) + 2*E^(I*a)*ArcTan[x/E^(I*a)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 381

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 4587

```
Int[Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[((I - I*E^(2
*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d)))^p, x] /; FreeQ[{a, b, d
, p}, x]
```

Rubi steps

$$\int \tan^2(a+i\log(x))\,dx=\int \tan^2(a+i\log(x))\,dx$$

Mathematica [A]
time = 0.10, size = 70, normalized size = 1.52

$$2\text{ArcTan}(x(\cos(a)-i\sin(a)))(\cos(a)+i\sin(a))+\frac{-x(3+x^2)\cos(a)+ix(-3+x^2)\sin(a)}{(1+x^2)\cos(a)-i\,(-1+x^2)\sin(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + I*Log[x]]^2,x]
```

```
[Out] 2*ArcTan[x*(Cos[a] - I*Sin[a])]*(Cos[a] + I*Sin[a]) + (-(x*(3 + x^2)*Cos[a]
) + I*x*(-3 + x^2)*Sin[a])/((1 + x^2)*Cos[a] - I*(-1 + x^2)*Sin[a])
```

Maple [A]
time = 0.04, size = 36, normalized size = 0.78

method	result	size
risch	$-3x+\frac{2x}{1+\frac{e^{2ia}}{x^2}}+2\arctan\left(x\,e^{-ia}\right)e^{ia}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a+I*ln(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3*x+2*x/(1+exp(2*I*a)/x^2)+2*arctan(x*exp(-I*a))*exp(I*a)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(34)=68$.
time = 0.51, size = 218, normalized size = 4.74

$$-\frac{2\,x^3+6\,x(\cos(2\,a)+i\,\sin(2\,a))+2\,(x^2(\cos(a)+i\,\sin(a))+(\cos(a)+i\,\sin(a))\cos(2\,a)-(-i\,\cos(a)+\sin(a))\sin(2\,a))\arctan\left(\frac{2\,x\cos(a)}{x^2+\cos(a)^2-2\,x\sin(a)+\sin(a)^2},\frac{x^2-\cos(a)^2-\sin(a)^2}{x^2+\cos(a)^2-2\,x\sin(a)+\sin(a)^2}\right)+(x^2(i\,\cos(a)-\sin(a))+(i\,\cos(a)-\sin(a))\cos(2\,a)-(\cos(a)+i\,\sin(a))\sin(2\,a))\log\left(\frac{x^2+\cos(a)^2+2\,x\sin(a)+\sin(a)^2}{x^2+\cos(a)^2-2\,x\sin(a)+\sin(a)^2}\right)}{2\,(x^2+\cos(2\,a)+i\,\sin(2\,a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*x^3 + 6*x*(cos(2*a) + I*sin(2*a)) + 2*(x^2*(cos(a) + I*sin(a)) + (c
os(a) + I*sin(a))*cos(2*a) - (-I*cos(a) + sin(a))*sin(2*a))*arctan2(2*x*cos
(a)/(x^2 + cos(a)^2 - 2*x*sin(a) + sin(a)^2), (x^2 - cos(a)^2 - sin(a)^2)/(
x^2 + cos(a)^2 - 2*x*sin(a) + sin(a)^2)) + (x^2*(I*cos(a) - sin(a)) + (I*co
s(a) - sin(a))*cos(2*a) - (cos(a) + I*sin(a))*sin(2*a))*log((x^2 + cos(a)^2
 + 2*x*sin(a) + sin(a)^2)/(x^2 + cos(a)^2 - 2*x*sin(a) + sin(a)^2)))/(x^2 +
 cos(2*a) + I*sin(2*a))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(34) = 68$.
time = 3.67, size = 77, normalized size = 1.67

$$-\frac{x^{3}+3\,xe^{(2i\,a)}-\left(i\,x^{2}e^{(i\,a)}+i\,e^{(3i\,a)}\right)\log\left(x+i\,e^{(i\,a)}\right)-\left(-i\,x^{2}e^{(i\,a)}-i\,e^{(3i\,a)}\right)\log\left(x-i\,e^{(i\,a)}\right)}{x^{2}+e^{(2i\,a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x))^2,x, algorithm="fricas")
```

```
[Out] -(x^3 + 3*x*e^(2*I*a) - (I*x^2*e^(I*a) + I*e^(3*I*a))*log(x + I*e^(I*a)) -
(-I*x^2*e^(I*a) - I*e^(3*I*a))*log(x - I*e^(I*a)))/(x^2 + e^(2*I*a))
```

Sympy [A]
time = 0.22, size = 51, normalized size = 1.11

$$-x-\frac{2xe^{2ia}}{x^{2}+e^{2ia}}-\left(i\log\left(x-ie^{ia}\right)-i\log\left(x+ie^{ia}\right)\right)e^{ia}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*ln(x))**2,x)
```

```
[Out] -x - 2*x*exp(2*I*a)/(x**2 + exp(2*I*a)) - (I*log(x - I*exp(I*a)) - I*log(x
+ I*exp(I*a)))*exp(I*a)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(34) = 68$.
time = 0.51, size = 114, normalized size = 2.48

$$-\frac{x^{3}}{x^{2}+\frac{e^{(4i\,a)}}{x^{2}}+2\,e^{(2i\,a)}}+2\left(\arctan\left(xe^{(-i\,a)}\right)e^{(-i\,a)}-\frac{x}{x^{2}+e^{(2i\,a)}}\right)e^{(2i\,a)}-\frac{6\,xe^{(2i\,a)}}{x^{2}+\frac{e^{(4i\,a)}}{x^{2}}+2\,e^{(2i\,a)}}-\frac{5\,e^{(4i\,a)}}{\left(x^{2}+\frac{e^{(4i\,a)}}{x^{2}}+2\,e^{(2i\,a)}\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x))^2,x, algorithm="giac")
```

```
[Out] -x^3/(x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a)) + 2*(arctan(x*e^(-I*a))*e^(-I*a) -
 x/(x^2 + e^(2*I*a)))*e^(2*I*a) - 6*x*e^(2*I*a)/(x^2 + e^(4*I*a)/x^2 + 2*e^
(2*I*a)) - 5*e^(4*I*a)/((x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a))*x)
```

Mupad [B]
time = 2.21, size = 42, normalized size = 0.91

$$-x+2\sqrt{\mathrm{e}^{a\,2\mathrm{i}}}\,\mathrm{atan}\left(\frac{x}{\sqrt{\mathrm{e}^{a\,2\mathrm{i}}}}\right)-\frac{2\,x\,\mathrm{e}^{a\,2\mathrm{i}}}{x^2+\mathrm{e}^{a\,2\mathrm{i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + log(x)*1i)^2,x)
```

```
[Out] 2*exp(a*2i)^(1/2)*atan(x/exp(a*2i)^(1/2)) - x - (2*x*exp(a*2i))/(exp(a*2i)
+ x^2)
```

3.147 $\int \frac{\tan^2(a+i\log(x))}{x}\,dx$

Optimal. Leaf size=18

$$-\log(x)-i\tan(a+i\log(x))$$

```
[Out] -ln(x)-I*tan(a+I*ln(x))
```

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = $\{3554, 8\}$

$$-\log(x)-i\tan(a+i\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + I*Log[x]]^2/x,x]
```

```
[Out] -Log[x] - I*Tan[a + I*Log[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(a+i\log(x))}{x}\,dx &= \text{Subst}\left(\int \tan^2(a+ix)\,dx, x, \log(x)\right)\\ &= -i\tan(a+i\log(x)) - \text{Subst}\left(\int 1\,dx, x, \log(x)\right)\\ &= -\log(x)-i\tan(a+i\log(x))\end{aligned}$$

Mathematica [A]
time = 0.05, size = 28, normalized size = 1.56

$$i\text{ArcTan}(\tan(a+i\log(x)))-i\tan(a+i\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + I*Log[x]]^2/x,x]
```

```
[Out] I*ArcTan[Tan[a + I*Log[x]]] - I*Tan[a + I*Log[x]]
```

Maple [A]
time = 0.04, size = 24, normalized size = 1.33

method	result	size
norman	$-\ln(x)-i\tan(a+i\ln(x))$	17
risch	$-\ln(x)+\frac{2}{1+\frac{e^{2ia}}{x^2}}$	21
derivativedivides	$-i(\tan(a+i\ln(x))-\arctan(\tan(a+i\ln(x))))$	24
default	$-i(\tan(a+i\ln(x))-\arctan(\tan(a+i\ln(x))))$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a+I*ln(x))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] -I*(tan(a+I*ln(x))-arctan(tan(a+I*ln(x))))
```

Maxima [A]
time = 0.47, size = 17, normalized size = 0.94

$$i\,a-\log(x)-i\,\tan(a+i\,\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x))^2/x,x, algorithm="maxima")
```

```
[Out] I*a - log(x) - I*tan(a + I*log(x))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(14)=28$.
time = 3.76, size = 30, normalized size = 1.67

$$-\frac{\left(x^2+e^{(2i\,a)}\right)\log(x)+2\,e^{(2i\,a)}}{x^2+e^{(2i\,a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x))^2/x,x, algorithm="fricas")
```

```
[Out] -((x^2 + e^(2*I*a))*log(x) + 2*e^(2*I*a))/(x^2 + e^(2*I*a))
```

Sympy [A]
time = 0.17, size = 22, normalized size = 1.22

$$-\log(x)-\frac{2e^{2ia}}{x^2+e^{2ia}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*ln(x))**2/x,x)
```

```
[Out] -log(x) - 2*exp(2*I*a)/(x**2 + exp(2*I*a))
```

Giac [A]
time = 0.43, size = 17, normalized size = 0.94

$$i\,a-\log\left(x\right)-i\,\tan\left(a+i\,\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x))^2/x,x, algorithm="giac")
```

```
[Out] I*a - log(x) - I*tan(a + I*log(x))
```

Mupad [B]
time = 2.38, size = 16, normalized size = 0.89

$$-\ln\left(x\right)-\tan(a+\ln\left(x\right)\,1\mathrm{i})\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + log(x)*1i)^2/x,x)
```

```
[Out] - tan(a + log(x)*1i)*1i - log(x)
```

3.148 $\int \frac{\tan^2(a+i\log(x))}{x^2}\,dx$

Optimal. Leaf size=60

$$\frac{e^{2ia}}{x\left(e^{2ia}+x^2\right)}+\frac{3x}{e^{2ia}+x^2}+2e^{-ia}\text{ArcTan}\left(e^{-ia}x\right)$$

```
[Out] exp(2*I*a)/x/(exp(2*I*a)+x^2)+3*x/(exp(2*I*a)+x^2)+2*arctan(x/exp(I*a))/exp
(I*a)
```

Rubi [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4591, 456, 473, 393, 209}

$$2e^{-ia}\text{ArcTan}\left(e^{-ia}x\right)+\frac{3x}{x^2+e^{2ia}}+\frac{e^{2ia}}{x\left(x^2+e^{2ia}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + I*Log[x]]^2/x^2,x]
```

```
[Out] E^((2*I)*a)/(x*(E^((2*I)*a) + x^2)) + (3*x)/(E^((2*I)*a) + x^2) + (2*ArcTan
[x/E^(I*a)])/E^(I*a)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 456

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int \frac{\tan^2(a+i\log(x))}{x^2}\,dx=\int \frac{\tan^2(a+i\log(x))}{x^2}\,dx$$

Mathematica [A]
time = 0.12, size = 72, normalized size = 1.20

$$\frac{1}{x}+2\text{ArcTan}(x(\cos(a)-i\sin(a)))\cos(a)-2i\text{ArcTan}(x(\cos(a)-i\sin(a)))\sin(a)+\frac{2x(\cos(a)-i\sin(a))}{(1+x^2)\cos(a)-i\,(-1+x^2)\sin(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + I*Log[x]]^2/x^2,x]
```

```
[Out] x^(-1) + 2*ArcTan[x*(Cos[a] - I*Sin[a])]*Cos[a] - (2*I)*ArcTan[x*(Cos[a] -
I*Sin[a])]*Sin[a] + (2*x*(Cos[a] - I*Sin[a]))/((1 + x^2)*Cos[a] - I*(-1 + x
^2)*Sin[a])
```

Maple [A]
time = 0.05, size = 38, normalized size = 0.63

method	result	size
risch	$\frac{1}{x}+\frac{2}{x\left(1+\frac{e^{2ia}}{x^2}\right)}+2\arctan\left(x\,e^{-ia}\right)e^{-ia}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a+I*ln(x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/x+2/x/(1+exp(2*I*a)/x^2)+2*arctan(x*exp(-I*a))*exp(-I*a)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(45) = 90$.
time = 0.55, size = 223, normalized size = 3.72

$$\frac{6\,x^2 - 2\left(x^3(\cos(a) - i\,\sin(a)) + ((\cos(a) - i\,\sin(a))\cos(2\,a) + (i\,\cos(a) + \sin(a))\sin(2\,a))x\right)\arctan\left(\frac{2\,x\cos(a)}{x^2+\cos(a)^2-2\,x\sin(a)+\sin(a)^2}, \frac{x^2-\cos(a)^2-\sin(a)^2}{x^2+\cos(a)^2-2\,x\sin(a)+\sin(a)^2}\right) + \left(x^3(-i\,\cos(a) - \sin(a)) + ((-i\,\cos(a) - \sin(a))\cos(2\,a) + (\cos(a) - i\,\sin(a))\sin(2\,a))x\right)\log\left(\frac{x^2+\cos(a)^2+2\,x\sin(a)+\sin(a)^2}{x^2+\cos(a)^2-2\,x\sin(a)+\sin(a)^2}\right) + 2\,\cos(2\,a) + 2i\,\sin(2\,a)}{2\left(x^3 + x(\cos(2\,a) + i\,\sin(2\,a))\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(6*x^2 - 2*(x^3*(cos(a) - I*sin(a)) + ((cos(a) - I*sin(a))*cos(2*a) + (
I*cos(a) + sin(a))*sin(2*a))*x)*arctan2(2*x*cos(a)/(x^2 + cos(a)^2 - 2*x*si
n(a) + sin(a)^2), (x^2 - cos(a)^2 - sin(a)^2)/(x^2 + cos(a)^2 - 2*x*sin(a)
+ sin(a)^2)) + (x^3*(-I*cos(a) - sin(a)) + ((-I*cos(a) - sin(a))*cos(2*a) +
 (cos(a) - I*sin(a))*sin(2*a))*x)*log((x^2 + cos(a)^2 + 2*x*sin(a) + sin(a)
^2)/(x^2 + cos(a)^2 - 2*x*sin(a) + sin(a)^2)) + 2*cos(2*a) + 2*I*sin(2*a))/
(x^3 + x*(cos(2*a) + I*sin(2*a)))
```

Fricas [A]
time = 3.27, size = 78, normalized size = 1.30

$$\frac{3\,x^2e^{(i\,a)} + \left(i\,x^3 + i\,xe^{(2i\,a)}\right)\log\left(x + i\,e^{(i\,a)}\right) + \left(-i\,x^3 - i\,xe^{(2i\,a)}\right)\log\left(x - i\,e^{(i\,a)}\right) + e^{(3i\,a)}}{x^3e^{(i\,a)} + xe^{(3i\,a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x))^2/x^2,x, algorithm="fricas")
```

```
[Out] (3*x^2*e^(I*a) + (I*x^3 + I*x*e^(2*I*a))*log(x + I*e^(I*a)) + (-I*x^3 - I*x
*e^(2*I*a))*log(x - I*e^(I*a)) + e^(3*I*a))/(x^3*e^(I*a) + x*e^(3*I*a))
```

Sympy [A]
time = 0.30, size = 54, normalized size = 0.90

$$-\frac{-3x^2 - e^{2ia}}{x^3 + xe^{2ia}} - \left(i\log\left(x - ie^{ia}\right) - i\log\left(x + ie^{ia}\right)\right)e^{-ia}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*ln(x))**2/x**2,x)
```

```
[Out] -(-3*x**2 - exp(2*I*a))/(x**3 + x*exp(2*I*a)) - (I*log(x - I*exp(I*a)) - I*
log(x + I*exp(I*a)))*exp(-I*a)
```

Giac [A]
time = 0.55, size = 73, normalized size = 1.22

$$2\left(\arctan\left(xe^{(-i\,a)}\right)e^{(-3i\,a)} + \frac{xe^{(-2i\,a)}}{x^2 + e^{(2i\,a)}}\right)e^{(2i\,a)} + \frac{5}{x\left(\frac{e^{(2i\,a)}}{x^2} + 1\right)} + \frac{e^{(2i\,a)}}{x^3\left(\frac{e^{(2i\,a)}}{x^2} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x))^2/x^2,x, algorithm="giac")
```

```
[Out] 2*(arctan(x*e^(-I*a))*e^(-3*I*a) + x*e^(-2*I*a)/(x^2 + e^(2*I*a)))*e^(2*I*a
) + 5/(x*(e^(2*I*a)/x^2 + 1)) + e^(2*I*a)/(x^3*(e^(2*I*a)/x^2 + 1))
```

Mupad [B]
time = 2.20, size = 45, normalized size = 0.75

$$\frac{2\,\mathrm{atan}\left(\frac{x}{\sqrt{\mathrm{e}^{a\,2\mathrm{i}}}}\right)}{\sqrt{\mathrm{e}^{a\,2\mathrm{i}}}}+\frac{3\,x^2+\mathrm{e}^{a\,2\mathrm{i}}}{x^3+\mathrm{e}^{a\,2\mathrm{i}}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + log(x)*1i)^2/x^2,x)
```

```
[Out] (2*atan(x/exp(a*2i)^(1/2)))/exp(a*2i)^(1/2) + (exp(a*2i) + 3*x^2)/(x^3 + x*
exp(a*2i))
```

3.149 $\int \frac{\tan^2(a+i\log(x))}{x^3}\,dx$

Optimal. Leaf size=55

$$-\frac{2e^{-2ia}}{1+\frac{e^{2ia}}{x^2}}+\frac{1}{2x^2}-2e^{-2ia}\log\left(1+\frac{e^{2ia}}{x^2}\right)$$

```
[Out] -2/exp(2*I*a)/(1+exp(2*I*a)/x^2)+1/2/x^2-2*ln(1+exp(2*I*a)/x^2)/exp(2*I*a)
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4591, 455, 45}

$$-\frac{2e^{-2ia}}{1+\frac{e^{2ia}}{x^2}}-2e^{-2ia}\log\left(1+\frac{e^{2ia}}{x^2}\right)+\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + I*Log[x]]^2/x^3,x]
```

```
[Out] -2/(E^((2*I)*a)*(1 + E^((2*I)*a)/x^2)) + 1/(2*x^2) - (2*Log[1 + E^((2*I)*a)
/x^2])/E^((2*I)*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int \frac{\tan^2(a + i\log(x))}{x^3}\,dx = \int \frac{\tan^2(a + i\log(x))}{x^3}\,dx$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 150 vs. $2(55) = 110$.
time = 0.21, size = 150, normalized size = 2.73

$$\frac{1}{2x^2} - 2i\text{ArcTan}\left(\frac{(1+x^2)\cot(a)}{-1+x^2}\right)\cos(2a) + 4\cos(2a)\log(x) - \cos(2a)\log\left(1+x^4+2x^2\cos(2a)\right) + \frac{2\cos(a) - 2i\sin(a)}{(1+x^2)\cos(a) - i(-1+x^2)\sin(a)} - 2\text{ArcTan}\left(\frac{(1+x^2)\cot(a)}{-1+x^2}\right)\sin(2a) - 4i\log(x)\sin(2a) + i\log\left(1+x^4+2x^2\cos(2a)\right)\sin(2a)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + I*Log[x]]^2/x^3,x]
```

```
[Out] 1/(2*x^2) - (2*I)*ArcTan[((1 + x^2)*Cot[a])/(-1 + x^2)]*Cos[2*a] + 4*Cos[2*
a]*Log[x] - Cos[2*a]*Log[1 + x^4 + 2*x^2*Cos[2*a]] + (2*Cos[a] - (2*I)*Sin[
a])/((1 + x^2)*Cos[a] - I*(-1 + x^2)*Sin[a]) - 2*ArcTan[((1 + x^2)*Cot[a])/
(-1 + x^2)]*Sin[2*a] - (4*I)*Log[x]*Sin[2*a] + I*Log[1 + x^4 + 2*x^2*Cos[2*
a]]*Sin[2*a]
```

Maple [**A**]
time = 0.05, size = 51, normalized size = 0.93

method	result	size
risch	$\frac{1}{2x^2} + \frac{2}{x^2\left(1+\frac{e^{2ia}}{x^2}\right)} + 4\,e^{-2ia}\ln(x) - 2\,e^{-2ia}\ln\left(e^{2ia} + x^2\right)$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a+I*ln(x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/x^2+2/x^2/(1+exp(2*I*a)/x^2)+4*exp(-2*I*a)*ln(x)-2*exp(-2*I*a)*ln(exp(2
*I*a)+x^2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x))^2/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 3.47, size = 74, normalized size = 1.35

$$\frac{5\,x^2e^{(2i\,a)}-4\left(x^4+x^2e^{(2i\,a)}\right)\log\left(x^2+e^{(2i\,a)}\right)+8\left(x^4+x^2e^{(2i\,a)}\right)\log\left(x\right)+e^{(4i\,a)}}{2\left(x^4e^{(2i\,a)}+x^2e^{(4i\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(5*x^2*e^(2*I*a) - 4*(x^4 + x^2*e^(2*I*a))*log(x^2 + e^(2*I*a)) + 8*(x^
4 + x^2*e^(2*I*a))*log(x) + e^(4*I*a))/(x^4*e^(2*I*a) + x^2*e^(4*I*a))
```

Sympy [A]
time = 0.42, size = 61, normalized size = 1.11

$$-\frac{-5x^2-e^{2ia}}{2x^4+2x^2e^{2ia}}+4e^{-2ia}\log\left(x\right)-2e^{-2ia}\log\left(x^2+e^{2ia}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*ln(x))**2/x**3,x)
```

```
[Out] -(-5*x**2 - exp(2*I*a))/(2*x**4 + 2*x**2*exp(2*I*a)) + 4*exp(-2*I*a)*log(x)
 - 2*exp(-2*I*a)*log(x**2 + exp(2*I*a))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(41) = 82.
time = 0.57, size = 178, normalized size = 3.24

$$-\frac{2\,\log\left(-x^2-e^{(2i\,a)}\right)}{\frac{e^{(4i\,a)}}{x^2}+e^{(2i\,a)}}+\frac{4\,\log\left(x\right)}{\frac{e^{(4i\,a)}}{x^2}+e^{(2i\,a)}}-\frac{2}{\frac{e^{(4i\,a)}}{x^2}+e^{(2i\,a)}}-\frac{2\,e^{(2i\,a)}\log\left(-x^2-e^{(2i\,a)}\right)}{x^2\left(\frac{e^{(4i\,a)}}{x^2}+e^{(2i\,a)}\right)}+\frac{4\,e^{(2i\,a)}\log\left(x\right)}{x^2\left(\frac{e^{(4i\,a)}}{x^2}+e^{(2i\,a)}\right)}+\frac{e^{(2i\,a)}}{2\,x^2\left(\frac{e^{(4i\,a)}}{x^2}+e^{(2i\,a)}\right)}+\frac{e^{(4i\,a)}}{2\,x^4\left(\frac{e^{(4i\,a)}}{x^2}+e^{(2i\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+I*log(x))^2/x^3,x, algorithm="giac")
```

```
[Out] -2*log(-x^2 - e^(2*I*a))/(e^(4*I*a)/x^2 + e^(2*I*a)) + 4*log(x)/(e^(4*I*a)/
x^2 + e^(2*I*a)) - 2/(e^(4*I*a)/x^2 + e^(2*I*a)) - 2*e^(2*I*a)*log(-x^2 - e
^(2*I*a))/(x^2*(e^(4*I*a)/x^2 + e^(2*I*a))) + 4*e^(2*I*a)*log(x)/(x^2*(e^(4
*I*a)/x^2 + e^(2*I*a))) + 1/2*e^(2*I*a)/(x^2*(e^(4*I*a)/x^2 + e^(2*I*a))) +
 1/2*e^(4*I*a)/(x^4*(e^(4*I*a)/x^2 + e^(2*I*a)))
```

Mupad [B]
time = 2.21, size = 56, normalized size = 1.02

$$-2\,\mathrm{e}^{-a\,2\mathrm{i}}\,\ln\left(x^2+\mathrm{e}^{a\,2\mathrm{i}}\right)+4\,\mathrm{e}^{-a\,2\mathrm{i}}\,\ln\left(x\right)+\frac{\frac{5\,x^2}{2}+\frac{\mathrm{e}^{a\,2\mathrm{i}}}{2}}{x^4+\mathrm{e}^{a\,2\mathrm{i}}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + log(x)*1i)^2/x^3,x)
```

```
[Out] 4*exp(-a*2i)*log(x) - 2*exp(-a*2i)*log(exp(a*2i) + x^2) + (exp(a*2i)/2 + (5
*x^2)/2)/(x^2*exp(a*2i) + x^4)
```

3.150 $\int (ex)^m \tan(a + i \log(x)) \, dx$

Optimal. Leaf size=71

$$-\frac{i(ex)^{1+m}}{e(1+m)} + \frac{2i(ex)^{1+m} \, _2F_1\left(1, \frac{1}{2}(-1-m); \frac{1-m}{2}; -\frac{e^{2ia}}{x^2}\right)}{e(1+m)}$$

```
[Out] -I*(e*x)^(1+m)/e/(1+m)+2*I*(e*x)^(1+m)*hypergeom([1, -1/2-1/2*m],[1/2-1/2*m
],-exp(2*I*a)/x^2)/e/(1+m)
```

Rubi [A]
time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {4591, 470, 346, 371}

$$\frac{2i(ex)^{m+1} \, _2F_1\left(1, \frac{1}{2}(-m-1); \frac{1-m}{2}; -\frac{e^{2ia}}{x^2}\right)}{e(m+1)} - \frac{i(ex)^{m+1}}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Tan[a + I*Log[x]],x]
```

```
[Out] ((-I)*(e*x)^(1 + m))/(e*(1 + m)) + ((2*I)*(e*x)^(1 + m)*Hypergeometric2F1[1
, (-1 - m)/2, (1 - m)/2, -(E^((2*I)*a)/x^2)])/(e*(1 + m))
```

Rule 346

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1), Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x
, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int (ex)^m \tan(a+i\log(x))\,dx = \int (ex)^m \tan(a+i\log(x))\,dx$$

Mathematica [A]
time = 0.22, size = 124, normalized size = 1.75

$$\frac{x(ex)^m(\cos(a)-i\sin(a))\left((3+m)\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-x^2(\cos(2a)-i\sin(2a))\right)(-i\cos(a)+\sin(a))+(1+m)x^2\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};-x^2(\cos(2a)-i\sin(2a))\right)(i\cos(a)+\sin(a))\right)}{(1+m)(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Tan[a + I*Log[x]],x]
[Out] (x*(e*x)^m*(Cos[a] - I*Sin[a])*((3 + m)*Hypergeometric2F1[1, (1 + m)/2, (3
+ m)/2, -(x^2*(Cos[2*a] - I*Sin[2*a]))]*((-I)*Cos[a] + Sin[a]) + (1 + m)*x^
2*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -(x^2*(Cos[2*a] - I*Sin[2*a]))
]*(I*Cos[a] + Sin[a])))/((1 + m)*(3 + m))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (ex)^m \tan\left(a+i\ln\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*tan(a+I*ln(x)),x)
[Out] int((e*x)^m*tan(a+I*ln(x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tan(a+I*log(x)),x, algorithm="maxima")
[Out] integrate((x*e)^m*tan(a + I*log(x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tan(a+I*log(x)),x, algorithm="fricas")
[Out] integral((I*x^2 - I*e^(2*I*a))*e^(m*log(x) + m)/(x^2 + e^(2*I*a)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \tan(a + i \log(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*tan(a+I*ln(x)),x)
[Out] Integral((e*x)**m*tan(a + I*log(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tan(a+I*log(x)),x, algorithm="giac")
[Out] integrate((e*x)^m*tan(a + I*log(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(a + \ln(x)\ 1\mathrm{i})\ (e\,x)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + log(x)*1i)*(e*x)^m,x)
[Out] int(tan(a + log(x)*1i)*(e*x)^m, x)
```

3.151 $\int (ex)^m \tan^2(a+i\log(x))\,dx$

Optimal. Leaf size=77

$$-\frac{x(ex)^m}{1+m}+\frac{2x(ex)^m}{1+\frac{e^{2ia}}{x^2}}-2x(ex)^m\,{}_2F_1\left(1,\frac{1}{2}(-1-m);\frac{1-m}{2};-\frac{e^{2ia}}{x^2}\right)$$

```
[Out] -x*(e*x)^m/(1+m)+2*x*(e*x)^m/(1+exp(2*I*a)/x^2)-2*x*(e*x)^m*hypergeom([1, -
1/2-1/2*m],[1/2-1/2*m],-exp(2*I*a)/x^2)
```

Rubi [A]
time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {4591, 511, 474, 470, 371}

$$-2x(ex)^m\,{}_2F_1\left(1,\frac{1}{2}(-m-1);\frac{1-m}{2};-\frac{e^{2ia}}{x^2}\right)+\frac{2x(ex)^m}{1+\frac{e^{2ia}}{x^2}}-\frac{x(ex)^m}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Tan[a + I*Log[x]]^2,x]
```

```
[Out] -((x*(e*x)^m)/(1 + m)) + (2*x*(e*x)^m)/(1 + E^((2*I)*a)/x^2) - 2*x*(e*x)^m*
Hypergeometric2F1[1, (-1 - m)/2, (1 - m)/2, -(E^((2*I)*a)/x^2)]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
```

```
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 511

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Dist[(-(e*x)^m)*(x^(-1))^m, Subst[Int[(a + b/x^n)^p*((
c + d/x^n)^q/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m, p, q},
 x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int (ex)^m \tan^2(a+i\log(x))\,dx=\int (ex)^m \tan^2(a+i\log(x))\,dx$$

Mathematica [A]
time = 0.18, size = 86, normalized size = 1.12

$$\frac{x(ex)^m\left(-1+4\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-x^2(\cos(2a)-i\sin(2a))\right)-4\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-x^2(\cos(2a)-i\sin(2a))\right)\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Tan[a + I*Log[x]]^2,x]
```

```
[Out] (x*(e*x)^m*(-1 + 4*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -(x^2*(Cos[2*
a] - I*Sin[2*a]))] - 4*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -(x^2*(Co
s[2*a] - I*Sin[2*a]))]))/(1 + m)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\tan^2\left(a+i\ln\left(x\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*tan(a+I*ln(x))^2,x)
```

```
[Out] int((e*x)^m*tan(a+I*ln(x))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tan(a+I*log(x))^2,x, algorithm="maxima")
[Out] integrate((x*e)^m*tan(a + I*log(x))^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tan(a+I*log(x))^2,x, algorithm="fricas")
[Out] integral(-(x^4 - 2*x^2*e^(2*I*a) + e^(4*I*a))*e^(m*log(x) + m)/(x^4 + 2*x^2
*e^(2*I*a) + e^(4*I*a)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \tan^2 (a + i \log (x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*tan(a+I*ln(x))**2,x)
[Out] Integral((e*x)**m*tan(a + I*log(x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tan(a+I*log(x))^2,x, algorithm="giac")
[Out] integrate((e*x)^m*tan(a + I*log(x))^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(a + \ln (x)\ 1\mathrm{i})^2\, (e\, x)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + log(x)*1i)^2*(e*x)^m,x)
[Out] int(tan(a + log(x)*1i)^2*(e*x)^m, x)
```

3.152 $\int (ex)^m \tan^3(a + i\log(x))\,dx$

Optimal. Leaf size=184

$$-\frac{i(1-m)mx(ex)^m}{2(1+m)}+\frac{i\left(1-\frac{e^{2ia}}{x^2}\right)^2 x(ex)^m}{2\left(1+\frac{e^{2ia}}{x^2}\right)^2}+\frac{ie^{-2ia}\left(e^{2ia}(3+m)+\frac{e^{4ia}(1-m)}{x^2}\right)x(ex)^m}{2\left(1+\frac{e^{2ia}}{x^2}\right)}-\frac{i(3+2m+m^2)\,x(ex)^n}{}$$

```
[Out] -1/2*I*(1-m)*m*x*(e*x)^m/(1+m)+1/2*I*(1-exp(2*I*a)/x^2)^2*x*(e*x)^m/(1+exp(
2*I*a)/x^2)^2+1/2*I*(exp(2*I*a)*(3+m)+exp(4*I*a)*(1-m)/x^2)*x*(e*x)^m/exp(2
*I*a)/(1+exp(2*I*a)/x^2)-I*(m^2+2*m+3)*x*(e*x)^m*hypergeom([1, -1/2-1/2*m],
[1/2-1/2*m],-exp(2*I*a)/x^2)/(1+m)
```

Rubi [A]
time = 0.18, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {4591, 511, 479, 591, 470, 371}

$$-\frac{i(m^2+2m+3)\,x(ex)^m\,{}_2F_1\left(1,\frac{1}{2}(-m-1);\frac{1-m}{2};-\frac{e^{2ia}}{x^2}\right)}{m+1}+\frac{ie^{-2ia}x\left(\frac{e^{4ia}(1-m)}{x^2}+e^{2ia}(m+3)\right)(ex)^m}{2\left(1+\frac{e^{2ia}}{x^2}\right)}+\frac{ix\left(1-\frac{e^{2ia}}{x^2}\right)^2(ex)^m}{2\left(1+\frac{e^{2ia}}{x^2}\right)^2}-\frac{i(1-m)mx(ex)^m}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Tan[a + I*Log[x]]^3,x]
```

```
[Out] ((-1/2*I)*(1 - m)*m*x*(e*x)^m)/(1 + m) + ((I/2)*(1 - E^((2*I)*a)/x^2)^2*x*(
e*x)^m)/(1 + E^((2*I)*a)/x^2)^2 + ((I/2)*(E^((2*I)*a)*(3 + m) + (E^((4*I)*a
)*(1 - m))/x^2)*x*(e*x)^m)/(E^((2*I)*a)*(1 + E^((2*I)*a)/x^2)) - (I*(3 + 2*
m + m^2)*x*(e*x)^m*Hypergeometric2F1[1, (-1 - m)/2, (1 - m)/2, -(E^((2*I)*a
)/x^2)])/(1 + m)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 511

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Dist[(-(e*x)^m)*(x^(-1))^m, Subst[Int[(a + b/x^n)^p*((
c + d/x^n)^q/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m, p, q},
 x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 591

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n,
 0] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e -
 a*f])
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int (ex)^m \tan^3(a+i\log(x))\,dx = \int (ex)^m \tan^3(a+i\log(x))\,dx$$

Mathematica [A]
time = 0.24, size = 125, normalized size = 0.68

$$\frac{ix(ex)^m\left(-1+6\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-x^2(\cos(2a)-i\sin(2a))\right)-12\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-x^2(\cos(2a)-i\sin(2a))\right)+8\,{}_2F_1\left(3,\frac{1+m}{2};\frac{3+m}{2};-x^2(\cos(2a)-i\sin(2a))\right)\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Tan[a + I*Log[x]]^3,x]
```

```
[Out] (I*x*(e*x)^m*(-1 + 6*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -(x^2*(Cos[
2*a] - I*Sin[2*a]))] - 12*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -(x^2*
(Cos[2*a] - I*Sin[2*a]))] + 8*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -(
x^2*(Cos[2*a] - I*Sin[2*a]))]))/(1 + m)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (ex)^{m} \left(\tan^{3}\left(a+i \ln \left(x\right)\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*tan(a+I*ln(x))^3,x)
```

```
[Out] int((e*x)^m*tan(a+I*ln(x))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tan(a+I*log(x))^3,x, algorithm="maxima")
```

```
[Out] integrate((x*e)^m*tan(a + I*log(x))^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tan(a+I*log(x))^3,x, algorithm="fricas")
```

```
[Out] integral((-I*x^6 + 3*I*x^4*e^(2*I*a) - 3*I*x^2*e^(4*I*a) + I*e^(6*I*a))*e^(
m*log(x) + m)/(x^6 + 3*x^4*e^(2*I*a) + 3*x^2*e^(4*I*a) + e^(6*I*a)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{m} \tan^{3}\left(a+i \log \left(x\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*tan(a+I*ln(x))**3,x)
```

```
[Out] Integral((e*x)**m*tan(a + I*log(x))**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tan(a+I*log(x))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*tan(a + I*log(x))^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(a+\ln(x)\ 1\mathrm{i})^3\,(e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + log(x)*1i)^3*(e*x)^m,x)
```

```
[Out] int(tan(a + log(x)*1i)^3*(e*x)^m, x)
```

3.153 $\int \tan^p(a+b\log(x))\,dx$

Optimal. Leaf size=142

$$x\left(1-e^{2ia}x^{2ib}\right)^{-p}\left(\frac{i\left(1-e^{2ia}x^{2ib}\right)}{1+e^{2ia}x^{2ib}}\right)^p\left(1+e^{2ia}x^{2ib}\right)^p F_1\left(-\frac{i}{2b};-p,p;1-\frac{i}{2b};e^{2ia}x^{2ib},-e^{2ia}x^{2ib}\right)$$

```
[Out] x*(I*(1-exp(2*I*a)*x^(2*I*b))/(1+exp(2*I*a)*x^(2*I*b)))^p*(1+exp(2*I*a)*x^(
2*I*b))^p*AppellF1(-1/2*I/b,-p,p,1-1/2*I/b,exp(2*I*a)*x^(2*I*b),-exp(2*I*a)
*x^(2*I*b))/((1-exp(2*I*a)*x^(2*I*b))^p)
```

Rubi [A]
time = 0.04, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {4587, 1986, 441, 440}

$$x\left(1-e^{2ia}x^{2ib}\right)^{-p}\left(\frac{i\left(1-e^{2ia}x^{2ib}\right)}{1+e^{2ia}x^{2ib}}\right)^p\left(1+e^{2ia}x^{2ib}\right)^p F_1\left(-\frac{i}{2b};-p,p;1-\frac{i}{2b};e^{2ia}x^{2ib},-e^{2ia}x^{2ib}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + b*Log[x]]^p,x]
```

```
[Out] (x*((I*(1 - E^((2*I)*a)*x^((2*I)*b)))/(1 + E^((2*I)*a)*x^((2*I)*b)))^p*(1 +
 E^((2*I)*a)*x^((2*I)*b))^p*AppellF1[(-1/2*I)/b, -p, p, 1 - (I/2)/b, E^((2*
I)*a)*x^((2*I)*b), -(E^((2*I)*a)*x^((2*I)*b))])/(1 - E^((2*I)*a)*x^((2*I)*b
))^p
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
```

```
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4587

```
Int[Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[((I - I*E^(2
*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d)))^p, x] /; FreeQ[{a, b, d
, p}, x]
```

Rubi steps

$$\int \tan^p(a+b\log(x))\,dx = \int \tan^p(a+b\log(x))\,dx$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 330 vs. 2(142) = 284.
time = 0.74, size = 330, normalized size = 2.32

$$\frac{(-i+2b)x\left(-\frac{i(-1+e^{2ia}x^{2ib})}{1+e^{2ia}x^{2ib}}\right)^p F_1\left(-\frac{i}{2b};-p,p;1-\frac{i}{2b};e^{2ia}x^{2ib},-e^{2ia}x^{2ib}\right)}{-2be^{2ia}px^{2ib}F_1\left(1-\frac{i}{2b};1-p,p;2-\frac{i}{2b};e^{2ia}x^{2ib},-e^{2ia}x^{2ib}\right)-2be^{2ia}px^{2ib}F_1\left(1-\frac{i}{2b};-p,1+p;2-\frac{i}{2b};e^{2ia}x^{2ib},-e^{2ia}x^{2ib}\right)+(-i+2b)F_1\left(-\frac{i}{2b};-p,p;1-\frac{i}{2b};e^{2ia}x^{2ib},-e^{2ia}x^{2ib}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[a + b*Log[x]]^p,x]
```

```
[Out] ((-I + 2*b)*x*(((-I)*(-1 + E^((2*I)*a)*x^((2*I)*b)))/(1 + E^((2*I)*a)*x^((2
*I)*b)))^p*AppellF1[(-1/2*I)/b, -p, p, 1 - (I/2)/b, E^((2*I)*a)*x^((2*I)*b)
, -(E^((2*I)*a)*x^((2*I)*b))])/(-2*b*E^((2*I)*a)*p*x^((2*I)*b)*AppellF1[1 -
 (I/2)/b, 1 - p, p, 2 - (I/2)/b, E^((2*I)*a)*x^((2*I)*b), -(E^((2*I)*a)*x^(
(2*I)*b))] - 2*b*E^((2*I)*a)*p*x^((2*I)*b)*AppellF1[1 - (I/2)/b, -p, 1 + p,
 2 - (I/2)/b, E^((2*I)*a)*x^((2*I)*b), -(E^((2*I)*a)*x^((2*I)*b))] + (-I +
2*b)*AppellF1[(-1/2*I)/b, -p, p, 1 - (I/2)/b, E^((2*I)*a)*x^((2*I)*b), -(E^
((2*I)*a)*x^((2*I)*b))])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \tan^{p}\left(a+b\ln\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a+b*ln(x))^p,x)
```

```
[Out] int(tan(a+b*ln(x))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(x))^p,x, algorithm="maxima")
[Out] integrate(tan(b*log(x) + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(x))^p,x, algorithm="fricas")
[Out] integral(tan(b*log(x) + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \tan^{p}\left(a + b\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*ln(x))**p,x)
[Out] Integral(tan(a + b*log(x))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(x))^p,x, algorithm="giac")
[Out] integrate(tan(b*log(x) + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{tan}\left(a+b\,\ln\left(x\right)\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*log(x))^p,x)
[Out] int(tan(a + b*log(x))^p, x)
```

3.154 $\int (ex)^m \tan^p(a + b\log(x))\,dx$

Optimal. Leaf size=162

$$\frac{(ex)^{1+m}\left(1-e^{2ia}x^{2ib}\right)^{-p}\left(\frac{i\left(1-e^{2ia}x^{2ib}\right)}{1+e^{2ia}x^{2ib}}\right)^{p}\left(1+e^{2ia}x^{2ib}\right)^{p}F_1\left(-\frac{i(1+m)}{2b};-p,p;1-\frac{i(1+m)}{2b};e^{2ia}x^{2ib},-e^{2ia}x^{2ib}\right)}{e(1+m)}$$

```
[Out] (e*x)^(1+m)*(I*(1-exp(2*I*a)*x^(2*I*b))/(1+exp(2*I*a)*x^(2*I*b)))^p*(1+exp(
2*I*a)*x^(2*I*b))^p*AppellF1(-1/2*I*(1+m)/b,-p,p,1-1/2*I*(1+m)/b,exp(2*I*a)
*x^(2*I*b),-exp(2*I*a)*x^(2*I*b))/e/(1+m)/((1-exp(2*I*a)*x^(2*I*b))^p)
```

Rubi [A]
time = 0.08, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {4591, 1986, 525, 524}

$$\frac{(ex)^{m+1}\left(1-e^{2ia}x^{2ib}\right)^{-p}\left(\frac{i\left(1-e^{2ia}x^{2ib}\right)}{1+e^{2ia}x^{2ib}}\right)^{p}\left(1+e^{2ia}x^{2ib}\right)^{p}F_1\left(-\frac{i(m+1)}{2b};-p,p;1-\frac{i(m+1)}{2b};e^{2ia}x^{2ib},-e^{2ia}x^{2ib}\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Tan[a + b*Log[x]]^p,x]
```

```
[Out] ((e*x)^(1 + m)*((I*(1 - E^((2*I)*a)*x^((2*I)*b)))/(1 + E^((2*I)*a)*x^((2*I)
*b)))^p*(1 + E^((2*I)*a)*x^((2*I)*b))^p*AppellF1[((-1/2*I)*(1 + m))/b, -p,
p, 1 - ((I/2)*(1 + m))/b, E^((2*I)*a)*x^((2*I)*b), -(E^((2*I)*a)*x^((2*I)*b
))])/(e*(1 + m)*(1 - E^((2*I)*a)*x^((2*I)*b))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
```

```
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int (ex)^m \tan^p(a+b\log(x))\,dx = \int (ex)^m \tan^p(a+b\log(x))\,dx$$

Mathematica [A]
time = 0.70, size = 157, normalized size = 0.97

$$\frac{x(ex)^m\left(1-e^{2ia}x^{2ib}\right)^{-p}\left(-\frac{i\left(-1+e^{2ia}x^{2ib}\right)}{1+e^{2ia}x^{2ib}}\right)^p\left(1+e^{2ia}x^{2ib}\right)^p F_1\left(-\frac{i(1+m)}{2b};-p,p;1-\frac{i(1+m)}{2b};e^{2ia}x^{2ib},-e^{2ia}x^{2ib}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Tan[a + b*Log[x]]^p,x]
```

```
[Out] (x*(e*x)^m*(((-I)*(-1 + E^((2*I)*a)*x^((2*I)*b)))/(1 + E^((2*I)*a)*x^((2*I)
*b)))^p*(1 + E^((2*I)*a)*x^((2*I)*b))^p*AppellF1[((-1/2*I)*(1 + m))/b, -p,
p, 1 - ((I/2)*(1 + m))/b, E^((2*I)*a)*x^((2*I)*b), -(E^((2*I)*a)*x^((2*I)*b
))])/((1 + m)*(1 - E^((2*I)*a)*x^((2*I)*b))^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\tan^p\left(a+b\ln\left(x\right)\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*tan(a+b*ln(x))^p,x)
```

```
[Out] int((e*x)^m*tan(a+b*ln(x))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tan(a+b*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e)^m*tan(b*log(x) + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tan(a+b*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral((x*e)^m*tan(b*log(x) + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \tan^p{(a + b \log{(x)})}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*tan(a+b*ln(x))**p,x)
```

```
[Out] Integral((e*x)**m*tan(a + b*log(x))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tan(a+b*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*tan(b*log(x) + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{tan}\left(a+b\,\ln\left(x\right)\right)}^p\,{\left(e\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*log(x))^p*(e*x)^m,x)
```

```
[Out] int(tan(a + b*log(x))^p*(e*x)^m, x)
```

3.155 $\int \tan^p(a+\log(x))\,dx$

Optimal. Leaf size=120

$$(1-e^{2ia}x^{2i})^{-p}\left(\frac{i(1-e^{2ia}x^{2i})}{1+e^{2ia}x^{2i}}\right)^p(1+e^{2ia}x^{2i})^p\,xF_1\left(-\frac{i}{2};-p,p;1-\frac{i}{2};e^{2ia}x^{2i},-e^{2ia}x^{2i}\right)$$

```
[Out] (I*(1-exp(2*I*a)*x^(2*I))/(1+exp(2*I*a)*x^(2*I)))^p*(1+exp(2*I*a)*x^(2*I))^
p*x*AppellF1(-1/2*I,-p,p,1-1/2*I,exp(2*I*a)*x^(2*I),-exp(2*I*a)*x^(2*I))/((
1-exp(2*I*a)*x^(2*I))^p)
```

Rubi [A]
time = 0.04, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4587, 1986, 441, 440}

$$x(1-e^{2ia}x^{2i})^{-p}\left(\frac{i(1-e^{2ia}x^{2i})}{1+e^{2ia}x^{2i}}\right)^p(1+e^{2ia}x^{2i})^p\,F_1\left(-\frac{i}{2};-p,p;1-\frac{i}{2};e^{2ia}x^{2i},-e^{2ia}x^{2i}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + Log[x]]^p,x]
```

```
[Out] (((I*(1 - E^((2*I)*a)*x^(2*I)))/(1 + E^((2*I)*a)*x^(2*I)))^p*(1 + E^((2*I)*
a)*x^(2*I))^p*x*AppellF1[-1/2*I, -p, p, 1 - I/2, E^((2*I)*a)*x^(2*I), -(E^(
(2*I)*a)*x^(2*I))])/(1 - E^((2*I)*a)*x^(2*I))^p
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4587

```
Int[Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[((I - I*E^(2
*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d)))^p, x] /; FreeQ[{a, b, d
, p}, x]
```

Rubi steps

$$\int \tan^p(a+\log(x))\,dx = \int \tan^p(a+\log(x))\,dx$$

Mathematica [A]
time = 0.58, size = 240, normalized size = 2.00

$$\frac{(1+2i)\left(-\frac{i(-1+e^{2ia}x^{2i})}{1+e^{2ia}x^{2i}}\right)^p xF_1\left(-\frac{i}{2};-p,p;1-\frac{i}{2};e^{2ia}x^{2i},-e^{2ia}x^{2i}\right)}{(1+2i)F_1\left(-\frac{i}{2};-p,p;1-\frac{i}{2};e^{2ia}x^{2i},-e^{2ia}x^{2i}\right)-2ie^{2ia}px^{2i}\left(F_1\left(1-\frac{i}{2};1-p,p;2-\frac{i}{2};e^{2ia}x^{2i},-e^{2ia}x^{2i}\right)+F_1\left(1-\frac{i}{2};-p,1+p;2-\frac{i}{2};e^{2ia}x^{2i},-e^{2ia}x^{2i}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[a + Log[x]]^p,x]
```

```
[Out] ((1 + 2*I)*(((-I)*(-1 + E^((2*I)*a)*x^(2*I)))/(1 + E^((2*I)*a)*x^(2*I)))^p*
x*AppellF1[-1/2*I, -p, p, 1 - I/2, E^((2*I)*a)*x^(2*I), -(E^((2*I)*a)*x^(2*
I))])/((1 + 2*I)*AppellF1[-1/2*I, -p, p, 1 - I/2, E^((2*I)*a)*x^(2*I), -(E^
((2*I)*a)*x^(2*I))] - (2*I)*E^((2*I)*a)*p*x^(2*I)*(AppellF1[1 - I/2, 1 - p,
 p, 2 - I/2, E^((2*I)*a)*x^(2*I), -(E^((2*I)*a)*x^(2*I))] + AppellF1[1 - I/
2, -p, 1 + p, 2 - I/2, E^((2*I)*a)*x^(2*I), -(E^((2*I)*a)*x^(2*I))]))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \tan^{p}\left(a+\ln\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a+ln(x))^p,x)
```

```
[Out] int(tan(a+ln(x))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(tan(a + log(x))^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(tan(a + log(x))^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \tan^{p}\left(a + \log\left(x\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+ln(x))**p,x)
```

```
[Out] Integral(tan(a + log(x))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(tan(a + log(x))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(a+\ln\left(x\right))^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + log(x))^p,x)
```

```
[Out] int(tan(a + log(x))^p, x)
```

3.156 $\int \tan^p(a+2\log(x))\,dx$

Optimal. Leaf size=120

$$\left(1-e^{2ia}x^{4i}\right)^{-p}\left(\frac{i(1-e^{2ia}x^{4i})}{1+e^{2ia}x^{4i}}\right)^p\left(1+e^{2ia}x^{4i}\right)^p xF_1\left(-\frac{i}{4};-p,p;1-\frac{i}{4};e^{2ia}x^{4i},-e^{2ia}x^{4i}\right)$$

```
[Out] (I*(1-exp(2*I*a)*x^(4*I))/(1+exp(2*I*a)*x^(4*I)))^p*(1+exp(2*I*a)*x^(4*I))^
p*x*AppellF1(-1/4*I,-p,p,1-1/4*I,exp(2*I*a)*x^(4*I),-exp(2*I*a)*x^(4*I))/((
1-exp(2*I*a)*x^(4*I))^p)
```

Rubi [A]
time = 0.04, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {4587, 1986, 441, 440}

$$x\left(1-e^{2ia}x^{4i}\right)^{-p}\left(\frac{i(1-e^{2ia}x^{4i})}{1+e^{2ia}x^{4i}}\right)^p\left(1+e^{2ia}x^{4i}\right)^p F_1\left(-\frac{i}{4};-p,p;1-\frac{i}{4};e^{2ia}x^{4i},-e^{2ia}x^{4i}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + 2*Log[x]]^p,x]
```

```
[Out] (((I*(1 - E^((2*I)*a)*x^(4*I)))/(1 + E^((2*I)*a)*x^(4*I)))^p*(1 + E^((2*I)*
a)*x^(4*I))^p*x*AppellF1[-1/4*I, -p, p, 1 - I/4, E^((2*I)*a)*x^(4*I), -(E^(
(2*I)*a)*x^(4*I))])/(1 - E^((2*I)*a)*x^(4*I))^p
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4587

```
Int[Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[((I - I*E^(2
*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d)))^p, x] /; FreeQ[{a, b, d
, p}, x]
```

Rubi steps

$$\int \tan^p(a+2\log(x))\,dx=\int \tan^p(a+2\log(x))\,dx$$

Mathematica [A]
time = 0.57, size = 240, normalized size = 2.00

$$\frac{(1+4i)\left(-\frac{i(-1+e^{2ia}x^{4i})}{1+e^{2ia}x^{4i}}\right)^p xF_1\left(-\frac{i}{4};-p,p;1-\frac{i}{4};e^{2ia}x^{4i},-e^{2ia}x^{4i}\right)}{(1+4i)F_1\left(-\frac{i}{4};-p,p;1-\frac{i}{4};e^{2ia}x^{4i},-e^{2ia}x^{4i}\right)-4ie^{2ia}px^{4i}\left(F_1\left(1-\frac{i}{4};1-p,p;2-\frac{i}{4};e^{2ia}x^{4i},-e^{2ia}x^{4i}\right)+F_1\left(1-\frac{i}{4};-p,1+p;2-\frac{i}{4};e^{2ia}x^{4i},-e^{2ia}x^{4i}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[a + 2*Log[x]]^p,x]
```

```
[Out] ((1 + 4*I)*(((-I)*(-1 + E^((2*I)*a)*x^(4*I)))/(1 + E^((2*I)*a)*x^(4*I)))^p*
x*AppellF1[-1/4*I, -p, p, 1 - I/4, E^((2*I)*a)*x^(4*I), -(E^((2*I)*a)*x^(4*
I))])/((1 + 4*I)*AppellF1[-1/4*I, -p, p, 1 - I/4, E^((2*I)*a)*x^(4*I), -(E^
((2*I)*a)*x^(4*I))] - (4*I)*E^((2*I)*a)*p*x^(4*I)*(AppellF1[1 - I/4, 1 - p,
 p, 2 - I/4, E^((2*I)*a)*x^(4*I), -(E^((2*I)*a)*x^(4*I))] + AppellF1[1 - I/
4, -p, 1 + p, 2 - I/4, E^((2*I)*a)*x^(4*I), -(E^((2*I)*a)*x^(4*I))]))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \tan^{p}\left(a+2\ln\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a+2*ln(x))^p,x)
```

```
[Out] int(tan(a+2*ln(x))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+2*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(tan(a + 2*log(x))^p, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+2*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(tan(a + 2*log(x))^p, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \tan^{p}\left(a+2\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+2*ln(x))**p,x)
```

```
[Out] Integral(tan(a + 2*log(x))**p, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+2*log(x))^p,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(a+2\,\ln\left(x\right))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + 2*log(x))^p,x)
```

```
[Out] int(tan(a + 2*log(x))^p, x)
```

3.157 $\int \tan^p(a+3\log(x))\,dx$

Optimal. Leaf size=120

$$\left(1-e^{2ia}x^{6i}\right)^{-p}\left(\frac{i(1-e^{2ia}x^{6i})}{1+e^{2ia}x^{6i}}\right)^p\left(1+e^{2ia}x^{6i}\right)^p xF_1\left(-\frac{i}{6};-p,p;1-\frac{i}{6};e^{2ia}x^{6i},-e^{2ia}x^{6i}\right)$$

```
[Out] (I*(1-exp(2*I*a)*x^(6*I))/(1+exp(2*I*a)*x^(6*I)))^p*(1+exp(2*I*a)*x^(6*I))^
p*x*AppellF1(-1/6*I,-p,p,1-1/6*I,exp(2*I*a)*x^(6*I),-exp(2*I*a)*x^(6*I))/((
1-exp(2*I*a)*x^(6*I))^p)
```

Rubi [A]
time = 0.04, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {4587, 1986, 441, 440}

$$x\left(1-e^{2ia}x^{6i}\right)^{-p}\left(\frac{i(1-e^{2ia}x^{6i})}{1+e^{2ia}x^{6i}}\right)^p\left(1+e^{2ia}x^{6i}\right)^p F_1\left(-\frac{i}{6};-p,p;1-\frac{i}{6};e^{2ia}x^{6i},-e^{2ia}x^{6i}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + 3*Log[x]]^p,x]
```

```
[Out] (((I*(1 - E^((2*I)*a)*x^(6*I)))/(1 + E^((2*I)*a)*x^(6*I)))^p*(1 + E^((2*I)*
a)*x^(6*I))^p*x*AppellF1[-1/6*I, -p, p, 1 - I/6, E^((2*I)*a)*x^(6*I), -(E^(
(2*I)*a)*x^(6*I))])/(1 - E^((2*I)*a)*x^(6*I))^p
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4587

```
Int[Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[((I - I*E^(2
*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d)))^p, x] /; FreeQ[{a, b, d
, p}, x]
```

Rubi steps

$$\int \tan^p(a+3\log(x))\,dx = \int \tan^p(a+3\log(x))\,dx$$

Mathematica [A]
time = 0.56, size = 240, normalized size = 2.00

$$\frac{(1+6i)\left(-\frac{i(-1+e^{2ia}x^{6i})}{1+e^{2ia}x^{6i}}\right)^p xF_1\left(-\frac{i}{6};-p,p;1-\frac{i}{6};e^{2ia}x^{6i},-e^{2ia}x^{6i}\right)}{(1+6i)F_1\left(-\frac{i}{6};-p,p;1-\frac{i}{6};e^{2ia}x^{6i},-e^{2ia}x^{6i}\right)-6ie^{2ia}px^{6i}\left(F_1\left(1-\frac{i}{6};1-p,p;2-\frac{i}{6};e^{2ia}x^{6i},-e^{2ia}x^{6i}\right)+F_1\left(1-\frac{i}{6};-p,1+p;2-\frac{i}{6};e^{2ia}x^{6i},-e^{2ia}x^{6i}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[a + 3*Log[x]]^p,x]
```

```
[Out] ((1 + 6*I)*(((-I)*(-1 + E^((2*I)*a)*x^(6*I)))/(1 + E^((2*I)*a)*x^(6*I)))^p*
x*AppellF1[-1/6*I, -p, p, 1 - I/6, E^((2*I)*a)*x^(6*I), -(E^((2*I)*a)*x^(6*
I))])/((1 + 6*I)*AppellF1[-1/6*I, -p, p, 1 - I/6, E^((2*I)*a)*x^(6*I), -(E^
((2*I)*a)*x^(6*I))] - (6*I)*E^((2*I)*a)*p*x^(6*I)*(AppellF1[1 - I/6, 1 - p,
 p, 2 - I/6, E^((2*I)*a)*x^(6*I), -(E^((2*I)*a)*x^(6*I))] + AppellF1[1 - I/
6, -p, 1 + p, 2 - I/6, E^((2*I)*a)*x^(6*I), -(E^((2*I)*a)*x^(6*I))]))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \tan^p\left(a+3\ln\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a+3*ln(x))^p,x)
```

```
[Out] int(tan(a+3*ln(x))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+3*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(tan(a + 3*log(x))^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+3*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(tan(a + 3*log(x))^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \tan^{p}\left(a + 3\log\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+3*ln(x))**p,x)
```

```
[Out] Integral(tan(a + 3*log(x))**p, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+3*log(x))^p,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(a + 3\,\ln\left(x\right))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + 3*log(x))^p,x)
```

```
[Out] int(tan(a + 3*log(x))^p, x)
```

3.158 $\int x^3 \tan\left(d(a+b\log\left(cx^n\right))\right)\,dx$

Optimal. Leaf size=71

$$-\frac{ix^4}{4}+\frac{1}{2}ix^4\,{}_2F_1\left(1,-\frac{2i}{bdn};1-\frac{2i}{bdn};-e^{2iad}\left(cx^n\right)^{2ibd}\right)$$

```
[Out] -1/4*I*x^4+1/2*I*x^4*hypergeom([1, -2*I/b/d/n],[1-2*I/b/d/n],-exp(2*I*a*d)*
(c*x^n)^(2*I*b*d))
```

Rubi [A]
time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4593, 4591, 470, 371}

$$\frac{1}{2}ix^4\,{}_2F_1\left(1,-\frac{2i}{bdn};1-\frac{2i}{bdn};-e^{2iad}\left(cx^n\right)^{2ibd}\right)-\frac{ix^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Tan[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (-1/4*I)*x^4 + (I/2)*x^4*Hypergeometric2F1[1, (-2*I)/(b*d*n), 1 - (2*I)/(b*
d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4593

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^3 \tan \left(d(a+b\log \left(cx^n\right))\right) \, dx = \int x^3 \tan \left(d(a+b\log \left(cx^n\right))\right) \, dx$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 146 vs. $2(71) = 142$.
time = 6.80, size = 146, normalized size = 2.06

$$\frac{x^4\left(2ie^{2id(a+b\log(cx^n))}\,{}_2F_1\left(1,1-\frac{2i}{bdn};2-\frac{2i}{bdn};-e^{2id(a+b\log(cx^n))}\right)+(-2i+bdn)\,{}_2F_1\left(1,-\frac{2i}{bdn};1-\frac{2i}{bdn};-e^{2id(a+b\log(cx^n))}\right)\right)}{-8-4ibdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Tan[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^4*((2*I)*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - (2*I)/(
b*d*n), 2 - (2*I)/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))] + (-2*I + b*d*n
)*Hypergeometric2F1[1, (-2*I)/(b*d*n), 1 - (2*I)/(b*d*n), -E^((2*I)*d*(a +
b*Log[c*x^n]))]))/(-8 - (4*I)*b*d*n)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^3 \tan \left(d(a+b\ln \left(c\,x^n\right))\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tan(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x^3*tan(d*(a+b*ln(c*x^n))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(x^3*tan((b*log(c*x^n) + a)*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(d*(a+b*log(c*x^n))),x, algorithm="fricas")
[Out] integral(x^3*tan(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \tan (ad + bd \log (cx^n))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*tan(d*(a+b*ln(c*x**n))),x)
[Out] Integral(x**3*tan(a*d + b*d*log(c*x**n)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \tan(d\,(a + b\,\ln(c\,x^n)))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tan(d*(a + b*log(c*x^n))),x)
[Out] int(x^3*tan(d*(a + b*log(c*x^n))), x)
```

3.159 $\int x^2 \tan(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=75

$$-\frac{ix^3}{3}+\frac{2}{3}ix^3\,{}_2F_1\left(1,-\frac{3i}{2bdn};1-\frac{3i}{2bdn};-e^{2iad}(cx^n)^{2ibd}\right)$$

```
[Out] -1/3*I*x^3+2/3*I*x^3*hypergeom([1, -3/2*I/b/d/n],[1-3/2*I/b/d/n],-exp(2*I*a
*d)*(c*x^n)^(2*I*b*d))
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {4593, 4591, 470, 371}

$$\frac{2}{3}ix^3\,{}_2F_1\left(1,-\frac{3i}{2bdn};1-\frac{3i}{2bdn};-e^{2iad}(cx^n)^{2ibd}\right)-\frac{ix^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Tan[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (-1/3*I)*x^3 + ((2*I)/3)*x^3*Hypergeometric2F1[1, ((-3*I)/2)/(b*d*n), 1 - (
(3*I)/2)/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4593

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^2 \tan\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x^2 \tan\left(d(a+b\log\left(cx^n\right))\right)\,dx$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 155 vs. 2(75) = 150.
time = 6.43, size = 155, normalized size = 2.07

$$\frac{x^3\left(3ie^{2id(a+b\log(cx^n))}\,{}_2F_1\left(1,1-\frac{3i}{2bdn};2-\frac{3i}{2bdn};-e^{2id(a+b\log(cx^n))}\right)+(-3i+2bdn)\,{}_2F_1\left(1,-\frac{3i}{2bdn};1-\frac{3i}{2bdn};-e^{2id(a+b\log(cx^n))}\right)\right)}{-9-6ibdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Tan[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^3*((3*I)*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - ((3*I)/
2)/(b*d*n), 2 - ((3*I)/2)/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))] + (-3*I
 + 2*b*d*n)*Hypergeometric2F1[1, ((-3*I)/2)/(b*d*n), 1 - ((3*I)/2)/(b*d*n),
 -E^((2*I)*d*(a + b*Log[c*x^n]))]))/(-9 - (6*I)*b*d*n)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^2 \tan\left(d(a+b\ln\left(c\,x^n\right))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*tan(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x^2*tan(d*(a+b*ln(c*x^n))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tan(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(x^2*tan((b*log(c*x^n) + a)*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tan(d*(a+b*log(c*x^n))),x, algorithm="fricas")
[Out] integral(x^2*tan(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \tan\left(ad + bd \log\left(cx^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*tan(d*(a+b*ln(c*x**n))),x)
[Out] Integral(x**2*tan(a*d + b*d*log(c*x**n)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tan(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \tan(d\left(a + b\,\ln\left(c\,x^n\right)\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*tan(d*(a + b*log(c*x^n))),x)
[Out] int(x^2*tan(d*(a + b*log(c*x^n))), x)
```

3.160 $\int x \tan(d(a + b\log(cx^n)))\,dx$

Optimal. Leaf size=69

$$-\frac{ix^2}{2} + ix^2\,{}_2F_1\left(1, -\frac{i}{bdn}; 1 - \frac{i}{bdn}; -e^{2iad}(cx^n)^{2ibd}\right)$$

```
[Out] -1/2*I*x^2+I*x^2*hypergeom([1, -I/b/d/n],[1-I/b/d/n],-exp(2*I*a*d)*(c*x^n)^
(2*I*b*d))
```

Rubi [A]
time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {4593, 4591, 470, 371}

$$ix^2\,{}_2F_1\left(1, -\frac{i}{bdn}; 1 - \frac{i}{bdn}; -e^{2iad}(cx^n)^{2ibd}\right) - \frac{ix^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Tan[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (-1/2*I)*x^2 + I*x^2*Hypergeometric2F1[1, (-I)/(b*d*n), 1 - I/(b*d*n), -(E^
((2*I)*a*d)*(c*x^n)^((2*I)*b*d))]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4593

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x \tan \left(d(a+b\log \left(cx^n\right))\right) \, dx = \int x \tan \left(d(a+b\log \left(cx^n\right))\right) \, dx$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 146 vs. $2(69) = 138$.
time = 6.66, size = 146, normalized size = 2.12

$$\frac{x^2\left(ie^{2id(a+b\log(cx^n))}\,{}_2F_1\left(1,1-\frac{i}{bdn};2-\frac{i}{bdn};-e^{2id(a+b\log(cx^n))}\right)+(-i+bdn)\,{}_2F_1\left(1,-\frac{i}{bdn};1-\frac{i}{bdn};-e^{2id(a+b\log(cx^n))}\right)\right)}{-2-2ibdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Tan[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^2*(I*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - I/(b*d*n),
2 - I/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))] + (-I + b*d*n)*Hypergeometr
ic2F1[1, (-I)/(b*d*n), 1 - I/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))]))/(-
2 - (2*I)*b*d*n)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x \tan \left(d(a+b\ln \left(c\,x^n\right))\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x*tan(d*(a+b*ln(c*x^n))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(x*tan((b*log(c*x^n) + a)*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(d*(a+b*log(c*x^n))),x, algorithm="fricas")
[Out] integral(x*tan(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\tan\left(ad + bd\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(d*(a+b*ln(c*x**n))),x)
[Out] Integral(x*tan(a*d + b*d*log(c*x**n)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\tan(d\left(a + b\,\ln\left(c\,x^n\right)\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(d*(a + b*log(c*x^n))),x)
[Out] int(x*tan(d*(a + b*log(c*x^n))), x)
```

3.161 $\int \tan(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=67

$$-ix+2ix\,{}_2F_1\left(1,-\frac{i}{2bdn};1-\frac{i}{2bdn};-e^{2iad}(cx^n)^{2ibd}\right)$$

```
[Out] -I*x+2*I*x*hypergeom([1, -1/2*I/b/d/n],[1-1/2*I/b/d/n],-exp(2*I*a*d)*(c*x^n
)^(2*I*b*d))
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {4589, 4591, 470, 371}

$$2ix\,{}_2F_1\left(1,-\frac{i}{2bdn};1-\frac{i}{2bdn};-e^{2iad}(cx^n)^{2ibd}\right)-ix$$

Antiderivative was successfully verified.

```
[In] Int[Tan[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (-I)*x + (2*I)*x*Hypergeometric2F1[1, (-1/2*I)/(b*d*n), 1 - (I/2)/(b*d*n),
-(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 4589

```
Int[Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Tan[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int \tan (d(a+b\log (cx^n)))\, dx = \int \tan (d(a+b\log (cx^n)))\, dx$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 151 vs. $2(67) = 134$.
time = 11.37, size = 151, normalized size = 2.25

$$\frac{x\left(-e^{2id(a+b\log(cx^n))}\,{}_2F_1\left(1,1-\frac{i}{2bdn};2-\frac{i}{2bdn};-e^{2id(a+b\log(cx^n))}\right)+(1+2ibdn)\,{}_2F_1\left(1,-\frac{i}{2bdn};1-\frac{i}{2bdn};-e^{2id(a+b\log(cx^n))}\right)\right)}{-i+2bdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x*(-(E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - (I/2)/(b*d*n)
, 2 - (I/2)/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))]) + (1 + (2*I)*b*d*n)*
Hypergeometric2F1[1, (-1/2*I)/(b*d*n), 1 - (I/2)/(b*d*n), -E^((2*I)*d*(a +
b*Log[c*x^n]))]))/(-I + 2*b*d*n)
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \tan (d(a+b\ln (c\,x^n)))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(tan(d*(a+b*ln(c*x^n))),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(tan((b*log(c*x^n) + a)*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*log(c*x^n))),x, algorithm="fricas")
[Out] integral(tan(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \tan \left(d(a + b \log \left(cx^{n}\right))\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*ln(c*x**n))),x)
[Out] Integral(tan(d*(a + b*log(c*x**n))), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(d\,(a + b\,\ln\left(c\,x^n\right)))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*(a + b*log(c*x^n))),x)
[Out] int(tan(d*(a + b*log(c*x^n))), x)
```

3.162 $\int \frac{\tan(d(a+b\log(cx^n)))}{x}\,dx$

Optimal. Leaf size=26

$$-\frac{\log\left(\cos\left(ad+bd\log\left(cx^n\right)\right)\right)}{bdn}$$

```
[Out] -ln(cos(a*d+b*d*ln(c*x^n)))/b/d/n
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{3556\}$

$$-\frac{\log\left(\cos\left(ad+bd\log\left(cx^n\right)\right)\right)}{bdn}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] -(Log[Cos[a*d + b*d*Log[c*x^n]]]/(b*d*n))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \tan(d(a+bx))\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= -\frac{\log\left(\cos\left(ad+bd\log\left(cx^n\right)\right)\right)}{bdn}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 25, normalized size = 0.96

$$-\frac{\log\left(\cos\left(d\left(a+b\log\left(cx^n\right)\right)\right)\right)}{bdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] -(Log[Cos[d*(a + b*Log[c*x^n])]]/(b*d*n))
```

Maple [A]
time = 0.06, size = 30, normalized size = 1.15

method	result
derivativedivides	$\frac{\ln\left(1+\tan^2\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)\right)}{2nbd}$
default	$\frac{\ln\left(1+\tan^2\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)\right)}{2nbd}$
risch	$-i\ln\left(x\right)+\frac{2ia}{nb}+\frac{2i\ln(c)}{n}+\frac{2i\ln(x^n)}{n}+\frac{\pi\mathrm{csgn}(ic\,x^n)^3}{n}-\frac{\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ic)}{n}-\frac{\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ix^n)}{n}+$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*(a+b*ln(c*x^n)))/x,x,method=_RETURNVERBOSE)

[Out] 1/2/n/b/d*ln(1+tan(d*(a+b*ln(c*x^n)))^2)

Maxima [A]
time = 0.28, size = 24, normalized size = 0.92

$$\frac{\log\left(\sec\left((b\log\left(cx^n\right)+a)d\right)\right)}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")

[Out] log(sec((b*log(c*x^n) + a)*d))/(b*d*n)

Fricas [A]
time = 2.80, size = 35, normalized size = 1.35

$$-\frac{\log\left(\frac{1}{2}\cos\left(2\,bdn\log\left(x\right)+2\,bd\log\left(c\right)+2\,ad\right)+\frac{1}{2}\right)}{2\,bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")

[Out] -1/2*log(1/2*cos(2*b*d*n*log(x) + 2*b*d*log(c) + 2*a*d) + 1/2)/(b*d*n)

Sympy [A]
time = 2.37, size = 44, normalized size = 1.69

$$\begin{cases}\log\left(x\right)\tan\left(ad\right) & \text{for}\ b=0\\ 0 & \text{for}\ d=0\\ \log\left(x\right)\tan\left(ad+bd\log\left(c\right)\right) & \text{for}\ n=0\\ -\frac{\log\left(\cos\left(ad+bd\log\left(cx^n\right)\right)\right)}{bdn} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*ln(c*x**n)))/x,x)
```

```
[Out] Piecewise((log(x)*tan(a*d), Eq(b, 0)), (0, Eq(d, 0)), (log(x)*tan(a*d + b*d
*log(c)), Eq(n, 0)), (-log(cos(a*d + b*d*log(c*x**n)))/(b*d*n), True))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 3.78, size = 38, normalized size = 1.46

$$\ln(x)\ 1\mathrm{i} - \frac{\ln\left(\mathrm{e}^{a\,d\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,d\,2\mathrm{i}}+1\right)}{b\,d\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*(a + b*log(c*x^n)))/x,x)
```

```
[Out] log(x)*1i - log(exp(a*d*2i)*(c*x^n)^(b*d*2i) + 1)/(b*d*n)
```

3.163 $\int \frac{\tan(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal. Leaf size=71

$$\frac{i}{x}-\frac{2i\,{}_2F_1\left(1,\frac{i}{2bdn};1+\frac{i}{2bdn};-e^{2iad}(cx^n)^{2ibd}\right)}{x}$$

```
[Out] I/x-2*I*hypergeom([1, 1/2*I/b/d/n],[1+1/2*I/b/d/n],-exp(2*I*a*d)*(c*x^n)^(2
*I*b*d))/x
```

Rubi [A]
time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4593, 4591, 470, 371}

$$\frac{i}{x}-\frac{2i\,{}_2F_1\left(1,\frac{i}{2bdn};1+\frac{i}{2bdn};-e^{2iad}(cx^n)^{2ibd}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] I/x - ((2*I)*Hypergeometric2F1[1, (I/2)/(b*d*n), 1 + (I/2)/(b*d*n), -(E^((2
*I)*a*d)*(c*x^n)^((2*I)*b*d))])/x
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4593

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\tan\left(d(a+b\log\left(cx^n\right))\right)}{x^2}\,dx=\int \frac{\tan\left(d(a+b\log\left(cx^n\right))\right)}{x^2}\,dx$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 153 vs. $2(71) = 142$.
time = 4.47, size = 153, normalized size = 2.15

$$\frac{-e^{2id(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{i}{2bdn};2+\frac{i}{2bdn};-e^{2id(a+b\log(cx^n))}\right)+(1-2ibdn)\,{}_2F_1\left(1,\frac{i}{2bdn};1+\frac{i}{2bdn};-e^{2id(a+b\log(cx^n))}\right)}{(i+2bdn)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] (-(E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + (I/2)/(b*d*n), 2
 + (I/2)/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))]) + (1 - (2*I)*b*d*n)*Hyp
ergeometric2F1[1, (I/2)/(b*d*n), 1 + (I/2)/(b*d*n), -E^((2*I)*d*(a + b*Log[
c*x^n]))])/((I + 2*b*d*n)*x)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(d(a+b\ln\left(c\,x^n\right))\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*(a+b*ln(c*x^n)))/x^2,x)
```

```
[Out] int(tan(d*(a+b*ln(c*x^n)))/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(tan((b*log(c*x^n) + a)*d)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

```
[Out] integral(tan(b*d*log(c*x^n) + a*d)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(ad + bd\log\left(cx^{n}\right)\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*ln(c*x**n)))/x**2,x)
```

```
[Out] Integral(tan(a*d + b*d*log(c*x**n))/x**2, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] int(tan(d*(a + b*log(c*x^n)))/x^2, x)
```

3.164 $\int \frac{\tan(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal. Leaf size=69

$$\frac{i}{2x^2}-\frac{i\,{}_2F_1\left(1,\frac{i}{bdn};1+\frac{i}{bdn};-e^{2iad}\left(cx^n\right)^{2ibd}\right)}{x^2}$$

```
[Out] 1/2*I/x^2-I*hypergeom([1, I/b/d/n],[1+I/b/d/n],-exp(2*I*a*d)*(c*x^n)^(2*I*b
*d))/x^2
```

Rubi [A]

time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4593, 4591, 470, 371}

$$\frac{i}{2x^2}-\frac{i\,{}_2F_1\left(1,\frac{i}{bdn};1+\frac{i}{bdn};-e^{2iad}\left(cx^n\right)^{2ibd}\right)}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] (I/2)/x^2 - (I*Hypergeometric2F1[1, I/(b*d*n), 1 + I/(b*d*n), -(E^((2*I)*a*
d)*(c*x^n)^((2*I)*b*d))])/x^2
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4593

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\tan\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx = \int \frac{\tan\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 147 vs. 2(69) = 138.
time = 4.19, size = 147, normalized size = 2.13

$$\frac{-e^{2id(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{i}{bdn};2+\frac{i}{bdn};-e^{2id(a+b\log(cx^n))}\right)+(1-ibdn)\,{}_2F_1\left(1,\frac{i}{bdn};1+\frac{i}{bdn};-e^{2id(a+b\log(cx^n))}\right)}{2(i+bdn)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] (-(E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + I/(b*d*n), 2 + I
/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))]) + (1 - I*b*d*n)*Hypergeometric2
F1[1, I/(b*d*n), 1 + I/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))])/(2*(I + b
*d*n)*x^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(d(a+b\ln\left(c\,x^n\right))\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*(a+b*ln(c*x^n)))/x^3,x)
```

```
[Out] int(tan(d*(a+b*ln(c*x^n)))/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

```
[Out] integrate(tan((b*log(c*x^n) + a)*d)/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

```
[Out] integral(tan(b*d*log(c*x^n) + a*d)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(ad + bd\log\left(cx^n\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*ln(c*x**n)))/x**3,x)
```

```
[Out] Integral(tan(a*d + b*d*log(c*x**n))/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
```

```
[Out] integrate(tan((b*log(c*x^n) + a)*d)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(d\left(a + b\,\ln\left(c\,x^n\right)\right))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] int(tan(d*(a + b*log(c*x^n)))/x^3, x)
```

3.165 $\int x^3 \tan^2(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=159

$$\frac{(4i-bdn)x^4}{4bdn}+\frac{ix^4\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bdn\left(1+e^{2iad}(cx^n)^{2ibd}\right)}-\frac{2ix^4\,{}_2F_1\left(1,-\frac{2i}{bdn};1-\frac{2i}{bdn};-e^{2iad}(cx^n)^{2ibd}\right)}{bdn}$$

```
[Out] 1/4*(4*I-b*d*n)*x^4/b/d/n+I*x^4*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/(1
+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))-2*I*x^4*hypergeom([1, -2*I/b/d/n],[1-2*I/b
/d/n],-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n
```

Rubi [A]

time = 0.12, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {4593, 4591, 516, 470, 371}

$$-\frac{2ix^4\,{}_2F_1\left(1,-\frac{2i}{bdn};1-\frac{2i}{bdn};-e^{2iad}(cx^n)^{2ibd}\right)}{bdn}+\frac{ix^4\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bdn\left(1+e^{2iad}(cx^n)^{2ibd}\right)}+\frac{x^4(-bdn+4i)}{4bdn}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Tan[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] ((4*I - b*d*n)*x^4)/(4*b*d*n) + (I*x^4*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*
d)))/(b*d*n*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - ((2*I)*x^4*Hypergeom
etric2F1[1, (-2*I)/(b*d*n), 1 - (2*I)/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*
I)*b*d))])/(b*d*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4593

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^3\tan^2\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x^3\tan^2\left(d(a+b\log\left(cx^n\right))\right)\,dx$$

Mathematica [A]
time = 7.08, size = 179, normalized size = 1.13

$$-\frac{x^4\left(-8e^{2id(a+b\log(cx^n))}\,{}_2F_1\left(1,1-\frac{2i}{bdn};2-\frac{2i}{bdn};-e^{2id(a+b\log(cx^n))}\right)+(-2i+bdn)\left(bdn+4i\,{}_2F_1\left(1,-\frac{2i}{bdn};1-\frac{2i}{bdn};-e^{2id(a+b\log(cx^n))}\right)-4\tan\left(d(a+b\log\left(cx^n\right))\right)\right)\right)}{4bdn(-2i+bdn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Tan[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] -1/4*(x^4*(-8*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - (2*I)
/(b*d*n), 2 - (2*I)/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))] + (-2*I + b*d
*n)*(b*d*n + (4*I)*Hypergeometric2F1[1, (-2*I)/(b*d*n), 1 - (2*I)/(b*d*n),
-E^((2*I)*d*(a + b*Log[c*x^n]))] - 4*Tan[d*(a + b*Log[c*x^n])])))/(b*d*n*(-
2*I + b*d*n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^3\left(\tan^2\left(d(a+b\ln\left(c\,x^n\right))\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tan(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(x^3*tan(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] -1/4*((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x^4*cos(2*b*d*l
og(x^n) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*
x^4*sin(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n*x^4 + 2*(b*d*n*cos(2*b*d*log(c))
- 4*sin(2*b*d*log(c)))*x^4*cos(2*b*d*log(x^n) + 2*a*d) - 2*(b*d*n*sin(2*b*d
*log(c)) + 4*cos(2*b*d*log(c)))*x^4*sin(2*b*d*log(x^n) + 2*a*d) + 32*(2*b^2
*d^2*n^2*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*sin(
2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) + b^2*d^2*n^2 + (b^2*d^2*cos(2*b*
d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) + 2*a*d)^
2 + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*sin(2*b
*d*log(x^n) + 2*a*d)^2)*integrate((x^3*cos(2*b*d*log(x^n) + 2*a*d)*sin(2*b*
d*log(c)) + x^3*cos(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d))/(2*b^2*d^2*n
^2*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*sin(2*b*d*
log(c))*sin(2*b*d*log(x^n) + 2*a*d) + b^2*d^2*n^2 + (b^2*d^2*cos(2*b*d*log(
c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) + 2*a*d)^2 + (b
^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*sin(2*b*d*log
(x^n) + 2*a*d)^2), x))/(2*b*d*n*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*
d) - 2*b*d*n*sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) + (b*d*cos(2*b*d
*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*cos(2*b*d*log(x^n) + 2*a*d)^2 + (b*
d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*sin(2*b*d*log(x^n) + 2*a
*d)^2 + b*d*n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*tan(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\tan^{2}\left(ad+bd\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*tan(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Integral(x**3*tan(a*d + b*d*log(c*x**n))**2, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\tan(d\,(a+b\,\ln\left(c\,x^n\right)))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tan(d*(a + b*log(c*x^n)))^2,x)
```

```
[Out] int(x^3*tan(d*(a + b*log(c*x^n)))^2, x)
```

3.166 $\int x^2 \tan^2(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=163

$$\frac{(3i-bdn)x^3}{3bdn}+\frac{ix^3\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bdn\left(1+e^{2iad}(cx^n)^{2ibd}\right)}-\frac{2ix^3\,{}_2F_1\left(1,-\frac{3i}{2bdn};1-\frac{3i}{2bdn};-e^{2iad}(cx^n)^{2ibd}\right)}{bdn}$$

```
[Out] 1/3*(3*I-b*d*n)*x^3/b/d/n+I*x^3*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/(1
+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))-2*I*x^3*hypergeom([1, -3/2*I/b/d/n],[1-3/2
*I/b/d/n],-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n
```

Rubi [A]
time = 0.12, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.263$, Rules used = {4593, 4591, 516, 470, 371}

$$-\frac{2ix^3\,{}_2F_1\left(1,-\frac{3i}{2bdn};1-\frac{3i}{2bdn};-e^{2iad}(cx^n)^{2ibd}\right)}{bdn}+\frac{ix^3\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bdn\left(1+e^{2iad}(cx^n)^{2ibd}\right)}+\frac{x^3(-bdn+3i)}{3bdn}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Tan[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] ((3*I - b*d*n)*x^3)/(3*b*d*n) + (I*x^3*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*
d)))/(b*d*n*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - ((2*I)*x^3*Hypergeom
etric2F1[1, ((-3*I)/2)/(b*d*n), 1 - ((3*I)/2)/(b*d*n), -(E^((2*I)*a*d)*(c*x
^n)^((2*I)*b*d))])/(b*d*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4593

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^2 \tan^2\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x^2 \tan^2\left(d(a+b\log\left(cx^n\right))\right)\,dx$$

Mathematica [A]
time = 6.87, size = 189, normalized size = 1.16

$$-\frac{x^3\left(-9e^{2id(a+b\log(cx^n))}\,{}_2F_1\left(1,1-\frac{3i}{2bdn};2-\frac{3i}{2bdn};-e^{2id(a+b\log(cx^n))}\right)+(-3i+2bdn)\left(bdn+3i\,{}_2F_1\left(1,-\frac{3i}{2bdn};1-\frac{3i}{2bdn};-e^{2id(a+b\log(cx^n))}\right)-3\tan\left(d(a+b\log\left(cx^n\right))\right)\right)\right)}{3bdn(-3i+2bdn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Tan[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] -1/3*(x^3*(-9*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - ((3*I
)/2)/(b*d*n), 2 - ((3*I)/2)/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))] + (-3
*I + 2*b*d*n)*(b*d*n + (3*I)*Hypergeometric2F1[1, ((-3*I)/2)/(b*d*n), 1 - (
(3*I)/2)/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))] - 3*Tan[d*(a + b*Log[c*x
^n])])))/(b*d*n*(-3*I + 2*b*d*n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^2\left(\tan^2\left(d(a+b\ln\left(c\,x^n\right))\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*tan(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(x^2*tan(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] -1/3*((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x^3*cos(2*b*d*l
og(x^n) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*
x^3*sin(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n*x^3 + 2*(b*d*n*cos(2*b*d*log(c))
- 3*sin(2*b*d*log(c)))*x^3*cos(2*b*d*log(x^n) + 2*a*d) - 2*(b*d*n*sin(2*b*d
*log(c)) + 3*cos(2*b*d*log(c)))*x^3*sin(2*b*d*log(x^n) + 2*a*d) + 18*(2*b^2
*d^2*n^2*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*sin(
2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) + b^2*d^2*n^2 + (b^2*d^2*cos(2*b*
d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) + 2*a*d)^
2 + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*sin(2*b
*d*log(x^n) + 2*a*d)^2)*integrate((x^2*cos(2*b*d*log(x^n) + 2*a*d)*sin(2*b*
d*log(c)) + x^2*cos(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d))/(2*b^2*d^2*n
^2*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*sin(2*b*d*
log(c))*sin(2*b*d*log(x^n) + 2*a*d) + b^2*d^2*n^2 + (b^2*d^2*cos(2*b*d*log(
c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) + 2*a*d)^2 + (b
^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*sin(2*b*d*log
(x^n) + 2*a*d)^2), x))/(2*b*d*n*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*
d) - 2*b*d*n*sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) + (b*d*cos(2*b*d
*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*cos(2*b*d*log(x^n) + 2*a*d)^2 + (b*
d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*sin(2*b*d*log(x^n) + 2*a
*d)^2 + b*d*n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*tan(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \tan^2 \left(ad + bd \log \left(cx^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*tan(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Integral(x**2*tan(a*d + b*d*log(c*x**n))**2, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \tan(d\,(a+b\,\ln{(c\,x^n)}))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*tan(d*(a + b*log(c*x^n)))^2,x)
```

```
[Out] int(x^2*tan(d*(a + b*log(c*x^n)))^2, x)
```

3.167 $\int x\tan^2(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=159

$$\frac{(2i-bdn)x^2}{2bdn}+\frac{ix^2\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bdn\left(1+e^{2iad}\,(cx^n)^{2ibd}\right)}-\frac{2ix^2\,{}_2F_1\left(1,-\frac{i}{bdn};1-\frac{i}{bdn};-e^{2iad}(cx^n)^{2ibd}\right)}{bdn}$$

```
[Out] 1/2*(2*I-b*d*n)*x^2/b/d/n+I*x^2*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/(1
+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))-2*I*x^2*hypergeom([1, -I/b/d/n],[1-I/b/d/n
],-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n
```

Rubi [A]
time = 0.12, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {4593, 4591, 516, 470, 371}

$$-\frac{2ix^2\,{}_2F_1\left(1,-\frac{i}{bdn};1-\frac{i}{bdn};-e^{2iad}(cx^n)^{2ibd}\right)}{bdn}+\frac{ix^2\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bdn\left(1+e^{2iad}\,(cx^n)^{2ibd}\right)}+\frac{x^2(-bdn+2i)}{2bdn}$$

Antiderivative was successfully verified.

```
[In] Int[x*Tan[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] ((2*I - b*d*n)*x^2)/(2*b*d*n) + (I*x^2*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*
d)))/(b*d*n*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - ((2*I)*x^2*Hypergeom
etric2F1[1, (-I)/(b*d*n), 1 - I/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d
))])/(b*d*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4593

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x\tan^2\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\tan^2\left(d(a+b\log\left(cx^n\right))\right)\,dx$$

Mathematica [A]
time = 6.97, size = 179, normalized size = 1.13

$$-\frac{x^2\left(-2e^{2id(a+b\log(cx^n))}\,{}_2F_1\left(1,1-\frac{i}{bdn};2-\frac{i}{bdn};-e^{2id(a+b\log(cx^n))}\right)+(-i+bdn)\left(bdn+2i\,{}_2F_1\left(1,-\frac{i}{bdn};1-\frac{i}{bdn};-e^{2id(a+b\log(cx^n))}\right)-2\tan\left(d(a+b\log\left(cx^n\right))\right)\right)\right)}{2bdn(-i+bdn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Tan[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] -1/2*(x^2*(-2*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - I/(b*
d*n), 2 - I/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))] + (-I + b*d*n)*(b*d*n
 + (2*I)*Hypergeometric2F1[1, (-I)/(b*d*n), 1 - I/(b*d*n), -E^((2*I)*d*(a +
 b*Log[c*x^n]))] - 2*Tan[d*(a + b*Log[c*x^n])])))/(b*d*n*(-I + b*d*n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x\left(\tan^2\left(d(a+b\ln\left(c\,x^n\right))\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(x*tan(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] -1/2*((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x^2*cos(2*b*d*l
og(x^n) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*
x^2*sin(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n*x^2 + 2*(b*d*n*cos(2*b*d*log(c))
- 2*sin(2*b*d*log(c)))*x^2*cos(2*b*d*log(x^n) + 2*a*d) - 2*(b*d*n*sin(2*b*d
*log(c)) + 2*cos(2*b*d*log(c)))*x^2*sin(2*b*d*log(x^n) + 2*a*d) + 8*(2*b^2*
d^2*n^2*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*sin(2
*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) + b^2*d^2*n^2 + (b^2*d^2*cos(2*b*d
*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) + 2*a*d)^2
 + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*sin(2*b*
d*log(x^n) + 2*a*d)^2)*integrate((x*cos(2*b*d*log(x^n) + 2*a*d)*sin(2*b*d*l
og(c)) + x*cos(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d))/(2*b^2*d^2*n^2*co
s(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*sin(2*b*d*log(c
))*sin(2*b*d*log(x^n) + 2*a*d) + b^2*d^2*n^2 + (b^2*d^2*cos(2*b*d*log(c))^2
 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) + 2*a*d)^2 + (b^2*d^
2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*sin(2*b*d*log(x^n)
 + 2*a*d)^2), x))/(2*b*d*n*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) -
2*b*d*n*sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) + (b*d*cos(2*b*d*log(
c))^2 + b*d*sin(2*b*d*log(c))^2)*n*cos(2*b*d*log(x^n) + 2*a*d)^2 + (b*d*cos
(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*sin(2*b*d*log(x^n) + 2*a*d)^2
 + b*d*n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(x*tan(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\tan^2\left(ad+bd\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Integral(x*tan(a*d + b*d*log(c*x**n))**2, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\tan(d\,(a+b\,\ln\left(c\,x^n\right)))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(d*(a + b*log(c*x^n)))^2,x)
```

```
[Out] int(x*tan(d*(a + b*log(c*x^n)))^2, x)
```

3.168 $\int \tan^2(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=154

$$\frac{(i-bdn)x}{bdn}+\frac{ix\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bdn\left(1+e^{2iad}\left(cx^n\right)^{2ibd}\right)}-\frac{2ix\,{}_2F_1\left(1,-\frac{i}{2bdn};1-\frac{i}{2bdn};-e^{2iad}(cx^n)^{2ibd}\right)}{bdn}$$

```
[Out] (I-b*d*n)*x/b/d/n+I*x*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/(1+exp(2*I*a
*d)*(c*x^n)^(2*I*b*d))-2*I*x*hypergeom([1, -1/2*I/b/d/n],[1-1/2*I/b/d/n],-e
xp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n
```

Rubi [A]
time = 0.11, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4589, 4591, 516, 470, 371}

$$-\frac{2ix\,{}_2F_1\left(1,-\frac{i}{2bdn};1-\frac{i}{2bdn};-e^{2iad}(cx^n)^{2ibd}\right)}{bdn}+\frac{ix\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bdn\left(1+e^{2iad}\left(cx^n\right)^{2ibd}\right)}+\frac{x(-bdn+i)}{bdn}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] ((I - b*d*n)*x)/(b*d*n) + (I*x*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*
d*n*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - ((2*I)*x*Hypergeometric2F1[1
, (-1/2*I)/(b*d*n), 1 - (I/2)/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))
])/(b*d*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 4589

```
Int[Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Tan[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int \tan^2\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int \tan^2\left(d(a+b\log\left(cx^n\right))\right)\,dx$$

Mathematica [A]
time = 11.71, size = 185, normalized size = 1.20

$$\frac{e^{2id(a+b\log(cx^n))}x\,{}_2F_1\left(1,1-\frac{i}{2bdn};2-\frac{i}{2bdn};-e^{2id(a+b\log(cx^n))}\right)-(-i+2bdn)x\left(bdn+i\,{}_2F_1\left(1,-\frac{i}{2bdn};1-\frac{i}{2bdn};-e^{2id(a+b\log(cx^n))}\right)-\tan\left(d(a+b\log\left(cx^n\right))\right)\right)}{bdn(-i+2bdn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (E^((2*I)*d*(a + b*Log[c*x^n]))*x*Hypergeometric2F1[1, 1 - (I/2)/(b*d*n), 2
 - (I/2)/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))] - (-I + 2*b*d*n)*x*(b*d*
n + I*Hypergeometric2F1[1, (-1/2*I)/(b*d*n), 1 - (I/2)/(b*d*n), -E^((2*I)*d
*(a + b*Log[c*x^n]))] - Tan[d*(a + b*Log[c*x^n])]))/(b*d*n*(-I + 2*b*d*n))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \tan^2\left(d(a+b\ln\left(c\,x^n\right))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(tan(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] -((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x*cos(2*b*d*log(x^n
) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x*sin(
2*b*d*log(x^n) + 2*a*d)^2 + b*d*n*x + 2*(b*d*n*cos(2*b*d*log(c)) - sin(2*b*
d*log(c)))*x*cos(2*b*d*log(x^n) + 2*a*d) - 2*(b*d*n*sin(2*b*d*log(c)) + cos
(2*b*d*log(c)))*x*sin(2*b*d*log(x^n) + 2*a*d) + 2*(2*b^2*d^2*n^2*cos(2*b*d*
log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*sin(2*b*d*log(c))*sin(2
*b*d*log(x^n) + 2*a*d) + b^2*d^2*n^2 + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d
^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) + 2*a*d)^2 + (b^2*d^2*cos(2*
b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*sin(2*b*d*log(x^n) + 2*a*d
)^2)*integrate((cos(2*b*d*log(x^n) + 2*a*d)*sin(2*b*d*log(c)) + cos(2*b*d*l
og(c))*sin(2*b*d*log(x^n) + 2*a*d))/(2*b^2*d^2*n^2*cos(2*b*d*log(c))*cos(2*
b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*sin(2*b*d*log(c))*sin(2*b*d*log(x^n)
+ 2*a*d) + b^2*d^2*n^2 + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*l
og(c))^2)*n^2*cos(2*b*d*log(x^n) + 2*a*d)^2 + (b^2*d^2*cos(2*b*d*log(c))^2
+ b^2*d^2*sin(2*b*d*log(c))^2)*n^2*sin(2*b*d*log(x^n) + 2*a*d)^2), x))/(2*b
*d*n*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b*d*n*sin(2*b*d*log(
c))*sin(2*b*d*log(x^n) + 2*a*d) + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*
log(c))^2)*n*cos(2*b*d*log(x^n) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d
*sin(2*b*d*log(c))^2)*n*sin(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(tan(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \tan^2\left(d(a+b\log\left(cx^n\right))\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Integral(tan(d*(a + b*log(c*x**n)))**2, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(d\,(a+b\,\ln\left(c\,x^n\right)))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*(a + b*log(c*x^n)))^2,x)
```

```
[Out] int(tan(d*(a + b*log(c*x^n)))^2, x)
```

3.169 $\int \frac{\tan^2(d(a+b\log(cx^n)))}{x}\,dx$

Optimal. Leaf size=29

$$-\log(x)+\frac{\tan(ad+bd\log(cx^n))}{bdn}$$

```
[Out] -ln(x)+tan(a*d+b*d*ln(c*x^n))/b/d/n
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3554, 8}

$$\frac{\tan(ad+bd\log(cx^n))}{bdn}-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[d*(a + b*Log[c*x^n])]^2/x,x]
```

```
[Out] -Log[x] + Tan[a*d + b*d*Log[c*x^n]]/(b*d*n)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(d(a+b\log(cx^n)))}{x}\,dx &= \frac{\text{Subst}\left(\int \tan^2(d(a+bx))\,dx,x,\log(cx^n)\right)}{n}\\ &= \frac{\tan(ad+bd\log(cx^n))}{bdn}-\frac{\text{Subst}\left(\int 1\,dx,x,\log(cx^n)\right)}{n}\\ &= -\log(x)+\frac{\tan(ad+bd\log(cx^n))}{bdn}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 51, normalized size = 1.76

$$-\frac{\text{ArcTan}(\tan(ad+bd\log(cx^n)))}{bdn}+\frac{\tan(ad+bd\log(cx^n))}{bdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[d*(a + b*Log[c*x^n])]^2/x,x]
```

```
[Out] -(ArcTan[Tan[a*d + b*d*Log[c*x^n]]]/(b*d*n)) + Tan[a*d + b*d*Log[c*x^n]]/(b
*d*n)
```

Maple [A]
time = 0.03, size = 41, normalized size = 1.41

method	result
derivativedivides	$\frac{\tan(d(a+b\ln(c\,x^{n})))-\arctan(\tan(d(a+b\ln(c\,x^{n}))))}{nbd}$
default	$\frac{\tan(d(a+b\ln(c\,x^{n})))-\arctan(\tan(d(a+b\ln(c\,x^{n}))))}{nbd}$
risch	$-\ln(x)+\frac{2i}{dbn\left(\mathrm{e}^{id\left(-ib\pi\mathrm{csgn}(ic\,x^{n})^{3}+ib\pi\mathrm{csgn}(ic\,x^{n})^{2}\mathrm{csgn}(ic)+ib\pi\mathrm{csgn}(ic\,x^{n})^{2}\mathrm{csgn}(ix^{n})-ib\pi\,\mathrm{csgn}(ic\,x^{n})\mathrm{csgn}(ic)\mathrm{csgn}(ix^{n})+\right.}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*(a+b*ln(c*x^n)))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b/d*(tan(d*(a+b*ln(c*x^n)))-arctan(tan(d*(a+b*ln(c*x^n)))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(29) = 58.
time = 0.29, size = 320, normalized size = 11.03

$$-\frac{(bd\cos(2bd\log(c))^{2}+bd\sin(2bd\log(c))^{2})n\cos(2bd\log(x^{n})+2ad)^{2}\log(x)+(bd\cos(2bd\log(c))^{2}+bd\sin(2bd\log(c))^{2})n\log(x)\sin(2bd\log(x^{n})+2ad)^{2}+bdn\log(x)+2(bdn\cos(2bd\log(c))\log(x)-\sin(2bd\log(c)))\cos(2bd\log(x^{n})+2ad)-2(bdn\log(x)\sin(2bd\log(c))+\cos(2bd\log(c)))\sin(2bd\log(x^{n})+2ad)}{2bdn\cos(2bd\log(c))\cos(2bd\log(x^{n})+2ad)-2bdn\sin(2bd\log(c))\sin(2bd\log(x^{n})+2ad)+(bd\cos(2bd\log(c))^{2}+bd\sin(2bd\log(c))^{2})n\cos(2bd\log(x^{n})+2ad)^{2}+(bd\cos(2bd\log(c))^{2}+bd\sin(2bd\log(c))^{2})n\sin(2bd\log(x^{n})+2ad)^{2}+bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*log(c*x^n)))^2/x,x, algorithm="maxima")
```

```
[Out] -((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*cos(2*b*d*log(x^n)
+ 2*a*d)^2*log(x) + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*l
og(x)*sin(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n*log(x) + 2*(b*d*n*cos(2*b*d*log
(c))*log(x) - sin(2*b*d*log(c)))*cos(2*b*d*log(x^n) + 2*a*d) - 2*(b*d*n*log
(x)*sin(2*b*d*log(c)) + cos(2*b*d*log(c)))*sin(2*b*d*log(x^n) + 2*a*d))/(2*
b*d*n*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b*d*n*sin(2*b*d*log
(c))*sin(2*b*d*log(x^n) + 2*a*d) + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d
*log(c))^2)*n*cos(2*b*d*log(x^n) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*
d*sin(2*b*d*log(c))^2)*n*sin(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(29) = 58.
time = 2.11, size = 85, normalized size = 2.93

$$-\frac{bdn\cos(2\,bdn\log(x)+2\,bd\log(c)+2\,ad)\log(x)+bdn\log(x)-\sin(2\,bdn\log(x)+2\,bd\log(c)+2\,ad)}{bdn\cos(2\,bdn\log(x)+2\,bd\log(c)+2\,ad)+bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*log(c*x^n)))^2/x,x, algorithm="fricas")
```

```
[Out] -(b*d*n*cos(2*b*d*n*log(x) + 2*b*d*log(c) + 2*a*d)*log(x) + b*d*n*log(x) -
sin(2*b*d*n*log(x) + 2*b*d*log(c) + 2*a*d))/(b*d*n*cos(2*b*d*n*log(x) + 2*b
*d*log(c) + 2*a*d) + b*d*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2\left(ad + bd\log\left(cx^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*ln(c*x**n)))**2/x,x)
```

```
[Out] Integral(tan(a*d + b*d*log(c*x**n))**2/x, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*log(c*x^n)))^2/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 3.84, size = 39, normalized size = 1.34

$$-\ln\left(x\right) + \frac{2\mathrm{i}}{b\,d\,n\,\left(\mathrm{e}^{a\,d\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,d\,2\mathrm{i}} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*(a + b*log(c*x^n)))^2/x,x)
```

```
[Out] 2i/(b*d*n*(exp(a*d*2i)*(c*x^n)^(b*d*2i) + 1)) - log(x)
```

3.170 $\int \frac{\tan^2(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal. Leaf size=157

$$\frac{1+\frac{i}{bdn}}{x}+\frac{i\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bdnx\left(1+e^{2iad}\left(cx^n\right)^{2ibd}\right)}-\frac{2i\,{}_2F_1\left(1,\frac{i}{2bdn};1+\frac{i}{2bdn};-e^{2iad}(cx^n)^{2ibd}\right)}{bdnx}$$

```
[Out] (1+I/b/d/n)/x+I*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/x/(1+exp(2*I*a*d)*
(c*x^n)^(2*I*b*d))-2*I*hypergeom([1, 1/2*I/b/d/n],[1+1/2*I/b/d/n],-exp(2*I*
a*d)*(c*x^n)^(2*I*b*d))/b/d/n/x
```

Rubi [A]
time = 0.13, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {4593, 4591, 516, 470, 371}

$$-\frac{2i\,{}_2F_1\left(1,\frac{i}{2bdn};1+\frac{i}{2bdn};-e^{2iad}(cx^n)^{2ibd}\right)}{bdnx}+\frac{i\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bdnx\left(1+e^{2iad}\left(cx^n\right)^{2ibd}\right)}+\frac{1+\frac{i}{bdn}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[d*(a + b*Log[c*x^n])]^2/x^2,x]
```

```
[Out] (1 + I/(b*d*n))/x + (I*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*d*n*x*(1
 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - ((2*I)*Hypergeometric2F1[1, (I/2)/
(b*d*n), 1 + (I/2)/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])/(b*d*n*x
)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4593

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\tan^2\left(d(a+b\log\left(cx^n\right))\right)}{x^2}\,dx = \int \frac{\tan^2\left(d(a+b\log\left(cx^n\right))\right)}{x^2}\,dx$$

Mathematica [A]
time = 4.89, size = 184, normalized size = 1.17

$$\frac{-e^{2id(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{i}{2bdn};2+\frac{i}{2bdn};-e^{2id(a+b\log(cx^n))}\right)+(i+2bdn)\left(bdn-i\,{}_2F_1\left(1,\frac{i}{2bdn};1+\frac{i}{2bdn};-e^{2id(a+b\log(cx^n))}\right)+\tan\left(d(a+b\log\left(cx^n\right))\right)\right)}{bdn(i+2bdn)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[d*(a + b*Log[c*x^n])]^2/x^2,x]
```

```
[Out] (-(E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + (I/2)/(b*d*n), 2
 + (I/2)/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))]) + (I + 2*b*d*n)*(b*d*n
- I*Hypergeometric2F1[1, (I/2)/(b*d*n), 1 + (I/2)/(b*d*n), -E^((2*I)*d*(a +
 b*Log[c*x^n]))] + Tan[d*(a + b*Log[c*x^n])]))/(b*d*n*(I + 2*b*d*n)*x)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\tan^2\left(d(a+b\ln\left(c\,x^n\right))\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*(a+b*ln(c*x^n)))^2/x^2,x)
```

```
[Out] int(tan(d*(a+b*ln(c*x^n)))^2/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="maxima")
```

```
[Out] ((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*cos(2*b*d*log(x^n) +
 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*sin(2*b*d
*log(x^n) + 2*a*d)^2 + b*d*n + 2*(b*d*n*cos(2*b*d*log(c)) + sin(2*b*d*log(c
)))*cos(2*b*d*log(x^n) + 2*a*d) + 2*(2*b^2*d^2*n^2*x*cos(2*b*d*log(c))*cos(
2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*x*sin(2*b*d*log(c))*sin(2*b*d*log(x
^n) + 2*a*d) + b^2*d^2*n^2*x + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2
*b*d*log(c))^2)*n^2*x*cos(2*b*d*log(x^n) + 2*a*d)^2 + (b^2*d^2*cos(2*b*d*lo
g(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*x*sin(2*b*d*log(x^n) + 2*a*d)^2)
*integrate((cos(2*b*d*log(x^n) + 2*a*d)*sin(2*b*d*log(c)) + cos(2*b*d*log(c
))*sin(2*b*d*log(x^n) + 2*a*d))/(2*b^2*d^2*n^2*x^2*cos(2*b*d*log(c))*cos(2*
b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*x^2*sin(2*b*d*log(c))*sin(2*b*d*log(x
^n) + 2*a*d) + b^2*d^2*n^2*x^2 + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin
(2*b*d*log(c))^2)*n^2*x^2*cos(2*b*d*log(x^n) + 2*a*d)^2 + (b^2*d^2*cos(2*b*
d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*x^2*sin(2*b*d*log(x^n) + 2*a
*d)^2), x) - 2*(b*d*n*sin(2*b*d*log(c)) - cos(2*b*d*log(c)))*sin(2*b*d*log(
x^n) + 2*a*d))/(2*b*d*n*x*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2
*b*d*n*x*sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) + (b*d*cos(2*b*d*log
(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x*cos(2*b*d*log(x^n) + 2*a*d)^2 + (b*d*
cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x*sin(2*b*d*log(x^n) + 2*a
*d)^2 + b*d*n*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(tan(b*d*log(c*x^n) + a*d)^2/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2\left(ad + bd\log\left(cx^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*ln(c*x**n)))**2/x**2,x)
```

```
[Out] Integral(tan(a*d + b*d*log(c*x**n))**2/x**2, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(d\,(a+b\,\ln\left(c\,x^n\right)))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*(a + b*log(c*x^n)))^2/x^2,x)
```

```
[Out] int(tan(d*(a + b*log(c*x^n)))^2/x^2, x)
```

3.171 $\int \frac{\tan^2(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal. Leaf size=156

$$\frac{1+\frac{2i}{bdn}}{2x^2}+\frac{i\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bdnx^2\left(1+e^{2iad}\left(cx^n\right)^{2ibd}\right)}-\frac{2i\,{}_2F_1\left(1,\frac{i}{bdn};1+\frac{i}{bdn};-e^{2iad}(cx^n)^{2ibd}\right)}{bdnx^2}$$

```
[Out] 1/2*(1+2*I/b/d/n)/x^2+I*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/x^2/(1+exp
(2*I*a*d)*(c*x^n)^(2*I*b*d))-2*I*hypergeom([1, I/b/d/n],[1+I/b/d/n],-exp(2*
I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/x^2
```

Rubi [A]
time = 0.13, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {4593, 4591, 516, 470, 371}

$$-\frac{2i\,{}_2F_1\left(1,\frac{i}{bdn};1+\frac{i}{bdn};-e^{2iad}(cx^n)^{2ibd}\right)}{bdnx^2}+\frac{i\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bdnx^2\left(1+e^{2iad}\left(cx^n\right)^{2ibd}\right)}+\frac{1+\frac{2i}{bdn}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[d*(a + b*Log[c*x^n])]^2/x^3,x]
```

```
[Out] (1 + (2*I)/(b*d*n))/(2*x^2) + (I*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(
b*d*n*x^2*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - ((2*I)*Hypergeometric2
F1[1, I/(b*d*n), 1 + I/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])/(b*d
*n*x^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4593

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\tan^2\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx=\int \frac{\tan^2\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx$$

Mathematica [A]
time = 4.18, size = 179, normalized size = 1.15

$$\frac{-2e^{2id(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{i}{bdn};2+\frac{i}{bdn};-e^{2id(a+b\log(cx^n))}\right)+(i+bdn)\left(bdn-2i\,{}_2F_1\left(1,\frac{i}{bdn};1+\frac{i}{bdn};-e^{2id(a+b\log(cx^n))}\right)+2\tan\left(d(a+b\log\left(cx^n\right))\right)\right)}{2bdn(i+bdn)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[d*(a + b*Log[c*x^n])]^2/x^3,x]
```

```
[Out] (-2*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + I/(b*d*n), 2 +
I/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))] + (I + b*d*n)*(b*d*n - (2*I)*Hy
pergeometric2F1[1, I/(b*d*n), 1 + I/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n])
)] + 2*Tan[d*(a + b*Log[c*x^n])]))/(2*b*d*n*(I + b*d*n)*x^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\tan^2\left(d(a+b\ln\left(c\,x^n\right))\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*(a+b*ln(c*x^n)))^2/x^3,x)
```

```
[Out] int(tan(d*(a+b*ln(c*x^n)))^2/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*cos(2*b*d*log(x^
n) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*sin(2
*b*d*log(x^n) + 2*a*d)^2 + b*d*n + 2*(b*d*n*cos(2*b*d*log(c)) + 2*sin(2*b*d
*log(c)))*cos(2*b*d*log(x^n) + 2*a*d) + 8*(2*b^2*d^2*n^2*x^2*cos(2*b*d*log(
c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*x^2*sin(2*b*d*log(c))*sin(2
*b*d*log(x^n) + 2*a*d) + b^2*d^2*n^2*x^2 + (b^2*d^2*cos(2*b*d*log(c))^2 + b
^2*d^2*sin(2*b*d*log(c))^2)*n^2*x^2*cos(2*b*d*log(x^n) + 2*a*d)^2 + (b^2*d^
2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*x^2*sin(2*b*d*log(
x^n) + 2*a*d)^2)*integrate((cos(2*b*d*log(x^n) + 2*a*d)*sin(2*b*d*log(c)) +
 cos(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d))/(2*b^2*d^2*n^2*x^3*cos(2*b*
d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*x^3*sin(2*b*d*log(c))
*sin(2*b*d*log(x^n) + 2*a*d) + b^2*d^2*n^2*x^3 + (b^2*d^2*cos(2*b*d*log(c))
^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*x^3*cos(2*b*d*log(x^n) + 2*a*d)^2 + (
b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*x^3*sin(2*b*
d*log(x^n) + 2*a*d)^2), x) - 2*(b*d*n*sin(2*b*d*log(c)) - 2*cos(2*b*d*log(c
)))*sin(2*b*d*log(x^n) + 2*a*d))/(2*b*d*n*x^2*cos(2*b*d*log(c))*cos(2*b*d*l
og(x^n) + 2*a*d) - 2*b*d*n*x^2*sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d
) + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x^2*cos(2*b*d*log
(x^n) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x^
2*sin(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n*x^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(tan(b*d*log(c*x^n) + a*d)^2/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2\left(ad + bd\log\left(cx^n\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*ln(c*x**n)))**2/x**3,x)
```

```
[Out] Integral(tan(a*d + b*d*log(c*x**n))**2/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(tan((b*log(c*x^n) + a)*d)^2/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(d\,(a+b\,\ln\left(c\,x^n\right)))^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*(a + b*log(c*x^n)))^2/x^3,x)
```

```
[Out] int(tan(d*(a + b*log(c*x^n)))^2/x^3, x)
```

3.172 $\int \frac{\tan^3(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=43

$$\frac{\log\left(\cos\left(a+b\log\left(cx^n\right)\right)\right)}{bn}+\frac{\tan^2\left(a+b\log\left(cx^n\right)\right)}{2bn}$$

```
[Out] ln(cos(a+b*ln(c*x^n)))/b/n+1/2*tan(a+b*ln(c*x^n))^2/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3554, 3556}

$$\frac{\tan^2\left(a+b\log\left(cx^n\right)\right)}{2bn}+\frac{\log\left(\cos\left(a+b\log\left(cx^n\right)\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] Log[Cos[a + b*Log[c*x^n]]]/(b*n) + Tan[a + b*Log[c*x^n]]^2/(2*b*n)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^3\left(a+b\log\left(cx^n\right)\right)}{x}\,dx&=\frac{\text{Subst}\left(\int \tan^3(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\&=\frac{\tan^2\left(a+b\log\left(cx^n\right)\right)}{2bn}-\frac{\text{Subst}\left(\int \tan(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\&=\frac{\log\left(\cos\left(a+b\log\left(cx^n\right)\right)\right)}{bn}+\frac{\tan^2\left(a+b\log\left(cx^n\right)\right)}{2bn}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 38, normalized size = 0.88

$$\frac{2\log\left(\cos\left(a+b\log\left(cx^n\right)\right)\right)+\tan^2\left(a+b\log\left(cx^n\right)\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] (2*Log[Cos[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]^2)/(2*b*n)
```

Maple [A]
time = 0.07, size = 42, normalized size = 0.98

method	result
derivativedivides	$\frac{\frac{\left(\tan^2(a+b\ln(c\,x^n))\right)}{2}-\frac{\ln\left(1+\tan^2(a+b\ln(c\,x^n))\right)}{2}}{nb}$
default	$\frac{\frac{\left(\tan^2(a+b\ln(c\,x^n))\right)}{2}-\frac{\ln\left(1+\tan^2(a+b\ln(c\,x^n))\right)}{2}}{nb}$
risch	$i\ln(x)-\frac{\pi\mathrm{csgn}(ic\,x^n)^3}{n}+\frac{\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ic)}{n}+\frac{\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ix^n)}{n}-\frac{\pi\,\mathrm{csgn}(ic\,x^n)\mathrm{csgn}(ic)\mathrm{csgn}(ix^n)}{n}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(1/2*tan(a+b*ln(c*x^n))^2-1/2*ln(1+tan(a+b*ln(c*x^n))^2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1242 vs. $2(41) = 82$.
time = 0.31, size = 1242, normalized size = 28.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] 1/2*(8*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*cos(2*b*log(x^n) + 2*a)^2 +
8*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*sin(2*b*log(x^n) + 2*a)^2 + 4*((c
os(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*cos(2*b*l
og(x^n) + 2*a) + (cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b
*log(c)))*sin(2*b*log(x^n) + 2*a))*cos(4*b*log(x^n) + 4*a) + 4*cos(2*b*log(
c))*cos(2*b*log(x^n) + 2*a) + ((cos(4*b*log(c))^2 + sin(4*b*log(c))^2)*cos(
4*b*log(x^n) + 4*a)^2 + 4*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*cos(2*b*l
og(x^n) + 2*a)^2 + (cos(4*b*log(c))^2 + sin(4*b*log(c))^2)*sin(4*b*log(x^n)
 + 4*a)^2 + 4*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*sin(2*b*log(x^n) + 2*
a)^2 + 2*(2*(cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(
c)))*cos(2*b*log(x^n) + 2*a) + 2*(cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b
*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) + cos(4*b*log(c)))*cos(4*
b*log(x^n) + 4*a) + 4*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) - 2*(2*(cos(2
*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x
```

```
^n) + 2*a) - 2*(cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*l
og(c)))*sin(2*b*log(x^n) + 2*a) + sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a)
- 4*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + 1)*log((cos(2*a)^2 + sin(2*a)
^2)*cos(2*b*log(c))^2 + (cos(2*a)^2 + sin(2*a)^2)*sin(2*b*log(c))^2 + 2*(co
s(2*b*log(c))*cos(2*a) - sin(2*b*log(c))*sin(2*a))*cos(2*b*log(x^n)) + cos(
2*b*log(x^n))^2 - 2*(cos(2*a)*sin(2*b*log(c)) + cos(2*b*log(c))*sin(2*a))*s
in(2*b*log(x^n)) + sin(2*b*log(x^n))^2) - 4*((cos(2*b*log(c))*sin(4*b*log(c
)) - cos(4*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) - (cos(4*b*lo
g(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) +
 2*a))*sin(4*b*log(x^n) + 4*a) - 4*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a))
/((b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*cos(4*b*log(x^n) + 4*a)^2 +
 4*b*n*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) + 4*(b*cos(2*b*log(c))^2 + b
*sin(2*b*log(c))^2)*n*cos(2*b*log(x^n) + 2*a)^2 + (b*cos(4*b*log(c))^2 + b*
sin(4*b*log(c))^2)*n*sin(4*b*log(x^n) + 4*a)^2 - 4*b*n*sin(2*b*log(c))*sin(
2*b*log(x^n) + 2*a) + 4*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*sin(2
*b*log(x^n) + 2*a)^2 + b*n + 2*(b*n*cos(4*b*log(c)) + 2*(b*cos(4*b*log(c))*
cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n) + 2
*a) + 2*(b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(
c)))*n*sin(2*b*log(x^n) + 2*a))*cos(4*b*log(x^n) + 4*a) - 2*(2*(b*cos(2*b*l
og(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*log(x
^n) + 2*a) + b*n*sin(4*b*log(c)) - 2*(b*cos(4*b*log(c))*cos(2*b*log(c)) + b
*sin(4*b*log(c))*sin(2*b*log(c)))*n*sin(2*b*log(x^n) + 2*a))*sin(4*b*log(x^
n) + 4*a))
```

Fricas [A]
time = 2.82, size = 69, normalized size = 1.60

$$\frac{(\cos(2\,bn\log(x)+2\,b\log(c)+2\,a)+1)\log\left(\frac{1}{2}\,\cos(2\,bn\log(x)+2\,b\log(c)+2\,a)+\frac{1}{2}\right)+2}{2\,(bn\cos(2\,bn\log(x)+2\,b\log(c)+2\,a)+bn)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] 1/2*((cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)*log(1/2*cos(2*b*n*log(x) +
2*b*log(c) + 2*a) + 1/2) + 2)/(b*n*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + b
*n)
```

Sympy [A]
time = 0.97, size = 63, normalized size = 1.47

$$\begin{cases} \log(x)\tan^3(a) & \text{for}\ b=0\wedge(b=0\vee n=0) \\ \log(x)\tan^3(a+b\log(c)) & \text{for}\ n=0 \\ -\frac{\log(\tan^2(a+b\log(cx^n))+1)}{2bn}+\frac{\tan^2(a+b\log(cx^n))}{2bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*ln(c*x**n))**3/x,x)
```

```
[Out] Piecewise((log(x)*tan(a)**3, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*tan
(a + b*log(c))**3, Eq(n, 0)), (-log(tan(a + b*log(c*x**n))**2 + 1)/(2*b*n)
+ tan(a + b*log(c*x**n))**2/(2*b*n), True))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 4.72, size = 105, normalized size = 2.44

$$-\ln(x)\ 1\mathrm{i}-\frac{2}{b\,n\,\left(2\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}+\mathrm{e}^{a\,4\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,4\mathrm{i}}+1\right)}+\frac{2}{b\,n\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}+1\right)}+\frac{\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}+1\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*log(c*x^n))^3/x,x)
```

```
[Out] 2/(b*n*(exp(a*2i)*(c*x^n)^(b*2i) + 1)) - 2/(b*n*(2*exp(a*2i)*(c*x^n)^(b*2i)
 + exp(a*4i)*(c*x^n)^(b*4i) + 1)) - log(x)*1i + log(exp(a*2i)*(c*x^n)^(b*2i
) + 1)/(b*n)
```

3.173 $\int \frac{\tan^4(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=45

$$\log(x)-\frac{\tan\left(a+b\log\left(cx^n\right)\right)}{bn}+\frac{\tan^3\left(a+b\log\left(cx^n\right)\right)}{3bn}$$

```
[Out] ln(x)-tan(a+b*ln(c*x^n))/b/n+1/3*tan(a+b*ln(c*x^n))^3/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3554, 8}

$$\frac{\tan^3\left(a+b\log\left(cx^n\right)\right)}{3bn}-\frac{\tan\left(a+b\log\left(cx^n\right)\right)}{bn}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] Log[x] - Tan[a + b*Log[c*x^n]]/(b*n) + Tan[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^4\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \tan^4(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\
&= \frac{\tan^3\left(a+b\log\left(cx^n\right)\right)}{3bn}-\frac{\text{Subst}\left(\int \tan^2(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\
&= -\frac{\tan\left(a+b\log\left(cx^n\right)\right)}{bn}+\frac{\tan^3\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{\text{Subst}\left(\int 1\,dx,x,\log\left(cx^n\right)\right)}{n}\\
&= \log(x)-\frac{\tan\left(a+b\log\left(cx^n\right)\right)}{bn}+\frac{\tan^3\left(a+b\log\left(cx^n\right)\right)}{3bn}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 62, normalized size = 1.38

$$\frac{\text{ArcTan}(\tan(a+b\log(cx^n)))}{bn}-\frac{\tan(a+b\log(cx^n))}{bn}+\frac{\tan^3(a+b\log(cx^n))}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] ArcTan[Tan[a + b*Log[c*x^n]]]/(b*n) - Tan[a + b*Log[c*x^n]]/(b*n) + Tan[a +
 b*Log[c*x^n]]^3/(3*b*n)
```

Maple [A]
time = 0.05, size = 49, normalized size = 1.09

method	result
derivativedivides	$\frac{\frac{\left(\tan^3(a+b\ln(c\,x^n))\right)}{3}-\tan(a+b\ln(c\,x^n))+\arctan(\tan(a+b\ln(c\,x^n)))}{nb}$
default	$\frac{\frac{\left(\tan^3(a+b\ln(c\,x^n))\right)}{3}-\tan(a+b\ln(c\,x^n))+\arctan(\tan(a+b\ln(c\,x^n)))}{nb}$
risch	$\ln(x)-\frac{4i\left(3(x^n)^{4ib}c^{4ib}e^{2b\pi\mathrm{csgn}(ic\,x^n)^3}e^{-2b\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ic)}e^{-2b\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ix^n)}e^{2b\pi\,\mathrm{csgn}(ic\,x^n)\mathrm{csgn}(ic)\mathrm{csgn}(}{3bn\left((x^n)^{2ib}c^{2ib}e^{b\pi\mathrm{csgn}(ic\,x^n)^3}e^{-b\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a+b*ln(c*x^n))^4/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(1/3*tan(a+b*ln(c*x^n))^3-tan(a+b*ln(c*x^n))+arctan(tan(a+b*ln(c*x^n)
)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2171 vs. $2(43) = 86$.
time = 0.36, size = 2171, normalized size = 48.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(b*cos(6*b*log(c))^2 + b*sin(6*b*log(c))^2)*n*cos(6*b*log(x^n) + 6*a
)^2*log(x) + 27*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*cos(4*b*log(x
^n) + 4*a)^2*log(x) + 27*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*cos(
2*b*log(x^n) + 2*a)^2*log(x) + 3*(b*cos(6*b*log(c))^2 + b*sin(6*b*log(c))^2
)*n*log(x)*sin(6*b*log(x^n) + 6*a)^2 + 27*(b*cos(4*b*log(c))^2 + b*sin(4*b*
log(c))^2)*n*log(x)*sin(4*b*log(x^n) + 4*a)^2 + 27*(b*cos(2*b*log(c))^2 + b
*sin(2*b*log(c))^2)*n*log(x)*sin(2*b*log(x^n) + 2*a)^2 + 3*b*n*log(x) + 2*(
```

```
3*b*n*cos(6*b*log(c))*log(x) + 3*(3*(b*cos(6*b*log(c))*cos(4*b*log(c)) + b*
sin(6*b*log(c))*sin(4*b*log(c)))*n*log(x) - 2*cos(4*b*log(c))*sin(6*b*log(c
)) + 2*cos(6*b*log(c))*sin(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) + 3*(3*(b*c
os(6*b*log(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n*log(x
) - 2*cos(2*b*log(c))*sin(6*b*log(c)) + 2*cos(6*b*log(c))*sin(2*b*log(c)))*
cos(2*b*log(x^n) + 2*a) + 3*(3*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6
*b*log(c))*sin(4*b*log(c)))*n*log(x) + 2*cos(6*b*log(c))*cos(4*b*log(c)) +
2*sin(6*b*log(c))*sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) + 3*(3*(b*cos(2*
b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(2*b*log(c)))*n*log(x) + 2
*cos(6*b*log(c))*cos(2*b*log(c)) + 2*sin(6*b*log(c))*sin(2*b*log(c)))*sin(2
*b*log(x^n) + 2*a) - 4*sin(6*b*log(c)))*cos(6*b*log(x^n) + 6*a) + 6*(3*b*n*
cos(4*b*log(c))*log(x) + 9*(b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*l
og(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n) + 2*a)*log(x) + 9*(b*cos(2*b*log
(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*n*log(x)*sin(2*b*
log(x^n) + 2*a) - 2*sin(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) + 6*(3*b*n*cos
(2*b*log(c))*log(x) - 2*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) - 2*(3*b*n
*log(x)*sin(6*b*log(c)) + 3*(3*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6
*b*log(c))*sin(4*b*log(c)))*n*log(x) + 2*cos(6*b*log(c))*cos(4*b*log(c)) +
2*sin(6*b*log(c))*sin(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) + 3*(3*(b*cos(2*
b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(2*b*log(c)))*n*log(x) + 2
*cos(6*b*log(c))*cos(2*b*log(c)) + 2*sin(6*b*log(c))*sin(2*b*log(c)))*cos(2
*b*log(x^n) + 2*a) - 3*(3*(b*cos(6*b*log(c))*cos(4*b*log(c)) + b*sin(6*b*lo
g(c))*sin(4*b*log(c)))*n*log(x) - 2*cos(4*b*log(c))*sin(6*b*log(c)) + 2*cos
(6*b*log(c))*sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) - 3*(3*(b*cos(6*b*log
(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n*log(x) - 2*cos(
2*b*log(c))*sin(6*b*log(c)) + 2*cos(6*b*log(c))*sin(2*b*log(c)))*sin(2*b*lo
g(x^n) + 2*a) + 4*cos(6*b*log(c)))*sin(6*b*log(x^n) + 6*a) - 6*(9*(b*cos(2*
b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*lo
g(x^n) + 2*a)*log(x) + 3*b*n*log(x)*sin(4*b*log(c)) - 9*(b*cos(4*b*log(c))*
cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log(c)))*n*log(x)*sin(2*b*log(x
^n) + 2*a) + 2*cos(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) - 6*(3*b*n*log(x)*s
in(2*b*log(c)) + 2*cos(2*b*log(c)))*sin(2*b*log(x^n) + 2*a))/((b*cos(6*b*lo
g(c))^2 + b*sin(6*b*log(c))^2)*n*cos(6*b*log(x^n) + 6*a)^2 + 9*(b*cos(4*b*l
og(c))^2 + b*sin(4*b*log(c))^2)*n*cos(4*b*log(x^n) + 4*a)^2 + 6*b*n*cos(2*b
*log(c))*cos(2*b*log(x^n) + 2*a) + 9*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c
))^2)*n*cos(2*b*log(x^n) + 2*a)^2 + (b*cos(6*b*log(c))^2 + b*sin(6*b*log(c)
)^2)*n*sin(6*b*log(x^n) + 6*a)^2 + 9*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c
))^2)*n*sin(4*b*log(x^n) + 4*a)^2 - 6*b*n*sin(2*b*log(c))*sin(2*b*log(x^n)
+ 2*a) + 9*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*sin(2*b*log(x^n) +
 2*a)^2 + b*n + 2*(b*n*cos(6*b*log(c)) + 3*(b*cos(6*b*log(c))*cos(4*b*log(c
)) + b*sin(6*b*log(c))*sin(4*b*log(c)))*n*cos(4*b*log(x^n) + 4*a) + 3*(b*co
s(6*b*log(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n*cos(2*
b*log(x^n) + 2*a) + 3*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c)
)*sin(4*b*log(c)))*n*sin(4*b*log(x^n) + 4*a) + 3*(b*cos(2*b*log(c))*sin(6*b
*log(c)) - b*cos(6*b*log(c))*sin(2*b*log(c)))*n*sin(2*b*log(x^n) + 2*a))*co
```

```
s(6*b*log(x^n) + 6*a) + 6*(b*n*cos(4*b*log(c)) + 3*(b*cos(4*b*log(c))*cos(2
*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n) + 2*a) +
 3*(b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*
n*sin(2*b*log(x^n) + 2*a))*cos(4*b*log(x^n) + 4*a) - 2*(3*(b*cos(4*b*log(c)
)*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b*log(c)))*n*cos(4*b*log(x^n) +
 4*a) + 3*(b*cos(2*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(2*b*lo
g(c)))*n*cos(2*b*log(x^n) + 2*a) + b*n*sin(6*b*log(c)) - 3*(b*cos(6*b*log(c
))*cos(4*b*log(c)) + b*sin(6*b*log(c))*sin(4*b*log(c)))*n*sin(4*b*log(x^n)
+ 4*a) - 3*(b*cos(6*b*log(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*l
og(c)))*n*sin(2*b*log(x^n) + 2*a))*sin(6*b*log(x^n) + 6*a) - 6*(3*(b*cos(2*
b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*lo
g(x^n) + 2*a) + b*n*sin(4*b*log(c)) - 3*(b*cos(4*b*log(c))*cos(2*b*log(c))
+ b*sin(4*b*log(c))*sin(2*b*log(c)))*n*sin(2*b*log(x^n) + 2*a))*sin(4*b*log
(x^n) + 4*a))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(43) = 86$.
time = 2.60, size = 140, normalized size = 3.11

$$\frac{3\,bn\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)^{2}\log\left(x\right)+6\,bn\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)\log\left(x\right)+3\,bn\log\left(x\right)-2\,\left(2\,\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)+1\right)\sin\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)}{3\,\left(bn\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)^{2}+2\,bn\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)+bn\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b*n*cos(2*b*n*log(x) + 2*b*log(c) + 2*a)^2*log(x) + 6*b*n*cos(2*b*n*
log(x) + 2*b*log(c) + 2*a)*log(x) + 3*b*n*log(x) - 2*(2*cos(2*b*n*log(x) +
2*b*log(c) + 2*a) + 1)*sin(2*b*n*log(x) + 2*b*log(c) + 2*a))/(b*n*cos(2*b*n
*log(x) + 2*b*log(c) + 2*a)^2 + 2*b*n*cos(2*b*n*log(x) + 2*b*log(c) + 2*a)
+ b*n)
```

Sympy [A]
time = 2.13, size = 65, normalized size = 1.44

$$\begin{cases} \log\left(x\right)\tan^{4}\left(a\right) & \text{for}\: b = 0 \wedge \left(b = 0 \vee n = 0\right) \\ \log\left(x\right)\tan^{4}\left(a + b\log\left(c\right)\right) & \text{for}\: n = 0 \\ \frac{\log\left(cx^{n}\right)}{n} + \frac{\tan^{3}\left(a + b\log\left(cx^{n}\right)\right)}{3bn} - \frac{\tan\left(a + b\log\left(cx^{n}\right)\right)}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Piecewise((log(x)*tan(a)**4, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*tan
(a + b*log(c))**4, Eq(n, 0)), (log(c*x**n)/n + tan(a + b*log(c*x**n))**3/(3
*b*n) - tan(a + b*log(c*x**n))/(b*n), True))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 8.04, size = 183, normalized size = 4.07

$$\ln(x) - \frac{\frac{4\mathrm{i}}{3\,b\,n} + \frac{\mathrm{e}^{a\,4\mathrm{i}}\,(c\,x^n)^{b\,4\mathrm{i}}\,4\mathrm{i}}{3\,b\,n}}{3\,\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}} + 3\,\mathrm{e}^{a\,4\mathrm{i}}\,(c\,x^n)^{b\,4\mathrm{i}} + \mathrm{e}^{a\,6\mathrm{i}}\,(c\,x^n)^{b\,6\mathrm{i}} + 1} - \frac{4\mathrm{i}}{3\,b\,n\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}} + 1\right)} - \frac{\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}}\,4\mathrm{i}}{3\,b\,n\,\left(2\,\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}} + \mathrm{e}^{a\,4\mathrm{i}}\,(c\,x^n)^{b\,4\mathrm{i}} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*log(c*x^n))^4/x,x)
```

```
[Out] log(x) - (4i/(3*b*n) + (exp(a*4i)*(c*x^n)^(b*4i)*4i)/(3*b*n))/(3*exp(a*2i)*
(c*x^n)^(b*2i) + 3*exp(a*4i)*(c*x^n)^(b*4i) + exp(a*6i)*(c*x^n)^(b*6i) + 1)
 - 4i/(3*b*n*(exp(a*2i)*(c*x^n)^(b*2i) + 1)) - (exp(a*2i)*(c*x^n)^(b*2i)*4i
)/(3*b*n*(2*exp(a*2i)*(c*x^n)^(b*2i) + exp(a*4i)*(c*x^n)^(b*4i) + 1))
```

3.174 $\int \frac{\tan^5(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=67

$$-\frac{\log\left(\cos\left(a+b\log\left(cx^n\right)\right)\right)}{bn}-\frac{\tan^2\left(a+b\log\left(cx^n\right)\right)}{2bn}+\frac{\tan^4\left(a+b\log\left(cx^n\right)\right)}{4bn}$$

```
[Out] -ln(cos(a+b*ln(c*x^n)))/b/n-1/2*tan(a+b*ln(c*x^n))^2/b/n+1/4*tan(a+b*ln(c*x
^n))^4/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3554, 3556}

$$\frac{\tan^4\left(a+b\log\left(cx^n\right)\right)}{4bn}-\frac{\tan^2\left(a+b\log\left(cx^n\right)\right)}{2bn}-\frac{\log\left(\cos\left(a+b\log\left(cx^n\right)\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] -(Log[Cos[a + b*Log[c*x^n]]]/(b*n)) - Tan[a + b*Log[c*x^n]]^2/(2*b*n) + Tan
[a + b*Log[c*x^n]]^4/(4*b*n)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^5\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \tan^5(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}\\
&= \frac{\tan^4\left(a+b\log\left(cx^n\right)\right)}{4bn}-\frac{\text{Subst}\left(\int \tan^3(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}\\
&= -\frac{\tan^2\left(a+b\log\left(cx^n\right)\right)}{2bn}+\frac{\tan^4\left(a+b\log\left(cx^n\right)\right)}{4bn}+\frac{\text{Subst}\left(\int \tan(a+bx)\,dx, x, \text{l}\right.}{n}\\
&= -\frac{\log\left(\cos\left(a+b\log\left(cx^n\right)\right)\right)}{bn}-\frac{\tan^2\left(a+b\log\left(cx^n\right)\right)}{2bn}+\frac{\tan^4\left(a+b\log\left(cx^n\right)\right)}{4bn}
\end{aligned}$$

Mathematica [A]
time = 0.19, size = 55, normalized size = 0.82

$$-\frac{4\log(\cos(a+b\log(cx^n)))+2\tan^2(a+b\log(cx^n))-\tan^4(a+b\log(cx^n))}{4bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] -1/4*(4*Log[Cos[a + b*Log[c*x^n]]] + 2*Tan[a + b*Log[c*x^n]]^2 - Tan[a + b*
Log[c*x^n]]^4)/(b*n)
```

Maple [A]
time = 0.14, size = 57, normalized size = 0.85

method	result
derivativedivides	$\frac{\frac{\left(\tan^4(a+b\ln(c\,x^n))\right)}{4}-\frac{\left(\tan^2(a+b\ln(c\,x^n))\right)}{2}+\frac{\ln\left(1+\tan^2(a+b\ln(c\,x^n))\right)}{2}}{nb}$
default	$\frac{\frac{\left(\tan^4(a+b\ln(c\,x^n))\right)}{4}-\frac{\left(\tan^2(a+b\ln(c\,x^n))\right)}{2}+\frac{\ln\left(1+\tan^2(a+b\ln(c\,x^n))\right)}{2}}{nb}$
risch	$-i\ln(x)+\frac{\pi\mathrm{csgn}(ic\,x^n)^3}{n}-\frac{\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ic)}{n}-\frac{\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ix^n)}{n}+\frac{\pi\,\mathrm{csgn}(ic\,x^n)\mathrm{csgn}(ic)\mathrm{csgn}(ix^n}{n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a+b*ln(c*x^n))^5/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(1/4*tan(a+b*ln(c*x^n))^4-1/2*tan(a+b*ln(c*x^n))^2+1/2*ln(1+tan(a+b*l
n(c*x^n))^2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 4466 vs. $2(63) = 126$.
time = 0.38, size = 4466, normalized size = 66.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

```
[Out] -1/2*(32*(cos(6*b*log(c))^2 + sin(6*b*log(c))^2)*cos(6*b*log(x^n) + 6*a)^2
+ 48*(cos(4*b*log(c))^2 + sin(4*b*log(c))^2)*cos(4*b*log(x^n) + 4*a)^2 + 32
*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*cos(2*b*log(x^n) + 2*a)^2 + 32*(co
s(6*b*log(c))^2 + sin(6*b*log(c))^2)*sin(6*b*log(x^n) + 6*a)^2 + 48*(cos(4*
b*log(c))^2 + sin(4*b*log(c))^2)*sin(4*b*log(x^n) + 4*a)^2 + 32*(cos(2*b*lo
g(c))^2 + sin(2*b*log(c))^2)*sin(2*b*log(x^n) + 2*a)^2 + 8*((cos(8*b*log(c)
)*cos(6*b*log(c)) + sin(8*b*log(c))*sin(6*b*log(c)))*cos(6*b*log(x^n) + 6*a
```

```
) + (cos(8*b*log(c))*cos(4*b*log(c)) + sin(8*b*log(c))*sin(4*b*log(c)))*cos
(4*b*log(x^n) + 4*a) + (cos(8*b*log(c))*cos(2*b*log(c)) + sin(8*b*log(c))*s
in(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + (cos(6*b*log(c))*sin(8*b*log(c))
- cos(8*b*log(c))*sin(6*b*log(c)))*sin(6*b*log(x^n) + 6*a) + (cos(4*b*log(c
))*sin(8*b*log(c)) - cos(8*b*log(c))*sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*
a) + (cos(2*b*log(c))*sin(8*b*log(c)) - cos(8*b*log(c))*sin(2*b*log(c)))*si
n(2*b*log(x^n) + 2*a))*cos(8*b*log(x^n) + 8*a) + 8*(10*(cos(6*b*log(c))*cos
(4*b*log(c)) + sin(6*b*log(c))*sin(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) + 8
*(cos(6*b*log(c))*cos(2*b*log(c)) + sin(6*b*log(c))*sin(2*b*log(c)))*cos(2*
b*log(x^n) + 2*a) + 10*(cos(4*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c))*s
in(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) + 8*(cos(2*b*log(c))*sin(6*b*log(c)
) - cos(6*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) + cos(6*b*log(
c)))*cos(6*b*log(x^n) + 6*a) + 8*(10*(cos(4*b*log(c))*cos(2*b*log(c)) + sin
(4*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 10*(cos(2*b*log(c))
*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a)
 + cos(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) + 8*cos(2*b*log(c))*cos(2*b*log
(x^n) + 2*a) + ((cos(8*b*log(c))^2 + sin(8*b*log(c))^2)*cos(8*b*log(x^n) +
8*a)^2 + 16*(cos(6*b*log(c))^2 + sin(6*b*log(c))^2)*cos(6*b*log(x^n) + 6*a)
^2 + 36*(cos(4*b*log(c))^2 + sin(4*b*log(c))^2)*cos(4*b*log(x^n) + 4*a)^2 +
 16*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*cos(2*b*log(x^n) + 2*a)^2 + (co
s(8*b*log(c))^2 + sin(8*b*log(c))^2)*sin(8*b*log(x^n) + 8*a)^2 + 16*(cos(6*
b*log(c))^2 + sin(6*b*log(c))^2)*sin(6*b*log(x^n) + 6*a)^2 + 36*(cos(4*b*lo
g(c))^2 + sin(4*b*log(c))^2)*sin(4*b*log(x^n) + 4*a)^2 + 16*(cos(2*b*log(c)
)^2 + sin(2*b*log(c))^2)*sin(2*b*log(x^n) + 2*a)^2 + 2*(4*(cos(8*b*log(c))*
cos(6*b*log(c)) + sin(8*b*log(c))*sin(6*b*log(c)))*cos(6*b*log(x^n) + 6*a)
+ 6*(cos(8*b*log(c))*cos(4*b*log(c)) + sin(8*b*log(c))*sin(4*b*log(c)))*cos
(4*b*log(x^n) + 4*a) + 4*(cos(8*b*log(c))*cos(2*b*log(c)) + sin(8*b*log(c))
*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 4*(cos(6*b*log(c))*sin(8*b*log(
c)) - cos(8*b*log(c))*sin(6*b*log(c)))*sin(6*b*log(x^n) + 6*a) + 6*(cos(4*b
*log(c))*sin(8*b*log(c)) - cos(8*b*log(c))*sin(4*b*log(c)))*sin(4*b*log(x^n
) + 4*a) + 4*(cos(2*b*log(c))*sin(8*b*log(c)) - cos(8*b*log(c))*sin(2*b*log
(c)))*sin(2*b*log(x^n) + 2*a) + cos(8*b*log(c)))*cos(8*b*log(x^n) + 8*a) +
8*(6*(cos(6*b*log(c))*cos(4*b*log(c)) + sin(6*b*log(c))*sin(4*b*log(c)))*co
s(4*b*log(x^n) + 4*a) + 4*(cos(6*b*log(c))*cos(2*b*log(c)) + sin(6*b*log(c)
)*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 6*(cos(4*b*log(c))*sin(6*b*log
(c)) - cos(6*b*log(c))*sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) + 4*(cos(2*
b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^
n) + 2*a) + cos(6*b*log(c)))*cos(6*b*log(x^n) + 6*a) + 12*(4*(cos(4*b*log(c
))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*
a) + 4*(cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)))*
sin(2*b*log(x^n) + 2*a) + cos(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) + 8*cos(
2*b*log(c))*cos(2*b*log(x^n) + 2*a) - 2*(4*(cos(6*b*log(c))*sin(8*b*log(c))
 - cos(8*b*log(c))*sin(6*b*log(c)))*cos(6*b*log(x^n) + 6*a) + 6*(cos(4*b*lo
g(c))*sin(8*b*log(c)) - cos(8*b*log(c))*sin(4*b*log(c)))*cos(4*b*log(x^n) +
 4*a) + 4*(cos(2*b*log(c))*sin(8*b*log(c)) - cos(8*b*log(c))*sin(2*b*log(c)
```

```
))*cos(2*b*log(x^n) + 2*a) - 4*(cos(8*b*log(c))*cos(6*b*log(c)) + sin(8*b*l
og(c))*sin(6*b*log(c)))*sin(6*b*log(x^n) + 6*a) - 6*(cos(8*b*log(c))*cos(4*
b*log(c)) + sin(8*b*log(c))*sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) - 4*(c
os(8*b*log(c))*cos(2*b*log(c)) + sin(8*b*log(c))*sin(2*b*log(c)))*sin(2*b*l
og(x^n) + 2*a) + sin(8*b*log(c)))*sin(8*b*log(x^n) + 8*a) - 8*(6*(cos(4*b*l
og(c))*sin(6*b*log(c)) - cos(6*b*log(c))*sin(4*b*log(c)))*cos(4*b*log(x^n)
+ 4*a) + 4*(cos(2*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c))*sin(2*b*log(c
)))*cos(2*b*log(x^n) + 2*a) - 6*(cos(6*b*log(c))*cos(4*b*log(c)) + sin(6*b*
log(c))*sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) - 4*(cos(6*b*log(c))*cos(2
*b*log(c)) + sin(6*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) + sin
(6*b*log(c)))*sin(6*b*log(x^n) + 6*a) - 12*(4*(cos(2*b*log(c))*sin(4*b*log(
c)) - cos(4*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) - 4*(cos(4*b
*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n
) + 2*a) + sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) - 8*sin(2*b*log(c))*sin
(2*b*log(x^n) + 2*a) + 1)*log((cos(2*a)^2 + sin...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(63) = 126$.
time = 3.30, size = 129, normalized size = 1.93

$$-\frac{\left(\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)^{2}+2\,\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)+1\right)\log\left(\frac{1}{2}\,\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)+\frac{1}{2}\right)+4\,\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)+2}{2\,\left(bn\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)^{2}+2\,bn\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)+bn\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

```
[Out] -1/2*((cos(2*b*n*log(x) + 2*b*log(c) + 2*a)^2 + 2*cos(2*b*n*log(x) + 2*b*lo
g(c) + 2*a) + 1)*log(1/2*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1/2) + 4*co
s(2*b*n*log(x) + 2*b*log(c) + 2*a) + 2)/(b*n*cos(2*b*n*log(x) + 2*b*log(c)
+ 2*a)^2 + 2*b*n*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + b*n)
```

Sympy [A]
time = 4.76, size = 82, normalized size = 1.22

$$\begin{cases} \log\left(x\right)\tan^{5}\left(a\right) & \text{for}\: b = 0 \wedge \left(b = 0 \vee n = 0\right) \\ \log\left(x\right)\tan^{5}\left(a + b\log\left(c\right)\right) & \text{for}\: n = 0 \\ \frac{\log\left(\tan^{2}\left(a + b\log\left(cx^{n}\right)\right) + 1\right)}{2bn} + \frac{\tan^{4}\left(a + b\log\left(cx^{n}\right)\right)}{4bn} - \frac{\tan^{2}\left(a + b\log\left(cx^{n}\right)\right)}{2bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*ln(c*x**n))**5/x,x)
```

```
[Out] Piecewise((log(x)*tan(a)**5, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*tan
(a + b*log(c))**5, Eq(n, 0)), (log(tan(a + b*log(c*x**n))**2 + 1)/(2*b*n) +
 tan(a + b*log(c*x**n))**4/(4*b*n) - tan(a + b*log(c*x**n))**2/(2*b*n), Tru
e))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 6.59, size = 247, normalized size = 3.69

$$\ln(x)\,1\mathrm{i}+\frac{8}{b\,n\,\left(2\,{\mathrm{e}}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}+{\mathrm{e}}^{a\,4\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,4\mathrm{i}}+1\right)}-\frac{4}{b\,n\,\left({\mathrm{e}}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}+1\right)}+\frac{4}{b\,n\,\left(4\,{\mathrm{e}}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}+6\,{\mathrm{e}}^{a\,4\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,4\mathrm{i}}+4\,{\mathrm{e}}^{a\,6\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,6\mathrm{i}}+{\mathrm{e}}^{a\,8\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,8\mathrm{i}}+1\right)}-\frac{\ln\left({\mathrm{e}}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}+1\right)}{b\,n}-\frac{8}{b\,n\,\left(3\,{\mathrm{e}}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}+3\,{\mathrm{e}}^{a\,4\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,4\mathrm{i}}+{\mathrm{e}}^{a\,6\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,6\mathrm{i}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*log(c*x^n))^5/x,x)
```

```
[Out] log(x)*1i + 8/(b*n*(2*exp(a*2i)*(c*x^n)^(b*2i) + exp(a*4i)*(c*x^n)^(b*4i) +
 1)) - 4/(b*n*(exp(a*2i)*(c*x^n)^(b*2i) + 1)) + 4/(b*n*(4*exp(a*2i)*(c*x^n)
^(b*2i) + 6*exp(a*4i)*(c*x^n)^(b*4i) + 4*exp(a*6i)*(c*x^n)^(b*6i) + exp(a*8
i)*(c*x^n)^(b*8i) + 1)) - log(exp(a*2i)*(c*x^n)^(b*2i) + 1)/(b*n) - 8/(b*n*
(3*exp(a*2i)*(c*x^n)^(b*2i) + 3*exp(a*4i)*(c*x^n)^(b*4i) + exp(a*6i)*(c*x^n
)^(b*6i) + 1))
```

3.175 $\int (ex)^m \tan(d(a + b\log(cx^n)))\,dx$

Optimal. Leaf size=101

$$-\frac{i(ex)^{1+m}}{e(1+m)}+\frac{2i(ex)^{1+m}\,{}_2F_1\left(1,-\frac{i(1+m)}{2bdn};1-\frac{i(1+m)}{2bdn};-e^{2iad}(cx^n)^{2ibd}\right)}{e(1+m)}$$

```
[Out] -I*(e*x)^(1+m)/e/(1+m)+2*I*(e*x)^(1+m)*hypergeom([1, -1/2*I*(1+m)/b/d/n],[1
-1/2*I*(1+m)/b/d/n],-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/e/(1+m)
```

Rubi [A]
time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {4593, 4591, 470, 371}

$$\frac{2i(ex)^{m+1}\,{}_2F_1\left(1,-\frac{i(m+1)}{2bdn};1-\frac{i(m+1)}{2bdn};-e^{2iad}(cx^n)^{2ibd}\right)}{e(m+1)}-\frac{i(ex)^{m+1}}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Tan[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((-I)*(e*x)^(1 + m))/(e*(1 + m)) + ((2*I)*(e*x)^(1 + m)*Hypergeometric2F1[1
, ((-1/2*I)*(1 + m))/(b*d*n), 1 - ((I/2)*(1 + m))/(b*d*n), -(E^((2*I)*a*d)*
(c*x^n)^((2*I)*b*d))])/(e*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4593

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int (ex)^m \tan \left(d(a+b\log \left(cx^n\right))\right)\,dx=\int (ex)^m \tan \left(d(a+b\log \left(cx^n\right))\right)\,dx$$

Mathematica [A]
time = 15.29, size = 186, normalized size = 1.84

$$\frac{ix(ex)^m\left({}_2F_1\left(1,-\frac{i(1+m)}{2bdn};1-\frac{i(1+m)}{2bdn};-e^{2id(a+b\log(cx^n))}\right)-\frac{e^{2iad}(1+m)(cx^n)^{2ibd}\,{}_2F_1\left(1,-\frac{i(1+m+2ibdn)}{2bdn};-\frac{i(1+m+4ibdn)}{2bdn};-e^{2iad}(cx^n)^{2ibd}\right)}{1+m+2ibdn}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Tan[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (I*x*(e*x)^m*(Hypergeometric2F1[1, ((-1/2*I)*(1 + m))/(b*d*n), 1 - ((I/2)*(
1 + m))/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))] - (E^((2*I)*a*d)*(1 + m)*
(c*x^n)^((2*I)*b*d)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m + (2*I)*b*d*n))/(
b*d*n), ((-1/2*I)*(1 + m + (4*I)*b*d*n))/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^(
(2*I)*b*d))])/(1 + m + (2*I)*b*d*n)))/(1 + m)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (ex)^m \tan \left(d(a+b\ln \left(c\,x^n\right))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*tan(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int((e*x)^m*tan(d*(a+b*ln(c*x^n))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tan(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate((x*e)^m*tan((b*log(c*x^n) + a)*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tan(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral((x*e)^m*tan(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \tan (ad + bd \log (cx^n))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*tan(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral((e*x)**m*tan(a*d + b*d*log(c*x**n)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tan(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(d\,(a + b\,\ln (c\,x^n)))\, (e\,x)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*(a + b*log(c*x^n)))*(e*x)^m,x)
```

```
[Out] int(tan(d*(a + b*log(c*x^n)))*(e*x)^m, x)
```

3.176 $\int (ex)^m \tan^2(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=196

$$\frac{(i(1+m)-bdn)(ex)^{1+m}}{bde(1+m)n}+\frac{i(ex)^{1+m}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bden\left(1+e^{2iad}(cx^n)^{2ibd}\right)}-\frac{2i(ex)^{1+m}\,{}_2F_1\left(1,-\frac{i(1+m)}{2bdn};1-\frac{i(1+m)}{2bdn};-e^{2iad}(cx^n)\right)}{bden}$$

```
[Out] (I*(1+m)-b*d*n)*(e*x)^(1+m)/b/d/e/(1+m)/n+I*(e*x)^(1+m)*(1-exp(2*I*a*d)*(c*
x^n)^(2*I*b*d))/b/d/e/n/(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))-2*I*(e*x)^(1+m)*
hypergeom([1, -1/2*I*(1+m)/b/d/n],[1-1/2*I*(1+m)/b/d/n],-exp(2*I*a*d)*(c*x^
n)^(2*I*b*d))/b/d/e/n
```

Rubi [A]

time = 0.15, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4593, 4591, 516, 470, 371}

$$-\frac{2i(ex)^{m+1}\,{}_2F_1\left(1,-\frac{i(m+1)}{2bdn};1-\frac{i(m+1)}{2bdn};-e^{2iad}(cx^n)^{2ibd}\right)}{bden}+\frac{i(ex)^{m+1}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bden\left(1+e^{2iad}(cx^n)^{2ibd}\right)}+\frac{(ex)^{m+1}(-bdn+i(m+1))}{bde(m+1)n}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Tan[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] ((I*(1 + m) - b*d*n)*(e*x)^(1 + m))/(b*d*e*(1 + m)*n) + (I*(e*x)^(1 + m)*(1
 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*d*e*n*(1 + E^((2*I)*a*d)*(c*x^n)^
((2*I)*b*d))) - ((2*I)*(e*x)^(1 + m)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m)
)/(b*d*n), 1 - ((I/2)*(1 + m))/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)
)])/(b*d*e*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4593

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int (ex)^m \tan^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int (ex)^m \tan^2\left(d(a+b\log\left(cx^n\right))\right)\,dx$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 550 vs. 2(196) = 392.
time = 17.76, size = 550, normalized size = 2.81

$$-\frac{x(ex)^m}{1+m}+\frac{x(ex)^m\sec\left(d(a+b(-n\log(x)+\log\left(cx^n\right)))\right)\sec\left(bdn\log(x)+d(a+b(-n\log(x)+\log\left(cx^n\right)))\right)\sin(bdn\log(x))}{bdn}-\frac{(1+m)x^{-m}(ex)^m\sec\left(d(a+b(-n\log(x)+\log\left(cx^n\right)))\right)\left(\frac{x^{1+m}\sec(d(a+b\log(cx^n)))\sin(bdn\log(x))}{1+m}-\frac{ie^{-\frac{(1+2m)(a+b(-n\log(x)+\log(cx^n)))}{bn}}\cos(d(a+b(-n\log(x)+\log(cx^n))))\left(-e^{\frac{a+2am+b(1+m)n\log(x)+b(1+2m)(-n\log(x)+\log(cx^n))}{bn}}(1+m+2ibdn)\,{}_2F_1\left(1,-\frac{i(1+m)}{2bdn};1-\frac{i(1+m)}{2bdn};-e^{2id(a+b\log(cx^n))}\right)+e^{\frac{a(1+2m+2ibdn)}{bn}+(1+m+2ibdn)\log(x)+\frac{(1+2m+2ibdn)(-n\log(x)+\log(cx^n))}{n}}(1+m)\,{}_2F_1\left(1,-\frac{i(1+m+2ibdn)}{2bdn};-\frac{i(1+m+4ibdn)}{2bdn};-e^{2id(a+b\log(cx^n))}\right)-ie^{\frac{a+2am+b(1+m)n\log(x)+b(1+2m)(-n\log(x)+\log(cx^n))}{bn}}(1+m+2ibdn)\tan(d(a+b\log(cx^n)))\right)}{(1+m)(1+m+2ibdn)}\right)}{bdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Tan[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] -((x*(e*x)^m)/(1 + m)) + (x*(e*x)^m*Sec[d*(a + b*(-(n*Log[x]) + Log[c*x^n])
)]*Sec[b*d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Sin[b*d*n*Log[x
]])/(b*d*n) - ((1 + m)*(e*x)^m*Sec[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*((
x^(1 + m)*Sec[d*(a + b*Log[c*x^n])]*Sin[b*d*n*Log[x]])/(1 + m) - (I*Cos[d*(
a + b*(-(n*Log[x]) + Log[c*x^n]))]*(-(E^((a + 2*a*m + b*(1 + m)*n*Log[x] +
b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n]))/(b*n))*(1 + m + (2*I)*b*d*n)*Hyperg
eometric2F1[1, ((-1/2*I)*(1 + m))/(b*d*n), 1 - ((I/2)*(1 + m))/(b*d*n), -E^
```

```
((2*I)*d*(a + b*Log[c*x^n]))]) + E^((a*(1 + 2*m + (2*I)*b*d*n))/(b*n) + (1
+ m + (2*I)*b*d*n)*Log[x] + ((1 + 2*m + (2*I)*b*d*n)*(-(n*Log[x]) + Log[c*x
^n]))/n)*(1 + m)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m + (2*I)*b*d*n))/(b*d
*n), ((-1/2*I)*(1 + m + (4*I)*b*d*n))/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n
]))] - I*E^((a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) + Lo
g[c*x^n]))/(b*n))*(1 + m + (2*I)*b*d*n)*Tan[d*(a + b*Log[c*x^n)]]))/(E^(((1
 + 2*m)*(a + b*(-(n*Log[x]) + Log[c*x^n])))/(b*n))*(1 + m)*(1 + m + (2*I)*b
*d*n))))/(b*d*n*x^m)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\tan^2\left(d(a+b\ln\left(c\,x^n\right))\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*tan(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int((e*x)^m*tan(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] -((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x*cos(2*b*d*log(x^n
) + 2*a*d)^2*e^(m*log(x) + m) + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*lo
g(c))^2)*n*x*e^(m*log(x) + m)*sin(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n*x*e^(m*
log(x) + m) + 2*(b*d*n*cos(2*b*d*log(c))*e^m - (m*sin(2*b*d*log(c)) + sin(2
*b*d*log(c)))*e^m)*x*x^m*cos(2*b*d*log(x^n) + 2*a*d) - 2*(b*d*n*e^m*sin(2*b
*d*log(c)) + (m*cos(2*b*d*log(c)) + cos(2*b*d*log(c)))*e^m)*x*x^m*sin(2*b*d
*log(x^n) + 2*a*d) + 2*((b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*lo
g(c))^2 + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*m^2 +
 2*(b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*m)*n^2*cos(2
*b*d*log(x^n) + 2*a*d)^2*e^m + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2
*b*d*log(c))^2 + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2
)*m^2 + 2*(b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*m)*n^
2*e^m*sin(2*b*d*log(x^n) + 2*a*d)^2 + 2*(b^2*d^2*m^2*cos(2*b*d*log(c)) + 2*
b^2*d^2*m*cos(2*b*d*log(c)) + b^2*d^2*cos(2*b*d*log(c)))*n^2*cos(2*b*d*log(
x^n) + 2*a*d)*e^m - 2*(b^2*d^2*m^2*sin(2*b*d*log(c)) + 2*b^2*d^2*m*sin(2*b*
d*log(c)) + b^2*d^2*sin(2*b*d*log(c)))*n^2*e^m*sin(2*b*d*log(x^n) + 2*a*d)
+ (b^2*d^2*m^2 + 2*b^2*d^2*m + b^2*d^2)*n^2*e^m)*integrate((x^m*cos(2*b*d*l
og(x^n) + 2*a*d)*sin(2*b*d*log(c)) + x^m*cos(2*b*d*log(c))*sin(2*b*d*log(x^
```

```
n) + 2*a*d))/(2*b^2*d^2*n^2*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) -
 2*b^2*d^2*n^2*sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) + b^2*d^2*n^2
+ (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d
*log(x^n) + 2*a*d)^2 + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log
(c))^2)*n^2*sin(2*b*d*log(x^n) + 2*a*d)^2), x))/((b*d*cos(2*b*d*log(c))^2 +
 b*d*sin(2*b*d*log(c))^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))
^2)*m)*n*cos(2*b*d*log(x^n) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin
(2*b*d*log(c))^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*m)*n
*sin(2*b*d*log(x^n) + 2*a*d)^2 + 2*(b*d*m*cos(2*b*d*log(c)) + b*d*cos(2*b*d
*log(c)))*n*cos(2*b*d*log(x^n) + 2*a*d) - 2*(b*d*m*sin(2*b*d*log(c)) + b*d*
sin(2*b*d*log(c)))*n*sin(2*b*d*log(x^n) + 2*a*d) + (b*d*m + b*d)*n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
[Out] integral((x*e)^m*tan(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \tan^2 (ad + bd \log (cx^n))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*tan(d*(a+b*ln(c*x**n)))**2,x)
[Out] Integral((e*x)**m*tan(a*d + b*d*log(c*x**n))**2, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(d\,(a + b\,\ln(c\,x^n)))^2\,(e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*(a + b*log(c*x^n)))^2*(e*x)^m,x)
[Out] int(tan(d*(a + b*log(c*x^n)))^2*(e*x)^m, x)
```

3.177 $\int (ex)^m \tan^3(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=351

$$-\frac{(i(1+m)-bdn)(1+m+2ibdn)(ex)^{1+m}}{2b^2d^2e(1+m)n^2}-\frac{(ex)^{1+m}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^2}{2bden\left(1+e^{2iad}(cx^n)^{2ibd}\right)^2}-\frac{ie^{-2iad}(ex)^{1+m}\left(\frac{e^{2iad}(1+m-2ibdn}{n}\right.}{2b^2d^2en\left(1+e^{2\right.}}$$

```
[Out] -1/2*(I*(1+m)-b*d*n)*(1+m+2*I*b*d*n)*(e*x)^(1+m)/b^2/d^2/e/(1+m)/n^2-1/2*(e
*x)^(1+m)*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^2/b/d/e/n/(1+exp(2*I*a*d)*(c*x
^n)^(2*I*b*d))^2-1/2*I*(e*x)^(1+m)*(exp(2*I*a*d)*(1+m-2*I*b*d*n)/n-exp(4*I*
a*d)*(1+m+2*I*b*d*n)*(c*x^n)^(2*I*b*d)/n)/b^2/d^2/e/exp(2*I*a*d)/n/(1+exp(2
*I*a*d)*(c*x^n)^(2*I*b*d))+I*(-2*b^2*d^2*n^2+m^2+2*m+1)*(e*x)^(1+m)*hyperge
om([1, -1/2*I*(1+m)/b/d/n],[1-1/2*I*(1+m)/b/d/n],-exp(2*I*a*d)*(c*x^n)^(2*I
*b*d))/b^2/d^2/e/(1+m)/n^2
```

Rubi [A]
time = 0.35, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4593, 4591, 516, 608, 470, 371}

$$\frac{i(ex)^{m+1}\left(-2b^2d^2n^2+m^2+2m+1\right){}_2F_1\left(1,-\frac{i(m+1)}{2bdn};1-\frac{i(m+1)}{2bdn};-e^{2iad}(cx^n)^{2ibd}\right)}{b^2d^2e(m+1)n^2}-\frac{ie^{-2iad}(ex)^{m+1}\left(\frac{e^{2iad}(-2ibdn+m+1)}{n}-\frac{e^{4iad}(2ibdn+m+1)(cx^n)^{2ibd}}{n}\right)}{2b^2d^2en\left(1+e^{2iad}(cx^n)^{2ibd}\right)}-\frac{(ex)^{m+1}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^2}{2bden\left(1+e^{2iad}(cx^n)^{2ibd}\right)^2}-\frac{(ex)^{m+1}(-bdn+i(m+1))(2ibdn+m+1)}{2b^2d^2e(m+1)n^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Tan[d*(a + b*Log[c*x^n])]^3,x]
```

```
[Out] -1/2*((I*(1 + m) - b*d*n)*(1 + m + (2*I)*b*d*n)*(e*x)^(1 + m))/(b^2*d^2*e*(
1 + m)*n^2) - ((e*x)^(1 + m)*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^2)/(2*
b*d*e*n*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^2) - ((I/2)*(e*x)^(1 + m)*(
(E^((2*I)*a*d)*(1 + m - (2*I)*b*d*n))/n - (E^((4*I)*a*d)*(1 + m + (2*I)*b*d
*n)*(c*x^n)^((2*I)*b*d))/n))/(b^2*d^2*e*E^((2*I)*a*d)*n*(1 + E^((2*I)*a*d)*
(c*x^n)^((2*I)*b*d))) + (I*(1 + 2*m + m^2 - 2*b^2*d^2*n^2)*(e*x)^(1 + m)*Hy
pergeometric2F1[1, ((-1/2*I)*(1 + m))/(b*d*n), 1 - ((I/2)*(1 + m))/(b*d*n),
 -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])/(b^2*d^2*e*(1 + m)*n^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
```

```
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 608

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && LtQ[
p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4593

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int (ex)^m \tan^3(d(a+b\log(cx^n)))\,dx = \int (ex)^m \tan^3(d(a+b\log(cx^n)))\,dx$$

Mathematica [A]
time = 18.01, size = 642, normalized size = 1.83

$$\frac{x(ex)^m\sec^2(bdn\log(x)+d(a+b(-n\log(x)+\log(cx^n))))}{2bdn} - \frac{(1+m)x(ex)^m\sec(d(a+b(-n\log(x)+\log(cx^n))))\sec(bdn\log(x)+d(a+b(-n\log(x)+\log(cx^n))))\sin(bdn\log(x))}{2b^2d^2n^2} - \frac{(-1-2m-m^2+2b^2d^2n^2)x^{-m}(ex)^m\sec(d(a+b(-n\log(x)+\log(cx^n))))\left(\frac{x^{1+m}\sec(d(a+b\log(cx^n)))\sin(bdn\log(x))}{1+m} - \frac{ie^{-\frac{(1+2m)(a+b(-n\log(x)+\log(cx^n)))}{bn}}\cos(d(a+b(-n\log(x)+\log(cx^n))))\left(-e^{\frac{a+2am+b(1+m)n\log(x)+b(1+2m)(-n\log(x)+\log(cx^n))}{bn}}(1+m+2ibdn)\,{}_2F_1\left(1,-\frac{i(1+m)}{2bdn};1-\frac{i(1+m)}{2bdn};-e^{2id(a+b\log(cx^n))}\right)+e^{\frac{a(1+2m+2ibdn)}{bn}+(1+m+2ibdn)\log(x)+\frac{(1+2m+2ibdn)(-n\log(x)+\log(cx^n))}{n}}(1+m)\,{}_2F_1\left(1,-\frac{i(1+m+2ibdn)}{2bdn};-\frac{i(1+m+4ibdn)}{2bdn};-e^{2id(a+b\log(cx^n))}\right)-ie^{\frac{a+2am+b(1+m)n\log(x)+b(1+2m)(-n\log(x)+\log(cx^n))}{bn}}(1+m+2ibdn)\tan(d(a+b\log(cx^n)))\right)}{(1+m)(1+m+2ibdn)}\right)}{2b^2d^2n^2} - \frac{x(ex)^m\tan(d(a+b(-n\log(x)+\log(cx^n))))}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Tan[d*(a + b*Log[c*x^n])]^3,x]
```

```
[Out] (x*(e*x)^m*Sec[b*d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]^2)/(2*b
*d*n) - ((1 + m)*x*(e*x)^m*Sec[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Sec[b*
d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Sin[b*d*n*Log[x]])/(2*b^
2*d^2*n^2) - ((-1 - 2*m - m^2 + 2*b^2*d^2*n^2)*(e*x)^m*Sec[d*(a + b*(-(n*Lo
g[x]) + Log[c*x^n]))]*((x^(1 + m)*Sec[d*(a + b*Log[c*x^n])]*Sin[b*d*n*Log[x
]])/(1 + m) - (I*Cos[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*(-(E^((a + 2*a*m
 + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n]))/(b*n))*(1 +
 m + (2*I)*b*d*n)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m))/(b*d*n), 1 - ((I/
2)*(1 + m))/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))]) + E^((a*(1 + 2*m + (
2*I)*b*d*n))/(b*n) + (1 + m + (2*I)*b*d*n)*Log[x] + ((1 + 2*m + (2*I)*b*d*n
)*(-(n*Log[x]) + Log[c*x^n]))/n)*(1 + m)*Hypergeometric2F1[1, ((-1/2*I)*(1
+ m + (2*I)*b*d*n))/(b*d*n), ((-1/2*I)*(1 + m + (4*I)*b*d*n))/(b*d*n), -E^(
(2*I)*d*(a + b*Log[c*x^n]))] - I*E^((a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1
+ 2*m)*(-(n*Log[x]) + Log[c*x^n]))/(b*n))*(1 + m + (2*I)*b*d*n)*Tan[d*(a +
b*Log[c*x^n])]))/(E^(((1 + 2*m)*(a + b*(-(n*Log[x]) + Log[c*x^n])))/(b*n))*
(1 + m)*(1 + m + (2*I)*b*d*n))))/(2*b^2*d^2*n^2*x^m) - (x*(e*x)^m*Tan[d*(a
+ b*(-(n*Log[x]) + Log[c*x^n]))])/(1 + m)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\tan^3\left(d(a+b\ln(c\,x^n))\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*tan(d*(a+b*ln(c*x^n)))^3,x)
```

```
[Out] int((e*x)^m*tan(d*(a+b*ln(c*x^n)))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tan(d*(a+b*log(c*x^n)))^3,x, algorithm="maxima")
```

```
[Out] (4*(b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x*cos(2*b*d*log(x^
n) + 2*a*d)^2*e^(m*log(x) + m) + 4*(b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d
*log(c))^2)*n*x*e^(m*log(x) + m)*sin(2*b*d*log(x^n) + 2*a*d)^2 + (2*b*d*n*c
os(2*b*d*log(c))*e^m - (m*sin(2*b*d*log(c)) + sin(2*b*d*log(c)))*e^m)*x*x^m
*cos(2*b*d*log(x^n) + 2*a*d) - (2*b*d*n*e^m*sin(2*b*d*log(c)) + (m*cos(2*b*
d*log(c)) + cos(2*b*d*log(c)))*e^m)*x*x^m*sin(2*b*d*log(x^n) + 2*a*d) + ((2
*(b*d*cos(4*b*d*log(c))*cos(2*b*d*log(c)) + b*d*sin(4*b*d*log(c))*sin(2*b*d
*log(c)))*n*e^m - ((cos(2*b*d*log(c))*sin(4*b*d*log(c)) - cos(4*b*d*log(c))
*sin(2*b*d*log(c)))*m + cos(2*b*d*log(c))*sin(4*b*d*log(c)) - cos(4*b*d*log
(c))*sin(2*b*d*log(c)))*e^m)*x*x^m*cos(2*b*d*log(x^n) + 2*a*d) + (2*(b*d*co
s(2*b*d*log(c))*sin(4*b*d*log(c)) - b*d*cos(4*b*d*log(c))*sin(2*b*d*log(c))
)*n*e^m + ((cos(4*b*d*log(c))*cos(2*b*d*log(c)) + sin(4*b*d*log(c))*sin(2*b
*d*log(c)))*m + cos(4*b*d*log(c))*cos(2*b*d*log(c)) + sin(4*b*d*log(c))*sin
(2*b*d*log(c)))*e^m)*x*x^m*sin(2*b*d*log(x^n) + 2*a*d) - (m*sin(4*b*d*log(c
)) + sin(4*b*d*log(c)))*x*e^(m*log(x) + m))*cos(4*b*d*log(x^n) + 4*a*d) - (
2*b^6*d^6*n^6*e^m - (b^4*d^4*m^2 + 2*b^4*d^4*m + b^4*d^4)*n^4*e^m + (2*(b^6
*d^6*cos(4*b*d*log(c))^2 + b^6*d^6*sin(4*b*d*log(c))^2)*n^6*e^m - (b^4*d^4*
cos(4*b*d*log(c))^2 + b^4*d^4*sin(4*b*d*log(c))^2 + (b^4*d^4*cos(4*b*d*log(
c))^2 + b^4*d^4*sin(4*b*d*log(c))^2)*m^2 + 2*(b^4*d^4*cos(4*b*d*log(c))^2 +
 b^4*d^4*sin(4*b*d*log(c))^2)*m)*n^4*e^m)*cos(4*b*d*log(x^n) + 4*a*d)^2 + 4
*(2*(b^6*d^6*cos(2*b*d*log(c))^2 + b^6*d^6*sin(2*b*d*log(c))^2)*n^6*e^m - (
b^4*d^4*cos(2*b*d*log(c))^2 + b^4*d^4*sin(2*b*d*log(c))^2 + (b^4*d^4*cos(2*
b*d*log(c))^2 + b^4*d^4*sin(2*b*d*log(c))^2)*m^2 + 2*(b^4*d^4*cos(2*b*d*log
(c))^2 + b^4*d^4*sin(2*b*d*log(c))^2)*m)*n^4*e^m)*cos(2*b*d*log(x^n) + 2*a*
d)^2 + (2*(b^6*d^6*cos(4*b*d*log(c))^2 + b^6*d^6*sin(4*b*d*log(c))^2)*n^6*e
^m - (b^4*d^4*cos(4*b*d*log(c))^2 + b^4*d^4*sin(4*b*d*log(c))^2 + (b^4*d^4*
cos(4*b*d*log(c))^2 + b^4*d^4*sin(4*b*d*log(c))^2)*m^2 + 2*(b^4*d^4*cos(4*b
*d*log(c))^2 + b^4*d^4*sin(4*b*d*log(c))^2)*m)*n^4*e^m)*sin(4*b*d*log(x^n)
+ 4*a*d)^2 + 4*(2*(b^6*d^6*cos(2*b*d*log(c))^2 + b^6*d^6*sin(2*b*d*log(c))^
2)*n^6*e^m - (b^4*d^4*cos(2*b*d*log(c))^2 + b^4*d^4*sin(2*b*d*log(c))^2 + (
b^4*d^4*cos(2*b*d*log(c))^2 + b^4*d^4*sin(2*b*d*log(c))^2)*m^2 + 2*(b^4*d^4
*cos(2*b*d*log(c))^2 + b^4*d^4*sin(2*b*d*log(c))^2)*m)*n^4*e^m)*sin(2*b*d*l
og(x^n) + 2*a*d)^2 + 2*(2*b^6*d^6*n^6*cos(4*b*d*log(c))*e^m - (b^4*d^4*m^2*
cos(4*b*d*log(c)) + 2*b^4*d^4*m*cos(4*b*d*log(c)) + b^4*d^4*cos(4*b*d*log(c
)))*n^4*e^m + 2*(2*(b^6*d^6*cos(4*b*d*log(c))*cos(2*b*d*log(c)) + b^6*d^6*s
in(4*b*d*log(c))*sin(2*b*d*log(c)))*n^6*e^m - (b^4*d^4*cos(4*b*d*log(c))*co
s(2*b*d*log(c)) + b^4*d^4*sin(4*b*d*log(c))*sin(2*b*d*log(c)) + (b^4*d^4*co
s(4*b*d*log(c))*cos(2*b*d*log(c)) + b^4*d^4*sin(4*b*d*log(c))*sin(2*b*d*log
(c)))*m^2 + 2*(b^4*d^4*cos(4*b*d*log(c))*cos(2*b*d*log(c)) + b^4*d^4*sin(4*
b*d*log(c))*sin(2*b*d*log(c)))*m)*n^4*e^m)*cos(2*b*d*log(x^n) + 2*a*d) + 2*
(2*(b^6*d^6*cos(2*b*d*log(c))*sin(4*b*d*log(c)) - b^6*d^6*cos(4*b*d*log(c))
*sin(2*b*d*log(c)))*n^6*e^m - (b^4*d^4*cos(2*b*d*log(c))*sin(4*b*d*log(c))
- b^4*d^4*cos(4*b*d*log(c))*sin(2*b*d*log(c)) + (b^4*d^4*cos(2*b*d*log(c))*
sin(4*b*d*log(c)) - b^4*d^4*cos(4*b*d*log(c))*sin(2*b*d*log(c)))*m^2 + 2*(b
^4*d^4*cos(2*b*d*log(c))*sin(4*b*d*log(c)) - b^4*d^4*cos(4*b*d*log(c))*sin(
```

```
2*b*d*log(c)))*m)*n^4*e^m)*sin(2*b*d*log(x^n) + 2*a*d))*cos(4*b*d*log(x^n)
+ 4*a*d) + 4*(2*b^6*d^6*n^6*cos(2*b*d*log(c))*e^m - (b^4*d^4*m^2*cos(2*b*d*
log(c)) + 2*b^4*d^4*m*cos(2*b*d*log(c)) + b^4*d^4*cos(2*b*d*log(c)))*n^4*e^
m)*cos(2*b*d*log(x^n) + 2*a*d) - 2*(2*b^6*d^6*n^6*e^m*sin(4*b*d*log(c)) - (
b^4*d^4*m^2*sin(4*b*d*log(c)) + 2*b^4*d^4*m*sin(4*b*d*log(c)) + b^4*d^4*sin
(4*b*d*log(c)))*n^4*e^m + 2*(2*(b^6*d^6*cos(2*b*d*log(c))*sin(4*b*d*log(c))
 - b^6*d^6*cos(4*b*d*log(c))*sin(2*b*d*log(c)))*n^6*e^m - (b^4*d^4*cos(2*b*
d*log(c))*sin(4*b*d*log(c)) - b^4*d^4*cos(4*b*d*log(c))*sin(2*b*d*log(c)) +
 (b^4*d^4*cos(2*b*d*log(c))*sin(4*b*d*log(c)) - b^4*d^4*cos(4*b*d*log(c))*s
in(2*b*d*log(c)))*m^2 + 2*(b^4*d^4*cos(2*b*d*log(c))*sin(4*b*d*log(c)) - b^
4*d^4*cos(4*b*d*log(c))*sin(2*b*d*log(c)))*m)*n^4*e^m)*cos(2*b*d*log(x^n) +
 2*a*d) - 2*(2*(b^6*d^6*cos(4*b*d*log(c))*cos(2*b*d*log(c)) + b^6*d^6*sin(4
*b*d*log(c))*sin(2*b*d*log(c)))*n^6*e^m - (b^4*d^4*cos(4*b*d*log(c))*cos(2*
b*d*log(c)) + b^4*d^4*sin(4*b*d*log(c))*sin(2*b*d*log(c)) + (b^4*d^4*cos(4*
b*d*log(c))*cos(2*b*d*log(c)) + b^4*d^4*sin(4*b*d*log(c))*sin(2*b*d*log(c))
)*m^2 + 2*(b^4*d^4*cos(4*b*d*log(c))*cos(2*b*d*log(c)) + b^4*d^4*sin(4*b*d*
log(c))*sin(2*b*d*log(c)))*m)*n^4*e^m)*sin(2*b*d*log(x^n) + 2*a*d))*sin(4*b
*d*log(x^n) + 4*a*d) - 4*(2*b^6*d^6*n^6*e^m*sin(2*b*d*log(c)) - (b^4*d^4*m^
2*sin(2*b*d*log(c)) + 2*b^4*d^4*m*sin(2*b*d*log(c)) + b^4*d^4*sin(2*b*d*log
(c)))*n^4*e^m)*sin(2*b*d*log(x^n) + 2*a*d))*integrate((x^m*cos(2*b*d*log(x^
n) + 2*a*d)*sin(2*b*d*log(c)) + x^m*cos(2*b*d*log(c))*sin(2*b*d*log(x^n) +
2*a*d))/(2*b^4*d^4*n^4*cos(2*b*d*log(c))*cos(2*...
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tan(d*(a+b*log(c*x^n)))^3,x, algorithm="fricas")
[Out] integral((x*e)^m*tan(b*d*log(c*x^n) + a*d)^3, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \tan^3 (ad + bd \log (cx^n))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*tan(d*(a+b*ln(c*x**n)))**3,x)
[Out] Integral((e*x)**m*tan(a*d + b*d*log(c*x**n))**3, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tan(d*(a+b*log(c*x^n)))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(d\,(a+b\,\ln\,(c\,x^n)))^3\,(e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*(a + b*log(c*x^n)))^3*(e*x)^m,x)
```

```
[Out] int(tan(d*(a + b*log(c*x^n)))^3*(e*x)^m, x)
```

3.178 $\int \tan^p(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=190

$$x\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(\frac{i\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{1+e^{2iad}(cx^n)^{2ibd}}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^p F_1\left(-\frac{i}{2bdn};-p,p;1-\frac{i}{2bdn};e^{2iad}(c$$

```
[Out] x*(I*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d)))
^p*(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^p*AppellF1(-1/2*I/b/d/n,-p,p,1-1/2*I/
b/d/n,exp(2*I*a*d)*(c*x^n)^(2*I*b*d),-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/((1-e
xp(2*I*a*d)*(c*x^n)^(2*I*b*d))^p)
```

Rubi [A]
time = 0.11, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4589, 4591, 1986, 525, 524}

$$x\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(\frac{i\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{1+e^{2iad}(cx^n)^{2ibd}}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^p F_1\left(-\frac{i}{2bdn};-p,p;1-\frac{i}{2bdn};e^{2iad}(cx^n)^{2ibd},-e^{2iad}(cx^n)^{2ibd}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] (x*((I*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(1 + E^((2*I)*a*d)*(c*x^n)^
((2*I)*b*d)))^p*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p*AppellF1[(-1/2*I)
/(b*d*n), -p, p, 1 - (I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d), -(E^
((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])/(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^
p
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4589

```
Int[Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Tan[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int \tan^p\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int \tan^p\left(d(a+b\log\left(cx^n\right))\right)\,dx$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 458 vs. $2(190) = 380$.
time = 1.49, size = 458, normalized size = 2.41

$$\frac{(-i+2bdn)x\left(-\frac{i\left(-1+e^{2iad}(cx^n)^{2ibd}\right)}{1+e^{2iad}(cx^n)^{2ibd}}\right)^p F_1\left(-\frac{i}{2bdn};-p,p;1-\frac{i}{2bdn};e^{2iad}(cx^n)^{2ibd},-e^{2iad}(cx^n)^{2ibd}\right)}{-2bde^{2iad}np\left(cx^n\right)^{2ibd}F_1\left(1-\frac{i}{2bdn};1-p,p;2-\frac{i}{2bdn};e^{2iad}\left(cx^n\right)^{2ibd},-e^{2iad}\left(cx^n\right)^{2ibd}\right)-2bde^{2iad}np\left(cx^n\right)^{2ibd}F_1\left(1-\frac{i}{2bdn};-p,1+p;2-\frac{i}{2bdn};e^{2iad}\left(cx^n\right)^{2ibd},-e^{2iad}\left(cx^n\right)^{2ibd}\right)+(-i+2bdn)F_1\left(-\frac{i}{2bdn};-p,p;1-\frac{i}{2bdn};e^{2iad}\left(cx^n\right)^{2ibd},-e^{2iad}\left(cx^n\right)^{2ibd}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] ((-I + 2*b*d*n)*x*(((-I)*(-1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(1 + E^(
(2*I)*a*d)*(c*x^n)^((2*I)*b*d)))^p*AppellF1[(-1/2*I)/(b*d*n), -p, p, 1 - (I
/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2
*I)*b*d))])/(-2*b*d*E^((2*I)*a*d)*n*p*(c*x^n)^((2*I)*b*d)*AppellF1[1 - (I/2
)/(b*d*n), 1 - p, p, 2 - (I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d),
-(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))] - 2*b*d*E^((2*I)*a*d)*n*p*(c*x^n)^((2
*I)*b*d)*AppellF1[1 - (I/2)/(b*d*n), -p, 1 + p, 2 - (I/2)/(b*d*n), E^((2*I)
*a*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))] + (-I + 2*
b*d*n)*AppellF1[(-1/2*I)/(b*d*n), -p, p, 1 - (I/2)/(b*d*n), E^((2*I)*a*d)*(
c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \tan^{p}\left(d(a+b\ln\left(c\,x^{n}\right))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*(a+b*ln(c*x^n)))^p,x)
```

```
[Out] int(tan(d*(a+b*ln(c*x^n)))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
```

```
[Out] integrate(tan((b*log(c*x^n) + a)*d)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

```
[Out] integral(tan(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \tan^{p}\left(d(a+b\log\left(cx^{n}\right))\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*ln(c*x**n)))**p,x)
```

```
[Out] Integral(tan(d*(a + b*log(c*x**n)))**p, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(d\,(a+b\,\ln\left(c\,x^n\right)))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*(a + b*log(c*x^n)))^p,x)
```

```
[Out] int(tan(d*(a + b*log(c*x^n)))^p, x)
```

3.179 $\int (ex)^m \tan^p(d(a+b\log(cx^n)))\, dx$

Optimal. Leaf size=210

$$\frac{(ex)^{1+m}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(\frac{i\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{1+e^{2iad}(cx^n)^{2ibd}}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^p F_1\left(-\frac{i(1+m)}{2bdn};-p,p;1-\frac{i(1+m)}{2bdn};e^{2iad}\right.}{e(1+m)}$$

```
[Out] (e*x)^(1+m)*(I*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/(1+exp(2*I*a*d)*(c*x^n)^(
2*I*b*d)))^p*(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^p*AppellF1(-1/2*I*(1+m)/b/d
/n,-p,p,1-1/2*I*(1+m)/b/d/n,exp(2*I*a*d)*(c*x^n)^(2*I*b*d),-exp(2*I*a*d)*(c
*x^n)^(2*I*b*d))/e/(1+m)/((1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^p)
```

Rubi [A]
time = 0.13, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4593, 4591, 1986, 525, 524}

$$\frac{(ex)^{m+1}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(\frac{i\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{1+e^{2iad}(cx^n)^{2ibd}}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^p F_1\left(-\frac{i(m+1)}{2bdn};-p,p;1-\frac{i(m+1)}{2bdn};e^{2iad}(cx^n)^{2ibd},-e^{2iad}(cx^n)^{2ibd}\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Tan[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] ((e*x)^(1 + m)*((I*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(1 + E^((2*I)*a
*d)*(c*x^n)^((2*I)*b*d)))^p*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p*Appel
lF1[((-1/2*I)*(1 + m))/(b*d*n), -p, p, 1 - ((I/2)*(1 + m))/(b*d*n), E^((2*I
)*a*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])/(e*(1 +
m)*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4593

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int (ex)^m \tan^p\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int (ex)^m \tan^p\left(d(a+b\log\left(cx^n\right))\right)\,dx$$

Mathematica [**A**]
time = 0.83, size = 205, normalized size = 0.98

$$\frac{x(ex)^m\left(1-e^{2iad}\left(cx^n\right)^{2ibd}\right)^{-p}\left(-\frac{i\left(-1+e^{2iad}\left(cx^n\right)^{2ibd}\right)}{1+e^{2iad}\left(cx^n\right)^{2ibd}}\right)^p\left(1+e^{2iad}\left(cx^n\right)^{2ibd}\right)^p F_1\left(-\frac{i(1+m)}{2bdn};-p,p;1-\frac{i(1+m)}{2bdn};e^{2iad}\left(cx^n\right)^{2ibd},-e^{2iad}\left(cx^n\right)^{2ibd}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Tan[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] (x*(e*x)^m*(((-I)*(-1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(1 + E^((2*I)*a
*d)*(c*x^n)^((2*I)*b*d)))^p*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p*Appel
lF1[((-1/2*I)*(1 + m))/(b*d*n), -p, p, 1 - ((I/2)*(1 + m))/(b*d*n), E^((2*I
)*a*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])/((1 + m)
*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p)
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\tan^p\left(d(a+b\ln\left(c\,x^n\right))\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*tan(d*(a+b*ln(c*x^n)))^p,x)
```

```
[Out] int((e*x)^m*tan(d*(a+b*ln(c*x^n)))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tan(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e)^m*tan((b*log(c*x^n) + a)*d)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tan(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

```
[Out] integral((x*e)^m*tan(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*tan(d*(a+b*ln(c*x**n)))**p,x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tan(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(d\,(a+b\,\ln\left(c\,x^n\right)))^p\,\left(e\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*(a + b*log(c*x^n)))^p*(e*x)^m,x)
[Out] int(tan(d*(a + b*log(c*x^n)))^p*(e*x)^m, x)
```

3.180 $\int \frac{\tan^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=201

$$\frac{\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}\right)}{\sqrt{2}\,bn}-\frac{\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}\right)}{\sqrt{2}\,bn}-\frac{\log\left(1-\sqrt{2}\,\sqrt{\text{ta}\right.}}{}$$

```
[Out] -1/2*arctan(-1+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)-1/2*arctan(1+2
^(1/2)*tan(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)-1/4*ln(1-2^(1/2)*tan(a+b*ln(c*
x^n))^(1/2)+tan(a+b*ln(c*x^n)))/b/n*2^(1/2)+1/4*ln(1+2^(1/2)*tan(a+b*ln(c*x
^n))^(1/2)+tan(a+b*ln(c*x^n)))/b/n*2^(1/2)+2/3*tan(a+b*ln(c*x^n))^(3/2)/b/n
```

Rubi [A]
time = 0.10, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3554, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}\right)}{\sqrt{2}\,bn}-\frac{\text{ArcTan}\left(\sqrt{2}\,\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}+1\right)}{\sqrt{2}\,bn}+\frac{2\tan^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{3bn}-\frac{\log\left(\tan\left(a+b\log\left(cx^n\right)\right)-\sqrt{2}\,\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}+1\right)}{2\sqrt{2}\,bn}+\frac{\log\left(\tan\left(a+b\log\left(cx^n\right)\right)+\sqrt{2}\,\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}+1\right)}{2\sqrt{2}\,bn}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) - ArcTan[1 +
Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) - Log[1 - Sqrt[2]*Sqrt[T
an[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) + Log[1 + Sq
rt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n)
+ (2*Tan[a + b*Log[c*x^n]]^(3/2))/(3*b*n)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{\tan^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\text{Subst}\left(\int \tan^{\frac{5}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2\tan^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} - \frac{\text{Subst}\left(\int \sqrt{\tan(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2\tan^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(a+b\log(cx^n))\right)}{bn}$$

$$= \frac{2\tan^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} - \frac{2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(a+b\log(cx^n))}\right)}{bn}$$

$$= \frac{2\tan^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(a+b\log(cx^n))}\right)}{bn} - \frac{\text{Su}}{}$$

$$= \frac{2\tan^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(a+b\log(cx^n))}\right.}{2bn}$$

$$= -\frac{\log\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+\tan(a+b\log(cx^n))\right)}{2\sqrt{2}bn} + \frac{\log\left(1+\sqrt{}\right.}{}$$

$$= \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}bn} - \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{\tan(a+b\log\right.}{\sqrt{2}bn}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.19, size = 50, normalized size = 0.25

$$-\frac{2\left(-1+{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(a+b\log(cx^n))\right)\right)\tan^{\frac{3}{2}}(a+b\log(cx^n))}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (-2*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Tan[a + b*Log[c*x^n]]^2])*Tan[a +
 b*Log[c*x^n]]^(3/2))/(3*b*n)
```

Maple [A]
time = 0.19, size = 139, normalized size = 0.69

method	result

derivativedivides	$\frac{\frac{2\left(\tan^{\frac{3}{2}}(a+b\ln(c\,x^n))\right)}{3}-\frac{\sqrt{2}\,\left(\ln\left(\frac{1-\sqrt{2}\,\left(\sqrt{\tan}\,(a+b\ln(c\,x^n))\right)+\tan(a+b\ln(c\,x^n))}{1+\sqrt{2}\,\left(\sqrt{\tan}\,(a+b\ln(c\,x^n))\right)+\tan(a+b\ln(c\,x^n))}\right)+2\arctan\left(1+\sqrt{2}\,\left(\sqrt{\tan}\,(a+\cdots\right.\right.\right.}{4}}{nb}$
default	$\frac{\frac{2\left(\tan^{\frac{3}{2}}(a+b\ln(c\,x^n))\right)}{3}-\frac{\sqrt{2}\,\left(\ln\left(\frac{1-\sqrt{2}\,\left(\sqrt{\tan}\,(a+b\ln(c\,x^n))\right)+\tan(a+b\ln(c\,x^n))}{1+\sqrt{2}\,\left(\sqrt{\tan}\,(a+b\ln(c\,x^n))\right)+\tan(a+b\ln(c\,x^n))}\right)+2\arctan\left(1+\sqrt{2}\,\left(\sqrt{\tan}\,(a+\cdots\right.\right.\right.}{4}}{nb}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a+b*ln(c*x^n))^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(2/3*tan(a+b*ln(c*x^n))^(3/2)-1/4*2^(1/2)*(ln((1-2^(1/2)*tan(a+b*ln(c
*x^n))^(1/2)+tan(a+b*ln(c*x^n)))/(1+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2)+tan(a+
b*ln(c*x^n))))+2*arctan(1+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2))+2*arctan(-1+2^(
1/2)*tan(a+b*ln(c*x^n))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(tan(b*log(c*x^n) + a)^(5/2)/x, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*ln(c*x**n))**(5/2)/x,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 3.39, size = 79, normalized size = 0.39

$$\frac{2\tan(a+b\,\ln(c\,x^n))^{3/2}}{3\,b\,n}-\frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{\tan\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}{b\,n}+\frac{(-1)^{1/4}\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{\tan\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*log(c*x^n))^(5/2)/x,x)
```

```
[Out] (2*tan(a + b*log(c*x^n))^(3/2))/(3*b*n) - ((-1)^(1/4)*atan((-1)^(1/4)*tan(a
 + b*log(c*x^n))^(1/2)))/(b*n) + ((-1)^(1/4)*atanh((-1)^(1/4)*tan(a + b*log
(c*x^n))^(1/2)))/(b*n)
```

3.181 $\int \frac{\tan^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=199

$$\frac{\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}\right)}{\sqrt{2}\,bn}-\frac{\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}\right)}{\sqrt{2}\,bn}+\frac{\log\left(1-\sqrt{2}\,\sqrt{\tan\right.}$$

```
[Out] -1/2*arctan(-1+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)-1/2*arctan(1+2
^(1/2)*tan(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)+1/4*ln(1-2^(1/2)*tan(a+b*ln(c*
x^n))^(1/2)+tan(a+b*ln(c*x^n)))/b/n*2^(1/2)-1/4*ln(1+2^(1/2)*tan(a+b*ln(c*x
^n))^(1/2)+tan(a+b*ln(c*x^n)))/b/n*2^(1/2)+2*tan(a+b*ln(c*x^n))^(1/2)/b/n
```

Rubi [A]

time = 0.09, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}\right)}{\sqrt{2}\,bn}-\frac{\text{ArcTan}\left(\sqrt{2}\,\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}+1\right)}{\sqrt{2}\,bn}+\frac{\log\left(\tan\left(a+b\log\left(cx^n\right)\right)-\sqrt{2}\,\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}+1\right)}{2\sqrt{2}\,bn}-\frac{\log\left(\tan\left(a+b\log\left(cx^n\right)\right)+\sqrt{2}\,\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}+1\right)}{2\sqrt{2}\,bn}+\frac{2\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) - ArcTan[1 +
Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) + Log[1 - Sqrt[2]*Sqrt[T
an[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) - Log[1 + Sq
rt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n)
+ (2*Sqrt[Tan[a + b*Log[c*x^n]]])/(b*n)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{\tan^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\text{Subst}\left(\int \tan^{\frac{3}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2\sqrt{\tan(a+b\log(cx^n))}}{bn} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\tan(a+bx)}}\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2\sqrt{\tan(a+b\log(cx^n))}}{bn} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx, x, \tan(a+b\log(cx^n))\right)}{bn}$$

$$= \frac{2\sqrt{\tan(a+b\log(cx^n))}}{bn} - \frac{2\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{\tan(a+b\log(cx^n))}\right)}{bn}$$

$$= \frac{2\sqrt{\tan(a+b\log(cx^n))}}{bn} - \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(a+b\log(cx^n))}\right)}{bn} - \frac{\text{S}}{}$$

$$= \frac{2\sqrt{\tan(a+b\log(cx^n))}}{bn} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{\tan(a+b\log(cx^n))}\right.}{2bn}$$

$$= \frac{\log\left(1-\sqrt{2}\,\sqrt{\tan(a+b\log(cx^n))}+\tan(a+b\log(cx^n))\right)}{2\sqrt{2}\,bn} - \frac{\log\left(1+\sqrt{2}\right.}{}$$

$$= \frac{\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}\,bn} - \frac{\tan^{-1}\left(1+\sqrt{2}\,\sqrt{\tan(a+b\log(c\right.}{\sqrt{2}\,bn}$$

Mathematica [A]
time = 0.19, size = 175, normalized size = 0.88

$$\frac{2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(a+b\log(cx^n))}\right)-2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(a+b\log(cx^n))}\right)+\sqrt{2}\,\log\left(1-\sqrt{2}\,\sqrt{\tan(a+b\log(cx^n))}+\tan(a+b\log(cx^n))\right)-\sqrt{2}\,\log\left(1+\sqrt{2}\,\sqrt{\tan(a+b\log(cx^n))}+\tan(a+b\log(cx^n))\right)+8\sqrt{\tan(a+b\log(cx^n))}}{4bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]] - 2*Sqrt[2]*ArcT
an[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[
Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]] - Sqrt[2]*Log[1 + Sqrt[2]*S
qrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]] + 8*Sqrt[Tan[a + b*Log[
c*x^n]]])/(4*b*n)
```

Maple [A]
time = 0.04, size = 139, normalized size = 0.70

method	result
derivativedivides	$\frac{2\left(\sqrt{\tan}\,(a+b\ln(c\,x^{n}))\right)-\frac{\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}\,(a+b\ln(c\,x^{n}))\right)+\tan(a+b\ln(c\,x^{n}))}{1-\sqrt{2}\left(\sqrt{\tan}\,(a+b\ln(c\,x^{n}))\right)+\tan(a+b\ln(c\,x^{n}))}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\mathrm{ta}}\right.\right.\right.}{4}}{nb}$
default	$\frac{2\left(\sqrt{\tan}\,(a+b\ln(c\,x^{n}))\right)-\frac{\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}\,(a+b\ln(c\,x^{n}))\right)+\tan(a+b\ln(c\,x^{n}))}{1-\sqrt{2}\left(\sqrt{\tan}\,(a+b\ln(c\,x^{n}))\right)+\tan(a+b\ln(c\,x^{n}))}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\mathrm{ta}}\right.\right.\right.}{4}}{nb}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(2*tan(a+b*ln(c*x^n))^(1/2)-1/4*2^(1/2)*(ln((1+2^(1/2)*tan(a+b*ln(c*x
^n))^(1/2)+tan(a+b*ln(c*x^n)))/(1-2^(1/2)*tan(a+b*ln(c*x^n))^(1/2)+tan(a+b*
ln(c*x^n))))+2*arctan(1+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2))+2*arctan(-1+2^(1/
2)*tan(a+b*ln(c*x^n))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(tan(b*log(c*x^n) + a)^(3/2)/x, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*ln(c*x**n))**(3/2)/x,x)
```

```
[Out] Integral(tan(a + b*log(c*x**n))**(3/2)/x, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 3.31, size = 78, normalized size = 0.39

$$\frac{2\sqrt{\tan\left(a+b\,\ln\left(c\,x^n\right)\right)}}{b\,n}+\frac{{\left(-1\right)}^{1/4}\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\sqrt{\tan\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)\,1{}\mathrm{i}}{b\,n}+\frac{{\left(-1\right)}^{1/4}\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,\sqrt{\tan\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)\,1{}\mathrm{i}}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*log(c*x^n))^(3/2)/x,x)
```

```
[Out] (2*tan(a + b*log(c*x^n))^(1/2))/(b*n) + ((-1)^(1/4)*atan((-1)^(1/4)*tan(a +
 b*log(c*x^n))^(1/2))*1i)/(b*n) + ((-1)^(1/4)*atanh((-1)^(1/4)*tan(a + b*lo
g(c*x^n))^(1/2))*1i)/(b*n)
```

3.182 $\int \frac{\sqrt{\tan(a+b\log(cx^n))}}{x}\,dx$

Optimal. Leaf size=176

$$-\frac{\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}\,bn}+\frac{\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}\,bn}+\frac{\log\left(1-\sqrt{2}\sqrt{\tan\right.}}{}$$

```
[Out] 1/2*arctan(-1+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)+1/2*arctan(1+2^
(1/2)*tan(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)+1/4*ln(1-2^(1/2)*tan(a+b*ln(c*x
^n))^(1/2)+tan(a+b*ln(c*x^n)))/b/n*2^(1/2)-1/4*ln(1+2^(1/2)*tan(a+b*ln(c*x^
n))^(1/2)+tan(a+b*ln(c*x^n)))/b/n*2^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}\,bn}+\frac{\mathrm{ArcTan}\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\sqrt{2}\,bn}+\frac{\log\left(\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{2\sqrt{2}\,bn}-\frac{\log\left(\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{2\sqrt{2}\,bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n)) + ArcTan[1
 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) + Log[1 - Sqrt[2]*Sqr
t[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) - Log[1 +
 Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*
n)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{\sqrt{\tan(a+b\log(cx^n))}}{x}\,dx = \frac{\text{Subst}\left(\int \sqrt{\tan(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(a+b\log(cx^n))\right)}{bn}$$
$$= \frac{2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(a+b\log(cx^n))}\right)}{bn}$$
$$= -\frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(a+b\log(cx^n))}\right)}{bn} + \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\text{ta}\right.}{bn}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{\tan(a+b\log(cx^n))}\right)}{2bn} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x\right.}{}$$
$$= \frac{\log\left(1-\sqrt{2}\,\sqrt{\tan(a+b\log(cx^n))}+\tan(a+b\log(cx^n))\right)}{2\sqrt{2}\,bn} - \frac{\log\left(1+\right.}{}$$
$$= -\frac{\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}\,bn} + \frac{\tan^{-1}\left(1+\sqrt{2}\,\sqrt{\tan(a+b\right.}{\sqrt{2}\,bn}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, size = 48, normalized size = 0.27

$$\frac{2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(a+b\log(cx^n))\right)\tan^{\frac{3}{2}}(a+b\log(cx^n))}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] (2*Hypergeometric2F1[3/4, 1, 7/4, -Tan[a + b*Log[c*x^n]]^2]*Tan[a + b*Log[c
*x^n]]^(3/2))/(3*b*n)
```

Maple [A]
time = 0.04, size = 122, normalized size = 0.69

method	result
derivativedivides	$\frac{\sqrt{2}\left(\ln\left(\frac{1-\sqrt{2}\left(\sqrt{\tan}(a+b\ln(c\,x^n))\right)+\tan(a+b\ln(c\,x^n))}{1+\sqrt{2}\left(\sqrt{\tan}(a+b\ln(c\,x^n))\right)+\tan(a+b\ln(c\,x^n))}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}(a+b\ln(c\,x^n))\right)\right)+2\,\text{a}\right.}{4nb}$

default	$\frac{\sqrt{2}\left(\ln\left(\frac{1-\sqrt{2}\left(\sqrt{\tan}\,(a+b\ln(c\,x^{n}))\right)+\tan(a+b\ln(c\,x^{n}))}{1+\sqrt{2}\left(\sqrt{\tan}\,(a+b\ln(c\,x^{n}))\right)+\tan(a+b\ln(c\,x^{n}))}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}\,(a+b\ln(c\,x^{n}))\right)\right)+2\operatorname{arc}\right.}{4nb}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/n/b*2^(1/2)*(ln((1-2^(1/2)*tan(a+b*ln(c*x^n))^(1/2)+tan(a+b*ln(c*x^n)))
/(1+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2)+tan(a+b*ln(c*x^n))))+2*arctan(1+2^(1/2
)*tan(a+b*ln(c*x^n))^(1/2))+2*arctan(-1+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tan(b*log(c*x^n) + a))/x, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\tan\left(a + b\log\left(cx^{n}\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*ln(c*x**n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(tan(a + b*log(c*x**n)))/x, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 2.63, size = 131, normalized size = 0.74

$$\frac{\sqrt{2}\,\left(\mathrm{atan}\left(\sqrt{2}\,\sqrt{\tan\left(a+b\ln\left(c\,x^n\right)\right)}-1\right)+\mathrm{atan}\left(\sqrt{2}\,\sqrt{\tan\left(a+b\ln\left(c\,x^n\right)\right)}+1\right)\right)}{2\,b\,n}+\frac{\sqrt{2}\,\left(\ln\left(\sqrt{2}\,\sqrt{\tan\left(a+b\ln\left(c\,x^n\right)\right)}-\tan(a+b\ln\left(c\,x^n\right))-1\right)-\ln\left(\tan(a+b\ln\left(c\,x^n\right))+\sqrt{2}\,\sqrt{\tan\left(a+b\ln\left(c\,x^n\right)\right)}+1\right)\right)}{4\,b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*log(c*x^n))^(1/2)/x,x)
```

```
[Out] (2^(1/2)*(atan(2^(1/2)*tan(a + b*log(c*x^n))^(1/2) - 1) + atan(2^(1/2)*tan(
a + b*log(c*x^n))^(1/2) + 1)))/(2*b*n) + (2^(1/2)*(log(2^(1/2)*tan(a + b*lo
g(c*x^n))^(1/2) - tan(a + b*log(c*x^n)) - 1) - log(tan(a + b*log(c*x^n)) +
2^(1/2)*tan(a + b*log(c*x^n))^(1/2) + 1)))/(4*b*n)
```

3.183 $\int \frac{1}{x\sqrt{\tan(a+b\log(cx^n))}}\,dx$

Optimal. Leaf size=176

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}bn}+\frac{\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}bn}-\frac{\log\left(1-\sqrt{2}\sqrt{\tan\right.}}{}$$

```
[Out] 1/2*arctan(-1+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)+1/2*arctan(1+2^
(1/2)*tan(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)-1/4*ln(1-2^(1/2)*tan(a+b*ln(c*x
^n))^(1/2)+tan(a+b*ln(c*x^n)))/b/n*2^(1/2)+1/4*ln(1+2^(1/2)*tan(a+b*ln(c*x^
n))^(1/2)+tan(a+b*ln(c*x^n)))/b/n*2^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}bn}+\frac{\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\sqrt{2}bn}-\frac{\log\left(\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{2\sqrt{2}bn}+\frac{\log\left(\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{2\sqrt{2}bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Tan[a + b*Log[c*x^n]]]),x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n)) + ArcTan[1
 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) - Log[1 - Sqrt[2]*Sqr
t[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) + Log[1 +
 Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*
n)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{x\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\tan(a+bx)}}\,dx, x, \log\left(cx^n\right)\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx, x, \tan\left(a+b\log\left(cx^n\right)\right)\right)}{bn}$$

$$= \frac{2\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

$$= \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} + \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan}\right.}{bn}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}\right)}{2bn} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x}\right.}{}$$

$$= -\frac{\log\left(1-\sqrt{2}\,\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}+\tan\left(a+b\log\left(cx^n\right)\right)\right)}{2\sqrt{2}\,bn} + \frac{\log\left(1+\right.}{}$$

$$= -\frac{\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}\right)}{\sqrt{2}\,bn} + \frac{\tan^{-1}\left(1+\sqrt{2}\,\sqrt{\tan\left(a+b\right.}\right.}{\sqrt{2}\,bn}$$

Mathematica [A]
time = 0.10, size = 142, normalized size = 0.81

$$\frac{-2\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}\right)+2\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}\right)-\log\left(1-\sqrt{2}\,\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}+\tan\left(a+b\log\left(cx^n\right)\right)\right)+\log\left(1+\sqrt{2}\,\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}+\tan\left(a+b\log\left(cx^n\right)\right)\right)}{2\sqrt{2}\,bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[Tan[a + b*Log[c*x^n]]]),x]
```

```
[Out] (-2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]] + 2*ArcTan[1 + Sqrt[2]*
Sqrt[Tan[a + b*Log[c*x^n]]]] - Log[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]
+ Tan[a + b*Log[c*x^n]]] + Log[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Ta
n[a + b*Log[c*x^n]]])/(2*Sqrt[2]*b*n)
```

Maple [A]
time = 0.05, size = 122, normalized size = 0.69

method	result

derivativedivides	$\frac{\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(a+b\ln(c\,x^n))\right)+\tan(a+b\ln(c\,x^n))}{1-\sqrt{2}\left(\sqrt{\tan}(a+b\ln(c\,x^n))\right)+\tan(a+b\ln(c\,x^n))}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}(a+b\ln(c\,x^n))\right)\right)+2\,\mathrm{a}\right.}{4nb}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(a+b\ln(c\,x^n))\right)+\tan(a+b\ln(c\,x^n))}{1-\sqrt{2}\left(\sqrt{\tan}(a+b\ln(c\,x^n))\right)+\tan(a+b\ln(c\,x^n))}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}(a+b\ln(c\,x^n))\right)\right)+2\,\mathrm{a}\right.}{4nb}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/tan(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/n/b*2^(1/2)*(ln((1+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2)+tan(a+b*ln(c*x^n)))
/(1-2^(1/2)*tan(a+b*ln(c*x^n))^(1/2)+tan(a+b*ln(c*x^n))))+2*arctan(1+2^(1/2
)*tan(a+b*ln(c*x^n))^(1/2))+2*arctan(-1+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/tan(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt(tan(b*log(c*x^n) + a))), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/tan(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\tan\left(a + b\log\left(cx^{n}\right)\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/tan(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(tan(a + b*log(c*x**n)))), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/tan(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 2.96, size = 59, normalized size = 0.34

$$-\frac{(-1)^{1/4}\operatorname{atan}\left((-1)^{1/4}\sqrt{\tan\left(a+b\ln\left(c\,x^n\right)\right)}\right)\,1\mathrm{i}}{b\,n}-\frac{(-1)^{1/4}\operatorname{atanh}\left((-1)^{1/4}\sqrt{\tan\left(a+b\ln\left(c\,x^n\right)\right)}\right)\,1\mathrm{i}}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*tan(a + b*log(c*x^n))^(1/2)),x)
```

```
[Out] - ((-1)^(1/4)*atan((-1)^(1/4)*tan(a + b*log(c*x^n))^(1/2))*1i)/(b*n) - ((-1
)^(1/4)*atanh((-1)^(1/4)*tan(a + b*log(c*x^n))^(1/2))*1i)/(b*n)
```

3.184 $\int \frac{1}{x \tan^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=199

$$\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}\right)}{\sqrt{2}\,bn}-\frac{\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}\right)}{\sqrt{2}\,bn}-\frac{\log\left(1-\sqrt{2}\sqrt{\text{ta}\right.}$$

```
[Out] -1/2*arctan(-1+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)-1/2*arctan(1+2
^(1/2)*tan(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)-1/4*ln(1-2^(1/2)*tan(a+b*ln(c*
x^n))^(1/2)+tan(a+b*ln(c*x^n)))/b/n*2^(1/2)+1/4*ln(1+2^(1/2)*tan(a+b*ln(c*x
^n))^(1/2)+tan(a+b*ln(c*x^n)))/b/n*2^(1/2)-2/b/n/tan(a+b*ln(c*x^n))^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}\,bn}-\frac{\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\sqrt{2}\,bn}-\frac{\log\left(\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{2\sqrt{2}\,bn}+\frac{\log\left(\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{2\sqrt{2}\,bn}-\frac{2}{bn\sqrt{\tan(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Tan[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) - ArcTan[1 +
Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) - Log[1 - Sqrt[2]*Sqrt[T
an[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) + Log[1 + Sq
rt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n)
- 2/(b*n*Sqrt[Tan[a + b*Log[c*x^n]]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{x\tan^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\tan^{\frac{3}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2}{bn\sqrt{\tan(a+b\log(cx^n))}} - \frac{\text{Subst}\left(\int \sqrt{\tan(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2}{bn\sqrt{\tan(a+b\log(cx^n))}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(a+b\log(cx^n))\right)}{bn}$$

$$= -\frac{2}{bn\sqrt{\tan(a+b\log(cx^n))}} - \frac{2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(a+b\log(cx^n))}\right.}{bn}$$

$$= -\frac{2}{bn\sqrt{\tan(a+b\log(cx^n))}} + \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(a+b\log(cx^n))}\right.}{bn}$$

$$= -\frac{2}{bn\sqrt{\tan(a+b\log(cx^n))}} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{\tan(a+b\log(\right.}{2bn}$$

$$= -\frac{\log\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+\tan(a+b\log(cx^n))\right)}{2\sqrt{2}\,bn} + \frac{\log\left(1+\right.}{}$$

$$= \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}\,bn} - \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{\tan(a+b\log\right.}{\sqrt{2}\,bn}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.09, size = 46, normalized size = 0.23

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\tan^2(a+b\log(cx^n))\right)}{bn\sqrt{\tan(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Tan[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[a + b*Log[c*x^n]]^2])/(b*n*Sqrt[Ta
n[a + b*Log[c*x^n]]])
```

Maple [A]
time = 0.05, size = 139, normalized size = 0.70

method	result
derivativedivides	$-\frac{\frac{\sqrt{2}\left(\ln\left(\frac{1-\sqrt{2}\left(\sqrt{\tan}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)+\tan\left(a+b\ln\left(c\,x^{n}\right)\right)}{1+\sqrt{2}\left(\sqrt{\tan}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)+\tan\left(a+b\ln\left(c\,x^{n}\right)\right)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)\right)+2\arctan\left(\right.}{4}}{nb}$
default	$-\frac{\frac{\sqrt{2}\left(\ln\left(\frac{1-\sqrt{2}\left(\sqrt{\tan}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)+\tan\left(a+b\ln\left(c\,x^{n}\right)\right)}{1+\sqrt{2}\left(\sqrt{\tan}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)+\tan\left(a+b\ln\left(c\,x^{n}\right)\right)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)\right)+2\arctan\left(\right.}{4}}{nb}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/tan(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(-1/4*2^(1/2)*(ln((1-2^(1/2)*tan(a+b*ln(c*x^n))^(1/2)+tan(a+b*ln(c*x^
n)))/(1+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2)+tan(a+b*ln(c*x^n))))+2*arctan(1+2^
(1/2)*tan(a+b*ln(c*x^n))^(1/2))+2*arctan(-1+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2
)))-2/tan(a+b*ln(c*x^n))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/tan(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*tan(b*log(c*x^n) + a)^(3/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/tan(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\tan^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/tan(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Integral(1/(x*tan(a + b*log(c*x**n))**(3/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/tan(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 2.92, size = 79, normalized size = 0.40

$$\frac{(-1)^{1/4}\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{\tan\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}{b\,n}-\frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{\tan\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}{b\,n}-\frac{2}{b\,n\,\sqrt{\tan\left(a+b\,\ln\left(c\,x^n\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*tan(a + b*log(c*x^n))^(3/2)),x)
```

```
[Out] ((-1)^(1/4)*atanh((-1)^(1/4)*tan(a + b*log(c*x^n))^(1/2)))/(b*n) - ((-1)^(1
/4)*atan((-1)^(1/4)*tan(a + b*log(c*x^n))^(1/2)))/(b*n) - 2/(b*n*tan(a + b*
log(c*x^n))^(1/2))
```

3.185 $\int \frac{1}{x\tan^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=201

$$\frac{\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}\right)}{\sqrt{2}\,bn}-\frac{\operatorname{ArcTan}\left(1+\sqrt{2}\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}\right)}{\sqrt{2}\,bn}+\frac{\log\left(1-\sqrt{2}\sqrt{\tan\right.}$$

```
[Out] -1/2*arctan(-1+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)-1/2*arctan(1+2
^(1/2)*tan(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)+1/4*ln(1-2^(1/2)*tan(a+b*ln(c*
x^n))^(1/2)+tan(a+b*ln(c*x^n)))/b/n*2^(1/2)-1/4*ln(1+2^(1/2)*tan(a+b*ln(c*x
^n))^(1/2)+tan(a+b*ln(c*x^n)))/b/n*2^(1/2)-2/3/b/n/tan(a+b*ln(c*x^n))^(3/2)
```

Rubi [A]
time = 0.09, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$,
Rules used = {3555, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}\right)}{\sqrt{2}\,bn}-\frac{\operatorname{ArcTan}\left(\sqrt{2}\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}+1\right)}{\sqrt{2}\,bn}-\frac{2}{3bn\tan^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}+\frac{\log\left(\tan\left(a+b\log\left(cx^n\right)\right)-\sqrt{2}\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}+1\right)}{2\sqrt{2}\,bn}-\frac{\log\left(\tan\left(a+b\log\left(cx^n\right)\right)+\sqrt{2}\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}+1\right)}{2\sqrt{2}\,bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Tan[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) - ArcTan[1 +
Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) + Log[1 - Sqrt[2]*Sqrt[T
an[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) - Log[1 + Sq
rt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n)
- 2/(3*b*n*Tan[a + b*Log[c*x^n]]^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{x\tan^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\tan^{\frac{5}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$
$$= -\frac{2}{3bn\tan^{\frac{3}{2}}(a+b\log(cx^n))} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\tan(a+bx)}}\,dx, x, \log(cx^n)\right)}{n}$$
$$= -\frac{2}{3bn\tan^{\frac{3}{2}}(a+b\log(cx^n))} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx, x, \tan(a+b\log(cx^n)\right)}{bn}$$
$$= -\frac{2}{3bn\tan^{\frac{3}{2}}(a+b\log(cx^n))} - \frac{2\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{\tan(a+b\log(cx^n))}\right)}{bn}$$
$$= -\frac{2}{3bn\tan^{\frac{3}{2}}(a+b\log(cx^n))} - \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(a+b\log(cx^n))}\right)}{bn}$$
$$= -\frac{2}{3bn\tan^{\frac{3}{2}}(a+b\log(cx^n))} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(a+b\log(c\right.}{2bn}$$
$$= \frac{\log\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+\tan(a+b\log(cx^n))\right)}{2\sqrt{2}bn} - \frac{\log\left(1+\sqrt{2}\right.}{}$$
$$= \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}bn} - \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{\tan(a+b\log\right.}{\sqrt{2}bn}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.16, size = 48, normalized size = 0.24

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\tan^2(a+b\log(cx^n))\right)}{3bn\tan^{\frac{3}{2}}(a+b\log(cx^n))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Tan[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 1, 1/4, -Tan[a + b*Log[c*x^n]]^2])/(3*b*n*Tan[a
 + b*Log[c*x^n]]^(3/2))
```

Maple [A]
time = 0.05, size = 139, normalized size = 0.69

method	result
derivativedivides	$\frac{-\frac{2}{3\tan(a+b\ln(c\,x^n))^{\frac{3}{2}}}-\frac{\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(a+b\ln(c\,x^n))\right)+\tan(a+b\ln(c\,x^n))}{1-\sqrt{2}\left(\sqrt{\tan}(a+b\ln(c\,x^n))\right)+\tan(a+b\ln(c\,x^n))}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}(a\right.\right.\right.}{4}}{nb}$
default	$\frac{-\frac{2}{3\tan(a+b\ln(c\,x^n))^{\frac{3}{2}}}-\frac{\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan}(a+b\ln(c\,x^n))\right)+\tan(a+b\ln(c\,x^n))}{1-\sqrt{2}\left(\sqrt{\tan}(a+b\ln(c\,x^n))\right)+\tan(a+b\ln(c\,x^n))}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}(a\right.\right.\right.}{4}}{nb}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/tan(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(-2/3/tan(a+b*ln(c*x^n))^(3/2)-1/4*2^(1/2)*(ln((1+2^(1/2)*tan(a+b*ln(
c*x^n))^(1/2)+tan(a+b*ln(c*x^n)))/(1-2^(1/2)*tan(a+b*ln(c*x^n))^(1/2)+tan(a
+b*ln(c*x^n))))+2*arctan(1+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2))+2*arctan(-1+2^
(1/2)*tan(a+b*ln(c*x^n))^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/tan(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*tan(b*log(c*x^n) + a)^(5/2)), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/tan(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\tan^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/tan(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Integral(1/(x*tan(a + b*log(c*x**n))**(5/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/tan(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 4.08, size = 78, normalized size = 0.39

$$-\frac{2}{3\,b\,n\,\tan\left(a+b\,\ln\left(c\,x^n\right)\right)^{3/2}}+\frac{{\left(-1\right)}^{1/4}\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\sqrt{\tan\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)\,1{}\mathrm{i}}{b\,n}+\frac{{\left(-1\right)}^{1/4}\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,\sqrt{\tan\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)\,1{}\mathrm{i}}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*tan(a + b*log(c*x^n))^(5/2)),x)
```

```
[Out] ((-1)^(1/4)*atan((-1)^(1/4)*tan(a + b*log(c*x^n))^(1/2))*1i)/(b*n) - 2/(3*b
*n*tan(a + b*log(c*x^n))^(3/2)) + ((-1)^(1/4)*atanh((-1)^(1/4)*tan(a + b*lo
g(c*x^n))^(1/2))*1i)/(b*n)
```

3.186 $\int x^3 \cot(a + i\log(x))\,dx$

Optimal. Leaf size=49

$$-ie^{2ia}x^2 - \frac{ix^4}{4} - ie^{4ia}\log\left(e^{2ia} - x^2\right)$$

[Out] -I*exp(2*I*a)*x^2-1/4*I*x^4-I*exp(4*I*a)*ln(exp(2*I*a)-x^2)

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {4592, 456, 457, 78}

$$-ie^{2ia}x^2 - ie^{4ia}\log\left(-x^2 + e^{2ia}\right) - \frac{ix^4}{4}$$

Antiderivative was successfully verified.

[In] Int[x^3*Cot[a + I*Log[x]],x]

[Out] (-I)*E^((2*I)*a)*x^2 - (I/4)*x^4 - I*E^((4*I)*a)*Log[E^((2*I)*a) - x^2]

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 456

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int x^3 \cot(a + i\log(x))\,dx = \int x^3 \cot(a + i\log(x))\,dx$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 137 vs. 2(49) = 98.
time = 0.03, size = 137, normalized size = 2.80

$$-\frac{ix^4}{4} - ix^2\cos(2a) - \mathrm{ArcTan}\left(\frac{(-1+x^2)\cos(a)}{-\sin(a) - x^2\sin(a)}\right)\cos(4a) - \frac{1}{2}i\cos(4a)\log\left(1 + x^4 - 2x^2\cos(2a)\right) + x^2\sin(2a) - i\mathrm{ArcTan}\left(\frac{(-1+x^2)\cos(a)}{-\sin(a) - x^2\sin(a)}\right)\sin(4a) + \frac{1}{2}\log\left(1 + x^4 - 2x^2\cos(2a)\right)\sin(4a)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cot[a + I*Log[x]],x]
```

```
[Out] (-1/4*I)*x^4 - I*x^2*Cos[2*a] - ArcTan[((-1 + x^2)*Cos[a])/(-Sin[a] - x^2*S
in[a])]*Cos[4*a] - (I/2)*Cos[4*a]*Log[1 + x^4 - 2*x^2*Cos[2*a]] + x^2*Sin[2
*a] - I*ArcTan[((-1 + x^2)*Cos[a])/(-Sin[a] - x^2*Sin[a])]*Sin[4*a] + (Log[
1 + x^4 - 2*x^2*Cos[2*a]]*Sin[4*a])/2
```

Maple [**A**]
time = 0.06, size = 39, normalized size = 0.80

method	result	size
risch	$-i\mathrm{e}^{2ia}x^2 - \frac{ix^4}{4} - i\mathrm{e}^{4ia}\ln\left(\mathrm{e}^{2ia} - x^2\right)$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cot(a+I*ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*exp(2*I*a)*x^2-1/4*I*x^4-I*exp(4*I*a)*ln(exp(2*I*a)-x^2)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(32) = 64.
time = 0.28, size = 131, normalized size = 2.67

$$-\frac{1}{4}i\,x^4 - x^2(i\cos(2a) - \sin(2a)) + (\cos(4a) + i\sin(4a))\arctan(\sin(a), x + \cos(a)) - (\cos(4a) + i\sin(4a))\arctan(\sin(a), x - \cos(a)) - \frac{1}{2}(i\cos(4a) - \sin(4a))\log\left(x^2 + 2x\cos(a) + \cos(a)^2 + \sin(a)^2\right) - \frac{1}{2}(i\cos(4a) - \sin(4a))\log\left(x^2 - 2x\cos(a) + \cos(a)^2 + \sin(a)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(a+I*log(x)),x, algorithm="maxima")
```

```
[Out] -1/4*I*x^4 - x^2*(I*cos(2*a) - sin(2*a)) + (cos(4*a) + I*sin(4*a))*arctan2(
sin(a), x + cos(a)) - (cos(4*a) + I*sin(4*a))*arctan2(sin(a), x - cos(a)) -
 1/2*(I*cos(4*a) - sin(4*a))*log(x^2 + 2*x*cos(a) + cos(a)^2 + sin(a)^2) -
1/2*(I*cos(4*a) - sin(4*a))*log(x^2 - 2*x*cos(a) + cos(a)^2 + sin(a)^2)
```

Fricas [A]
time = 3.78, size = 32, normalized size = 0.65

$$-\frac{1}{4}i\,x^4 - i\,x^2e^{(2i\,a)} - i\,e^{(4i\,a)}\log\left(x^2 - e^{(2i\,a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(a+I*log(x)),x, algorithm="fricas")
[Out] -1/4*I*x^4 - I*x^2*e^(2*I*a) - I*e^(4*I*a)*log(x^2 - e^(2*I*a))
```

Sympy [A]
time = 0.11, size = 39, normalized size = 0.80

$$-\frac{ix^4}{4} - ix^2e^{2ia} - ie^{4ia}\log\left(x^2 - e^{2ia}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cot(a+I*ln(x)),x)
[Out] -I*x**4/4 - I*x**2*exp(2*I*a) - I*exp(4*I*a)*log(x**2 - exp(2*I*a))
```

Giac [A]
time = 0.44, size = 50, normalized size = 1.02

$$-\frac{1}{4}i\,x^4 - i\,x^2e^{(2i\,a)} + \frac{1}{2}\pi e^{(4i\,a)} - i\,e^{(4i\,a)}\log\left(x + e^{(i\,a)}\right) - i\,e^{(4i\,a)}\log\left(-x + e^{(i\,a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(a+I*log(x)),x, algorithm="giac")
[Out] -1/4*I*x^4 - I*x^2*e^(2*I*a) + 1/2*pi*e^(4*I*a) - I*e^(4*I*a)*log(x + e^(I*
a)) - I*e^(4*I*a)*log(-x + e^(I*a))
```

Mupad [B]
time = 2.22, size = 38, normalized size = 0.78

$$-x^2\,\mathrm{e}^{a\,2\mathrm{i}}\,1\mathrm{i} - \ln\left(x^2 - \mathrm{e}^{a\,2\mathrm{i}}\right)\,\mathrm{e}^{a\,4\mathrm{i}}\,1\mathrm{i} - \frac{x^4\,1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cot(a + log(x)*1i),x)
[Out] - x^2*exp(a*2i)*1i - log(x^2 - exp(a*2i))*exp(a*4i)*1i - (x^4*1i)/4
```

3.187 $\int x^2 \cot(a + i \log(x)) \, dx$

Optimal. Leaf size=43

$$-2ie^{2ia}x - \frac{ix^3}{3} + 2ie^{3ia} \tanh^{-1}\left(e^{-ia}x\right)$$

```
[Out] -2*I*exp(2*I*a)*x-1/3*I*x^3+2*I*exp(3*I*a)*arctanh(x/exp(I*a))
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {4592, 456, 470, 327, 213}

$$-2ie^{2ia}x + 2ie^{3ia} \tanh^{-1}\left(e^{-ia}x\right) - \frac{ix^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cot[a + I*Log[x]],x]
```

```
[Out] (-2*I)*E^((2*I)*a)*x - (I/3)*x^3 + (2*I)*E^((3*I)*a)*ArcTanh[x/E^(I*a)]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 456

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
```

```
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int x^2 \cot(a+i\log(x))\,dx = \int x^2 \cot(a+i\log(x))\,dx$$

Mathematica [A]
time = 0.01, size = 66, normalized size = 1.53

$$-\frac{ix^3}{3}-2ix\cos(2a)+2i\tanh^{-1}(x\cos(a)-ix\sin(a))\cos(3a)+2x\sin(2a)-2\tanh^{-1}(x\cos(a)-ix\sin(a))\sin(3a)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cot[a + I*Log[x]],x]
```

```
[Out] (-1/3*I)*x^3 - (2*I)*x*Cos[2*a] + (2*I)*ArcTanh[x*Cos[a] - I*x*Sin[a]]*Cos[
3*a] + 2*x*Sin[2*a] - 2*ArcTanh[x*Cos[a] - I*x*Sin[a]]*Sin[3*a]
```

Maple [A]
time = 0.07, size = 33, normalized size = 0.77

method	result	size
risch	$-\frac{ix^3}{3}-2i\,\mathrm{e}^{2ia}x+2i\,\mathrm{arctanh}\left(x\,\mathrm{e}^{-ia}\right)\mathrm{e}^{3ia}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cot(a+I*ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I*x^3-2*I*exp(2*I*a)*x+2*I*arctanh(x*exp(-I*a))*exp(3*I*a)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(26) = 52.
time = 0.28, size = 126, normalized size = 2.93

$$-\frac{1}{3}ix^3+2x(-i\cos(2a)+\sin(2a))-(\cos(3a)+i\sin(3a))\arctan(\sin(a),x+\cos(a))-(\cos(3a)+i\sin(3a))\arctan(\sin(a),x-\cos(a))+\frac{1}{2}(i\cos(3a)-\sin(3a))\log\left(x^2+2x\cos(a)+\cos(a)^2+\sin(a)^2\right)+\frac{1}{2}(-i\cos(3a)+\sin(3a))\log\left(x^2-2x\cos(a)+\cos(a)^2+\sin(a)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(a+I*log(x)),x, algorithm="maxima")
```

```
[Out] -1/3*I*x^3 + 2*x*(-I*cos(2*a) + sin(2*a)) - (cos(3*a) + I*sin(3*a))*arctan2
(sin(a), x + cos(a)) - (cos(3*a) + I*sin(3*a))*arctan2(sin(a), x - cos(a))
+ 1/2*(I*cos(3*a) - sin(3*a))*log(x^2 + 2*x*cos(a) + cos(a)^2 + sin(a)^2) +
 1/2*(-I*cos(3*a) + sin(3*a))*log(x^2 - 2*x*cos(a) + cos(a)^2 + sin(a)^2)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(26) = 52.
time = 4.30, size = 78, normalized size = 1.81

$$-\frac{1}{3}i\,x^3-2i\,xe^{(2i\,a)}-\sqrt{-e^{(6i\,a)}}\,\log\left(\left(xe^{(2i\,a)}+i\,\sqrt{-e^{(6i\,a)}}\right)e^{(-2i\,a)}\right)+\sqrt{-e^{(6i\,a)}}\,\log\left(\left(xe^{(2i\,a)}-i\,\sqrt{-e^{(6i\,a)}}\right)e^{(-2i\,a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(a+I*log(x)),x, algorithm="fricas")
```

```
[Out] -1/3*I*x^3 - 2*I*x*e^(2*I*a) - sqrt(-e^(6*I*a))*log((x*e^(2*I*a) + I*sqrt(-
e^(6*I*a)))*e^(-2*I*a)) + sqrt(-e^(6*I*a))*log((x*e^(2*I*a) - I*sqrt(-e^(6*
I*a)))*e^(-2*I*a))
```

Sympy [A]
time = 0.11, size = 63, normalized size = 1.47

$$-\frac{ix^3}{3}-2ixe^{2ia}-\left(i\log\left(xe^{2ia}-e^{3ia}\right)-i\log\left(xe^{2ia}+e^{3ia}\right)\right)e^{3ia}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cot(a+I*ln(x)),x)
```

```
[Out] -I*x**3/3 - 2*I*x*exp(2*I*a) - (I*log(x*exp(2*I*a) - exp(3*I*a)) - I*log(x*
exp(2*I*a) + exp(3*I*a)))*exp(3*I*a)
```

Giac [A]
time = 0.43, size = 47, normalized size = 1.09

$$-\frac{1}{3}i\,x^3-2i\,xe^{(2i\,a)}+i\,e^{(3i\,a)}\log\left(i\,x+i\,e^{(i\,a)}\right)-i\,e^{(3i\,a)}\log\left(-i\,x+i\,e^{(i\,a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(a+I*log(x)),x, algorithm="giac")
```

```
[Out] -1/3*I*x^3 - 2*I*x*e^(2*I*a) + I*e^(3*I*a)*log(I*x + I*e^(I*a)) - I*e^(3*I*
a)*log(-I*x + I*e^(I*a))
```

Mupad [B]
time = 2.20, size = 40, normalized size = 0.93

$$-\mathrm{atan}\left(\frac{x}{\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}}}\right)\,{\left(-\mathrm{e}^{a\,2\mathrm{i}}\right)}^{3/2}\,2\mathrm{i}-\frac{x^3\,1\mathrm{i}}{3}-x\,\mathrm{e}^{a\,2\mathrm{i}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cot(a + log(x)*1i),x)
```

```
[Out] - atan(x/(-exp(a*2i))^(1/2))*(-exp(a*2i))^(3/2)*2i - (x^3*1i)/3 - x*exp(a*2
i)*2i
```

3.188 $\int x \cot(a+i\log(x))\,dx$

Optimal. Leaf size=35

$$-\frac{ix^2}{2}-ie^{2ia}\log\left(e^{2ia}-x^2\right)$$

```
[Out] -1/2*I*x^2-I*exp(2*I*a)*ln(exp(2*I*a)-x^2)
```

Rubi [A]

time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.364$, Rules used = {4592, 456, 455, 45}

$$-ie^{2ia}\log\left(-x^2+e^{2ia}\right)-\frac{ix^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cot[a + I*Log[x]],x]
```

```
[Out] (-1/2*I)*x^2 - I*E^((2*I)*a)*Log[E^((2*I)*a) - x^2]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 456

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int x\cot(a+i\log(x))\,dx=\int x\cot(a+i\log(x))\,dx$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 118 vs. $2(35)=70$.
time = 0.02, size = 118, normalized size = 3.37

$$-\frac{ix^2}{2}-\text{ArcTan}\left(\frac{(-1+x^2)\cos(a)}{-\sin(a)-x^2\sin(a)}\right)\cos(2a)-\frac{1}{2}i\cos(2a)\log\left(1+x^4-2x^2\cos(2a)\right)-i\text{ArcTan}\left(\frac{(-1+x^2)\cos(a)}{-\sin(a)-x^2\sin(a)}\right)\sin(2a)+\frac{1}{2}\log\left(1+x^4-2x^2\cos(2a)\right)\sin(2a)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cot[a + I*Log[x]],x]
```

```
[Out] (-1/2*I)*x^2 - ArcTan[((-1 + x^2)*Cos[a])/(-Sin[a] - x^2*Sin[a])]*Cos[2*a]
- (I/2)*Cos[2*a]*Log[1 + x^4 - 2*x^2*Cos[2*a]] - I*ArcTan[((-1 + x^2)*Cos[a
])/(-Sin[a] - x^2*Sin[a])]*Sin[2*a] + (Log[1 + x^4 - 2*x^2*Cos[2*a]]*Sin[2*
a])/2
```

Maple [A]
time = 0.05, size = 28, normalized size = 0.80

method	result	size
risch	$-\frac{ix^2}{2}-ie^{2ia}\ln\left(e^{2ia}-x^2\right)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(a+I*ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*x^2-I*exp(2*I*a)*ln(exp(2*I*a)-x^2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(23)=46$.
time = 0.27, size = 109, normalized size = 3.11

$$-\frac{1}{2}ix^2+(\cos(2a)+i\sin(2a))\arctan(\sin(a),x+\cos(a))-(\cos(2a)+i\sin(2a))\arctan(\sin(a),x-\cos(a))+\frac{1}{2}(-i\cos(2a)+\sin(2a))\log\left(x^2+2x\cos(a)+\cos(a)^2+\sin(a)^2\right)+\frac{1}{2}(-i\cos(2a)+\sin(2a))\log\left(x^2-2x\cos(a)+\cos(a)^2+\sin(a)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(a+I*log(x)),x, algorithm="maxima")
```

```
[Out] -1/2*I*x^2 + (cos(2*a) + I*sin(2*a))*arctan2(sin(a), x + cos(a)) - (cos(2*a
) + I*sin(2*a))*arctan2(sin(a), x - cos(a)) + 1/2*(-I*cos(2*a) + sin(2*a))*
log(x^2 + 2*x*cos(a) + cos(a)^2 + sin(a)^2) + 1/2*(-I*cos(2*a) + sin(2*a))*
log(x^2 - 2*x*cos(a) + cos(a)^2 + sin(a)^2)
```

Fricas [A]
time = 2.77, size = 23, normalized size = 0.66

$$-\frac{1}{2} i x^2 - i e^{(2 i a)} \log\left(x^2 - e^{(2 i a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(a+I*log(x)),x, algorithm="fricas")
```

```
[Out] -1/2*I*x^2 - I*e^(2*I*a)*log(x^2 - e^(2*I*a))
```

Sympy [A]
time = 0.11, size = 27, normalized size = 0.77

$$-\frac{ix^2}{2} - ie^{2ia} \log\left(x^2 - e^{2ia}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(a+I*ln(x)),x)
```

```
[Out] -I*x**2/2 - I*exp(2*I*a)*log(x**2 - exp(2*I*a))
```

Giac [A]
time = 0.42, size = 41, normalized size = 1.17

$$-\frac{1}{2} i x^2 + \frac{1}{2} \pi e^{(2 i a)} - i e^{(2 i a)} \log\left(x + e^{(i a)}\right) - i e^{(2 i a)} \log\left(-x + e^{(i a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(a+I*log(x)),x, algorithm="giac")
```

```
[Out] -1/2*I*x^2 + 1/2*pi*e^(2*I*a) - I*e^(2*I*a)*log(x + e^(I*a)) - I*e^(2*I*a)*
log(-x + e^(I*a))
```

Mupad [B]
time = 2.20, size = 27, normalized size = 0.77

$$-\ln\left(x^2 - \mathrm{e}^{a\,2\mathrm{i}}\right)\,\mathrm{e}^{a\,2\mathrm{i}}\,1\mathrm{i} - \frac{x^2\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(a + log(x)*1i),x)
```

```
[Out] - log(x^2 - exp(a*2i))*exp(a*2i)*1i - (x^2*1i)/2
```

3.189 $\int \cot(a+i\log(x))\,dx$

Optimal. Leaf size=27

$$-ix+2ie^{ia}\tanh^{-1}\left(e^{-ia}x\right)$$

```
[Out] -I*x+2*I*exp(I*a)*arctanh(x/exp(I*a))
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {4588, 381, 396, 213}

$$2ie^{ia}\tanh^{-1}\left(e^{-ia}x\right)-ix$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + I*Log[x]],x]
```

```
[Out] (-I)*x + (2*I)*E^(I*a)*ArcTanh[x/E^(I*a)]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 381

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4588

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[((-I - I*E^(
2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*d)))^p, x] /; FreeQ[{a, b,
d, p}, x]
```

Rubi steps

$$\int \cot(a+i\log(x))\,dx = \int \cot(a+i\log(x))\,dx$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 1.56

$$-ix+2i\tanh^{-1}(x\cos(a)-ix\sin(a))\cos(a)-2\tanh^{-1}(x\cos(a)-ix\sin(a))\sin(a)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + I*Log[x]],x]
```

```
[Out] (-I)*x + (2*I)*ArcTanh[x*Cos[a] - I*x*Sin[a]]*Cos[a] - 2*ArcTanh[x*Cos[a] -
 I*x*Sin[a]]*Sin[a]
```

Maple [A]
time = 0.05, size = 22, normalized size = 0.81

method	result	size
risch	$-ix+2i\,\mathrm{arctanh}\left(x\,\mathrm{e}^{-ia}\right)\mathrm{e}^{ia}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a+I*ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*x+2*I*arctanh(x*exp(-I*a))*exp(I*a)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(17) = 34.
time = 0.28, size = 94, normalized size = 3.48

$$-(\cos(a)+i\sin(a))\arctan(\sin(a),x+\cos(a))-(\cos(a)+i\sin(a))\arctan(\sin(a),x-\cos(a))-\frac{1}{2}(-i\cos(a)+\sin(a))\log\left(x^2+2x\cos(a)+\cos(a)^2+\sin(a)^2\right)-\frac{1}{2}(i\cos(a)-\sin(a))\log\left(x^2-2x\cos(a)+\cos(a)^2+\sin(a)^2\right)-ix$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x)),x, algorithm="maxima")
```

```
[Out] -(cos(a) + I*sin(a))*arctan2(sin(a), x + cos(a)) - (cos(a) + I*sin(a))*arct
an2(sin(a), x - cos(a)) - 1/2*(-I*cos(a) + sin(a))*log(x^2 + 2*x*cos(a) + c
os(a)^2 + sin(a)^2) - 1/2*(I*cos(a) - sin(a))*log(x^2 - 2*x*cos(a) + cos(a)
^2 + sin(a)^2) - I*x
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(17) = 34.
time = 3.21, size = 49, normalized size = 1.81

$$-\sqrt{-e^{(2ia)}}\log\left(x+i\sqrt{-e^{(2ia)}}\right)+\sqrt{-e^{(2ia)}}\log\left(x-i\sqrt{-e^{(2ia)}}\right)-ix$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x)),x, algorithm="fricas")
```

```
[Out] -sqrt(-e^(2*I*a))*log(x + I*sqrt(-e^(2*I*a))) + sqrt(-e^(2*I*a))*log(x - I*
sqrt(-e^(2*I*a))) - I*x
```

Sympy [A]
time = 0.10, size = 29, normalized size = 1.07

$$-ix - \left(i\log\left(x - e^{ia}\right) - i\log\left(x + e^{ia}\right)\right)e^{ia}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*ln(x)),x)
```

```
[Out] -I*x - (I*log(x - exp(I*a)) - I*log(x + exp(I*a)))*exp(I*a)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(17) = 34$.
time = 0.44, size = 38, normalized size = 1.41

$$i\,e^{(i\,a)}\log\left(i\,x + i\,e^{(i\,a)}\right) - i\,e^{(i\,a)}\log\left(-i\,x + i\,e^{(i\,a)}\right) - i\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x)),x, algorithm="giac")
```

```
[Out] I*e^(I*a)*log(I*x + I*e^(I*a)) - I*e^(I*a)*log(-I*x + I*e^(I*a)) - I*x
```

Mupad [B]
time = 2.18, size = 29, normalized size = 1.07

$$-x\,1\mathrm{i} + \mathrm{atan}\left(\frac{x}{\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}}}\right)\,\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + log(x)*1i),x)
```

```
[Out] atan(x/(-exp(a*2i))^(1/2))*(-exp(a*2i))^(1/2)*2i - x*1i
```

3.190 $\int \frac{\cot(a+i\log(x))}{x}\,dx$

Optimal. Leaf size=14

$$-i\log(\sin(a+i\log(x)))$$

```
[Out] -I*ln(sin(a+I*ln(x)))
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{3556\}$

$$-i\log(\sin(a+i\log(x)))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + I*Log[x]]/x,x]
[Out] (-I)*Log[Sin[a + I*Log[x]]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(a+i\log(x))}{x}\,dx &= \text{Subst}\left(\int \cot(a+ix)\,dx, x, \log(x)\right)\\ &= -i\log(\sin(a+i\log(x)))\end{aligned}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 1.79

$$-i(\log(\cos(a+i\log(x)))+\log(\tan(a+i\log(x))))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + I*Log[x]]/x,x]
[Out] (-I)*(Log[Cos[a + I*Log[x]]] + Log[Tan[a + I*Log[x]]])
```

Maple [A]
time = 0.04, size = 17, normalized size = 1.21

method	result	size
derivativedivides	$\frac{i\ln\left(\cot^2(a+i\ln(x))+1\right)}{2}$	17
default	$\frac{i\ln\left(\cot^2(a+i\ln(x))+1\right)}{2}$	17
risch	$-i\ln(x)-2a-i\ln\left(\frac{e^{2ia}}{x^2}-1\right)$	25
norman	$-i\ln(\tan(a+i\ln(x)))+\frac{i\ln\left(1+\tan^2(a+i\ln(x))\right)}{2}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a+I*ln(x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*ln(cot(a+I*ln(x))^2+1)
```

Maxima [A]
time = 0.27, size = 10, normalized size = 0.71

$$-i\log(\sin(a+i\log(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x))/x,x, algorithm="maxima")
```

```
[Out] -I*log(sin(a + I*log(x)))
```

Fricas [A]
time = 2.73, size = 18, normalized size = 1.29

$$-i\log\left(x^2-e^{(2i\,a)}\right)+i\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x))/x,x, algorithm="fricas")
```

```
[Out] -I*log(x^2 - e^(2*I*a)) + I*log(x)
```

Sympy [A]
time = 0.15, size = 17, normalized size = 1.21

$$i\log(x)-i\log\left(x^2-e^{2ia}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*ln(x))/x,x)
```

```
[Out] I*log(x) - I*log(x**2 - exp(2*I*a))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(10) = 20$.
time = 0.43, size = 75, normalized size = 5.36

$$-i \log\left(\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\left(\frac{\left(|x|^2+1\right)^2}{|x|^2}-\frac{\left(|x|^2-1\right)^2}{|x|^2}\right)\cos\left(\pi\mathrm{sgn}\left(x\right)+2\,a\right)+\frac{\left(|x|^2+1\right)^2}{|x|^2}+\frac{\left(|x|^2-1\right)^2}{|x|^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x))/x,x, algorithm="giac")
```

```
[Out] -I*log(1/2*sqrt(1/2)*sqrt(((abs(x)^2 + 1)^2/abs(x)^2 - (abs(x)^2 - 1)^2/abs
(x)^2)*cos(pi*sgn(x) + 2*a) + (abs(x)^2 + 1)^2/abs(x)^2 + (abs(x)^2 - 1)^2/
abs(x)^2))
```

Mupad [B]
time = 2.25, size = 21, normalized size = 1.50

$$-\ln\left(x^2-\mathrm{e}^{a\,2\mathrm{i}}\right)\,1\mathrm{i}+\ln\left(x\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + log(x)*1i)/x,x)
```

```
[Out] log(x)*1i - log(x^2 - exp(a*2i))*1i
```

3.191 $\int \frac{\cot(a+i\log(x))}{x^2}\,dx$

Optimal. Leaf size=29

$$-\frac{i}{x}+2ie^{-ia}\tanh^{-1}\left(e^{-ia}x\right)$$

```
[Out] -I/x+2*I*arctanh(x/exp(I*a))/exp(I*a)
```

Rubi [A]

time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {4592, 456, 464, 213}

$$2ie^{-ia}\tanh^{-1}\left(e^{-ia}x\right)-\frac{i}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + I*Log[x]]/x^2,x]
```

```
[Out] (-I)/x + ((2*I)*ArcTanh[x/E^(I*a)])/E^(I*a)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 456

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
```

```
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int \frac{\cot(a+i\log(x))}{x^2}\,dx = \int \frac{\cot(a+i\log(x))}{x^2}\,dx$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 1.52

$$-\frac{i}{x} + 2i\tanh^{-1}(x\cos(a) - ix\sin(a))\cos(a) + 2\tanh^{-1}(x\cos(a) - ix\sin(a))\sin(a)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + I*Log[x]]/x^2,x]
[Out] (-I)/x + (2*I)*ArcTanh[x*Cos[a] - I*x*Sin[a]]*Cos[a] + 2*ArcTanh[x*Cos[a] -
 I*x*Sin[a]]*Sin[a]
```

Maple [A]
time = 0.05, size = 24, normalized size = 0.83

method	result	size
risch	$-\frac{i}{x}+2i\,\mathrm{arctanh}\left(x\,\mathrm{e}^{-ia}\right)\mathrm{e}^{-ia}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a+I*ln(x))/x^2,x,method=_RETURNVERBOSE)
[Out] -I/x+2*I*arctanh(x*exp(-I*a))*exp(-I*a)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(19) = 38$.
time = 0.28, size = 99, normalized size = 3.41

$$\frac{x(i\,\cos(a)+\sin(a))\log\left(x^2+2\,x\cos(a)+\cos(a)^2+\sin(a)^2\right)+x(-i\,\cos(a)-\sin(a))\log\left(x^2-2\,x\cos(a)+\cos(a)^2+\sin(a)^2\right)-2\left((\cos(a)-i\,\sin(a))\arctan\left(\sin(a),x+\cos(a)\right)+(\cos(a)-i\,\sin(a))\arctan\left(\sin(a),x-\cos(a)\right)\right)x-2i}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x))/x^2,x, algorithm="maxima")
[Out] 1/2*(x*(I*cos(a) + sin(a))*log(x^2 + 2*x*cos(a) + cos(a)^2 + sin(a)^2) + x*
(-I*cos(a) - sin(a))*log(x^2 - 2*x*cos(a) + cos(a)^2 + sin(a)^2) - 2*((cos(
a) - I*sin(a))*arctan2(sin(a), x + cos(a)) + (cos(a) - I*sin(a))*arctan2(si
n(a), x - cos(a)))*x - 2*I)/x
```

Fricas [A]
time = 2.69, size = 36, normalized size = 1.24

$$\frac{i\,xe^{(-i\,a)}\log\left(x+e^{(i\,a)}\right)-i\,xe^{(-i\,a)}\log\left(x-e^{(i\,a)}\right)-i}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x))/x^2,x, algorithm="fricas")
```

```
[Out] (I*x*e^(-I*a)*log(x + e^(I*a)) - I*x*e^(-I*a)*log(x - e^(I*a)) - I)/x
```

Sympy [A]
time = 0.13, size = 29, normalized size = 1.00

$$-\left(i\log\left(x-e^{ia}\right)-i\log\left(x+e^{ia}\right)\right)e^{-ia}-\frac{i}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*ln(x))/x**2,x)
```

```
[Out] -(I*log(x - exp(I*a)) - I*log(x + exp(I*a)))*exp(-I*a) - I/x
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(19) = 38.
time = 0.42, size = 40, normalized size = 1.38

$$i\,e^{(-i\,a)}\log\left(i\,x+i\,e^{(i\,a)}\right)-i\,e^{(-i\,a)}\log\left(-i\,x+i\,e^{(i\,a)}\right)-\frac{i}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x))/x^2,x, algorithm="giac")
```

```
[Out] I*e^(-I*a)*log(I*x + I*e^(I*a)) - I*e^(-I*a)*log(-I*x + I*e^(I*a)) - I/x
```

Mupad [B]
time = 2.21, size = 31, normalized size = 1.07

$$-\frac{\mathrm{atan}\left(\frac{x}{\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}}}\right)\,2\mathrm{i}}{\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}}}-\frac{1\mathrm{i}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + log(x)*1i)/x^2,x)
```

```
[Out] - (atan(x/(-exp(a*2i))^(1/2))*2i)/(-exp(a*2i))^(1/2) - 1i/x
```

3.192 $\int \frac{\cot(a+i\log(x))}{x^3}\,dx$

Optimal. Leaf size=36

$$-\frac{i}{2x^2} - ie^{-2ia}\log\left(1-\frac{e^{2ia}}{x^2}\right)$$

```
[Out] -1/2*I/x^2-I*ln(1-exp(2*I*a)/x^2)/exp(2*I*a)
```

Rubi [A]
time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4592, 455, 45}

$$-ie^{-2ia}\log\left(1-\frac{e^{2ia}}{x^2}\right)-\frac{i}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + I*Log[x]]/x^3,x]
[Out] (-1/2*I)/x^2 - (I*Log[1 - E^((2*I)*a)/x^2])/E^((2*I)*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int \frac{\cot(a+i\log(x))}{x^3}\,dx = \int \frac{\cot(a+i\log(x))}{x^3}\,dx$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 136 vs. $2(36) = 72$.
time = 0.02, size = 136, normalized size = 3.78

$$-\frac{i}{2x^2} - \text{ArcTan}\left(\frac{(-1+x^2)\cos(a)}{-\sin(a) - x^2\sin(a)}\right)\cos(2a) + 2i\cos(2a)\log(x) - \frac{1}{2}i\cos(2a)\log\left(1+x^4-2x^2\cos(2a)\right) + i\text{ArcTan}\left(\frac{(-1+x^2)\cos(a)}{-\sin(a) - x^2\sin(a)}\right)\sin(2a) + 2\log(x)\sin(2a) - \frac{1}{2}\log\left(1+x^4-2x^2\cos(2a)\right)\sin(2a)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + I*Log[x]]/x^3,x]
```

```
[Out] (-1/2*I)/x^2 - ArcTan[((-1 + x^2)*Cos[a])/(-Sin[a] - x^2*Sin[a])]*Cos[2*a]
+ (2*I)*Cos[2*a]*Log[x] - (I/2)*Cos[2*a]*Log[1 + x^4 - 2*x^2*Cos[2*a]] + I*
ArcTan[((-1 + x^2)*Cos[a])/(-Sin[a] - x^2*Sin[a])]*Sin[2*a] + 2*Log[x]*Sin[
2*a] - (Log[1 + x^4 - 2*x^2*Cos[2*a]]*Sin[2*a])/2
```

Maple [**A**]
time = 0.06, size = 38, normalized size = 1.06

method	result	size
risch	$-\frac{i}{2x^2} - i\mathrm{e}^{-2ia}\ln\left(\mathrm{e}^{2ia} - x^2\right) + 2i\mathrm{e}^{-2ia}\ln\left(x\right)$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a+I*ln(x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/x^2-I*exp(-2*I*a)*ln(exp(2*I*a)-x^2)+2*I*exp(-2*I*a)*ln(x)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(24) = 48$.
time = 0.28, size = 135, normalized size = 3.75

$$-\frac{x^2(i\cos(2a)+\sin(2a))\log\left(x^2+2x\cos(a)+\cos(a)^2+\sin(a)^2\right) + x^2(i\cos(2a)+\sin(2a))\log\left(x^2-2x\cos(a)+\cos(a)^2+\sin(a)^2\right) - 2\left((\cos(2a)-i\sin(2a))\arctan(\sin(a), x+\cos(a)) - (\cos(2a)-i\sin(2a))\arctan(\sin(a), x-\cos(a)) + 2(i\cos(2a)+\sin(2a))\log(x)\right)x^2 + i}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(x^2*(I*cos(2*a) + sin(2*a))*log(x^2 + 2*x*cos(a) + cos(a)^2 + sin(a)^
2) + x^2*(I*cos(2*a) + sin(2*a))*log(x^2 - 2*x*cos(a) + cos(a)^2 + sin(a)^2
) - 2*((cos(2*a) - I*sin(2*a))*arctan2(sin(a), x + cos(a)) - (cos(2*a) - I*
sin(2*a))*arctan2(sin(a), x - cos(a)) + 2*(I*cos(2*a) + sin(2*a))*log(x))*x
^2 + I)/x^2
```

Fricas [**A**]
time = 2.37, size = 39, normalized size = 1.08

$$\frac{\left(-2i\,x^2\log\left(x^2 - e^{(2i\,a)}\right) + 4i\,x^2\log\left(x\right) - i\,e^{(2i\,a)}\right)e^{(-2i\,a)}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x))/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(-2*I*x^2*log(x^2 - e^(2*I*a)) + 4*I*x^2*log(x) - I*e^(2*I*a))*e^(-2*I*
a)/x^2
```

Sympy [A]
time = 0.21, size = 39, normalized size = 1.08

$$2ie^{-2ia}\log\left(x\right)-ie^{-2ia}\log\left(x^2-e^{2ia}\right)-\frac{i}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*ln(x))/x**3,x)
```

```
[Out] 2*I*exp(-2*I*a)*log(x) - I*exp(-2*I*a)*log(x**2 - exp(2*I*a)) - I/(2*x**2)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(24) = 48.
time = 0.43, size = 49, normalized size = 1.36

$$\frac{1}{2}\pi e^{(-2i\,a)}-i\,e^{(-2i\,a)}\log\left(x+e^{(i\,a)}\right)+2i\,e^{(-2i\,a)}\log\left(x\right)-i\,e^{(-2i\,a)}\log\left(-x+e^{(i\,a)}\right)-\frac{i}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x))/x^3,x, algorithm="giac")
```

```
[Out] 1/2*pi*e^(-2*I*a) - I*e^(-2*I*a)*log(x + e^(I*a)) + 2*I*e^(-2*I*a)*log(x) -
 I*e^(-2*I*a)*log(-x + e^(I*a)) - 1/2*I/x^2
```

Mupad [B]
time = 2.23, size = 37, normalized size = 1.03

$$\mathrm{e}^{-a\,2\mathrm{i}}\,\ln\left(x\right)\,2\mathrm{i}-\ln\left(x^2-\mathrm{e}^{a\,2\mathrm{i}}\right)\,\mathrm{e}^{-a\,2\mathrm{i}}\,1\mathrm{i}-\frac{1\mathrm{i}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + log(x)*1i)/x^3,x)
```

```
[Out] exp(-a*2i)*log(x)*2i - log(x^2 - exp(a*2i))*exp(-a*2i)*1i - 1i/(2*x^2)
```

3.193 $\int \frac{\cot(a+i\log(x))}{x^4}\,dx$

Optimal. Leaf size=45

$$-\frac{i}{3x^3}-\frac{2ie^{-2ia}}{x}+2ie^{-3ia}\tanh^{-1}\left(e^{-ia}x\right)$$

```
[Out] -1/3*I/x^3-2*I/exp(2*I*a)/x+2*I*arctanh(x/exp(I*a))/exp(3*I*a)
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {4592, 456, 464, 331, 213}

$$-\frac{2ie^{-2ia}}{x}+2ie^{-3ia}\tanh^{-1}\left(e^{-ia}x\right)-\frac{i}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + I*Log[x]]/x^4,x]
```

```
[Out] (-1/3*I)/x^3 - (2*I)/(E^((2*I)*a)*x) + ((2*I)*ArcTanh[x/E^(I*a)])/E^((3*I)*
a)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 456

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int \frac{\cot(a+i\log(x))}{x^4}\,dx = \int \frac{\cot(a+i\log(x))}{x^4}\,dx$$

Mathematica [A]
time = 0.02, size = 70, normalized size = 1.56

$$-\frac{i}{3x^3}-\frac{2i\cos(2a)}{x}+2i\tanh^{-1}(x\cos(a)-ix\sin(a))\cos(3a)-\frac{2\sin(2a)}{x}+2\tanh^{-1}(x\cos(a)-ix\sin(a))\sin(3a)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + I*Log[x]]/x^4,x]
```

```
[Out] (-1/3*I)/x^3 - ((2*I)*Cos[2*a])/x + (2*I)*ArcTanh[x*Cos[a] - I*x*Sin[a]]*Co
s[3*a] - (2*Sin[2*a])/x + 2*ArcTanh[x*Cos[a] - I*x*Sin[a]]*Sin[3*a]
```

Maple [A]
time = 0.06, size = 35, normalized size = 0.78

method	result	size
risch	$-\frac{i}{3x^3}-\frac{2i\,\mathrm{e}^{-2ia}}{x}+2i\,\mathrm{arctanh}\left(x\,\mathrm{e}^{-ia}\right)\mathrm{e}^{-3ia}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a+I*ln(x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I/x^3-2*I*exp(-2*I*a)/x+2*I*arctanh(x*exp(-I*a))*exp(-3*I*a)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(28) = 56.
time = 0.28, size = 139, normalized size = 3.09

$$-\frac{3x^3(-i\cos(3a)-\sin(3a))\log\left(x^2+2x\cos(a)+\cos(a)^2+\sin(a)^2\right)+3x^3(i\cos(3a)+\sin(3a))\log\left(x^2-2x\cos(a)+\cos(a)^2+\sin(a)^2\right)+6\left((\cos(3a)-i\sin(3a))\arctan(\sin(a),x+\cos(a))+(\cos(3a)-i\sin(3a))\arctan(\sin(a),x-\cos(a))\right)x^3+12x^2(i\cos(2a)+\sin(2a))+2i}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*x^3*(-I*cos(3*a) - sin(3*a))*log(x^2 + 2*x*cos(a) + cos(a)^2 + sin(
a)^2) + 3*x^3*(I*cos(3*a) + sin(3*a))*log(x^2 - 2*x*cos(a) + cos(a)^2 + sin
(a)^2) + 6*((cos(3*a) - I*sin(3*a))*arctan2(sin(a), x + cos(a)) + (cos(3*a)
 - I*sin(3*a))*arctan2(sin(a), x - cos(a)))*x^3 + 12*x^2*(I*cos(2*a) + sin(
2*a)) + 2*I)/x^3
```

Fricas [A]
time = 2.25, size = 55, normalized size = 1.22

$$\frac{\left(3i\,x^{3}e^{(-i\,a)}\log\left(x+e^{(i\,a)}\right)-3i\,x^{3}e^{(-i\,a)}\log\left(x-e^{(i\,a)}\right)-6i\,x^{2}-i\,e^{(2i\,a)}\right)e^{(-2i\,a)}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*I*x^3*e^(-I*a)*log(x + e^(I*a)) - 3*I*x^3*e^(-I*a)*log(x - e^(I*a))
- 6*I*x^2 - I*e^(2*I*a))*e^(-2*I*a)/x^3
```

Sympy [A]
time = 0.17, size = 54, normalized size = 1.20

$$-\left(i\log\left(x-e^{ia}\right)-i\log\left(x+e^{ia}\right)\right)e^{-3ia}-\frac{\left(6ix^{2}+ie^{2ia}\right)e^{-2ia}}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*ln(x))/x**4,x)
```

```
[Out] -(I*log(x - exp(I*a)) - I*log(x + exp(I*a)))*exp(-3*I*a) - (6*I*x**2 + I*ex
p(2*I*a))*exp(-2*I*a)/(3*x**3)
```

Giac [A]
time = 0.45, size = 49, normalized size = 1.09

$$i\,e^{(-3i\,a)}\log\left(i\,x+i\,e^{(i\,a)}\right)-i\,e^{(-3i\,a)}\log\left(-i\,x+i\,e^{(i\,a)}\right)-\frac{2i\,e^{(-2i\,a)}}{x}-\frac{i}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x))/x^4,x, algorithm="giac")
```

```
[Out] I*e^(-3*I*a)*log(I*x + I*e^(I*a)) - I*e^(-3*I*a)*log(-I*x + I*e^(I*a)) - 2*
I*e^(-2*I*a)/x - 1/3*I/x^3
```

Mupad [B]
time = 2.21, size = 44, normalized size = 0.98

$$\frac{\operatorname{atan}\left(\frac{x}{\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}}}\right)\,2\mathrm{i}}{\left(-\mathrm{e}^{a\,2\mathrm{i}}\right)^{3/2}}-\frac{2\mathrm{i}\,\mathrm{e}^{-a\,2\mathrm{i}}\,x^2+\frac{1}{3}\mathrm{i}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + log(x)*1i)/x^4,x)
```

```
[Out] (atan(x/(-exp(a*2i))^(1/2))*2i)/(-exp(a*2i))^(3/2) - (x^2*exp(-a*2i)*2i + 1
i/3)/x^3
```

3.194 $\int x^3 \cot^2(a+i\log(x))\,dx$

Optimal. Leaf size=67

$$-2e^{2ia}x^2-\frac{x^4}{4}-\frac{2e^{6ia}}{e^{2ia}-x^2}-4e^{4ia}\log\left(e^{2ia}-x^2\right)$$

```
[Out] -2*exp(2*I*a)*x^2-1/4*x^4-2*exp(6*I*a)/(exp(2*I*a)-x^2)-4*exp(4*I*a)*ln(exp
(2*I*a)-x^2)
```

Rubi [A]
time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {4592, 456, 457, 78}

$$-2e^{2ia}x^2-\frac{2e^{6ia}}{-x^2+e^{2ia}}-4e^{4ia}\log\left(-x^2+e^{2ia}\right)-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cot[a + I*Log[x]]^2,x]
```

```
[Out] -2*E^((2*I)*a)*x^2 - x^4/4 - (2*E^((6*I)*a))/(E^((2*I)*a) - x^2) - 4*E^((4*
I)*a)*Log[E^((2*I)*a) - x^2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 456

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int x^3 \cot^2(a + i\log(x))\,dx = \int x^3 \cot^2(a + i\log(x))\,dx$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 162 vs. 2(67) = 134.
time = 0.14, size = 162, normalized size = 2.42

$$-\frac{x^4}{4} - 2x^2\cos(2a) + 4i\text{ArcTan}\left(\frac{\cot(a) - x^2\cot(a)}{1 + x^2}\right)\cos(4a) - 2\cos(4a)\log\left(1 + x^4 - 2x^2\cos(2a)\right) - 2ix^2\sin(2a) - 4\text{ArcTan}\left(\frac{\cot(a) - x^2\cot(a)}{1 + x^2}\right)\sin(4a) - 2i\log\left(1 + x^4 - 2x^2\cos(2a)\right)\sin(4a) + \frac{2\cos(5a) + 2i\sin(5a)}{(-1 + x^2)\cos(a) - i(1 + x^2)\sin(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cot[a + I*Log[x]]^2,x]
```

```
[Out] -1/4*x^4 - 2*x^2*Cos[2*a] + (4*I)*ArcTan[(Cot[a] - x^2*Cot[a])/(1 + x^2)]*C
os[4*a] - 2*Cos[4*a]*Log[1 + x^4 - 2*x^2*Cos[2*a]] - (2*I)*x^2*Sin[2*a] - 4
*ArcTan[(Cot[a] - x^2*Cot[a])/(1 + x^2)]*Sin[4*a] - (2*I)*Log[1 + x^4 - 2*x
^2*Cos[2*a]]*Sin[4*a] + (2*Cos[5*a] + (2*I)*Sin[5*a])/((-1 + x^2)*Cos[a] -
I*(1 + x^2)*Sin[a])
```

Maple [**A**]
time = 0.06, size = 54, normalized size = 0.81

method	result	size
risch	$-\frac{9x^4}{4} - \frac{2x^4}{\frac{e^{2ia}}{x^2} - 1} - 4\,e^{2ia}x^2 - 4\,e^{4ia}\ln\left(e^{2ia} - x^2\right)$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cot(a+I*ln(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -9/4*x^4-2*x^4/(exp(2*I*a)/x^2-1)-4*exp(2*I*a)*x^2-4*exp(4*I*a)*ln(exp(2*I*
a)-x^2)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(50) = 100.
time = 0.29, size = 345, normalized size = 5.15

$$-\frac{x^6 + 7x^4(\cos(2a) + i\sin(2a)) - 8\left(2(-i\cos(4a) + \sin(4a))\arctan(\sin(a), x + \cos(a)) + 2(i\cos(4a) - \sin(4a))\arctan(\sin(a), x - \cos(a)) + \cos(4a) + i\sin(4a)\right)x^2 - 16\left((i\cos(2a) - \sin(2a))\cos(4a) - (\cos(2a) + i\sin(2a))\sin(4a)\right)\arctan(\sin(a), x + \cos(a)) - 16\left((-i\cos(2a) + \sin(2a))\cos(4a) + (\cos(2a) + i\sin(2a))\sin(4a)\right)\arctan(\sin(a), x - \cos(a)) + 8\left(x^2(\cos(4a) + i\sin(4a)) - (\cos(2a) + i\sin(2a))\cos(4a) - (i\cos(2a) - \sin(2a))\sin(4a)\right)\log\left(x^2 + 2x\cos(a) + \cos(a)^2 + \sin(a)^2\right) + 8\left(x^2(\cos(4a) + i\sin(4a)) - (\cos(2a) + i\sin(2a))\cos(4a) - (i\cos(2a) - \sin(2a))\sin(4a)\right)\log\left(x^2 - 2x\cos(a) + \cos(a)^2 + \sin(a)^2\right) - 8\cos(6a) - 8i\sin(6a)}{4\left(x^2 - \cos(2a) - i\sin(2a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(a+I*log(x))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(x^6 + 7*x^4*(cos(2*a) + I*sin(2*a)) - 8*(2*(-I*cos(4*a) + sin(4*a))*a
rctan2(sin(a), x + cos(a)) + 2*(I*cos(4*a) - sin(4*a))*arctan2(sin(a), x -
cos(a)) + cos(4*a) + I*sin(4*a))*x^2 - 16*((I*cos(2*a) - sin(2*a))*cos(4*a)
 - (cos(2*a) + I*sin(2*a))*sin(4*a))*arctan2(sin(a), x + cos(a)) - 16*((-I*
cos(2*a) + sin(2*a))*cos(4*a) + (cos(2*a) + I*sin(2*a))*sin(4*a))*arctan2(s
in(a), x - cos(a)) + 8*(x^2*(cos(4*a) + I*sin(4*a)) - (cos(2*a) + I*sin(2*a
))*cos(4*a) - (I*cos(2*a) - sin(2*a))*sin(4*a))*log(x^2 + 2*x*cos(a) + cos(
a)^2 + sin(a)^2) + 8*(x^2*(cos(4*a) + I*sin(4*a)) - (cos(2*a) + I*sin(2*a))
*cos(4*a) - (I*cos(2*a) - sin(2*a))*sin(4*a))*log(x^2 - 2*x*cos(a) + cos(a)
^2 + sin(a)^2) - 8*cos(6*a) - 8*I*sin(6*a))/(x^2 - cos(2*a) - I*sin(2*a))
```

Fricas [A]
time = 2.70, size = 70, normalized size = 1.04

$$-\frac{x^{6}+7\,x^{4}e^{(2i\,a)}-8\,x^{2}e^{(4i\,a)}+16\,\left(x^{2}e^{(4i\,a)}-e^{(6i\,a)}\right)\log\left(x^{2}-e^{(2i\,a)}\right)-8\,e^{(6i\,a)}}{4\,\left(x^{2}-e^{(2i\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(a+I*log(x))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(x^6 + 7*x^4*e^(2*I*a) - 8*x^2*e^(4*I*a) + 16*(x^2*e^(4*I*a) - e^(6*I*
a))*log(x^2 - e^(2*I*a)) - 8*e^(6*I*a))/(x^2 - e^(2*I*a))
```

Sympy [A]
time = 0.18, size = 54, normalized size = 0.81

$$-\frac{x^{4}}{4}-2x^{2}e^{2ia}-4e^{4ia}\log\left(x^{2}-e^{2ia}\right)+\frac{2e^{6ia}}{x^{2}-e^{2ia}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cot(a+I*ln(x))**2,x)
```

```
[Out] -x**4/4 - 2*x**2*exp(2*I*a) - 4*exp(4*I*a)*log(x**2 - exp(2*I*a)) + 2*exp(6
*I*a)/(x**2 - exp(2*I*a))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(50) = 100$.
time = 0.46, size = 139, normalized size = 2.07

$$-\frac{x^{6}}{4\,(x^{2}-e^{(2i\,a)})}-\frac{7\,x^{4}e^{(2i\,a)}}{4\,(x^{2}-e^{(2i\,a)})}-\frac{4\,x^{2}e^{(4i\,a)}\log\left(-x^{2}+e^{(2i\,a)}\right)}{x^{2}-e^{(2i\,a)}}+\frac{2\,x^{2}e^{(4i\,a)}}{x^{2}-e^{(2i\,a)}}+\frac{4\,e^{(6i\,a)}\log\left(-x^{2}+e^{(2i\,a)}\right)}{x^{2}-e^{(2i\,a)}}+\frac{2\,e^{(6i\,a)}}{x^{2}-e^{(2i\,a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(a+I*log(x))^2,x, algorithm="giac")
```

```
[Out] -1/4*x^6/(x^2 - e^(2*I*a)) - 7/4*x^4*e^(2*I*a)/(x^2 - e^(2*I*a)) - 4*x^2*e^
(4*I*a)*log(-x^2 + e^(2*I*a))/(x^2 - e^(2*I*a)) + 2*x^2*e^(4*I*a)/(x^2 - e^
(2*I*a)) + 4*e^(6*I*a)*log(-x^2 + e^(2*I*a))/(x^2 - e^(2*I*a)) + 2*e^(6*I*a
)/(x^2 - e^(2*I*a))
```

Mupad [B]
time = 2.23, size = 55, normalized size = 0.82

$$-2\,x^2\,{\mathrm{e}}^{a\,2{}\mathrm{i}}-\frac{2\,{\mathrm{e}}^{a\,6{}\mathrm{i}}}{{\mathrm{e}}^{a\,2{}\mathrm{i}}-x^2}-4\,\ln\left(x^2-{\mathrm{e}}^{a\,2{}\mathrm{i}}\right)\,{\mathrm{e}}^{a\,4{}\mathrm{i}}-\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cot(a + log(x)*1i)^2,x)
```

```
[Out] - 2*x^2*exp(a*2i) - (2*exp(a*6i))/(exp(a*2i) - x^2) - 4*log(x^2 - exp(a*2i)
)*exp(a*4i) - x^4/4
```

3.195 $\int x^2 \cot^2(a+i\log(x))\,dx$

Optimal. Leaf size=64

$$-6e^{2ia}x-\frac{x^3}{3}-\frac{2e^{2ia}x^3}{e^{2ia}-x^2}+6e^{3ia}\tanh^{-1}\left(e^{-ia}x\right)$$

```
[Out] -6*exp(2*I*a)*x-1/3*x^3-2*exp(2*I*a)*x^3/(exp(2*I*a)-x^2)+6*exp(3*I*a)*arct
anh(x/exp(I*a))
```

Rubi [A]
time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.400$, Rules used = {4592, 456, 474, 470, 327, 213}

$$-\frac{2e^{2ia}x^3}{-x^2+e^{2ia}}-6e^{2ia}x+6e^{3ia}\tanh^{-1}\left(e^{-ia}x\right)-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cot[a + I*Log[x]]^2,x]
```

```
[Out] -6*E^((2*I)*a)*x - x^3/3 - (2*E^((2*I)*a)*x^3)/(E^((2*I)*a) - x^2) + 6*E^((
3*I)*a)*ArcTanh[x/E^(I*a)]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 456

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int x^2 \cot^2(a+i\log(x))\,dx = \int x^2 \cot^2(a+i\log(x))\,dx$$

Mathematica [A]
time = 0.09, size = 100, normalized size = 1.56

$$-\frac{x^3}{3}-4x\cos(2a)+6\tanh^{-1}(x(\cos(a)-i\sin(a)))\cos(3a)-4ix\sin(2a)+\frac{2x(\cos(3a)+i\sin(3a))}{(-1+x^2)\cos(a)-i(1+x^2)\sin(a)}+6i\tanh^{-1}(x(\cos(a)-i\sin(a)))\sin(3a)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cot[a + I*Log[x]]^2,x]
```

```
[Out] -1/3*x^3 - 4*x*Cos[2*a] + 6*ArcTanh[x*(Cos[a] - I*Sin[a])]*Cos[3*a] - (4*I)
*x*Sin[2*a] + (2*x*(Cos[3*a] + I*Sin[3*a]))/((-1 + x^2)*Cos[a] - I*(1 + x^2
)*Sin[a]) + (6*I)*ArcTanh[x*(Cos[a] - I*Sin[a])]*Sin[3*a]
```

Maple [A]
time = 0.06, size = 48, normalized size = 0.75

method	result	size

risch	$-\frac{7x^3}{3}-\frac{2x^3}{\frac{e^{2ia}}{x^2}-1}-6\,e^{2ia}x+6\,\mathrm{arctanh}\left(x\,e^{-ia}\right)e^{3ia}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cot(a+I*ln(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -7/3*x^3-2*x^3/(exp(2*I*a)/x^2-1)-6*exp(2*I*a)*x+6*arctanh(x*exp(-I*a))*exp
(3*I*a)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(47) = 94$.
time = 0.30, size = 335, normalized size = 5.23

$$-\frac{2x^5+22x^3(\cos(2a)+i\sin(2a))+18((-i\cos(3a)+\sin(3a))\arctan(\sin(a),x+\cos(a))+(-i\cos(3a)+\sin(3a))\arctan(\sin(a),x-\cos(a)))x^2-36x(\cos(4a)+i\sin(4a))+18((i\cos(2a)-\sin(2a))\cos(3a)-(\cos(2a)+i\sin(2a))\sin(3a))\arctan(\sin(a),x+\cos(a))+18((i\cos(2a)-\sin(2a))\cos(3a)-(\cos(2a)+i\sin(2a))\sin(3a))\arctan(\sin(a),x-\cos(a))-9(x^2(\cos(3a)+i\sin(3a))-(\cos(2a)+i\sin(2a))\cos(3a)-(i\cos(2a)-\sin(2a))\sin(3a))\log(x^2+2x\cos(a)+\cos(a)^2+\sin(a)^2)+9(x^2(\cos(3a)+i\sin(3a))-(\cos(2a)+i\sin(2a))\cos(3a)+(-i\cos(2a)+\sin(2a))\sin(3a))\log(x^2-2x\cos(a)+\cos(a)^2+\sin(a)^2)}{6(x^2-\cos(2a)-i\sin(2a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(a+I*log(x))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*x^5 + 22*x^3*(cos(2*a) + I*sin(2*a)) + 18*((-I*cos(3*a) + sin(3*a))
*arctan2(sin(a), x + cos(a)) + (-I*cos(3*a) + sin(3*a))*arctan2(sin(a), x -
 cos(a)))*x^2 - 36*x*(cos(4*a) + I*sin(4*a)) + 18*((I*cos(2*a) - sin(2*a))*
cos(3*a) - (cos(2*a) + I*sin(2*a))*sin(3*a))*arctan2(sin(a), x + cos(a)) +
18*((I*cos(2*a) - sin(2*a))*cos(3*a) - (cos(2*a) + I*sin(2*a))*sin(3*a))*ar
ctan2(sin(a), x - cos(a)) - 9*(x^2*(cos(3*a) + I*sin(3*a)) - (cos(2*a) + I*
sin(2*a))*cos(3*a) - (I*cos(2*a) - sin(2*a))*sin(3*a))*log(x^2 + 2*x*cos(a)
 + cos(a)^2 + sin(a)^2) + 9*(x^2*(cos(3*a) + I*sin(3*a)) - (cos(2*a) + I*si
n(2*a))*cos(3*a) + (-I*cos(2*a) + sin(2*a))*sin(3*a))*log(x^2 - 2*x*cos(a)
+ cos(a)^2 + sin(a)^2))/(x^2 - cos(2*a) - I*sin(2*a))
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(47) = 94$.
time = 2.81, size = 102, normalized size = 1.59

$$-\frac{x^5+11\,x^3e^{(2ia)}-9\left(x^2-e^{(2ia)}\right)e^{(3ia)}\log\left(\left(xe^{(2ia)}+e^{(3ia)}\right)e^{(-2ia)}\right)+9\left(x^2-e^{(2ia)}\right)e^{(3ia)}\log\left(\left(xe^{(2ia)}-e^{(3ia)}\right)e^{(-2ia)}\right)-18\,xe^{(4ia)}}{3\left(x^2-e^{(2ia)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(a+I*log(x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(x^5 + 11*x^3*e^(2*I*a) - 9*(x^2 - e^(2*I*a))*e^(3*I*a)*log((x*e^(2*I*
a) + e^(3*I*a))*e^(-2*I*a)) + 9*(x^2 - e^(2*I*a))*e^(3*I*a)*log((x*e^(2*I*a
) - e^(3*I*a))*e^(-2*I*a)) - 18*x*e^(4*I*a))/(x^2 - e^(2*I*a))
```

Sympy [**A**]
time = 0.20, size = 60, normalized size = 0.94

$$-\frac{x^3}{3}-4xe^{2ia}+\frac{2xe^{4ia}}{x^2-e^{2ia}}-3\left(\log\left(x-e^{ia}\right)-\log\left(x+e^{ia}\right)\right)e^{3ia}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cot(a+I*ln(x))**2,x)
```

```
[Out] -x**3/3 - 4*x*exp(2*I*a) + 2*x*exp(4*I*a)/(x**2 - exp(2*I*a)) - 3*(log(x - exp(I*a)) - log(x + exp(I*a)))*exp(3*I*a)
```

Giac [A]
time = 0.48, size = 83, normalized size = 1.30

$$-\frac{x^5}{3\,(x^2-e^{(2i\,a)})}-\frac{11\,x^3e^{(2i\,a)}}{3\,(x^2-e^{(2i\,a)})}-\frac{6\,\arctan\left(\frac{x}{\sqrt{-e^{(2i\,a)}}}\right)e^{(4i\,a)}}{\sqrt{-e^{(2i\,a)}}}+\frac{10\,xe^{(4i\,a)}}{x^2-e^{(2i\,a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(a+I*log(x))^2,x, algorithm="giac")
```

```
[Out] -1/3*x^5/(x^2 - e^(2*I*a)) - 11/3*x^3*e^(2*I*a)/(x^2 - e^(2*I*a)) - 6*arctan(x/sqrt(-e^(2*I*a)))*e^(4*I*a)/sqrt(-e^(2*I*a)) + 10*x*e^(4*I*a)/(x^2 - e^(2*I*a))
```

Mupad [B]
time = 2.22, size = 57, normalized size = 0.89

$$-\left(\mathrm{e}^{a\,2\mathrm{i}}\right)^{3/2}\mathrm{atan}\left(\frac{x\,1\mathrm{i}}{\sqrt{\mathrm{e}^{a\,2\mathrm{i}}}}\right)\,6\mathrm{i}-\frac{x^3}{3}-4\,x\,\mathrm{e}^{a\,2\mathrm{i}}-\frac{2\,x\,\mathrm{e}^{a\,4\mathrm{i}}}{\mathrm{e}^{a\,2\mathrm{i}}-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cot(a + log(x)*1i)^2,x)
```

```
[Out] - exp(a*2i)^(3/2)*atan((x*1i)/exp(a*2i)^(1/2))*6i - x^3/3 - 4*x*exp(a*2i) - (2*x*exp(a*4i))/(exp(a*2i) - x^2)
```

3.196 $\int x \cot^2(a + i \log(x)) \, dx$

Optimal. Leaf size=55

$$-\frac{x^2}{2} - \frac{2e^{4ia}}{e^{2ia} - x^2} - 2e^{2ia} \log\left(e^{2ia} - x^2\right)$$

```
[Out] -1/2*x^2-2*exp(4*I*a)/(exp(2*I*a)-x^2)-2*exp(2*I*a)*ln(exp(2*I*a)-x^2)
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {4592, 456, 455, 45}

$$-\frac{2e^{4ia}}{-x^2 + e^{2ia}} - 2e^{2ia} \log\left(-x^2 + e^{2ia}\right) - \frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cot[a + I*Log[x]]^2,x]
```

```
[Out] -1/2*x^2 - (2*E^((4*I)*a))/(E^((2*I)*a) - x^2) - 2*E^((2*I)*a)*Log[E^((2*I)
*a) - x^2]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 456

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
```

```
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int x\cot^2(a+i\log(x))\,dx=\int x\cot^2(a+i\log(x))\,dx$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 142 vs. $2(55)=110$.
time $=0.09$, size $=142$, normalized size $=2.58$

$$-\frac{x^2}{2}+2i\text{ArcTan}\left(\frac{\cot(a)-x^2\cot(a)}{1+x^2}\right)\cos(2a)-\cos(2a)\log\left(1+x^4-2x^2\cos(2a)\right)-4\text{ArcTan}\left(\frac{\cot(a)-x^2\cot(a)}{1+x^2}\right)\cos(a)\sin(a)-i\log\left(1+x^4-2x^2\cos(2a)\right)\sin(2a)+\frac{2\cos(3a)+2i\sin(3a)}{(-1+x^2)\cos(a)-i(1+x^2)\sin(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cot[a + I*Log[x]]^2,x]
```

```
[Out] -1/2*x^2 + (2*I)*ArcTan[(Cot[a] - x^2*Cot[a])/(1 + x^2)]*Cos[2*a] - Cos[2*a
]*Log[1 + x^4 - 2*x^2*Cos[2*a]] - 4*ArcTan[(Cot[a] - x^2*Cot[a])/(1 + x^2)]
*Cos[a]*Sin[a] - I*Log[1 + x^4 - 2*x^2*Cos[2*a]]*Sin[2*a] + (2*Cos[3*a] + (
2*I)*Sin[3*a])/((-1 + x^2)*Cos[a] - I*(1 + x^2)*Sin[a])
```

Maple [**A**]
time $=0.05$, size $=44$, normalized size $=0.80$

method	result	size
risch	$-\frac{5x^2}{2}-\frac{2x^2}{\frac{e^{2ia}}{x^2}-1}-2\,e^{2ia}\ln\left(e^{2ia}-x^2\right)$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(a+I*ln(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -5/2*x^2-2*x^2/(exp(2*I*a)/x^2-1)-2*exp(2*I*a)*ln(exp(2*I*a)-x^2)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(41)=82$.
time $=0.30$, size $=290$, normalized size $=5.27$

$$-\frac{x^4-\left(4\left(-i\cos(2a)+\sin(2a)\right)\arctan\left(\sin(a),x+\cos(a)\right)+4\left(i\cos(2a)-\sin(2a)\right)\arctan\left(\sin(a),x-\cos(a)\right)+\cos(2a)+i\sin(2a)\right)x^2-4\left(i\cos(2a)^2-2\cos(2a)\sin(2a)-i\sin(2a)^2\right)\arctan\left(\sin(a),x+\cos(a)\right)-4\left(-i\cos(2a)^2+2\cos(2a)\sin(2a)+i\sin(2a)^2\right)\arctan\left(\sin(a),x-\cos(a)\right)+2\left(x^2(\cos(2a)+i\sin(2a))-\cos(2a)^2-2i\cos(2a)\sin(2a)+\sin(2a)^2\right)\log\left(x^2+2x\cos(a)+\cos(a)^2+\sin(a)^2\right)+2\left(x^2(\cos(2a)+i\sin(2a))-\cos(2a)^2-2i\cos(2a)\sin(2a)+\sin(2a)^2\right)\log\left(x^2-2x\cos(a)+\cos(a)^2+\sin(a)^2\right)-4\cos(4a)-4i\sin(4a)}{2\left(x^2-\cos(2a)-i\sin(2a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(a+I*log(x))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x^4 - (4*(-I*cos(2*a) + sin(2*a))*arctan2(sin(a), x + cos(a)) + 4*(I*
cos(2*a) - sin(2*a))*arctan2(sin(a), x - cos(a)) + cos(2*a) + I*sin(2*a))*x
^2 - 4*(I*cos(2*a)^2 - 2*cos(2*a)*sin(2*a) - I*sin(2*a)^2)*arctan2(sin(a),
x + cos(a)) - 4*(-I*cos(2*a)^2 + 2*cos(2*a)*sin(2*a) + I*sin(2*a)^2)*arctan
2(sin(a), x - cos(a)) + 2*(x^2*(cos(2*a) + I*sin(2*a)) - cos(2*a)^2 - 2*I*c
os(2*a)*sin(2*a) + sin(2*a)^2)*log(x^2 + 2*x*cos(a) + cos(a)^2 + sin(a)^2)
+ 2*(x^2*(cos(2*a) + I*sin(2*a)) - cos(2*a)^2 - 2*I*cos(2*a)*sin(2*a) + sin
(2*a)^2)*log(x^2 - 2*x*cos(a) + cos(a)^2 + sin(a)^2) - 4*cos(4*a) - 4*I*sin
(4*a))/(x^2 - cos(2*a) - I*sin(2*a))
```

Fricas [A]
time = 2.48, size = 61, normalized size = 1.11

$$-\frac{x^{4}-x^{2}e^{(2ia)}+4\left(x^{2}e^{(2ia)}-e^{(4ia)}\right)\log\left(x^{2}-e^{(2ia)}\right)-4\,e^{(4ia)}}{2\left(x^{2}-e^{(2ia)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(a+I*log(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(x^4 - x^2*e^(2*I*a) + 4*(x^2*e^(2*I*a) - e^(4*I*a))*log(x^2 - e^(2*I*
a)) - 4*e^(4*I*a))/(x^2 - e^(2*I*a))
```

Sympy [A]
time = 0.16, size = 42, normalized size = 0.76

$$-\frac{x^{2}}{2}-2e^{2ia}\log\left(x^{2}-e^{2ia}\right)+\frac{2e^{4ia}}{x^{2}-e^{2ia}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(a+I*ln(x))**2,x)
```

```
[Out] -x**2/2 - 2*exp(2*I*a)*log(x**2 - exp(2*I*a)) + 2*exp(4*I*a)/(x**2 - exp(2*
I*a))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(41) = 82.
time = 0.46, size = 118, normalized size = 2.15

$$-\frac{x^{4}}{2\left(x^{2}-e^{(2ia)}\right)}-\frac{2\,x^{2}e^{(2ia)}\log\left(-x^{2}+e^{(2ia)}\right)}{x^{2}-e^{(2ia)}}+\frac{x^{2}e^{(2ia)}}{2\left(x^{2}-e^{(2ia)}\right)}+\frac{2\,e^{(4ia)}\log\left(-x^{2}+e^{(2ia)}\right)}{x^{2}-e^{(2ia)}}+\frac{2\,e^{(4ia)}}{x^{2}-e^{(2ia)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(a+I*log(x))^2,x, algorithm="giac")
```

```
[Out] -1/2*x^4/(x^2 - e^(2*I*a)) - 2*x^2*e^(2*I*a)*log(-x^2 + e^(2*I*a))/(x^2 - e
^(2*I*a)) + 1/2*x^2*e^(2*I*a)/(x^2 - e^(2*I*a)) + 2*e^(4*I*a)*log(-x^2 + e^
(2*I*a))/(x^2 - e^(2*I*a)) + 2*e^(4*I*a)/(x^2 - e^(2*I*a))
```

Mupad [B]
time = 2.19, size = 45, normalized size = 0.82

$$-\frac{2\,{\mathrm{e}}^{a\,4{}\mathrm{i}}}{{\mathrm{e}}^{a\,2{}\mathrm{i}}-x^2}-2\,\ln\left(x^2-{\mathrm{e}}^{a\,2{}\mathrm{i}}\right)\,{\mathrm{e}}^{a\,2{}\mathrm{i}}-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(a + log(x)*1i)^2,x)
[Out] - (2*exp(a*4i))/(exp(a*2i) - x^2) - 2*log(x^2 - exp(a*2i))*exp(a*2i) - x^2/
2
```

3.197 $\int \cot^2(a+i\log(x))\,dx$

Optimal. Leaf size=48

$$-x-\frac{2e^{2ia}x}{e^{2ia}-x^2}+2e^{ia}\tanh^{-1}\left(e^{-ia}x\right)$$

```
[Out] -x-2*exp(2*I*a)*x/(exp(2*I*a)-x^2)+2*exp(I*a)*arctanh(x/exp(I*a))
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {4588, 381, 398, 294, 213}

$$-\frac{2e^{2ia}x}{-x^2+e^{2ia}}+2e^{ia}\tanh^{-1}\left(e^{-ia}x\right)-x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + I*Log[x]]^2,x]
```

```
[Out] -x - (2*E^((2*I)*a)*x)/(E^((2*I)*a) - x^2) + 2*E^(I*a)*ArcTanh[x/E^(I*a)]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 381

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 4588

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[((-I - I*E^(
2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*d)))^p, x] /; FreeQ[{a, b,
d, p}, x]
```

Rubi steps

$$\int \cot^2(a+i\log(x))\,dx=\int \cot^2(a+i\log(x))\,dx$$

Mathematica [A]
time = 0.06, size = 70, normalized size = 1.46

$$2\tanh^{-1}(x(\cos(a)-i\sin(a)))(\cos(a)+i\sin(a))+\frac{-x(-3+x^2)\cos(a)+ix(3+x^2)\sin(a)}{(-1+x^2)\cos(a)-i\,(1+x^2)\sin(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + I*Log[x]]^2,x]
[Out] 2*ArcTanh[x*(Cos[a] - I*Sin[a])]*(Cos[a] + I*Sin[a]) + (-(x*(-3 + x^2)*Cos[
a]) + I*x*(3 + x^2)*Sin[a])/((-1 + x^2)*Cos[a] - I*(1 + x^2)*Sin[a])
```

Maple [A]
time = 0.05, size = 36, normalized size = 0.75

method	result	size
risch	$-3x-\frac{2x}{\frac{e^{2ia}}{x^2}-1}+2\,\mathrm{arctanh}\left(x\,e^{-ia}\right)e^{ia}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a+I*ln(x))^2,x,method=_RETURNVERBOSE)
[Out] -3*x-2*x/(exp(2*I*a)/x^2-1)+2*arctanh(x*exp(-I*a))*exp(I*a)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(36) = 72.
time = 0.30, size = 270, normalized size = 5.62

$$-\frac{2\left((-i\cos(a)+\sin(a))\arctan\left(\sin(a),x+\cos(a)\right)+(-i\cos(a)+\sin(a))\arctan\left(\sin(a),x-\cos(a)\right)\right)x^2+2x^3-6x(\cos(2a)+i\sin(2a))+2\left((i\cos(a)-\sin(a))\cos(2a)-(\cos(a)+i\sin(a))\sin(2a)\right)\arctan\left(\sin(a),x+\cos(a)\right)+2\left((i\cos(a)-\sin(a))\cos(2a)-(\cos(a)+i\sin(a))\sin(2a)\right)\arctan\left(\sin(a),x-\cos(a)\right)-\left(x^2(\cos(a)+i\sin(a))-(\cos(a)+i\sin(a))\cos(2a)+(-i\cos(a)+\sin(a))\sin(2a)\right)\log\left(x^2+2x\cos(a)+\cos(a)^2+\sin(a)^2\right)+\left(x^2(\cos(a)+i\sin(a))-(\cos(a)+i\sin(a))\cos(2a)-(i\cos(a)-\sin(a))\sin(2a)\right)\log\left(x^2-2x\cos(a)+\cos(a)^2+\sin(a)^2\right)}{2\left(x^2-\cos(2a)-i\sin(2a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*((-I*cos(a) + sin(a))*arctan2(sin(a), x + cos(a)) + (-I*cos(a) + si
n(a))*arctan2(sin(a), x - cos(a)))*x^2 + 2*x^3 - 6*x*(cos(2*a) + I*sin(2*a)
) + 2*((I*cos(a) - sin(a))*cos(2*a) - (cos(a) + I*sin(a))*sin(2*a))*arctan2
(sin(a), x + cos(a)) + 2*((I*cos(a) - sin(a))*cos(2*a) - (cos(a) + I*sin(a)
)*sin(2*a))*arctan2(sin(a), x - cos(a)) - (x^2*(cos(a) + I*sin(a)) - (cos(a
) + I*sin(a))*cos(2*a) + (-I*cos(a) + sin(a))*sin(2*a))*log(x^2 + 2*x*cos(a
) + cos(a)^2 + sin(a)^2) + (x^2*(cos(a) + I*sin(a)) - (cos(a) + I*sin(a))*c
os(2*a) - (I*cos(a) - sin(a))*sin(2*a))*log(x^2 - 2*x*cos(a) + cos(a)^2 + s
in(a)^2))/(x^2 - cos(2*a) - I*sin(2*a))
```

Fricas [A]
time = 3.20, size = 72, normalized size = 1.50

$$-\frac{x^{3}-\left(x^{2}-e^{(2ia)}\right)e^{(ia)}\log\left(x+e^{(ia)}\right)+\left(x^{2}-e^{(2ia)}\right)e^{(ia)}\log\left(x-e^{(ia)}\right)-3\,xe^{(2ia)}}{x^{2}-e^{(2ia)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x))^2,x, algorithm="fricas")
```

```
[Out] -(x^3 - (x^2 - e^(2*I*a))*e^(I*a)*log(x + e^(I*a)) + (x^2 - e^(2*I*a))*e^(I
*a)*log(x - e^(I*a)) - 3*x*e^(2*I*a))/(x^2 - e^(2*I*a))
```

Sympy [A]
time = 0.16, size = 42, normalized size = 0.88

$$-x+\frac{2xe^{2ia}}{x^{2}-e^{2ia}}-\left(\log\left(x-e^{ia}\right)-\log\left(x+e^{ia}\right)\right)e^{ia}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*ln(x))**2,x)
```

```
[Out] -x + 2*x*exp(2*I*a)/(x**2 - exp(2*I*a)) - (log(x - exp(I*a)) - log(x + exp(
I*a)))*exp(I*a)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(36) = 72$.
time = 0.43, size = 79, normalized size = 1.65

$$-\frac{x^{3}}{x^{2}-e^{(2ia)}}-2\left(\frac{\arctan\left(\frac{x}{\sqrt{-e^{(2ia)}}}\right)}{\sqrt{-e^{(2ia)}}}-\frac{x}{x^{2}-e^{(2ia)}}\right)e^{(2ia)}+\frac{5\,xe^{(2ia)}}{x^{2}-e^{(2ia)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x))^2,x, algorithm="giac")
```

```
[Out] -x^3/(x^2 - e^(2*I*a)) - 2*(arctan(x/sqrt(-e^(2*I*a)))/sqrt(-e^(2*I*a)) - x
/(x^2 - e^(2*I*a)))*e^(2*I*a) + 5*x*e^(2*I*a)/(x^2 - e^(2*I*a))
```

Mupad [B]
time = 2.19, size = 44, normalized size = 0.92

$$-x+2\sqrt{\mathrm{e}^{a\,2\mathrm{i}}}\,\mathrm{atanh}\left(\frac{x}{\sqrt{\mathrm{e}^{a\,2\mathrm{i}}}}\right)-\frac{2\,x\,\mathrm{e}^{a\,2\mathrm{i}}}{\mathrm{e}^{a\,2\mathrm{i}}-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + log(x)*1i)^2,x)
```

```
[Out] 2*exp(a*2i)^(1/2)*atanh(x/exp(a*2i)^(1/2)) - x - (2*x*exp(a*2i))/(exp(a*2i)
 - x^2)
```

3.198 $\int \frac{\cot^2(a+i\log(x))}{x} \, dx$

Optimal. Leaf size=18

$$i\cot(a+i\log(x))-\log(x)$$

```
[Out] I*cot(a+I*ln(x))-ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3554, 8}

$$-\log(x)+i\cot(a+i\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + I*Log[x]]^2/x,x]
```

```
[Out] I*Cot[a + I*Log[x]] - Log[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^2(a+i\log(x))}{x} \, dx &= \text{Subst}\left(\int \cot^2(a+ix) \, dx, x, \log(x)\right)\\ &= i\cot(a+i\log(x)) - \text{Subst}\left(\int 1 \, dx, x, \log(x)\right)\\ &= i\cot(a+i\log(x)) - \log(x)\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 34, normalized size = 1.89

$$i\cot(a+i\log(x))\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(a+i\log(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + I*Log[x]]^2/x,x]
[Out] I*Cot[a + I*Log[x]]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[a + I*Log[x]]^2]
```

Maple [A]
time = 0.04, size = 29, normalized size = 1.61

method	result	size
risch	$-\ln(x)-\frac{2}{\frac{e^{2ia}}{x^{2}}-1}$	21
norman	$\frac{-\ln(x)\tan(a+i\ln(x))+i}{\tan(a+i\ln(x))}$	27
derivativedivides	$-i\left(-\cot(a+i\ln(x))+\frac{\pi}{2}-\operatorname{arccot}(\cot(a+i\ln(x)))\right)$	29
default	$-i\left(-\cot(a+i\ln(x))+\frac{\pi}{2}-\operatorname{arccot}(\cot(a+i\ln(x)))\right)$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a+I*ln(x))^2/x,x,method=_RETURNVERBOSE)
[Out] -I*(-cot(a+I*ln(x))+1/2*Pi-arccot(cot(a+I*ln(x))))
```

Maxima [A]
time = 0.49, size = 19, normalized size = 1.06

$$i\,a+\frac{i}{\tan\left(a+i\,\log\left(x\right)\right)}-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x))^2/x,x, algorithm="maxima")
[Out] I*a + I/tan(a + I*log(x)) - log(x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(14) = 28.
time = 3.22, size = 34, normalized size = 1.89

$$-\frac{\left(x^{2}-e^{(2i\,a)}\right)\log\left(x\right)-2\,e^{(2i\,a)}}{x^{2}-e^{(2i\,a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x))^2/x,x, algorithm="fricas")
[Out] -((x^2 - e^(2*I*a))*log(x) - 2*e^(2*I*a))/(x^2 - e^(2*I*a))
```

Sympy [A]
time = 0.17, size = 20, normalized size = 1.11

$$-\log\left(x\right)+\frac{2e^{2ia}}{x^{2}-e^{2ia}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*ln(x))**2/x,x)
```

```
[Out] -log(x) + 2*exp(2*I*a)/(x**2 - exp(2*I*a))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(14) = 28$.
time = 0.42, size = 32, normalized size = 1.78

$$ia + \frac{i}{2\,\tan\left(\frac{1}{2}\,a + \frac{1}{2}i\,\log\left(x\right)\right)} - \log\left(x\right) - \frac{1}{2}i\,\tan\left(\frac{1}{2}\,a + \frac{1}{2}i\,\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x))^2/x,x, algorithm="giac")
```

```
[Out] I*a + 1/2*I/tan(1/2*a + 1/2*I*log(x)) - log(x) - 1/2*I*tan(1/2*a + 1/2*I*lo
g(x))
```

Mupad [B]
time = 2.49, size = 16, normalized size = 0.89

$$-\ln\left(x\right) + \cot(a + \ln\left(x\right)\ 1\mathrm{i})\ 1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + log(x)*1i)^2/x,x)
```

```
[Out] cot(a + log(x)*1i)*1i - log(x)
```

3.199 $\int \frac{\cot^2(a+i\log(x))}{x^2}\,dx$

Optimal. Leaf size=64

$$\frac{e^{2ia}}{x\left(e^{2ia}-x^2\right)}-\frac{3x}{e^{2ia}-x^2}-2e^{-ia}\tanh^{-1}\left(e^{-ia}x\right)$$

```
[Out] exp(2*I*a)/x/(exp(2*I*a)-x^2)-3*x/(exp(2*I*a)-x^2)-2*arctanh(x/exp(I*a))/ex
p(I*a)
```

Rubi [A]
time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$,
Rules used = {4592, 456, 473, 393, 213}

$$-\frac{3x}{-x^2+e^{2ia}}+\frac{e^{2ia}}{x\left(-x^2+e^{2ia}\right)}-2e^{-ia}\tanh^{-1}\left(e^{-ia}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + I*Log[x]]^2/x^2,x]
```

```
[Out] E^((2*I)*a)/(x*(E^((2*I)*a) - x^2)) - (3*x)/(E^((2*I)*a) - x^2) - (2*ArcTan
h[x/E^(I*a)])/E^(I*a)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 456

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int \frac{\cot^2(a+i\log(x))}{x^2}\,dx=\int \frac{\cot^2(a+i\log(x))}{x^2}\,dx$$

Mathematica [A]
time = 0.09, size = 72, normalized size = 1.12

$$\frac{1}{x}-2\tanh^{-1}(x(\cos(a)-i\sin(a)))\cos(a)+2i\tanh^{-1}(x(\cos(a)-i\sin(a)))\sin(a)+\frac{2x(\cos(a)-i\sin(a))}{(-1+x^2)\cos(a)-i(1+x^2)\sin(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + I*Log[x]]^2/x^2,x]
[Out] x^(-1) - 2*ArcTanh[x*(Cos[a] - I*Sin[a])]*Cos[a] + (2*I)*ArcTanh[x*(Cos[a]
- I*Sin[a])]*Sin[a] + (2*x*(Cos[a] - I*Sin[a]))/((-1 + x^2)*Cos[a] - I*(1 +
 x^2)*Sin[a])
```

Maple [A]
time = 0.06, size = 38, normalized size = 0.59

method	result	size
risch	$\frac{1}{x}-\frac{2}{x\left(\frac{e^{2ia}}{x^2}-1\right)}-2\,\mathrm{arctanh}\left(x\,e^{-ia}\right)e^{-ia}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a+I*ln(x))^2/x^2,x,method=_RETURNVERBOSE)
[Out] 1/x-2/x/(exp(2*I*a)/x^2-1)-2*arctanh(x*exp(-I*a))*exp(-I*a)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(50) = 100$.
time = 0.31, size = 276, normalized size = 4.31

$$-\frac{2\,\left(\left(i\,\cos\left(a\right)+\sin\left(a\right)\right)\arctan\left(\sin\left(a\right),x+\cos\left(a\right)\right)+\left(i\,\cos\left(a\right)+\sin\left(a\right)\right)\arctan\left(\sin\left(a\right),x-\cos\left(a\right)\right)\right)x^{3}+2\,\left(\left(\left(-i\,\cos\left(a\right)-\sin\left(a\right)\right)\cos\left(2\,a\right)+\left(\cos\left(a\right)-i\,\sin\left(a\right)\right)\sin\left(2\,a\right)\right)\arctan\left(\sin\left(a\right),x+\cos\left(a\right)\right)+\left(\left(-i\,\cos\left(a\right)-\sin\left(a\right)\right)\cos\left(2\,a\right)+\left(\cos\left(a\right)-i\,\sin\left(a\right)\right)\sin\left(2\,a\right)\right)\arctan\left(\sin\left(a\right),x-\cos\left(a\right)\right)\right)x-6\,x^{2}+\left(x^{3}\left(\cos\left(a\right)-i\,\sin\left(a\right)\right)-\left(\left(\cos\left(a\right)-i\,\sin\left(a\right)\right)\cos\left(2\,a\right)+\left(i\,\cos\left(a\right)+\sin\left(a\right)\right)\sin\left(2\,a\right)\right)x\right)\log\left(x^{2}+2\,x\cos\left(a\right)+\cos\left(a\right)^{2}+\sin\left(a\right)^{2}\right)-\left(x^{3}\left(\cos\left(a\right)-i\,\sin\left(a\right)\right)-\left(\left(\cos\left(a\right)-i\,\sin\left(a\right)\right)\cos\left(2\,a\right)-\left(-i\,\cos\left(a\right)-\sin\left(a\right)\right)\sin\left(2\,a\right)\right)x\right)\log\left(x^{2}-2\,x\cos\left(a\right)+\cos\left(a\right)^{2}+\sin\left(a\right)^{2}\right)+2\,\cos\left(2\,a\right)+2i\,\sin\left(2\,a\right)}{2\,\left(x^{3}-x\left(\cos\left(2\,a\right)+i\,\sin\left(2\,a\right)\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x))^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*((I*cos(a) + sin(a))*arctan2(sin(a), x + cos(a)) + (I*cos(a) + sin(
a))*arctan2(sin(a), x - cos(a)))*x^3 + 2*(((-I*cos(a) - sin(a))*cos(2*a) +
(cos(a) - I*sin(a))*sin(2*a))*arctan2(sin(a), x + cos(a)) + ((-I*cos(a) - s
in(a))*cos(2*a) + (cos(a) - I*sin(a))*sin(2*a))*arctan2(sin(a), x - cos(a))
)*x - 6*x^2 + (x^3*(cos(a) - I*sin(a)) - ((cos(a) - I*sin(a))*cos(2*a) + (I
*cos(a) + sin(a))*sin(2*a))*x)*log(x^2 + 2*x*cos(a) + cos(a)^2 + sin(a)^2)
- (x^3*(cos(a) - I*sin(a)) - ((cos(a) - I*sin(a))*cos(2*a) - (-I*cos(a) - s
in(a))*sin(2*a))*x)*log(x^2 - 2*x*cos(a) + cos(a)^2 + sin(a)^2) + 2*cos(2*a
) + 2*I*sin(2*a))/(x^3 - x*(cos(2*a) + I*sin(2*a)))
```

Fricas [A]
time = 2.10, size = 74, normalized size = 1.16

$$-\frac{\left(x^{3}-xe^{\left(2 i a\right)}\right)e^{\left(-i a\right)}\log\left(x+e^{\left(i a\right)}\right)-\left(x^{3}-xe^{\left(2 i a\right)}\right)e^{\left(-i a\right)}\log\left(x-e^{\left(i a\right)}\right)-3\,x^{2}+e^{\left(2 i a\right)}}{x^{3}-xe^{\left(2 i a\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x))^2/x^2,x, algorithm="fricas")
```

```
[Out] -((x^3 - x*e^(2*I*a))*e^(-I*a)*log(x + e^(I*a)) - (x^3 - x*e^(2*I*a))*e^(-I
*a)*log(x - e^(I*a)) - 3*x^2 + e^(2*I*a))/(x^3 - x*e^(2*I*a))
```

Sympy [A]
time = 0.22, size = 46, normalized size = 0.72

$$-\frac{-3x^{2}+e^{2ia}}{x^{3}-xe^{2ia}}-\left(-\log\left(x-e^{ia}\right)+\log\left(x+e^{ia}\right)\right)e^{-ia}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*ln(x))**2/x**2,x)
```

```
[Out] -(-3*x**2 + exp(2*I*a))/(x**3 - x*exp(2*I*a)) - (-log(x - exp(I*a)) + log(x
 + exp(I*a)))*exp(-I*a)
```

Giac [A]
time = 0.46, size = 87, normalized size = 1.36

$$2\left(\frac{\arctan\left(\frac{x}{\sqrt{-e^{\left(2 i a\right)}}}\right)e^{\left(-2 i a\right)}}{\sqrt{-e^{\left(2 i a\right)}}}+\frac{xe^{\left(-2 i a\right)}}{x^{2}-e^{\left(2 i a\right)}}\right)e^{\left(2 i a\right)}+\frac{5\,x^{2}}{x^{3}-xe^{\left(2 i a\right)}}-\frac{e^{\left(2 i a\right)}}{x^{3}-xe^{\left(2 i a\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x))^2/x^2,x, algorithm="giac")
```

```
[Out] 2*(arctan(x/sqrt(-e^(2*I*a)))*e^(-2*I*a)/sqrt(-e^(2*I*a)) + x*e^(-2*I*a)/(x
^2 - e^(2*I*a)))*e^(2*I*a) + 5*x^2/(x^3 - x*e^(2*I*a)) - e^(2*I*a)/(x^3 - x
*e^(2*I*a))
```

Mupad [B]
time = 2.21, size = 47, normalized size = 0.73

$$-\frac{2\,\mathrm{atanh}\left(\frac{x}{\sqrt{\mathrm{e}^{a\,2\mathrm{i}}}}\right)}{\sqrt{\mathrm{e}^{a\,2\mathrm{i}}}}-\frac{\mathrm{e}^{a\,2\mathrm{i}}-3\,x^2}{x^3-x\,\mathrm{e}^{a\,2\mathrm{i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + log(x)*1i)^2/x^2,x)
```

```
[Out] - (2*atanh(x/exp(a*2i)^(1/2)))/exp(a*2i)^(1/2) - (exp(a*2i) - 3*x^2)/(x^3 -
 x*exp(a*2i))
```

3.200 $\int \frac{\cot^2(a+i\log(x))}{x^3}\,dx$

Optimal. Leaf size=57

$$\frac{2e^{-2ia}}{1-\frac{e^{2ia}}{x^2}}+\frac{1}{2x^2}+2e^{-2ia}\log\left(1-\frac{e^{2ia}}{x^2}\right)$$

```
[Out] 2/exp(2*I*a)/(1-exp(2*I*a)/x^2)+1/2/x^2+2*ln(1-exp(2*I*a)/x^2)/exp(2*I*a)
```

Rubi [A]
time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4592, 455, 45}

$$\frac{2e^{-2ia}}{1-\frac{e^{2ia}}{x^2}}+2e^{-2ia}\log\left(1-\frac{e^{2ia}}{x^2}\right)+\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + I*Log[x]]^2/x^3,x]
```

```
[Out] 2/(E^((2*I)*a)*(1 - E^((2*I)*a)/x^2)) + 1/(2*x^2) + (2*Log[1 - E^((2*I)*a)/
x^2])/E^((2*I)*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int \frac{\cot^2(a+i\log(x))}{x^3}\,dx = \int \frac{\cot^2(a+i\log(x))}{x^3}\,dx$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 153 vs. $2(57) = 114$.
time = 0.17, size = 153, normalized size = 2.68

$$\frac{1}{2x^2} + \cos(2a)\left(-4\log(x) + \log\left(1 + x^4 - 2x^2\cos(2a)\right)\right) + \frac{2\cos(a)}{(-1+x^2)\cos(a) - i(1+x^2)\sin(a)} + \frac{2\sin(a)}{i(-1+x^2)\cos(a) + (1+x^2)\sin(a)} + \mathrm{ArcTan}\left(\frac{\cot(a) - x^2\cot(a)}{1+x^2}\right)(-2i\cos(2a) - 4\cos(a)\sin(a)) + 4i\log(x)\sin(2a) - i\log\left(1 + x^4 - 2x^2\cos(2a)\right)\sin(2a)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + I*Log[x]]^2/x^3,x]
```

```
[Out] 1/(2*x^2) + Cos[2*a]*(-4*Log[x] + Log[1 + x^4 - 2*x^2*Cos[2*a]]) + (2*Cos[a
])/((-1 + x^2)*Cos[a] - I*(1 + x^2)*Sin[a]) + (2*Sin[a])/(I*(-1 + x^2)*Cos[
a] + (1 + x^2)*Sin[a]) + ArcTan[(Cot[a] - x^2*Cot[a])/(1 + x^2)]*((-2*I)*Co
s[2*a] - 4*Cos[a]*Sin[a]) + (4*I)*Log[x]*Sin[2*a] - I*Log[1 + x^4 - 2*x^2*C
os[2*a]]*Sin[2*a]
```

Maple [A]
time = 0.06, size = 53, normalized size = 0.93

method	result	size
risch	$\frac{1}{2x^2} - \frac{2}{x^2\left(\frac{e^{2ia}}{x^2}-1\right)} + 2\,e^{-2ia}\ln\left(e^{2ia} - x^2\right) - 4\,e^{-2ia}\ln(x)$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a+I*ln(x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/x^2-2/x^2/(exp(2*I*a)/x^2-1)+2*exp(-2*I*a)*ln(exp(2*I*a)-x^2)-4*exp(-2*
I*a)*ln(x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x))^2/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 2.87, size = 81, normalized size = 1.42

$$\frac{5\,x^2e^{(2i\,a)}+4\left(x^4-x^2e^{(2i\,a)}\right)\log\left(x^2-e^{(2i\,a)}\right)-8\left(x^4-x^2e^{(2i\,a)}\right)\log\left(x\right)-e^{(4i\,a)}}{2\left(x^4e^{(2i\,a)}-x^2e^{(4i\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(5*x^2*e^(2*I*a) + 4*(x^4 - x^2*e^(2*I*a))*log(x^2 - e^(2*I*a)) - 8*(x^
4 - x^2*e^(2*I*a))*log(x) - e^(4*I*a))/(x^4*e^(2*I*a) - x^2*e^(4*I*a))
```

Sympy [A]
time = 0.28, size = 60, normalized size = 1.05

$$-\frac{-5x^2+e^{2ia}}{2x^4-2x^2e^{2ia}}-4e^{-2ia}\log\left(x\right)+2e^{-2ia}\log\left(x^2-e^{2ia}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*ln(x))**2/x**3,x)
```

```
[Out] -(-5*x**2 + exp(2*I*a))/(2*x**4 - 2*x**2*exp(2*I*a)) - 4*exp(-2*I*a)*log(x)
 + 2*exp(-2*I*a)*log(x**2 - exp(2*I*a))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(42) = 84.
time = 0.45, size = 190, normalized size = 3.33

$$\frac{2\,x^4\log\left(x^2-e^{(2i\,a)}\right)}{x^4e^{(2i\,a)}-x^2e^{(4i\,a)}}-\frac{4\,x^4\log\left(x\right)}{x^4e^{(2i\,a)}-x^2e^{(4i\,a)}}-\frac{2\,x^2e^{(2i\,a)}\log\left(x^2-e^{(2i\,a)}\right)}{x^4e^{(2i\,a)}-x^2e^{(4i\,a)}}+\frac{4\,x^2e^{(2i\,a)}\log\left(x\right)}{x^4e^{(2i\,a)}-x^2e^{(4i\,a)}}+\frac{5\,x^2e^{(2i\,a)}}{2\left(x^4e^{(2i\,a)}-x^2e^{(4i\,a)}\right)}-\frac{e^{(4i\,a)}}{2\left(x^4e^{(2i\,a)}-x^2e^{(4i\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x))^2/x^3,x, algorithm="giac")
```

```
[Out] 2*x^4*log(x^2 - e^(2*I*a))/(x^4*e^(2*I*a) - x^2*e^(4*I*a)) - 4*x^4*log(x)/(
x^4*e^(2*I*a) - x^2*e^(4*I*a)) - 2*x^2*e^(2*I*a)*log(x^2 - e^(2*I*a))/(x^4*
e^(2*I*a) - x^2*e^(4*I*a)) + 4*x^2*e^(2*I*a)*log(x)/(x^4*e^(2*I*a) - x^2*e^
(4*I*a)) + 5/2*x^2*e^(2*I*a)/(x^4*e^(2*I*a) - x^2*e^(4*I*a)) - 1/2*e^(4*I*a
)/(x^4*e^(2*I*a) - x^2*e^(4*I*a))
```

Mupad [B]
time = 2.23, size = 60, normalized size = 1.05

$$-4\,{\mathrm{e}}^{-a\,2\mathrm{i}}\,\ln\left(x\right)+2\,\ln\left(x^2-{\mathrm{e}}^{a\,2\mathrm{i}}\right)\,{\mathrm{e}}^{-a\,2\mathrm{i}}+\frac{\frac{{\mathrm{e}}^{a\,2\mathrm{i}}}{2}-\frac{5\,x^2}{2}}{x^2\,{\mathrm{e}}^{a\,2\mathrm{i}}-x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + log(x)*1i)^2/x^3,x)
```

```
[Out] 2*log(x^2 - exp(a*2i))*exp(-a*2i) - 4*exp(-a*2i)*log(x) + (exp(a*2i)/2 - (5
*x^2)/2)/(x^2*exp(a*2i) - x^4)
```

3.201 $\int (ex)^m \cot(a + i\log(x))\,dx$

Optimal. Leaf size=70

$$\frac{i(ex)^{1+m}}{e(1+m)} - \frac{2i(ex)^{1+m}\,{}_2F_1\left(1,\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{e^{2ia}}{x^2}\right)}{e(1+m)}$$

```
[Out] I*(e*x)^(1+m)/e/(1+m)-2*I*(e*x)^(1+m)*hypergeom([1, -1/2-1/2*m],[1/2-1/2*m]
,exp(2*I*a)/x^2)/e/(1+m)
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {4592, 470, 346, 371}

$$\frac{i(ex)^{m+1}}{e(m+1)} - \frac{2i(ex)^{m+1}\,{}_2F_1\left(1,\frac{1}{2}(-m-1);\frac{1-m}{2};\frac{e^{2ia}}{x^2}\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Cot[a + I*Log[x]],x]
```

```
[Out] (I*(e*x)^(1 + m))/(e*(1 + m)) - ((2*I)*(e*x)^(1 + m)*Hypergeometric2F1[1, (
-1 - m)/2, (1 - m)/2, E^((2*I)*a)/x^2])/(e*(1 + m))
```

Rule 346

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1), Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x
, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int (ex)^m \cot(a + i \log(x))\, dx = \int (ex)^m \cot(a + i \log(x))\, dx$$

Mathematica [A]
time = 0.19, size = 103, normalized size = 1.47

$$ix(ex)^m \left(\frac{{}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; x^2(\cos(2a) - i\sin(2a))\right)}{1+m} + \frac{x^2\, {}_2F_1\left(1, \frac{3+m}{2}; \frac{5+m}{2}; x^2(\cos(2a) - i\sin(2a))\right)(\cos(a) - i\sin(a))^2}{3+m} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Cot[a + I*Log[x]],x]
```

```
[Out] I*x*(e*x)^m*(Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, x^2*(Cos[2*a] - I*S
in[2*a])]/(1 + m) + (x^2*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, x^2*(Co
s[2*a] - I*Sin[2*a])]*(Cos[a] - I*Sin[a])^2)/(3 + m))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (ex)^m \cot\left(a + i\ln\left(x\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*cot(a+I*ln(x)),x)
```

```
[Out] int((e*x)^m*cot(a+I*ln(x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(a+I*log(x)),x, algorithm="maxima")
```

```
[Out] integrate((x*e)^m*cot(a + I*log(x)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(a+I*log(x)),x, algorithm="fricas")
[Out] integral(-(I*x^2 + I*e^(2*I*a))*e^(m*log(x) + m)/(x^2 - e^(2*I*a)), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \cot (a + i \log (x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*cot(a+I*ln(x)),x)
[Out] Integral((e*x)**m*cot(a + I*log(x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(a+I*log(x)),x, algorithm="giac")
[Out] integrate((e*x)^m*cot(a + I*log(x)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(a + \ln (x)\ 1\mathrm{i})\ (e\,x)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + log(x)*1i)*(e*x)^m,x)
[Out] int(cot(a + log(x)*1i)*(e*x)^m, x)
```

3.202 $\int (ex)^m \cot^2(a+i\log(x))\,dx$

Optimal. Leaf size=77

$$-\frac{x(ex)^m}{1+m}+\frac{2x(ex)^m}{1-\frac{e^{2ia}}{x^2}}-2x(ex)^m\,{}_2F_1\left(1,\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{e^{2ia}}{x^2}\right)$$

```
[Out] -x*(e*x)^m/(1+m)+2*x*(e*x)^m/(1-exp(2*I*a)/x^2)-2*x*(e*x)^m*hypergeom([1, -
1/2-1/2*m],[1/2-1/2*m],exp(2*I*a)/x^2)
```

Rubi [A]
time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {4592, 511, 474, 470, 371}

$$-2x(ex)^m\,{}_2F_1\left(1,\frac{1}{2}(-m-1);\frac{1-m}{2};\frac{e^{2ia}}{x^2}\right)+\frac{2x(ex)^m}{1-\frac{e^{2ia}}{x^2}}-\frac{x(ex)^m}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Cot[a + I*Log[x]]^2,x]
```

```
[Out] -((x*(e*x)^m)/(1 + m)) + (2*x*(e*x)^m)/(1 - E^((2*I)*a)/x^2) - 2*x*(e*x)^m*
Hypergeometric2F1[1, (-1 - m)/2, (1 - m)/2, E^((2*I)*a)/x^2]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
```

```
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 511

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Dist[(-(e*x)^m)*(x^(-1))^m, Subst[Int[(a + b/x^n)^p*((
c + d/x^n)^q/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m, p, q},
 x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int (ex)^m \cot^2(a+i\log(x))\,dx = \int (ex)^m \cot^2(a+i\log(x))\,dx$$

Mathematica [A]
time = 0.16, size = 84, normalized size = 1.09

$$\frac{x(ex)^m \left(-1+4\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};x^2(\cos(2a)-i\sin(2a))\right)-4\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};x^2(\cos(2a)-i\sin(2a))\right)\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Cot[a + I*Log[x]]^2,x]
```

```
[Out] (x*(e*x)^m*(-1 + 4*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, x^2*(Cos[2*a]
 - I*Sin[2*a])] - 4*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, x^2*(Cos[2*a
] - I*Sin[2*a])]))/(1 + m)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\cot^2\left(a+i\ln\left(x\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*cot(a+I*ln(x))^2,x)
```

```
[Out] int((e*x)^m*cot(a+I*ln(x))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(a+I*log(x))^2,x, algorithm="maxima")
[Out] integrate((x*e)^m*cot(a + I*log(x))^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(a+I*log(x))^2,x, algorithm="fricas")
[Out] integral(-(x^4 + 2*x^2*e^(2*I*a) + e^(4*I*a))*e^(m*log(x) + m)/(x^4 - 2*x^2
*e^(2*I*a) + e^(4*I*a)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \cot^2(a + i\log(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*cot(a+I*ln(x))**2,x)
[Out] Integral((e*x)**m*cot(a + I*log(x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(a+I*log(x))^2,x, algorithm="giac")
[Out] integrate((e*x)^m*cot(a + I*log(x))^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{cot}(a + \ln(x)\ 1\mathrm{i})^2\, (e\, x)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + log(x)*1i)^2*(e*x)^m,x)
[Out] int(cot(a + log(x)*1i)^2*(e*x)^m, x)
```

3.203 $\int (ex)^m \cot^3(a+i\log(x))\,dx$

Optimal. Leaf size=169

$$\frac{i(1-m)mx(ex)^m}{2(1+m)}-\frac{i\left(1+\frac{e^{2ia}}{x^2}\right)^2 x(ex)^m}{2\left(1-\frac{e^{2ia}}{x^2}\right)^2}-\frac{i\left(3+m-\frac{e^{2ia}(1-m)}{x^2}\right)x(ex)^m}{2\left(1-\frac{e^{2ia}}{x^2}\right)}+\frac{i(3+2m+m^2)\,x(ex)^m\,{}_2F_1\left(1,\frac{1}{2}(\right.}{1+m}$$

```
[Out] 1/2*I*(1-m)*m*x*(e*x)^m/(1+m)-1/2*I*(1+exp(2*I*a)/x^2)^2*x*(e*x)^m/(1-exp(2
*I*a)/x^2)^2-1/2*I*(3+m-exp(2*I*a)*(1-m)/x^2)*x*(e*x)^m/(1-exp(2*I*a)/x^2)+
I*(m^2+2*m+3)*x*(e*x)^m*hypergeom([1, -1/2-1/2*m],[1/2-1/2*m],exp(2*I*a)/x^
2)/(1+m)
```

Rubi [A]
time = 0.17, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {4592, 511, 479, 591, 470, 371}

$$\frac{i(m^2+2m+3)\,x(ex)^m\,{}_2F_1\left(1,\frac{1}{2}(-m-1);\frac{1-m}{2};\frac{e^{2ia}}{x^2}\right)}{m+1}-\frac{ix\left(1+\frac{e^{2ia}}{x^2}\right)^2(ex)^m}{2\left(1-\frac{e^{2ia}}{x^2}\right)^2}-\frac{ix\left(-\frac{e^{2ia}(1-m)}{x^2}+m+3\right)(ex)^m}{2\left(1-\frac{e^{2ia}}{x^2}\right)}+\frac{i(1-m)mx(ex)^m}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Cot[a + I*Log[x]]^3,x]
```

```
[Out] ((I/2)*(1 - m)*m*x*(e*x)^m)/(1 + m) - ((I/2)*(1 + E^((2*I)*a)/x^2)^2*x*(e*x
)^m)/(1 - E^((2*I)*a)/x^2)^2 - ((I/2)*(3 + m - (E^((2*I)*a)*(1 - m))/x^2)*x
*(e*x)^m)/(1 - E^((2*I)*a)/x^2) + (I*(3 + 2*m + m^2)*x*(e*x)^m*Hypergeometr
ic2F1[1, (-1 - m)/2, (1 - m)/2, E^((2*I)*a)/x^2])/(1 + m)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 511

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Dist[(-(e*x)^m)*(x^(-1))^m, Subst[Int[(a + b/x^n)^p*((
c + d/x^n)^q/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m, p, q},
 x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 591

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n,
 0] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e -
 a*f])
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int (ex)^m \cot^3(a+i\log(x))\,dx = \int (ex)^m \cot^3(a+i\log(x))\,dx$$

Mathematica [A]
time = 0.23, size = 122, normalized size = 0.72

$$-\frac{ix(ex)^m\left(-1+6\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};x^2(\cos(2a)-i\sin(2a))\right)-12\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};x^2(\cos(2a)-i\sin(2a))\right)+8\,{}_2F_1\left(3,\frac{1+m}{2};\frac{3+m}{2};x^2(\cos(2a)-i\sin(2a))\right)\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Cot[a + I*Log[x]]^3,x]
```

```
[Out] ((-I)*x*(e*x)^m*(-1 + 6*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, x^2*(Cos
[2*a] - I*Sin[2*a])] - 12*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, x^2*(C
os[2*a] - I*Sin[2*a])] + 8*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, x^2*(
Cos[2*a] - I*Sin[2*a])]))/(1 + m)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\cot^3 (a + i \ln (x))\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*cot(a+I*ln(x))^3,x)
```

```
[Out] int((e*x)^m*cot(a+I*ln(x))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(a+I*log(x))^3,x, algorithm="maxima")
```

```
[Out] integrate((x*e)^m*cot(a + I*log(x))^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(a+I*log(x))^3,x, algorithm="fricas")
```

```
[Out] integral(-(-I*x^6 - 3*I*x^4*e^(2*I*a) - 3*I*x^2*e^(4*I*a) - I*e^(6*I*a))*e^
(m*log(x) + m)/(x^6 - 3*x^4*e^(2*I*a) + 3*x^2*e^(4*I*a) - e^(6*I*a)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \cot^3 (a + i \log (x)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*cot(a+I*ln(x))**3,x)
```

```
[Out] Integral((e*x)**m*cot(a + I*log(x))**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(a+I*log(x))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*cot(a + I*log(x))^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(a + \ln(x)\ 1\mathrm{i})^3\, (e\,x)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + log(x)*1i)^3*(e*x)^m,x)
```

```
[Out] int(cot(a + log(x)*1i)^3*(e*x)^m, x)
```

3.204 $\int \cot^p(a + b\log(x))\,dx$

Optimal. Leaf size=142

$$x\left(1-e^{2ia}x^{2ib}\right)^p\left(1+e^{2ia}x^{2ib}\right)^{-p}\left(-\frac{i\left(1+e^{2ia}x^{2ib}\right)}{1-e^{2ia}x^{2ib}}\right)^p F_1\left(-\frac{i}{2b};p,-p;1-\frac{i}{2b};e^{2ia}x^{2ib},-e^{2ia}x^{2ib}\right)$$

```
[Out] x*(1-exp(2*I*a)*x^(2*I*b))^p*(-I*(1+exp(2*I*a)*x^(2*I*b))/(1-exp(2*I*a)*x^(
2*I*b)))^p*AppellF1(-1/2*I/b,p,-p,1-1/2*I/b,exp(2*I*a)*x^(2*I*b),-exp(2*I*a
)*x^(2*I*b))/((1+exp(2*I*a)*x^(2*I*b))^p)
```

Rubi [A]
time = 0.04, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {4588, 1986, 441, 440}

$$x\left(1-e^{2ia}x^{2ib}\right)^p\left(1+e^{2ia}x^{2ib}\right)^{-p}\left(-\frac{i\left(1+e^{2ia}x^{2ib}\right)}{1-e^{2ia}x^{2ib}}\right)^p F_1\left(-\frac{i}{2b};p,-p;1-\frac{i}{2b};e^{2ia}x^{2ib},-e^{2ia}x^{2ib}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*Log[x]]^p,x]
```

```
[Out] (x*(1 - E^((2*I)*a)*x^((2*I)*b))^p*(((-I)*(1 + E^((2*I)*a)*x^((2*I)*b)))/(1
 - E^((2*I)*a)*x^((2*I)*b)))^p*AppellF1[(-1/2*I)/b, p, -p, 1 - (I/2)/b, E^(
(2*I)*a)*x^((2*I)*b), -(E^((2*I)*a)*x^((2*I)*b))])/(1 + E^((2*I)*a)*x^((2*I
)*b))^p
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
```

```
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4588

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[((-I - I*E^(
2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*d)))^p, x] /; FreeQ[{a, b,
d, p}, x]
```

Rubi steps

$$\int \cot^p(a+b\log(x))\,dx = \int \cot^p(a+b\log(x))\,dx$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 330 vs. $2(142) = 284$.
time = 0.67, size = 330, normalized size = 2.32

$$\frac{(-i+2b)x\left(\frac{i(1+e^{2ia}x^{2ib})}{-1+e^{2ia}x^{2ib}}\right)^p F_1\left(-\frac{i}{2b};p,-p;1-\frac{i}{2b};e^{2ia}x^{2ib},-e^{2ia}x^{2ib}\right)}{2be^{2ia}px^{2ib}F_1\left(1-\frac{i}{2b};p,1-p;2-\frac{i}{2b};e^{2ia}x^{2ib},-e^{2ia}x^{2ib}\right)+2be^{2ia}px^{2ib}F_1\left(1-\frac{i}{2b};1+p,-p;2-\frac{i}{2b};e^{2ia}x^{2ib},-e^{2ia}x^{2ib}\right)+(-i+2b)F_1\left(-\frac{i}{2b};p,-p;1-\frac{i}{2b};e^{2ia}x^{2ib},-e^{2ia}x^{2ib}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[a + b*Log[x]]^p,x]
[Out] ((-I + 2*b)*x*((I*(1 + E^((2*I)*a)*x^((2*I)*b)))/(-1 + E^((2*I)*a)*x^((2*I)
*b)))^p*AppellF1[(-1/2*I)/b, p, -p, 1 - (I/2)/b, E^((2*I)*a)*x^((2*I)*b), -
(E^((2*I)*a)*x^((2*I)*b))])/(2*b*E^((2*I)*a)*p*x^((2*I)*b)*AppellF1[1 - (I/
2)/b, p, 1 - p, 2 - (I/2)/b, E^((2*I)*a)*x^((2*I)*b), -(E^((2*I)*a)*x^((2*I
)*b))] + 2*b*E^((2*I)*a)*p*x^((2*I)*b)*AppellF1[1 - (I/2)/b, 1 + p, -p, 2 -
 (I/2)/b, E^((2*I)*a)*x^((2*I)*b), -(E^((2*I)*a)*x^((2*I)*b))] + (-I + 2*b)
*AppellF1[(-1/2*I)/b, p, -p, 1 - (I/2)/b, E^((2*I)*a)*x^((2*I)*b), -(E^((2*
I)*a)*x^((2*I)*b))])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \cot^p\left(a+b\ln(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a+b*ln(x))^p,x)
[Out] int(cot(a+b*ln(x))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*log(x))^p,x, algorithm="maxima")
[Out] integrate(cot(b*log(x) + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*log(x))^p,x, algorithm="fricas")
[Out] integral(cot(b*log(x) + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cot^{p}\left(a + b \log\left(x\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*ln(x))**p,x)
[Out] Integral(cot(a + b*log(x))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*log(x))^p,x, algorithm="giac")
[Out] integrate(cot(b*log(x) + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(a + b\,\ln\left(x\right))^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*log(x))^p,x)
[Out] int(cot(a + b*log(x))^p, x)
```

3.205 $\int (ex)^m \cot^p(a + b\log(x))\,dx$

Optimal. Leaf size=162

$$\frac{(ex)^{1+m}\left(1-e^{2ia}x^{2ib}\right)^p\left(1+e^{2ia}x^{2ib}\right)^{-p}\left(-\frac{i(1+e^{2ia}x^{2ib})}{1-e^{2ia}x^{2ib}}\right)^p F_1\left(-\frac{i(1+m)}{2b};p,-p;1-\frac{i(1+m)}{2b};e^{2ia}x^{2ib},-e^{2ia}x^{2ib}\right)}{e(1+m)}$$

```
[Out] (e*x)^(1+m)*(1-exp(2*I*a)*x^(2*I*b))^p*(-I*(1+exp(2*I*a)*x^(2*I*b))/(1-exp(
2*I*a)*x^(2*I*b)))^p*AppellF1(-1/2*I*(1+m)/b,p,-p,1-1/2*I*(1+m)/b,exp(2*I*a
)*x^(2*I*b),-exp(2*I*a)*x^(2*I*b))/e/(1+m)/((1+exp(2*I*a)*x^(2*I*b))^p)
```

Rubi [A]
time = 0.08, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {4592, 1986, 525, 524}

$$\frac{(ex)^{m+1}\left(1-e^{2ia}x^{2ib}\right)^p\left(1+e^{2ia}x^{2ib}\right)^{-p}\left(-\frac{i(1+e^{2ia}x^{2ib})}{1-e^{2ia}x^{2ib}}\right)^p F_1\left(-\frac{i(m+1)}{2b};p,-p;1-\frac{i(m+1)}{2b};e^{2ia}x^{2ib},-e^{2ia}x^{2ib}\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Cot[a + b*Log[x]]^p,x]
```

```
[Out] ((e*x)^(1 + m)*(1 - E^((2*I)*a)*x^((2*I)*b))^p*(((-I)*(1 + E^((2*I)*a)*x^((
2*I)*b)))/(1 - E^((2*I)*a)*x^((2*I)*b)))^p*AppellF1[((-1/2*I)*(1 + m))/b, p
, -p, 1 - ((I/2)*(1 + m))/b, E^((2*I)*a)*x^((2*I)*b), -(E^((2*I)*a)*x^((2*I
)*b))])/(e*(1 + m)*(1 + E^((2*I)*a)*x^((2*I)*b))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
```

```
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int (ex)^m \cot^p(a+b\log(x))\,dx = \int (ex)^m \cot^p(a+b\log(x))\,dx$$

Mathematica [A]
time = 0.71, size = 157, normalized size = 0.97

$$\frac{x(ex)^m \left(1-e^{2ia}x^{2ib}\right)^p \left(1+e^{2ia}x^{2ib}\right)^{-p} \left(\frac{i(1+e^{2ia}x^{2ib})}{-1+e^{2ia}x^{2ib}}\right)^p F_1\left(-\frac{i(1+m)}{2b};p,-p;1-\frac{i(1+m)}{2b};e^{2ia}x^{2ib},-e^{2ia}x^{2ib}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Cot[a + b*Log[x]]^p,x]
[Out] (x*(e*x)^m*(1 - E^((2*I)*a)*x^((2*I)*b))^p*((I*(1 + E^((2*I)*a)*x^((2*I)*b)
))/(-1 + E^((2*I)*a)*x^((2*I)*b)))^p*AppellF1[((-1/2*I)*(1 + m))/b, p, -p,
1 - ((I/2)*(1 + m))/b, E^((2*I)*a)*x^((2*I)*b), -(E^((2*I)*a)*x^((2*I)*b))]
)/((1 + m)*(1 + E^((2*I)*a)*x^((2*I)*b))^p)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\cot^p\left(a+b\ln\left(x\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*cot(a+b*ln(x))^p,x)
[Out] int((e*x)^m*cot(a+b*ln(x))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(a+b*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e)^m*cot(b*log(x) + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(a+b*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral((x*e)^m*cot(b*log(x) + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \cot^p (a + b\log (x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*cot(a+b*ln(x))**p,x)
```

```
[Out] Integral((e*x)**m*cot(a + b*log(x))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(a+b*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*cot(b*log(x) + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{cot}(a+b\,\ln (x))^p\,{(e\,x)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*log(x))^p*(e*x)^m,x)
```

```
[Out] int(cot(a + b*log(x))^p*(e*x)^m, x)
```

3.206 $\int \cot^p(a+\log(x))\,dx$

Optimal. Leaf size=120

$$\left(1-e^{2ia}x^{2i}\right)^p\left(1+e^{2ia}x^{2i}\right)^{-p}\left(-\frac{i(1+e^{2ia}x^{2i})}{1-e^{2ia}x^{2i}}\right)^p xF_1\left(-\frac{i}{2};p,-p;1-\frac{i}{2};e^{2ia}x^{2i},-e^{2ia}x^{2i}\right)$$

```
[Out] (1-exp(2*I*a)*x^(2*I))^p*(-I*(1+exp(2*I*a)*x^(2*I))/(1-exp(2*I*a)*x^(2*I)))
^p*x*AppellF1(-1/2*I,p,-p,1-1/2*I,exp(2*I*a)*x^(2*I),-exp(2*I*a)*x^(2*I))/(
(1+exp(2*I*a)*x^(2*I))^p)
```

Rubi [A]
time = 0.04, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4588, 1986, 441, 440}

$$x\left(1-e^{2ia}x^{2i}\right)^p\left(1+e^{2ia}x^{2i}\right)^{-p}\left(-\frac{i(1+e^{2ia}x^{2i})}{1-e^{2ia}x^{2i}}\right)^p F_1\left(-\frac{i}{2};p,-p;1-\frac{i}{2};e^{2ia}x^{2i},-e^{2ia}x^{2i}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + Log[x]]^p,x]
```

```
[Out] ((1 - E^((2*I)*a)*x^(2*I))^p*(((-I)*(1 + E^((2*I)*a)*x^(2*I)))/(1 - E^((2*I
)*a)*x^(2*I)))^p*x*AppellF1[-1/2*I, p, -p, 1 - I/2, E^((2*I)*a)*x^(2*I), -(
E^((2*I)*a)*x^(2*I))])/(1 + E^((2*I)*a)*x^(2*I))^p
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4588

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[((-I - I*E^(
2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*d)))^p, x] /; FreeQ[{a, b,
d, p}, x]
```

Rubi steps

$$\int \cot^p(a+\log(x))\,dx=\int \cot^p(a+\log(x))\,dx$$

Mathematica [A]
time = 0.56, size = 238, normalized size = 1.98

$$\frac{(2-i)\left(\frac{i(1+e^{2ia}x^{2i})}{-1+e^{2ia}x^{2i}}\right)^p xF_1\left(-\frac{i}{2};p,-p;1-\frac{i}{2};e^{2ia}x^{2i},-e^{2ia}x^{2i}\right)}{(2-i)F_1\left(-\frac{i}{2};p,-p;1-\frac{i}{2};e^{2ia}x^{2i},-e^{2ia}x^{2i}\right)+2e^{2ia}px^{2i}\left(F_1\left(1-\frac{i}{2};p,1-p;2-\frac{i}{2};e^{2ia}x^{2i},-e^{2ia}x^{2i}\right)+F_1\left(1-\frac{i}{2};1+p,-p;2-\frac{i}{2};e^{2ia}x^{2i},-e^{2ia}x^{2i}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[a + Log[x]]^p,x]
```

```
[Out] ((2 - I)*((I*(1 + E^((2*I)*a)*x^(2*I)))/(-1 + E^((2*I)*a)*x^(2*I)))^p*x*App
ellF1[-1/2*I, p, -p, 1 - I/2, E^((2*I)*a)*x^(2*I), -(E^((2*I)*a)*x^(2*I))])
/((2 - I)*AppellF1[-1/2*I, p, -p, 1 - I/2, E^((2*I)*a)*x^(2*I), -(E^((2*I)*
a)*x^(2*I))] + 2*E^((2*I)*a)*p*x^(2*I)*(AppellF1[1 - I/2, p, 1 - p, 2 - I/2
, E^((2*I)*a)*x^(2*I), -(E^((2*I)*a)*x^(2*I))] + AppellF1[1 - I/2, 1 + p, -
p, 2 - I/2, E^((2*I)*a)*x^(2*I), -(E^((2*I)*a)*x^(2*I))]))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \cot^{p}\left(a+\ln\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a+ln(x))^p,x)
```

```
[Out] int(cot(a+ln(x))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(cot(a + log(x))^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(cot(a + log(x))^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cot^{p}\left(a+\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+ln(x))**p,x)
```

```
[Out] Integral(cot(a + log(x))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(cot(a + log(x))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{cot}\left(a+\ln\left(x\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + log(x))^p,x)
```

```
[Out] int(cot(a + log(x))^p, x)
```

3.207 $\int \cot^p(a+2\log(x))\,dx$

Optimal. Leaf size=120

$$\left(1-e^{2ia}x^{4i}\right)^p\left(1+e^{2ia}x^{4i}\right)^{-p}\left(-\frac{i(1+e^{2ia}x^{4i})}{1-e^{2ia}x^{4i}}\right)^p xF_1\left(-\frac{i}{4};p,-p;1-\frac{i}{4};e^{2ia}x^{4i},-e^{2ia}x^{4i}\right)$$

```
[Out] (1-exp(2*I*a)*x^(4*I))^p*(-I*(1+exp(2*I*a)*x^(4*I))/(1-exp(2*I*a)*x^(4*I)))
^p*x*AppellF1(-1/4*I,p,-p,1-1/4*I,exp(2*I*a)*x^(4*I),-exp(2*I*a)*x^(4*I))/(
(1+exp(2*I*a)*x^(4*I))^p)
```

Rubi [A]
time = 0.04, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {4588, 1986, 441, 440}

$$x\left(1-e^{2ia}x^{4i}\right)^p\left(1+e^{2ia}x^{4i}\right)^{-p}\left(-\frac{i(1+e^{2ia}x^{4i})}{1-e^{2ia}x^{4i}}\right)^p F_1\left(-\frac{i}{4};p,-p;1-\frac{i}{4};e^{2ia}x^{4i},-e^{2ia}x^{4i}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + 2*Log[x]]^p,x]
```

```
[Out] ((1 - E^((2*I)*a)*x^(4*I))^p*(((-I)*(1 + E^((2*I)*a)*x^(4*I)))/(1 - E^((2*I
)*a)*x^(4*I)))^p*x*AppellF1[-1/4*I, p, -p, 1 - I/4, E^((2*I)*a)*x^(4*I), -(
E^((2*I)*a)*x^(4*I))])/(1 + E^((2*I)*a)*x^(4*I))^p
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4588

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[((-I - I*E^(
2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*d)))^p, x] /; FreeQ[{a, b,
d, p}, x]
```

Rubi steps

$$\int \cot^p(a+2\log(x))\,dx=\int \cot^p(a+2\log(x))\,dx$$

Mathematica [A]
time = 0.54, size = 238, normalized size = 1.98

$$\frac{(4-i)\left(\frac{i(1+e^{2ia}x^{4i})}{-1+e^{2ia}x^{4i}}\right)^p xF_1\left(-\frac{i}{4};p,-p;1-\frac{i}{4};e^{2ia}x^{4i},-e^{2ia}x^{4i}\right)}{(4-i)F_1\left(-\frac{i}{4};p,-p;1-\frac{i}{4};e^{2ia}x^{4i},-e^{2ia}x^{4i}\right)+4e^{2ia}px^{4i}\left(F_1\left(1-\frac{i}{4};p,1-p;2-\frac{i}{4};e^{2ia}x^{4i},-e^{2ia}x^{4i}\right)+F_1\left(1-\frac{i}{4};1+p,-p;2-\frac{i}{4};e^{2ia}x^{4i},-e^{2ia}x^{4i}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[a + 2*Log[x]]^p,x]
```

```
[Out] ((4 - I)*((I*(1 + E^((2*I)*a)*x^(4*I)))/(-1 + E^((2*I)*a)*x^(4*I)))^p*x*App
ellF1[-1/4*I, p, -p, 1 - I/4, E^((2*I)*a)*x^(4*I), -(E^((2*I)*a)*x^(4*I))])
/((4 - I)*AppellF1[-1/4*I, p, -p, 1 - I/4, E^((2*I)*a)*x^(4*I), -(E^((2*I)*
a)*x^(4*I))] + 4*E^((2*I)*a)*p*x^(4*I)*(AppellF1[1 - I/4, p, 1 - p, 2 - I/4
, E^((2*I)*a)*x^(4*I), -(E^((2*I)*a)*x^(4*I))] + AppellF1[1 - I/4, 1 + p, -
p, 2 - I/4, E^((2*I)*a)*x^(4*I), -(E^((2*I)*a)*x^(4*I))]))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \cot^{p}\left(a+2\ln\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a+2*ln(x))^p,x)
```

```
[Out] int(cot(a+2*ln(x))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+2*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(cot(a + 2*log(x))^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+2*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(cot(a + 2*log(x))^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cot^{p}\left(a + 2\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+2*ln(x))**p,x)
```

```
[Out] Integral(cot(a + 2*log(x))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+2*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(cot(a + 2*log(x))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{cot}\left(a+2\,\ln\left(x\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + 2*log(x))^p,x)
```

```
[Out] int(cot(a + 2*log(x))^p, x)
```

3.208 $\int \cot^p(a+3\log(x))\,dx$

Optimal. Leaf size=120

$$\left(1-e^{2ia}x^{6i}\right)^p\left(1+e^{2ia}x^{6i}\right)^{-p}\left(-\frac{i(1+e^{2ia}x^{6i})}{1-e^{2ia}x^{6i}}\right)^p xF_1\left(-\frac{i}{6};p,-p;1-\frac{i}{6};e^{2ia}x^{6i},-e^{2ia}x^{6i}\right)$$

```
[Out] (1-exp(2*I*a)*x^(6*I))^p*(-I*(1+exp(2*I*a)*x^(6*I))/(1-exp(2*I*a)*x^(6*I)))
^p*x*AppellF1(-1/6*I,p,-p,1-1/6*I,exp(2*I*a)*x^(6*I),-exp(2*I*a)*x^(6*I))/(
(1+exp(2*I*a)*x^(6*I))^p)
```

Rubi [A]
time = 0.04, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {4588, 1986, 441, 440}

$$x\left(1-e^{2ia}x^{6i}\right)^p\left(1+e^{2ia}x^{6i}\right)^{-p}\left(-\frac{i(1+e^{2ia}x^{6i})}{1-e^{2ia}x^{6i}}\right)^p F_1\left(-\frac{i}{6};p,-p;1-\frac{i}{6};e^{2ia}x^{6i},-e^{2ia}x^{6i}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + 3*Log[x]]^p,x]
```

```
[Out] ((1 - E^((2*I)*a)*x^(6*I))^p*(((-I)*(1 + E^((2*I)*a)*x^(6*I)))/(1 - E^((2*I
)*a)*x^(6*I)))^p*x*AppellF1[-1/6*I, p, -p, 1 - I/6, E^((2*I)*a)*x^(6*I), -(
E^((2*I)*a)*x^(6*I))])/(1 + E^((2*I)*a)*x^(6*I))^p
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4588

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[((-I - I*E^(
2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*d)))^p, x] /; FreeQ[{a, b,
d, p}, x]
```

Rubi steps

$$\int \cot^p(a+3\log(x))\,dx = \int \cot^p(a+3\log(x))\,dx$$

Mathematica [A]
time = 0.54, size = 238, normalized size = 1.98

$$\frac{(6-i)\left(\frac{i(1+e^{2ia}x^{6i})}{-1+e^{2ia}x^{6i}}\right)^p xF_1\left(-\frac{i}{6};p,-p;1-\frac{i}{6};e^{2ia}x^{6i},-e^{2ia}x^{6i}\right)}{(6-i)F_1\left(-\frac{i}{6};p,-p;1-\frac{i}{6};e^{2ia}x^{6i},-e^{2ia}x^{6i}\right)+6e^{2ia}px^{6i}\left(F_1\left(1-\frac{i}{6};p,1-p;2-\frac{i}{6};e^{2ia}x^{6i},-e^{2ia}x^{6i}\right)+F_1\left(1-\frac{i}{6};1+p,-p;2-\frac{i}{6};e^{2ia}x^{6i},-e^{2ia}x^{6i}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[a + 3*Log[x]]^p,x]
```

```
[Out] ((6 - I)*((I*(1 + E^((2*I)*a)*x^(6*I)))/(-1 + E^((2*I)*a)*x^(6*I)))^p*x*App
ellF1[-1/6*I, p, -p, 1 - I/6, E^((2*I)*a)*x^(6*I), -(E^((2*I)*a)*x^(6*I))])
/((6 - I)*AppellF1[-1/6*I, p, -p, 1 - I/6, E^((2*I)*a)*x^(6*I), -(E^((2*I)*
a)*x^(6*I))] + 6*E^((2*I)*a)*p*x^(6*I)*(AppellF1[1 - I/6, p, 1 - p, 2 - I/6
, E^((2*I)*a)*x^(6*I), -(E^((2*I)*a)*x^(6*I))] + AppellF1[1 - I/6, 1 + p, -
p, 2 - I/6, E^((2*I)*a)*x^(6*I), -(E^((2*I)*a)*x^(6*I))]))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \cot^p\left(a+3\ln\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a+3*ln(x))^p,x)
```

```
[Out] int(cot(a+3*ln(x))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+3*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(cot(a + 3*log(x))^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+3*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(cot(a + 3*log(x))^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cot^{p}\left(a+3\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+3*ln(x))**p,x)
```

```
[Out] Integral(cot(a + 3*log(x))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+3*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(cot(a + 3*log(x))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{cot}\left(a+3\,\ln\left(x\right)\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + 3*log(x))^p,x)
```

```
[Out] int(cot(a + 3*log(x))^p, x)
```

3.209 $\int x^3 \cot(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=70

$$\frac{ix^4}{4} - \frac{1}{2}ix^4\,{}_2F_1\left(1,-\frac{2i}{bdn};1-\frac{2i}{bdn};e^{2iad}(cx^n)^{2ibd}\right)$$

```
[Out] 1/4*I*x^4-1/2*I*x^4*hypergeom([1, -2*I/b/d/n],[1-2*I/b/d/n],exp(2*I*a*d)*(c
*x^n)^(2*I*b*d))
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {4594, 4592, 470, 371}

$$\frac{ix^4}{4} - \frac{1}{2}ix^4\,{}_2F_1\left(1,-\frac{2i}{bdn};1-\frac{2i}{bdn};e^{2iad}(cx^n)^{2ibd}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cot[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (I/4)*x^4 - (I/2)*x^4*Hypergeometric2F1[1, (-2*I)/(b*d*n), 1 - (2*I)/(b*d*n
), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4594

```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^3 \cot\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int x^3 \cot\left(d(a+b\log\left(cx^n\right))\right)\,dx$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 220 vs. $2(70)=140$.
time = 5.68, size = 220, normalized size = 3.14

$$-\frac{x^4\left(2e^{2id(a+b\log(cx^n))}\,{}_2F_1\left(1,1-\frac{2i}{bdn};2-\frac{2i}{bdn};e^{2id(a+b\log(cx^n))}\right)+(-2i+bdn)\left(\cot\left(d(a+b\log\left(cx^n\right))\right)-\cot\left(d(a-bn\log(x)+b\log\left(cx^n\right))\right)+i\,{}_2F_1\left(1,-\frac{2i}{bdn};1-\frac{2i}{bdn};e^{2id(a+b\log(cx^n))}\right)+\csc\left(d(a+b\log\left(cx^n\right))\right)\csc\left(d(a-bn\log(x)+b\log\left(cx^n\right))\right)\sin(bdn\log(x))\right)\right)}{-8i+4bdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cot[d*(a + b*Log[c*x^n])],x]
```

```
[Out] -((x^4*(2*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - (2*I)/(b*
d*n), 2 - (2*I)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + (-2*I + b*d*n)*(
Cot[d*(a + b*Log[c*x^n])] - Cot[d*(a - b*n*Log[x] + b*Log[c*x^n])] + I*Hype
rgeometric2F1[1, (-2*I)/(b*d*n), 1 - (2*I)/(b*d*n), E^((2*I)*d*(a + b*Log[c
*x^n]))] + Csc[d*(a + b*Log[c*x^n])]*Csc[d*(a - b*n*Log[x] + b*Log[c*x^n])]
*Sin[b*d*n*Log[x]])))/(-8*I + 4*b*d*n))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^3 \cot\left(d(a+b\ln\left(c\,x^n\right))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cot(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x^3*cot(d*(a+b*ln(c*x^n))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(x^3*cot((b*log(c*x^n) + a)*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(x^3*cot(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cot (ad + bd \log (cx^n))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cot(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x**3*cot(a*d + b*d*log(c*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(x^3*cot((b*log(c*x^n) + a)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \cot(d\,(a + b\,\ln(c\,x^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cot(d*(a + b*log(c*x^n))),x)
```

```
[Out] int(x^3*cot(d*(a + b*log(c*x^n))), x)
```

3.210 $\int x^2 \cot(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=74

$$\frac{ix^3}{3} - \frac{2}{3}ix^3\,{}_2F_1\left(1,-\frac{3i}{2bdn};1-\frac{3i}{2bdn};e^{2iad}(cx^n)^{2ibd}\right)$$

```
[Out] 1/3*I*x^3-2/3*I*x^3*hypergeom([1, -3/2*I/b/d/n],[1-3/2*I/b/d/n],exp(2*I*a*d
)*(c*x^n)^(2*I*b*d))
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {4594, 4592, 470, 371}

$$\frac{ix^3}{3} - \frac{2}{3}ix^3\,{}_2F_1\left(1,-\frac{3i}{2bdn};1-\frac{3i}{2bdn};e^{2iad}(cx^n)^{2ibd}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cot[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (I/3)*x^3 - ((2*I)/3)*x^3*Hypergeometric2F1[1, ((-3*I)/2)/(b*d*n), 1 - ((3*
I)/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4594

```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^2 \cot\left(d(a + b \log\left(cx^n\right))\right)\,dx = \int x^2 \cot\left(d(a + b \log\left(cx^n\right))\right)\,dx$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 229 vs. $2(74) = 148$.
time = 6.18, size = 229, normalized size = 3.09

$$-\frac{x^3\left(3e^{2id(a+b\log(cx^n))}\,{}_2F_1\left(1,1-\frac{3i}{2bdn};2-\frac{3i}{2bdn};e^{2id(a+b\log(cx^n))}\right)+(-3i+2bdn)\left(\cot\left(d(a+b\log\left(cx^n\right))\right)-\cot\left(d(a-bn\log(x)+b\log\left(cx^n\right))\right)+i\,{}_2F_1\left(1,-\frac{3i}{2bdn};1-\frac{3i}{2bdn};e^{2id(a+b\log(cx^n))}\right)+\csc\left(d(a+b\log\left(cx^n\right))\right)\csc\left(d(a-bn\log(x)+b\log\left(cx^n\right))\right)\sin(bdn\log(x))\right)\right)}{-9i+6bdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cot[d*(a + b*Log[c*x^n])],x]
```

```
[Out] -((x^3*(3*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - ((3*I)/2)
/(b*d*n), 2 - ((3*I)/2)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + (-3*I +
2*b*d*n)*(Cot[d*(a + b*Log[c*x^n])] - Cot[d*(a - b*n*Log[x] + b*Log[c*x^n])
] + I*Hypergeometric2F1[1, ((-3*I)/2)/(b*d*n), 1 - ((3*I)/2)/(b*d*n), E^((2
*I)*d*(a + b*Log[c*x^n]))] + Csc[d*(a + b*Log[c*x^n])]*Csc[d*(a - b*n*Log[x
] + b*Log[c*x^n])]*Sin[b*d*n*Log[x]])))/(-9*I + 6*b*d*n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^2 \cot\left(d(a + b \ln\left(c\,x^n\right))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cot(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x^2*cot(d*(a+b*ln(c*x^n))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(x^2*cot((b*log(c*x^n) + a)*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(x^2*cot(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cot\left(ad + bd \log\left(cx^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cot(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x**2*cot(a*d + b*d*log(c*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(x^2*cot((b*log(c*x^n) + a)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \cot(d\left(a + b\ \ln\left(c\,x^n\right)\right))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cot(d*(a + b*log(c*x^n))),x)
```

```
[Out] int(x^2*cot(d*(a + b*log(c*x^n))), x)
```

3.211 $\int x \cot(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=68

$$\frac{ix^2}{2}-ix^2\,{}_2F_1\left(1,-\frac{i}{bdn};1-\frac{i}{bdn};e^{2iad}(cx^n)^{2ibd}\right)$$

```
[Out] 1/2*I*x^2-I*x^2*hypergeom([1, -I/b/d/n],[1-I/b/d/n],exp(2*I*a*d)*(c*x^n)^(2
*I*b*d))
```

Rubi [A]
time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4594, 4592, 470, 371}

$$\frac{ix^2}{2}-ix^2\,{}_2F_1\left(1,-\frac{i}{bdn};1-\frac{i}{bdn};e^{2iad}(cx^n)^{2ibd}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cot[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (I/2)*x^2 - I*x^2*Hypergeometric2F1[1, (-I)/(b*d*n), 1 - I/(b*d*n), E^((2*I
)*a*d)*(c*x^n)^((2*I)*b*d)]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4594

```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x\cot\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\cot\left(d(a+b\log\left(cx^n\right))\right)\,dx$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 219 vs. $2(68)=136$.
time = 6.18, size = 219, normalized size = 3.22

$$-\frac{x^2\left(e^{2id(a+b\log(cx^n))}\,{}_2F_1\left(1,1-\frac{i}{bdn};2-\frac{i}{bdn};e^{2id(a+b\log(cx^n))}\right)+(-i+bdn)\left(\cot\left(d(a+b\log\left(cx^n\right))\right)-\cot\left(d(a-bn\log(x)+b\log\left(cx^n\right))\right)+i\,{}_2F_1\left(1,-\frac{i}{bdn};1-\frac{i}{bdn};e^{2id(a+b\log(cx^n))}\right)+\csc\left(d(a+b\log\left(cx^n\right))\right)\csc\left(d(a-bn\log(x)+b\log\left(cx^n\right))\right)\sin(bdn\log(x))\right)\right)}{-2i+2bdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cot[d*(a + b*Log[c*x^n])],x]
```

```
[Out] -((x^2*(E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - I/(b*d*n),
2 - I/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + (-I + b*d*n)*(Cot[d*(a + b
*Log[c*x^n])] - Cot[d*(a - b*n*Log[x] + b*Log[c*x^n])] + I*Hypergeometric2F
1[1, (-I)/(b*d*n), 1 - I/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + Csc[d*(
a + b*Log[c*x^n])]*Csc[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sin[b*d*n*Log[x]]
)))/(-2*I + 2*b*d*n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x\cot\left(d(a+b\ln\left(c\,x^n\right))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x*cot(d*(a+b*ln(c*x^n))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(x*cot((b*log(c*x^n) + a)*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(x*cot(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\cot\left(ad+bd\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x*cot(a*d + b*d*log(c*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(x*cot((b*log(c*x^n) + a)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\cot(d\left(a+b\,\ln\left(c\,x^{n}\right)\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(d*(a + b*log(c*x^n))),x)
```

```
[Out] int(x*cot(d*(a + b*log(c*x^n))), x)
```

3.212 $\int \cot(d(a + b\log(cx^n)))\,dx$

Optimal. Leaf size=66

$$ix - 2ix\,{}_2F_1\left(1, -\frac{i}{2bdn}; 1 - \frac{i}{2bdn}; e^{2iad}(cx^n)^{2ibd}\right)$$

```
[Out] I*x-2*I*x*hypergeom([1, -1/2*I/b/d/n],[1-1/2*I/b/d/n],exp(2*I*a*d)*(c*x^n)^
(2*I*b*d))
```

Rubi [A]
time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {4590, 4592, 470, 371}

$$ix - 2ix\,{}_2F_1\left(1, -\frac{i}{2bdn}; 1 - \frac{i}{2bdn}; e^{2iad}(cx^n)^{2ibd}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[d*(a + b*Log[c*x^n])],x]
```

```
[Out] I*x - (2*I)*x*Hypergeometric2F1[1, (-1/2*I)/(b*d*n), 1 - (I/2)/(b*d*n), E^(
(2*I)*a*d)*(c*x^n)^((2*I)*b*d)]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 4590

```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Cot[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int \cot (d(a+b\log (cx^n)))\ dx=\int \cot (d(a+b\log (cx^n)))\ dx$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 141 vs. $2(66) = 132$.
time = 10.91, size = 141, normalized size = 2.14

$$x\left(-\frac{e^{2id(a+b\log(cx^n))}\,{}_2F_1\left(1,1-\frac{i}{2bdn};2-\frac{i}{2bdn};e^{2id(a+b\log(cx^n))}\right)}{-i+2bdn}-i\,{}_2F_1\left(1,-\frac{i}{2bdn};1-\frac{i}{2bdn};e^{2id(a+b\log(cx^n))}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[d*(a + b*Log[c*x^n])],x]
```

```
[Out] x*(-((E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - (I/2)/(b*d*n)
, 2 - (I/2)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))])/(-I + 2*b*d*n)) - I*H
ypergeometric2F1[1, (-1/2*I)/(b*d*n), 1 - (I/2)/(b*d*n), E^((2*I)*d*(a + b*
Log[c*x^n]))])
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \cot (d(a+b\ln (c\,x^n)))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(cot(d*(a+b*ln(c*x^n))),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(cot((b*log(c*x^n) + a)*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n))),x, algorithm="fricas")
[Out] integral(cot(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cot\left(d(a + b\log\left(cx^n\right))\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*ln(c*x**n))),x)
[Out] Integral(cot(d*(a + b*log(c*x**n))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(cot((b*log(c*x^n) + a)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(d\left(a + b\,\ln\left(c\,x^n\right)\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*(a + b*log(c*x^n))),x)
[Out] int(cot(d*(a + b*log(c*x^n))), x)
```

3.213 $\int \frac{\cot(d(a+b\log(cx^n)))}{x}\,dx$

Optimal. Leaf size=25

$$\frac{\log\left(\sin\left(ad+bd\log\left(cx^n\right)\right)\right)}{bdn}$$

```
[Out] ln(sin(a*d+b*d*ln(c*x^n)))/b/d/n
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{3556\}$

$$\frac{\log\left(\sin\left(ad+bd\log\left(cx^n\right)\right)\right)}{bdn}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] Log[Sin[a*d + b*d*Log[c*x^n]]]/(b*d*n)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \cot(d(a+bx))\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= \frac{\log\left(\sin\left(ad+bd\log\left(cx^n\right)\right)\right)}{bdn}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 40, normalized size = 1.60

$$\frac{\log\left(\cos\left(d\left(a+b\log\left(cx^n\right)\right)\right)\right)+\log\left(\tan\left(ad+bd\log\left(cx^n\right)\right)\right)}{bdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] (Log[Cos[d*(a + b*Log[c*x^n])]] + Log[Tan[a*d + b*d*Log[c*x^n]]])/(b*d*n)
```

Maple [A]
time = 0.06, size = 30, normalized size = 1.20

method	result
derivativedivides	$-\frac{\ln(\cot^2(d(a+b\ln(c\,x^n)))+1)}{2nbd}$
default	$-\frac{\ln(\cot^2(d(a+b\ln(c\,x^n)))+1)}{2nbd}$
risch	$i\ln(x)-\frac{2ia}{nb}-\frac{2i\ln(c)}{n}-\frac{2i\ln(x^n)}{n}-\frac{\pi\mathrm{csgn}(ic\,x^n)^3}{n}+\frac{\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ic)}{n}+\frac{\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ix^n)}{n}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*(a+b*ln(c*x^n)))/x,x,method=_RETURNVERBOSE)
[Out] -1/2/n/b/d*ln(cot(d*(a+b*ln(c*x^n)))^2+1)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.96

$$\frac{\log\left(\sin\left((b\log\left(cx^n\right)+a)d\right)\right)}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
[Out] log(sin((b*log(c*x^n) + a)*d))/(b*d*n)
```

Fricas [A]
time = 2.66, size = 35, normalized size = 1.40

$$\frac{\log\left(-\frac{1}{2}\cos\left(2\,bdn\log\left(x\right)+2\,bd\log\left(c\right)+2\,ad\right)+\frac{1}{2}\right)}{2\,bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
[Out] 1/2*log(-1/2*cos(2*b*d*n*log(x) + 2*b*d*log(c) + 2*a*d) + 1/2)/(b*d*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(20) = 40.
time = 2.56, size = 46, normalized size = 1.84

$$\begin{cases}\log\left(x\right)\cot\left(ad\right) & \text{for}\: b=0\\\tilde{\infty}\log\left(x\right) & \text{for}\: d=0\\\log\left(x\right)\cot\left(ad+bd\log\left(c\right)\right) & \text{for}\: n=0\\\frac{\log\left(\sin\left(ad+bd\log\left(cx^n\right)\right)\right)}{bdn} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*ln(c*x**n)))/x,x)
```

```
[Out] Piecewise((log(x)*cot(a*d), Eq(b, 0)), (zoo*log(x), Eq(d, 0)), (log(x)*cot(
a*d + b*d*log(c)), Eq(n, 0)), (log(sin(a*d + b*d*log(c*x**n)))/(b*d*n), Tru
e))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 3.80, size = 37, normalized size = 1.48

$$-\ln(x)\ 1\mathrm{i} + \frac{\ln\left(\mathrm{e}^{a\,d\,2\mathrm{i}}\,(c\,x^n)^{b\,d\,2\mathrm{i}} - 1\right)}{b\,d\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*(a + b*log(c*x^n)))/x,x)
```

```
[Out] log(exp(a*d*2i)*(c*x^n)^(b*d*2i) - 1)/(b*d*n) - log(x)*1i
```

3.214 $\int \frac{\cot(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal. Leaf size=70

$$-\frac{i}{x}+\frac{2i\,{}_2F_1\left(1,\frac{i}{2bdn};1+\frac{i}{2bdn};e^{2iad}(cx^n)^{2ibd}\right)}{x}$$

```
[Out] -I/x+2*I*hypergeom([1, 1/2*I/b/d/n],[1+1/2*I/b/d/n],exp(2*I*a*d)*(c*x^n)^(2
*I*b*d))/x
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4594, 4592, 470, 371}

$$\frac{2i\,{}_2F_1\left(1,\frac{i}{2bdn};1+\frac{i}{2bdn};e^{2iad}(cx^n)^{2ibd}\right)}{x}-\frac{i}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] (-I)/x + ((2*I)*Hypergeometric2F1[1, (I/2)/(b*d*n), 1 + (I/2)/(b*d*n), E^((
2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/x
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4594

```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\cot(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\cot(d(a+b\log(cx^n)))}{x^2}\,dx$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 217 vs. $2(70) = 140$.
time = 5.18, size = 217, normalized size = 3.10

$$\frac{\cot(d(a+b\log(cx^n))) - \cot(d(a-bn\log(x)+b\log(cx^n))) - \frac{e^{2id(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{i}{2bdn};2+\frac{i}{2bdn};e^{2id(a+b\log(cx^n))}\right)}{i+2bdn} + i\,{}_2F_1\left(1,\frac{i}{2bdn};1+\frac{i}{2bdn};e^{2id(a+b\log(cx^n))}\right) + \csc(d(a+b\log(cx^n)))\csc(d(a-bn\log(x)+b\log(cx^n)))\sin(bdn\log(x))}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] (Cot[d*(a + b*Log[c*x^n])] - Cot[d*(a - b*n*Log[x] + b*Log[c*x^n])] - (E^((
2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + (I/2)/(b*d*n), 2 + (I/2
)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))])/(I + 2*b*d*n) + I*Hypergeometri
c2F1[1, (I/2)/(b*d*n), 1 + (I/2)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] +
 Csc[d*(a + b*Log[c*x^n])]*Csc[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sin[b*d*n
*Log[x]])/x
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\cot(d(a+b\ln(c\,x^n)))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*(a+b*ln(c*x^n)))/x^2,x)
```

```
[Out] int(cot(d*(a+b*ln(c*x^n)))/x^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(cot((b*log(c*x^n) + a)*d)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

```
[Out] integral(cot(b*d*log(c*x^n) + a*d)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(ad + bd\log\left(cx^{n}\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*ln(c*x**n)))/x**2,x)
```

```
[Out] Integral(cot(a*d + b*d*log(c*x**n))/x**2, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot\left(d\left(a+b\,\ln\left(c\,x^{n}\right)\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] int(cot(d*(a + b*log(c*x^n)))/x^2, x)
```

3.215 $\int \frac{\cot(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal. Leaf size=68

$$-\frac{i}{2x^2}+\frac{i\,{}_2F_1\left(1,\frac{i}{bdn};1+\frac{i}{bdn};e^{2iad}(cx^n)^{2ibd}\right)}{x^2}$$

```
[Out] -1/2*I/x^2+I*hypergeom([1, I/b/d/n],[1+I/b/d/n],exp(2*I*a*d)*(c*x^n)^(2*I*b
*d))/x^2
```

Rubi [A]
time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4594, 4592, 470, 371}

$$\frac{i\,{}_2F_1\left(1,\frac{i}{bdn};1+\frac{i}{bdn};e^{2iad}(cx^n)^{2ibd}\right)}{x^2}-\frac{i}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] (-1/2*I)/x^2 + (I*Hypergeometric2F1[1, I/(b*d*n), 1 + I/(b*d*n), E^((2*I)*a
*d)*(c*x^n)^((2*I)*b*d)])/x^2
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4594

```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\cot(d(a+b\log(cx^n)))}{x^3}\,dx=\int \frac{\cot(d(a+b\log(cx^n)))}{x^3}\,dx$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 211 vs. $2(68)=136$.
time = 4.58, size = 211, normalized size = 3.10

$$\frac{\cot(d(a+b\log(cx^n)))-\cot(d(a-bn\log(x)+b\log(cx^n)))-\frac{e^{2id(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{i}{bdn};2+\frac{i}{bdn};e^{2id(a+b\log(cx^n))}\right)}{i+bdn}+i\,{}_2F_1\left(1,\frac{i}{bdn};1+\frac{i}{bdn};e^{2id(a+b\log(cx^n))}\right)+\csc(d(a+b\log(cx^n)))\csc(d(a-bn\log(x)+b\log(cx^n)))\sin(bdn\log(x))}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] (Cot[d*(a + b*Log[c*x^n])] - Cot[d*(a - b*n*Log[x] + b*Log[c*x^n])] - (E^((
2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + I/(b*d*n), 2 + I/(b*d*n
), E^((2*I)*d*(a + b*Log[c*x^n]))])/(I + b*d*n) + I*Hypergeometric2F1[1, I/
(b*d*n), 1 + I/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + Csc[d*(a + b*Log[
c*x^n])]*Csc[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sin[b*d*n*Log[x]])/(2*x^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\cot(d(a+b\ln(c\,x^n)))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*(a+b*ln(c*x^n)))/x^3,x)
```

```
[Out] int(cot(d*(a+b*ln(c*x^n)))/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

```
[Out] integrate(cot((b*log(c*x^n) + a)*d)/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

```
[Out] integral(cot(b*d*log(c*x^n) + a*d)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(ad + bd\log\left(cx^{n}\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*ln(c*x**n)))/x**3,x)
```

```
[Out] Integral(cot(a*d + b*d*log(c*x**n))/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
```

```
[Out] integrate(cot((b*log(c*x^n) + a)*d)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(d\left(a + b\,\ln\left(c\,x^{n}\right)\right))}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] int(cot(d*(a + b*log(c*x^n)))/x^3, x)
```

3.216 $\int x^3 \cot^2\left(d(a+b\log\left(cx^n\right))\right)\,dx$

Optimal. Leaf size=158

$$\frac{(4i-bdn)x^4}{4bdn}+\frac{ix^4\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bdn\left(1-e^{2iad}\left(cx^n\right)^{2ibd}\right)}-\frac{2ix^4\,{}_2F_1\left(1,-\frac{2i}{bdn};1-\frac{2i}{bdn};e^{2iad}(cx^n)^{2ibd}\right)}{bdn}$$

```
[Out] 1/4*(4*I-b*d*n)*x^4/b/d/n+I*x^4*(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/(1
-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))-2*I*x^4*hypergeom([1, -2*I/b/d/n],[1-2*I/b
/d/n],exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n
```

Rubi [A]
time = 0.13, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {4594, 4592, 516, 470, 371}

$$-\frac{2ix^4\,{}_2F_1\left(1,-\frac{2i}{bdn};1-\frac{2i}{bdn};e^{2iad}(cx^n)^{2ibd}\right)}{bdn}+\frac{ix^4\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bdn\left(1-e^{2iad}\left(cx^n\right)^{2ibd}\right)}+\frac{x^4(-bdn+4i)}{4bdn}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cot[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] ((4*I - b*d*n)*x^4)/(4*b*d*n) + (I*x^4*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*
d)))/(b*d*n*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - ((2*I)*x^4*Hypergeom
etric2F1[1, (-2*I)/(b*d*n), 1 - (2*I)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)
*b*d)])/(b*d*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4594

```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^3 \cot^2(d(a+b\log(cx^n)))\,dx = \int x^3 \cot^2(d(a+b\log(cx^n)))\,dx$$

Mathematica [A]
time = 5.13, size = 175, normalized size = 1.11

$$-\frac{x^4\left(8e^{2id(a+b\log(cx^n))}\,{}_2F_1\left(1,1-\frac{2i}{bdn};2-\frac{2i}{bdn};e^{2id(a+b\log(cx^n))}\right)+(-2i+bdn)\left(bdn+4\cot(d(a+b\log(cx^n)))+4i\,{}_2F_1\left(1,-\frac{2i}{bdn};1-\frac{2i}{bdn};e^{2id(a+b\log(cx^n))}\right)\right)\right)}{4bdn(-2i+bdn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cot[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] -1/4*(x^4*(8*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - (2*I)/
(b*d*n), 2 - (2*I)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + (-2*I + b*d*n
)*(b*d*n + 4*Cot[d*(a + b*Log[c*x^n])] + (4*I)*Hypergeometric2F1[1, (-2*I)/
(b*d*n), 1 - (2*I)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))])))/(b*d*n*(-2*I
 + b*d*n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^3\left(\cot^2(d(a+b\ln(c\,x^n)))\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cot(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(x^3*cot(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] 1/4*((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x^4*cos(2*b*d*lo
g(x^n) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x
^4*sin(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n*x^4 - 2*(b*d*n*cos(2*b*d*log(c)) -
 4*sin(2*b*d*log(c)))*x^4*cos(2*b*d*log(x^n) + 2*a*d) + 2*(b*d*n*sin(2*b*d*
log(c)) + 4*cos(2*b*d*log(c)))*x^4*sin(2*b*d*log(x^n) + 2*a*d) - 16*(2*b^2*
d^2*n^2*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*sin(2
*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) - b^2*d^2*n^2 - (b^2*d^2*cos(2*b*d
*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) + 2*a*d)^2
 - (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*sin(2*b*
d*log(x^n) + 2*a*d)^2)*integrate((x^3*cos(b*d*log(x^n) + a*d)*sin(b*d*log(c
)) + x^3*cos(b*d*log(c))*sin(b*d*log(x^n) + a*d))/(2*b^2*d^2*n^2*cos(b*d*lo
g(c))*cos(b*d*log(x^n) + a*d) - 2*b^2*d^2*n^2*sin(b*d*log(c))*sin(b*d*log(x
^n) + a*d) + b^2*d^2*n^2 + (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*sin(b*d*log
(c))^2)*n^2*cos(b*d*log(x^n) + a*d)^2 + (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^
2*sin(b*d*log(c))^2)*n^2*sin(b*d*log(x^n) + a*d)^2), x) + 16*(2*b^2*d^2*n^2
*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*sin(2*b*d*lo
g(c))*sin(2*b*d*log(x^n) + 2*a*d) - b^2*d^2*n^2 - (b^2*d^2*cos(2*b*d*log(c)
)^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) + 2*a*d)^2 - (b^2
*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*sin(2*b*d*log(x
^n) + 2*a*d)^2)*integrate(-(x^3*cos(b*d*log(x^n) + a*d)*sin(b*d*log(c)) + x
^3*cos(b*d*log(c))*sin(b*d*log(x^n) + a*d))/(2*b^2*d^2*n^2*cos(b*d*log(c))*
cos(b*d*log(x^n) + a*d) - 2*b^2*d^2*n^2*sin(b*d*log(c))*sin(b*d*log(x^n) +
a*d) - b^2*d^2*n^2 - (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*sin(b*d*log(c))^2
)*n^2*cos(b*d*log(x^n) + a*d)^2 - (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*sin(
b*d*log(c))^2)*n^2*sin(b*d*log(x^n) + a*d)^2), x))/(2*b*d*n*cos(2*b*d*log(c
))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b*d*n*sin(2*b*d*log(c))*sin(2*b*d*log(x^
n) + 2*a*d) - (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*cos(2*b
*d*log(x^n) + 2*a*d)^2 - (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2
)*n*sin(2*b*d*log(x^n) + 2*a*d)^2 - b*d*n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*cot(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cot(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \cot(d\,(a+b\,\ln(c\,x^n)))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cot(d*(a + b*log(c*x^n)))^2,x)
```

```
[Out] int(x^3*cot(d*(a + b*log(c*x^n)))^2, x)
```

3.217 $\int x^2 \cot^2(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=162

$$\frac{(3i-bdn)x^3}{3bdn}+\frac{ix^3\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bdn\left(1-e^{2iad}(cx^n)^{2ibd}\right)}-\frac{2ix^3\,{}_2F_1\left(1,-\frac{3i}{2bdn};1-\frac{3i}{2bdn};e^{2iad}(cx^n)^{2ibd}\right)}{bdn}$$

```
[Out] 1/3*(3*I-b*d*n)*x^3/b/d/n+I*x^3*(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/(1
-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))-2*I*x^3*hypergeom([1, -3/2*I/b/d/n],[1-3/2
*I/b/d/n],exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n
```

Rubi [A]

time = 0.12, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {4594, 4592, 516, 470, 371}

$$-\frac{2ix^3\,{}_2F_1\left(1,-\frac{3i}{2bdn};1-\frac{3i}{2bdn};e^{2iad}(cx^n)^{2ibd}\right)}{bdn}+\frac{ix^3\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bdn\left(1-e^{2iad}(cx^n)^{2ibd}\right)}+\frac{x^3(-bdn+3i)}{3bdn}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cot[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] ((3*I - b*d*n)*x^3)/(3*b*d*n) + (I*x^3*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*
d)))/(b*d*n*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - ((2*I)*x^3*Hypergeom
etric2F1[1, ((-3*I)/2)/(b*d*n), 1 - ((3*I)/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n
)^((2*I)*b*d)])/(b*d*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4594

```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^2 \cot^2(d(a+b\log(cx^n)))\,dx = \int x^2 \cot^2(d(a+b\log(cx^n)))\,dx$$

Mathematica [A]
time = 5.80, size = 185, normalized size = 1.14

$$-\frac{x^3\left(9e^{2id(a+b\log(cx^n))}\,{}_2F_1\left(1,1-\frac{3i}{2bdn};2-\frac{3i}{2bdn};e^{2id(a+b\log(cx^n))}\right)+(-3i+2bdn)\left(bdn+3\cot(d(a+b\log(cx^n)))+3i\,{}_2F_1\left(1,-\frac{3i}{2bdn};1-\frac{3i}{2bdn};e^{2id(a+b\log(cx^n))}\right)\right)\right)}{3bdn(-3i+2bdn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cot[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] -1/3*(x^3*(9*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - ((3*I)
/2)/(b*d*n), 2 - ((3*I)/2)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + (-3*I
 + 2*b*d*n)*(b*d*n + 3*Cot[d*(a + b*Log[c*x^n])] + (3*I)*Hypergeometric2F1[
1, ((-3*I)/2)/(b*d*n), 1 - ((3*I)/2)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n])
)])))/(b*d*n*(-3*I + 2*b*d*n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^2\left(\cot^2(d(a+b\ln(c\,x^n)))\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cot(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(x^2*cot(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x^3*cos(2*b*d*lo
g(x^n) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x
^3*sin(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n*x^3 - 2*(b*d*n*cos(2*b*d*log(c)) -
 3*sin(2*b*d*log(c)))*x^3*cos(2*b*d*log(x^n) + 2*a*d) + 2*(b*d*n*sin(2*b*d*
log(c)) + 3*cos(2*b*d*log(c)))*x^3*sin(2*b*d*log(x^n) + 2*a*d) - 9*(2*b^2*d
^2*n^2*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*sin(2*
b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) - b^2*d^2*n^2 - (b^2*d^2*cos(2*b*d*
log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) + 2*a*d)^2
- (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*sin(2*b*d
*log(x^n) + 2*a*d)^2)*integrate((x^2*cos(b*d*log(x^n) + a*d)*sin(b*d*log(c)
) + x^2*cos(b*d*log(c))*sin(b*d*log(x^n) + a*d))/(2*b^2*d^2*n^2*cos(b*d*log
(c))*cos(b*d*log(x^n) + a*d) - 2*b^2*d^2*n^2*sin(b*d*log(c))*sin(b*d*log(x^
n) + a*d) + b^2*d^2*n^2 + (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*sin(b*d*log(
c))^2)*n^2*cos(b*d*log(x^n) + a*d)^2 + (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2
*sin(b*d*log(c))^2)*n^2*sin(b*d*log(x^n) + a*d)^2), x) + 9*(2*b^2*d^2*n^2*c
os(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*sin(2*b*d*log(
c))*sin(2*b*d*log(x^n) + 2*a*d) - b^2*d^2*n^2 - (b^2*d^2*cos(2*b*d*log(c))^
2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) + 2*a*d)^2 - (b^2*d
^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*sin(2*b*d*log(x^n
) + 2*a*d)^2)*integrate(-(x^2*cos(b*d*log(x^n) + a*d)*sin(b*d*log(c)) + x^2
*cos(b*d*log(c))*sin(b*d*log(x^n) + a*d))/(2*b^2*d^2*n^2*cos(b*d*log(c))*co
s(b*d*log(x^n) + a*d) - 2*b^2*d^2*n^2*sin(b*d*log(c))*sin(b*d*log(x^n) + a*
d) - b^2*d^2*n^2 - (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*sin(b*d*log(c))^2)*
n^2*cos(b*d*log(x^n) + a*d)^2 - (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*sin(b*
d*log(c))^2)*n^2*sin(b*d*log(x^n) + a*d)^2), x))/(2*b*d*n*cos(2*b*d*log(c))
*cos(2*b*d*log(x^n) + 2*a*d) - 2*b*d*n*sin(2*b*d*log(c))*sin(2*b*d*log(x^n)
 + 2*a*d) - (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*cos(2*b*d
*log(x^n) + 2*a*d)^2 - (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*
n*sin(2*b*d*log(x^n) + 2*a*d)^2 - b*d*n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*cot(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cot^2 \left(ad + bd \log \left(cx^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cot(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Integral(x**2*cot(a*d + b*d*log(c*x**n))**2, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \cot(d\left(a + b\,\ln\left(c\,x^n\right)\right))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cot(d*(a + b*log(c*x^n)))^2,x)
```

```
[Out] int(x^2*cot(d*(a + b*log(c*x^n)))^2, x)
```

3.218 $\int x \cot^2(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=158

$$\frac{(2i-bdn)x^2}{2bdn}+\frac{ix^2\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bdn\left(1-e^{2iad}(cx^n)^{2ibd}\right)}-\frac{2ix^2\,{}_2F_1\left(1,-\frac{i}{bdn};1-\frac{i}{bdn};e^{2iad}(cx^n)^{2ibd}\right)}{bdn}$$

```
[Out] 1/2*(2*I-b*d*n)*x^2/b/d/n+I*x^2*(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/(1
-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))-2*I*x^2*hypergeom([1, -I/b/d/n],[1-I/b/d/n
],exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n
```

Rubi [A]
time = 0.12, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {4594, 4592, 516, 470, 371}

$$-\frac{2ix^2\,{}_2F_1\left(1,-\frac{i}{bdn};1-\frac{i}{bdn};e^{2iad}(cx^n)^{2ibd}\right)}{bdn}+\frac{ix^2\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bdn\left(1-e^{2iad}(cx^n)^{2ibd}\right)}+\frac{x^2(-bdn+2i)}{2bdn}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cot[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] ((2*I - b*d*n)*x^2)/(2*b*d*n) + (I*x^2*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*
d)))/(b*d*n*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - ((2*I)*x^2*Hypergeom
etric2F1[1, (-I)/(b*d*n), 1 - I/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)]
)/(b*d*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4594

```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x\cot^2\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\cot^2\left(d(a+b\log\left(cx^n\right))\right)\,dx$$

Mathematica [A]
time = 5.66, size = 175, normalized size = 1.11

$$-\frac{x^2\left(2e^{2id(a+b\log(cx^n))}\,{}_2F_1\left(1,1-\frac{i}{bdn};2-\frac{i}{bdn};e^{2id(a+b\log(cx^n))}\right)+(-i+bdn)\left(bdn+2\cot\left(d(a+b\log\left(cx^n\right))\right)+2i\,{}_2F_1\left(1,-\frac{i}{bdn};1-\frac{i}{bdn};e^{2id(a+b\log(cx^n))}\right)\right)\right)}{2bdn(-i+bdn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cot[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] -1/2*(x^2*(2*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - I/(b*d
*n), 2 - I/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + (-I + b*d*n)*(b*d*n +
 2*Cot[d*(a + b*Log[c*x^n])] + (2*I)*Hypergeometric2F1[1, (-I)/(b*d*n), 1 -
 I/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))])))/(b*d*n*(-I + b*d*n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x\left(\cot^2\left(d(a+b\ln\left(c\,x^n\right))\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(x*cot(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x^2*cos(2*b*d*lo
g(x^n) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x
^2*sin(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n*x^2 - 2*(b*d*n*cos(2*b*d*log(c)) -
 2*sin(2*b*d*log(c)))*x^2*cos(2*b*d*log(x^n) + 2*a*d) + 2*(b*d*n*sin(2*b*d*
log(c)) + 2*cos(2*b*d*log(c)))*x^2*sin(2*b*d*log(x^n) + 2*a*d) - 4*(2*b^2*d
^2*n^2*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*sin(2*
b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) - b^2*d^2*n^2 - (b^2*d^2*cos(2*b*d*
log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) + 2*a*d)^2
- (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*sin(2*b*d
*log(x^n) + 2*a*d)^2)*integrate((x*cos(b*d*log(x^n) + a*d)*sin(b*d*log(c))
+ x*cos(b*d*log(c))*sin(b*d*log(x^n) + a*d))/(2*b^2*d^2*n^2*cos(b*d*log(c))
*cos(b*d*log(x^n) + a*d) - 2*b^2*d^2*n^2*sin(b*d*log(c))*sin(b*d*log(x^n) +
 a*d) + b^2*d^2*n^2 + (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*sin(b*d*log(c))^
2)*n^2*cos(b*d*log(x^n) + a*d)^2 + (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*sin
(b*d*log(c))^2)*n^2*sin(b*d*log(x^n) + a*d)^2), x) + 4*(2*b^2*d^2*n^2*cos(2
*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*sin(2*b*d*log(c))*
sin(2*b*d*log(x^n) + 2*a*d) - b^2*d^2*n^2 - (b^2*d^2*cos(2*b*d*log(c))^2 +
b^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) + 2*a*d)^2 - (b^2*d^2*c
os(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*sin(2*b*d*log(x^n) +
2*a*d)^2)*integrate(-(x*cos(b*d*log(x^n) + a*d)*sin(b*d*log(c)) + x*cos(b*d
*log(c))*sin(b*d*log(x^n) + a*d))/(2*b^2*d^2*n^2*cos(b*d*log(c))*cos(b*d*lo
g(x^n) + a*d) - 2*b^2*d^2*n^2*sin(b*d*log(c))*sin(b*d*log(x^n) + a*d) - b^2
*d^2*n^2 - (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*sin(b*d*log(c))^2)*n^2*cos(
b*d*log(x^n) + a*d)^2 - (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*sin(b*d*log(c)
)^2)*n^2*sin(b*d*log(x^n) + a*d)^2), x))/(2*b*d*n*cos(2*b*d*log(c))*cos(2*b
*d*log(x^n) + 2*a*d) - 2*b*d*n*sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d
) - (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*cos(2*b*d*log(x^n
) + 2*a*d)^2 - (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*sin(2*
b*d*log(x^n) + 2*a*d)^2 - b*d*n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(x*cot(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\cot^2\left(ad+bd\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Integral(x*cot(a*d + b*d*log(c*x**n))**2, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{cot}{\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(d*(a + b*log(c*x^n)))^2,x)
```

```
[Out] int(x*cot(d*(a + b*log(c*x^n)))^2, x)
```

3.219 $\int \cot^2(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=153

$$\frac{(i-bdn)x}{bdn}+\frac{ix\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bdn\left(1-e^{2iad}(cx^n)^{2ibd}\right)}-\frac{2ix\,{}_2F_1\left(1,-\frac{i}{2bdn};1-\frac{i}{2bdn};e^{2iad}(cx^n)^{2ibd}\right)}{bdn}$$

```
[Out] (I-b*d*n)*x/b/d/n+I*x*(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/(1-exp(2*I*a
*d)*(c*x^n)^(2*I*b*d))-2*I*x*hypergeom([1, -1/2*I/b/d/n],[1-1/2*I/b/d/n],ex
p(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n
```

Rubi [A]
time = 0.11, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4590, 4592, 516, 470, 371}

$$-\frac{2ix\,{}_2F_1\left(1,-\frac{i}{2bdn};1-\frac{i}{2bdn};e^{2iad}(cx^n)^{2ibd}\right)}{bdn}+\frac{ix\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bdn\left(1-e^{2iad}(cx^n)^{2ibd}\right)}+\frac{x(-bdn+i)}{bdn}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] ((I - b*d*n)*x)/(b*d*n) + (I*x*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*
d*n*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - ((2*I)*x*Hypergeometric2F1[1
, (-1/2*I)/(b*d*n), 1 - (I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/
(b*d*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 4590

```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Cot[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int \cot^2\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int \cot^2\left(d(a+b\log\left(cx^n\right))\right)\,dx$$

Mathematica [A]
time = 9.98, size = 178, normalized size = 1.16

$$-\frac{x\left(e^{2id(a+b\log(cx^n))}\,{}_2F_1\left(1,1-\frac{i}{2bdn};2-\frac{i}{2bdn};e^{2id(a+b\log(cx^n))}\right)+(-i+2bdn)\left(bdn+\cot\left(d(a+b\log\left(cx^n\right))\right)+i\,{}_2F_1\left(1,-\frac{i}{2bdn};1-\frac{i}{2bdn};e^{2id(a+b\log(cx^n))}\right)\right)\right)}{bdn(-i+2bdn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[d*(a + b*Log[c*x^n])]^2,x]
[Out] -((x*(E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - (I/2)/(b*d*n)
, 2 - (I/2)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + (-I + 2*b*d*n)*(b*d*
n + Cot[d*(a + b*Log[c*x^n])] + I*Hypergeometric2F1[1, (-1/2*I)/(b*d*n), 1
- (I/2)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))])))/(b*d*n*(-I + 2*b*d*n)))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \cot^2\left(d(a+b\ln\left(c\,x^n\right))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(cot(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] ((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x*cos(2*b*d*log(x^n)
 + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x*sin(2
*b*d*log(x^n) + 2*a*d)^2 + b*d*n*x - 2*(b*d*n*cos(2*b*d*log(c)) - sin(2*b*d
*log(c)))*x*cos(2*b*d*log(x^n) + 2*a*d) + 2*(b*d*n*sin(2*b*d*log(c)) + cos(
2*b*d*log(c)))*x*sin(2*b*d*log(x^n) + 2*a*d) - (2*b^2*d^2*n^2*cos(2*b*d*log
(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*sin(2*b*d*log(c))*sin(2*b*
d*log(x^n) + 2*a*d) - b^2*d^2*n^2 - (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*
sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) + 2*a*d)^2 - (b^2*d^2*cos(2*b*d
*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*sin(2*b*d*log(x^n) + 2*a*d)^2
)*integrate((cos(b*d*log(x^n) + a*d)*sin(b*d*log(c)) + cos(b*d*log(c))*sin(
b*d*log(x^n) + a*d))/(2*b^2*d^2*n^2*cos(b*d*log(c))*cos(b*d*log(x^n) + a*d)
 - 2*b^2*d^2*n^2*sin(b*d*log(c))*sin(b*d*log(x^n) + a*d) + b^2*d^2*n^2 + (b
^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*sin(b*d*log(c))^2)*n^2*cos(b*d*log(x^n)
+ a*d)^2 + (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*sin(b*d*log(c))^2)*n^2*sin(
b*d*log(x^n) + a*d)^2), x) + (2*b^2*d^2*n^2*cos(2*b*d*log(c))*cos(2*b*d*log
(x^n) + 2*a*d) - 2*b^2*d^2*n^2*sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d
) - b^2*d^2*n^2 - (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^
2)*n^2*cos(2*b*d*log(x^n) + 2*a*d)^2 - (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d
^2*sin(2*b*d*log(c))^2)*n^2*sin(2*b*d*log(x^n) + 2*a*d)^2)*integrate(-(cos(
b*d*log(x^n) + a*d)*sin(b*d*log(c)) + cos(b*d*log(c))*sin(b*d*log(x^n) + a*
d))/(2*b^2*d^2*n^2*cos(b*d*log(c))*cos(b*d*log(x^n) + a*d) - 2*b^2*d^2*n^2*
sin(b*d*log(c))*sin(b*d*log(x^n) + a*d) - b^2*d^2*n^2 - (b^2*d^2*cos(b*d*lo
g(c))^2 + b^2*d^2*sin(b*d*log(c))^2)*n^2*cos(b*d*log(x^n) + a*d)^2 - (b^2*d
^2*cos(b*d*log(c))^2 + b^2*d^2*sin(b*d*log(c))^2)*n^2*sin(b*d*log(x^n) + a*
d)^2), x))/(2*b*d*n*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b*d*n
*sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) - (b*d*cos(2*b*d*log(c))^2 +
 b*d*sin(2*b*d*log(c))^2)*n*cos(2*b*d*log(x^n) + 2*a*d)^2 - (b*d*cos(2*b*d*
log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*sin(2*b*d*log(x^n) + 2*a*d)^2 - b*d*
n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(cot(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cot^2\left(d(a+b\log\left(cx^n\right))\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Integral(cot(d*(a + b*log(c*x**n)))**2, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(d\left(a+b\,\ln\left(c\,x^n\right)\right))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*(a + b*log(c*x^n)))^2,x)
```

```
[Out] int(cot(d*(a + b*log(c*x^n)))^2, x)
```

3.220 $\int \frac{\cot^2(d(a+b\log(cx^n)))}{x}\,dx$

Optimal. Leaf size=30

$$-\frac{\cot(ad+bd\log(cx^n))}{bdn}-\log(x)$$

[Out] -cot(a*d+b*d*ln(c*x^n))/b/d/n-ln(x)

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3554, 8}

$$-\frac{\cot(ad+bd\log(cx^n))}{bdn}-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[d*(a + b*Log[c*x^n])]^2/x,x]
```

```
[Out] -(Cot[a*d + b*d*Log[c*x^n]]/(b*d*n)) - Log[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^2(d(a+b\log(cx^n)))}{x}\,dx &= \frac{\text{Subst}\left(\int \cot^2(d(a+bx))\,dx, x, \log(cx^n)\right)}{n}\\
&= -\frac{\cot(ad+bd\log(cx^n))}{bdn}-\frac{\text{Subst}\left(\int 1\,dx, x, \log(cx^n)\right)}{n}\\
&= -\frac{\cot(ad+bd\log(cx^n))}{bdn}-\log(x)
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.13, size = 51, normalized size = 1.70

$$-\frac{\cot(ad+bd\log(cx^n))\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(ad+bd\log(cx^n))\right)}{bdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[d*(a + b*Log[c*x^n])]^2/x,x]
```

```
[Out] -((Cot[a*d + b*d*Log[c*x^n]]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[a*d + b*d
*Log[c*x^n]]^2])/(b*d*n))
```

Maple [A]
time = 0.04, size = 46, normalized size = 1.53

method	result
derivativedivides	$\frac{-\cot(d(a+b\ln(c\,x^{n})))+\frac{\pi}{2}-\operatorname{arccot}(\cot(d(a+b\ln(c\,x^{n}))))}{nbd}$
default	$\frac{-\cot(d(a+b\ln(c\,x^{n})))+\frac{\pi}{2}-\operatorname{arccot}(\cot(d(a+b\ln(c\,x^{n}))))}{nbd}$
risch	$-\ln(x)-\frac{2i}{dbn\Big(\mathrm{e}^{id\big(-ib\pi\,\mathrm{csgn}(ic\,x^{n})^{3}+ib\pi\,\mathrm{csgn}(ic\,x^{n})^{2}\mathrm{csgn}(ic)+ib\pi\,\mathrm{csgn}(ic\,x^{n})^{2}\mathrm{csgn}(ix^{n})-ib\pi\,\mathrm{csgn}(ic\,x^{n})\mathrm{csgn}(ic)\mathrm{csgn}(ix^{n})+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*(a+b*ln(c*x^n)))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b/d*(-cot(d*(a+b*ln(c*x^n)))+1/2*Pi-arccot(cot(d*(a+b*ln(c*x^n)))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(30) = 60$.
time = 0.29, size = 322, normalized size = 10.73

$$\frac{(bd\cos(2\,bd\log(c))^{2}+bd\sin(2\,bd\log(c))^{2})n\cos(2\,bd\log(x^{n})+2\,ad)^{2}\log(x)+(bd\cos(2\,bd\log(c))^{2}+bd\sin(2\,bd\log(c))^{2})n\log(x)\sin(2\,bd\log(x^{n})+2\,ad)^{2}+bdn\log(x)-2\,(bdn\cos(2\,bd\log(c))\log(x)-\sin(2\,bd\log(c)))\cos(2\,bd\log(x^{n})+2\,ad)+2\,(bdn\log(x)\sin(2\,bd\log(c))+\cos(2\,bd\log(c)))\sin(2\,bd\log(x^{n})+2\,ad)}{2\,bdn\cos(2\,bd\log(c))\cos(2\,bd\log(x^{n})+2\,ad)-2\,bdn\sin(2\,bd\log(c))\sin(2\,bd\log(x^{n})+2\,ad)-(bd\cos(2\,bd\log(c))^{2}+bd\sin(2\,bd\log(c))^{2})n\cos(2\,bd\log(x^{n})+2\,ad)^{2}-(bd\cos(2\,bd\log(c))^{2}+bd\sin(2\,bd\log(c))^{2})n\sin(2\,bd\log(x^{n})+2\,ad)^{2}-bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n)))^2/x,x, algorithm="maxima")
```

```
[Out] ((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*cos(2*b*d*log(x^n) +
 2*a*d)^2*log(x) + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*lo
g(x)*sin(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n*log(x) - 2*(b*d*n*cos(2*b*d*log(
c))*log(x) - sin(2*b*d*log(c)))*cos(2*b*d*log(x^n) + 2*a*d) + 2*(b*d*n*log(
x)*sin(2*b*d*log(c)) + cos(2*b*d*log(c)))*sin(2*b*d*log(x^n) + 2*a*d))/(2*b
*d*n*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b*d*n*sin(2*b*d*log(
c))*sin(2*b*d*log(x^n) + 2*a*d) - (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*
log(c))^2)*n*cos(2*b*d*log(x^n) + 2*a*d)^2 - (b*d*cos(2*b*d*log(c))^2 + b*d
*sin(2*b*d*log(c))^2)*n*sin(2*b*d*log(x^n) + 2*a*d)^2 - b*d*n)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(30) = 60$.
time = 3.18, size = 78, normalized size = 2.60

$$-\frac{bdn\log(x)\sin(2\,bdn\log(x)+2\,bd\log(c)+2\,ad)+\cos(2\,bdn\log(x)+2\,bd\log(c)+2\,ad)+1}{bdn\sin(2\,bdn\log(x)+2\,bd\log(c)+2\,ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n)))^2/x,x, algorithm="fricas")
```

```
[Out] -(b*d*n*log(x)*sin(2*b*d*n*log(x) + 2*b*d*log(c) + 2*a*d) + cos(2*b*d*n*log
(x) + 2*b*d*log(c) + 2*a*d) + 1)/(b*d*n*sin(2*b*d*n*log(x) + 2*b*d*log(c) +
 2*a*d))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2\left(ad + bd\log\left(cx^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*ln(c*x**n)))**2/x,x)
```

```
[Out] Integral(cot(a*d + b*d*log(c*x**n))**2/x, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n)))^2/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 3.86, size = 39, normalized size = 1.30

$$-\ln\left(x\right)-\frac{2\mathrm{i}}{b\,d\,n\,\left({\mathrm{e}}^{a\,d\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,d\,2\mathrm{i}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*(a + b*log(c*x^n)))^2/x,x)
```

```
[Out] - log(x) - 2i/(b*d*n*(exp(a*d*2i)*(c*x^n)^(b*d*2i) - 1))
```

3.221 $\int \frac{\cot^2(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal. Leaf size=156

$$\frac{1+\frac{i}{bdn}}{x}+\frac{i\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bdnx\left(1-e^{2iad}\left(cx^n\right)^{2ibd}\right)}-\frac{2i\,{}_2F_1\left(1,\frac{i}{2bdn};1+\frac{i}{2bdn};e^{2iad}(cx^n)^{2ibd}\right)}{bdnx}$$

```
[Out] (1+I/b/d/n)/x+I*(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/x/(1-exp(2*I*a*d)*
(c*x^n)^(2*I*b*d))-2*I*hypergeom([1, 1/2*I/b/d/n],[1+1/2*I/b/d/n],exp(2*I*a
*d)*(c*x^n)^(2*I*b*d))/b/d/n/x
```

Rubi [A]

time = 0.11, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {4594, 4592, 516, 470, 371}

$$-\frac{2i\,{}_2F_1\left(1,\frac{i}{2bdn};1+\frac{i}{2bdn};e^{2iad}(cx^n)^{2ibd}\right)}{bdnx}+\frac{i\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bdnx\left(1-e^{2iad}\left(cx^n\right)^{2ibd}\right)}+\frac{1+\frac{i}{bdn}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[d*(a + b*Log[c*x^n])]^2/x^2,x]
```

```
[Out] (1 + I/(b*d*n))/x + (I*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*d*n*x*(1
 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - ((2*I)*Hypergeometric2F1[1, (I/2)/
(b*d*n), 1 + (I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(b*d*n*x)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
```

```
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4594

```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\cot^2(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\cot^2(d(a+b\log(cx^n)))}{x^2}\,dx$$

Mathematica [A]
time = 4.88, size = 181, normalized size = 1.16

$$\frac{e^{2id(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{i}{2bdn};2+\frac{i}{2bdn};e^{2id(a+b\log(cx^n))}\right)+(i+2bdn)\left(bdn-\cot(d(a+b\log(cx^n)))-i\,{}_2F_1\left(1,\frac{i}{2bdn};1+\frac{i}{2bdn};e^{2id(a+b\log(cx^n))}\right)\right)}{bdn(i+2bdn)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[d*(a + b*Log[c*x^n])]^2/x^2,x]
```

```
[Out] (E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + (I/2)/(b*d*n), 2 +
 (I/2)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + (I + 2*b*d*n)*(b*d*n - Co
t[d*(a + b*Log[c*x^n])] - I*Hypergeometric2F1[1, (I/2)/(b*d*n), 1 + (I/2)/(
b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))]))/(b*d*n*(I + 2*b*d*n)*x)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(d(a+b\ln(c\,x^n)))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*(a+b*ln(c*x^n)))^2/x^2,x)
```

```
[Out] int(cot(d*(a+b*ln(c*x^n)))^2/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="maxima")
```

```
[Out] -((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*cos(2*b*d*log(x^n)
+ 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*sin(2*b*
d*log(x^n) + 2*a*d)^2 + b*d*n - 2*(b*d*n*cos(2*b*d*log(c)) + sin(2*b*d*log(
c)))*cos(2*b*d*log(x^n) + 2*a*d) - (2*b^2*d^2*n^2*x*cos(2*b*d*log(c))*cos(2
*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*x*sin(2*b*d*log(c))*sin(2*b*d*log(x^
n) + 2*a*d) - b^2*d^2*n^2*x - (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*
b*d*log(c))^2)*n^2*x*cos(2*b*d*log(x^n) + 2*a*d)^2 - (b^2*d^2*cos(2*b*d*log
(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*x*sin(2*b*d*log(x^n) + 2*a*d)^2)*
integrate((cos(b*d*log(x^n) + a*d)*sin(b*d*log(c)) + cos(b*d*log(c))*sin(b*
d*log(x^n) + a*d))/(2*b^2*d^2*n^2*x^2*cos(b*d*log(c))*cos(b*d*log(x^n) + a*
d) - 2*b^2*d^2*n^2*x^2*sin(b*d*log(c))*sin(b*d*log(x^n) + a*d) + b^2*d^2*n^
2*x^2 + (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*sin(b*d*log(c))^2)*n^2*x^2*cos
(b*d*log(x^n) + a*d)^2 + (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*sin(b*d*log(c
))^2)*n^2*x^2*sin(b*d*log(x^n) + a*d)^2), x) + (2*b^2*d^2*n^2*x*cos(2*b*d*l
og(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*x*sin(2*b*d*log(c))*sin(
2*b*d*log(x^n) + 2*a*d) - b^2*d^2*n^2*x - (b^2*d^2*cos(2*b*d*log(c))^2 + b^
2*d^2*sin(2*b*d*log(c))^2)*n^2*x*cos(2*b*d*log(x^n) + 2*a*d)^2 - (b^2*d^2*c
os(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*x*sin(2*b*d*log(x^n)
+ 2*a*d)^2)*integrate(-(cos(b*d*log(x^n) + a*d)*sin(b*d*log(c)) + cos(b*d*l
og(c))*sin(b*d*log(x^n) + a*d))/(2*b^2*d^2*n^2*x^2*cos(b*d*log(c))*cos(b*d*
log(x^n) + a*d) - 2*b^2*d^2*n^2*x^2*sin(b*d*log(c))*sin(b*d*log(x^n) + a*d)
 - b^2*d^2*n^2*x^2 - (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*sin(b*d*log(c))^2
)*n^2*x^2*cos(b*d*log(x^n) + a*d)^2 - (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*
sin(b*d*log(c))^2)*n^2*x^2*sin(b*d*log(x^n) + a*d)^2), x) + 2*(b*d*n*sin(2*
b*d*log(c)) - cos(2*b*d*log(c)))*sin(2*b*d*log(x^n) + 2*a*d))/(2*b*d*n*x*co
s(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b*d*n*x*sin(2*b*d*log(c))*s
in(2*b*d*log(x^n) + 2*a*d) - (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c
))^2)*n*x*cos(2*b*d*log(x^n) + 2*a*d)^2 - (b*d*cos(2*b*d*log(c))^2 + b*d*si
n(2*b*d*log(c))^2)*n*x*sin(2*b*d*log(x^n) + 2*a*d)^2 - b*d*n*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(cot(b*d*log(c*x^n) + a*d)^2/x^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2\left(ad + bd\log\left(cx^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*ln(c*x**n)))**2/x**2,x)
```

```
[Out] Integral(cot(a*d + b*d*log(c*x**n))**2/x**2, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cot\left(d\left(a+b\,\ln\left(c\,x^n\right)\right)\right)}^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*(a + b*log(c*x^n)))^2/x^2,x)
```

```
[Out] int(cot(d*(a + b*log(c*x^n)))^2/x^2, x)
```

3.222 $\int \frac{\cot^2(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal. Leaf size=155

$$\frac{1+\frac{2i}{bdn}}{2x^2}+\frac{i\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bdnx^2\left(1-e^{2iad}\left(cx^n\right)^{2ibd}\right)}-\frac{2i\,{}_2F_1\left(1,\frac{i}{bdn};1+\frac{i}{bdn};e^{2iad}(cx^n)^{2ibd}\right)}{bdnx^2}$$

[Out] 1/2*(1+2*I/b/d/n)/x^2+I*(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/x^2/(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))-2*I*hypergeom([1, I/b/d/n],[1+I/b/d/n],exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/x^2

Rubi [A]
time = 0.12, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.263$, Rules used = {4594, 4592, 516, 470, 371}

$$-\frac{2i\,{}_2F_1\left(1,\frac{i}{bdn};1+\frac{i}{bdn};e^{2iad}(cx^n)^{2ibd}\right)}{bdnx^2}+\frac{i\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bdnx^2\left(1-e^{2iad}\left(cx^n\right)^{2ibd}\right)}+\frac{1+\frac{2i}{bdn}}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[Cot[d*(a + b*Log[c*x^n])]^2/x^3,x]

[Out] (1 + (2*I)/(b*d*n))/(2*x^2) + (I*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*d*n*x^2*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - ((2*I)*Hypergeometric2F1[1, I/(b*d*n), 1 + I/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(b*d*n*x^2)

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4594

```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\cot^2(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\cot^2(d(a+b\log(cx^n)))}{x^3}\,dx$$

Mathematica [A]
time = 4.36, size = 175, normalized size = 1.13

$$\frac{2e^{2id(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{i}{bdn};2+\frac{i}{bdn};e^{2id(a+b\log(cx^n))}\right)+(i+bdn)\left(bdn-2\cot\left(d(a+b\log(cx^n))\right)-2i\,{}_2F_1\left(1,\frac{i}{bdn};1+\frac{i}{bdn};e^{2id(a+b\log(cx^n))}\right)\right)}{2bdn(i+bdn)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[d*(a + b*Log[c*x^n])]^2/x^3,x]
```

```
[Out] (2*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + I/(b*d*n), 2 + I
/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + (I + b*d*n)*(b*d*n - 2*Cot[d*(a
 + b*Log[c*x^n])] - (2*I)*Hypergeometric2F1[1, I/(b*d*n), 1 + I/(b*d*n), E^
((2*I)*d*(a + b*Log[c*x^n]))]))/(2*b*d*n*(I + b*d*n)*x^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(d(a+b\ln(c\,x^n)))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*(a+b*ln(c*x^n)))^2/x^3,x)
```

```
[Out] int(cot(d*(a+b*ln(c*x^n)))^2/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*cos(2*b*d*log(x
^n) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*sin(
2*b*d*log(x^n) + 2*a*d)^2 + b*d*n - 2*(b*d*n*cos(2*b*d*log(c)) + 2*sin(2*b*
d*log(c)))*cos(2*b*d*log(x^n) + 2*a*d) - 4*(2*b^2*d^2*n^2*x^2*cos(2*b*d*log
(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*x^2*sin(2*b*d*log(c))*sin(
2*b*d*log(x^n) + 2*a*d) - b^2*d^2*n^2*x^2 - (b^2*d^2*cos(2*b*d*log(c))^2 +
b^2*d^2*sin(2*b*d*log(c))^2)*n^2*x^2*cos(2*b*d*log(x^n) + 2*a*d)^2 - (b^2*d
^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*x^2*sin(2*b*d*log
(x^n) + 2*a*d)^2)*integrate((cos(b*d*log(x^n) + a*d)*sin(b*d*log(c)) + cos(
b*d*log(c))*sin(b*d*log(x^n) + a*d))/(2*b^2*d^2*n^2*x^3*cos(b*d*log(c))*cos
(b*d*log(x^n) + a*d) - 2*b^2*d^2*n^2*x^3*sin(b*d*log(c))*sin(b*d*log(x^n) +
 a*d) + b^2*d^2*n^2*x^3 + (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*sin(b*d*log(
c))^2)*n^2*x^3*cos(b*d*log(x^n) + a*d)^2 + (b^2*d^2*cos(b*d*log(c))^2 + b^2
*d^2*sin(b*d*log(c))^2)*n^2*x^3*sin(b*d*log(x^n) + a*d)^2), x) + 4*(2*b^2*d
^2*n^2*x^2*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*x^
2*sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) - b^2*d^2*n^2*x^2 - (b^2*d^
2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*x^2*cos(2*b*d*log(
x^n) + 2*a*d)^2 - (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^
2)*n^2*x^2*sin(2*b*d*log(x^n) + 2*a*d)^2)*integrate(-(cos(b*d*log(x^n) + a*
d)*sin(b*d*log(c)) + cos(b*d*log(c))*sin(b*d*log(x^n) + a*d))/(2*b^2*d^2*n^
2*x^3*cos(b*d*log(c))*cos(b*d*log(x^n) + a*d) - 2*b^2*d^2*n^2*x^3*sin(b*d*l
og(c))*sin(b*d*log(x^n) + a*d) - b^2*d^2*n^2*x^3 - (b^2*d^2*cos(b*d*log(c))
^2 + b^2*d^2*sin(b*d*log(c))^2)*n^2*x^3*cos(b*d*log(x^n) + a*d)^2 - (b^2*d^
2*cos(b*d*log(c))^2 + b^2*d^2*sin(b*d*log(c))^2)*n^2*x^3*sin(b*d*log(x^n) +
 a*d)^2), x) + 2*(b*d*n*sin(2*b*d*log(c)) - 2*cos(2*b*d*log(c)))*sin(2*b*d*
log(x^n) + 2*a*d))/(2*b*d*n*x^2*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*
d) - 2*b*d*n*x^2*sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) - (b*d*cos(2
*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x^2*cos(2*b*d*log(x^n) + 2*a*d)
^2 - (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x^2*sin(2*b*d*lo
g(x^n) + 2*a*d)^2 - b*d*n*x^2)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(cot(b*d*log(c*x^n) + a*d)^2/x^3, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2\left(ad + bd\log\left(cx^n\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*ln(c*x**n)))**2/x**3,x)
```

```
[Out] Integral(cot(a*d + b*d*log(c*x**n))**2/x**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(cot((b*log(c*x^n) + a)*d)^2/x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{cot}\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)}^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*(a + b*log(c*x^n)))^2/x^3,x)
```

```
[Out] int(cot(d*(a + b*log(c*x^n)))^2/x^3, x)
```

3.223 $\int \frac{\cot^3(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=44

$$-\frac{\cot^2\left(a+b\log\left(cx^n\right)\right)}{2bn}-\frac{\log\left(\sin\left(a+b\log\left(cx^n\right)\right)\right)}{bn}$$

```
[Out] -1/2*cot(a+b*ln(c*x^n))^2/b/n-ln(sin(a+b*ln(c*x^n)))/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3554, 3556}

$$-\frac{\log\left(\sin\left(a+b\log\left(cx^n\right)\right)\right)}{bn}-\frac{\cot^2\left(a+b\log\left(cx^n\right)\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] -1/2*Cot[a + b*Log[c*x^n]]^2/(b*n) - Log[Sin[a + b*Log[c*x^n]]]/(b*n)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^3\left(a+b\log\left(cx^n\right)\right)}{x}\,dx&=\frac{\text{Subst}\left(\int \cot^3(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\&=-\frac{\cot^2\left(a+b\log\left(cx^n\right)\right)}{2bn}-\frac{\text{Subst}\left(\int \cot(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\&=-\frac{\cot^2\left(a+b\log\left(cx^n\right)\right)}{2bn}-\frac{\log\left(\sin\left(a+b\log\left(cx^n\right)\right)\right)}{bn}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 52, normalized size = 1.18

$$-\frac{\cot^2\left(a+b\log\left(cx^n\right)\right)+2\log\left(\cos\left(a+b\log\left(cx^n\right)\right)\right)+2\log\left(\tan\left(a+b\log\left(cx^n\right)\right)\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] -1/2*(Cot[a + b*Log[c*x^n]]^2 + 2*Log[Cos[a + b*Log[c*x^n]]] + 2*Log[Tan[a
+ b*Log[c*x^n]]])/(b*n)
```

Maple [A]
time = 0.08, size = 42, normalized size = 0.95

method	result
derivativedivides	$\frac{-\frac{\left(\cot^2(a+b\ln(c\,x^n))\right)}{2}+\frac{\ln\left(\cot^2(a+b\ln(c\,x^n))+1\right)}{2}}{nb}$
default	$\frac{-\frac{\left(\cot^2(a+b\ln(c\,x^n))\right)}{2}+\frac{\ln\left(\cot^2(a+b\ln(c\,x^n))+1\right)}{2}}{nb}$
risch	$-i\ln(x)+\frac{\pi\mathrm{csgn}(ic\,x^n)^3}{n}-\frac{\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ic)}{n}-\frac{\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ix^n)}{n}+\frac{\pi\,\mathrm{csgn}(ic\,x^n)\mathrm{csgn}(ic)\mathrm{csgn}(ix^n)}{n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(-1/2*cot(a+b*ln(c*x^n))^2+1/2*ln(cot(a+b*ln(c*x^n))^2+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1713 vs. $2(42) = 84$.
time = 0.34, size = 1713, normalized size = 38.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] -1/2*(8*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*cos(2*b*log(x^n) + 2*a)^2 +
 8*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*sin(2*b*log(x^n) + 2*a)^2 - 4*((
cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*cos(2*b*
log(x^n) + 2*a) + (cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*
b*log(c)))*sin(2*b*log(x^n) + 2*a))*cos(4*b*log(x^n) + 4*a) - 4*cos(2*b*log
(c))*cos(2*b*log(x^n) + 2*a) + ((cos(4*b*log(c))^2 + sin(4*b*log(c))^2)*cos
(4*b*log(x^n) + 4*a)^2 + 4*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*cos(2*b*
log(x^n) + 2*a)^2 + (cos(4*b*log(c))^2 + sin(4*b*log(c))^2)*sin(4*b*log(x^n
) + 4*a)^2 + 4*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*sin(2*b*log(x^n) + 2
*a)^2 - 2*(2*(cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*log
(c)))*cos(2*b*log(x^n) + 2*a) + 2*(cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*
b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) - cos(4*b*log(c)))*cos(4
*b*log(x^n) + 4*a) - 4*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) + 2*(2*(cos(
```

```
2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(
x^n) + 2*a) - 2*(cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*
log(c)))*sin(2*b*log(x^n) + 2*a) - sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a)
 + 4*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + 1)*log((cos(a)^2 + sin(a)^2)
*cos(b*log(c))^2 + (cos(a)^2 + sin(a)^2)*sin(b*log(c))^2 + 2*(cos(b*log(c))
*cos(a) - sin(b*log(c))*sin(a))*cos(b*log(x^n)) + cos(b*log(x^n))^2 - 2*(co
s(a)*sin(b*log(c)) + cos(b*log(c))*sin(a))*sin(b*log(x^n)) + sin(b*log(x^n)
)^2) + ((cos(4*b*log(c))^2 + sin(4*b*log(c))^2)*cos(4*b*log(x^n) + 4*a)^2 +
 4*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*cos(2*b*log(x^n) + 2*a)^2 + (cos
(4*b*log(c))^2 + sin(4*b*log(c))^2)*sin(4*b*log(x^n) + 4*a)^2 + 4*(cos(2*b*
log(c))^2 + sin(2*b*log(c))^2)*sin(2*b*log(x^n) + 2*a)^2 - 2*(2*(cos(4*b*lo
g(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) +
 2*a) + 2*(cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)
))*sin(2*b*log(x^n) + 2*a) - cos(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) - 4*c
os(2*b*log(c))*cos(2*b*log(x^n) + 2*a) + 2*(2*(cos(2*b*log(c))*sin(4*b*log(
c)) - cos(4*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) - 2*(cos(4*b
*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n
) + 2*a) - sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) + 4*sin(2*b*log(c))*sin
(2*b*log(x^n) + 2*a) + 1)*log((cos(a)^2 + sin(a)^2)*cos(b*log(c))^2 + (cos(
a)^2 + sin(a)^2)*sin(b*log(c))^2 - 2*(cos(b*log(c))*cos(a) - sin(b*log(c))*
sin(a))*cos(b*log(x^n)) + cos(b*log(x^n))^2 + 2*(cos(a)*sin(b*log(c)) + cos
(b*log(c))*sin(a))*sin(b*log(x^n)) + sin(b*log(x^n))^2) + 4*((cos(2*b*log(c
))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*
a) - (cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*si
n(2*b*log(x^n) + 2*a))*sin(4*b*log(x^n) + 4*a) + 4*sin(2*b*log(c))*sin(2*b*
log(x^n) + 2*a))/((b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*cos(4*b*log
(x^n) + 4*a)^2 - 4*b*n*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) + 4*(b*cos(2
*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*cos(2*b*log(x^n) + 2*a)^2 + (b*cos(4*
b*log(c))^2 + b*sin(4*b*log(c))^2)*n*sin(4*b*log(x^n) + 4*a)^2 + 4*b*n*sin(
2*b*log(c))*sin(2*b*log(x^n) + 2*a) + 4*(b*cos(2*b*log(c))^2 + b*sin(2*b*lo
g(c))^2)*n*sin(2*b*log(x^n) + 2*a)^2 + b*n + 2*(b*n*cos(4*b*log(c)) - 2*(b*
cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log(c)))*n*cos(
2*b*log(x^n) + 2*a) - 2*(b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(
c))*sin(2*b*log(c)))*n*sin(2*b*log(x^n) + 2*a))*cos(4*b*log(x^n) + 4*a) + 2
*(2*(b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))
*n*cos(2*b*log(x^n) + 2*a) - b*n*sin(4*b*log(c)) - 2*(b*cos(4*b*log(c))*cos
(2*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log(c)))*n*sin(2*b*log(x^n) + 2*a)
)*sin(4*b*log(x^n) + 4*a))
```

Fricas [A]
time = 3.61, size = 70, normalized size = 1.59

$$-\frac{(\cos(2\,bn\log(x)+2\,b\log(c)+2\,a)-1)\log\left(-\frac{1}{2}\,\cos(2\,bn\log(x)+2\,b\log(c)+2\,a)+\frac{1}{2}\right)-2}{2\,(bn\cos(2\,bn\log(x)+2\,b\log(c)+2\,a)-bn)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] -1/2*((cos(2*b*n*log(x) + 2*b*log(c) + 2*a) - 1)*log(-1/2*cos(2*b*n*log(x)
+ 2*b*log(c) + 2*a) + 1/2) - 2)/(b*n*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) -
 b*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(36) = 72$.
time = 4.94, size = 97, normalized size = 2.20

$$\begin{cases} \tilde{\infty}\log(x) & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ \log(x)\cot^3(a) & \text{for } b = 0 \\ \log(x)\cot^3(a + b\log(c)) & \text{for } n = 0 \\ \tilde{\infty}\log(x) & \text{for } a = -b\log(cx^n) \\ \frac{\log(\tan^2(a+b\log(cx^n))+1)}{2bn} - \frac{\log(\tan(a+b\log(cx^n)))}{bn} - \frac{1}{2bn\tan^2(a+b\log(cx^n))} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*ln(c*x**n))**3/x,x)
```

```
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)*cot(a)**3,
Eq(b, 0)), (log(x)*cot(a + b*log(c))**3, Eq(n, 0)), (zoo*log(x), Eq(a, -b*l
og(c*x**n))), (log(tan(a + b*log(c*x**n))**2 + 1)/(2*b*n) - log(tan(a + b*l
og(c*x**n)))/(b*n) - 1/(2*b*n*tan(a + b*log(c*x**n))**2), True))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.69, size = 106, normalized size = 2.41

$$\ln(x)\ 1\mathrm{i} + \frac{2}{b\,n\,\left(1 + \mathrm{e}^{a\,4\mathrm{i}}\,(c\,x^n)^{b\,4\mathrm{i}} - 2\,\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}}\right)} + \frac{2}{b\,n\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}} - 1\right)} - \frac{\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}} - 1\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*log(c*x^n))^3/x,x)
```

```
[Out] log(x)*1i + 2/(b*n*(exp(a*4i)*(c*x^n)^(b*4i) - 2*exp(a*2i)*(c*x^n)^(b*2i) +
 1)) + 2/(b*n*(exp(a*2i)*(c*x^n)^(b*2i) - 1)) - log(exp(a*2i)*(c*x^n)^(b*2i
) - 1)/(b*n)
```

3.224 $\int \frac{\cot^4(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=44

$$\frac{\cot\left(a+b\log\left(cx^n\right)\right)}{bn}-\frac{\cot^3\left(a+b\log\left(cx^n\right)\right)}{3bn}+\log(x)$$

```
[Out] cot(a+b*ln(c*x^n))/b/n-1/3*cot(a+b*ln(c*x^n))^3/b/n+ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3554, 8}

$$-\frac{\cot^3\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{\cot\left(a+b\log\left(cx^n\right)\right)}{bn}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] Cot[a + b*Log[c*x^n]]/(b*n) - Cot[a + b*Log[c*x^n]]^3/(3*b*n) + Log[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^4\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \cot^4(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\
&= -\frac{\cot^3\left(a+b\log\left(cx^n\right)\right)}{3bn}-\frac{\text{Subst}\left(\int \cot^2(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\
&= \frac{\cot\left(a+b\log\left(cx^n\right)\right)}{bn}-\frac{\cot^3\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{\text{Subst}\left(\int 1\,dx,x,\log\left(cx^n\right)\right)}{n}\\
&= \frac{\cot\left(a+b\log\left(cx^n\right)\right)}{bn}-\frac{\cot^3\left(a+b\log\left(cx^n\right)\right)}{3bn}+\log(x)
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.13, size = 46, normalized size = 1.05

$$-\frac{\cot^3\left(a+b\log\left(cx^n\right)\right)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2\left(a+b\log\left(cx^n\right)\right)\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] -1/3*(Cot[a + b*Log[c*x^n]]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[a + b*L
og[c*x^n]]^2])/(b*n)
```

Maple [A]
time = 0.06, size = 50, normalized size = 1.14

method	result
derivativedivides	$\frac{-\frac{\left(\cot^3(a+b\ln(c\,x^n))\right)}{3}+\cot(a+b\ln(c\,x^n))-\frac{\pi}{2}+\operatorname{arccot}(\cot(a+b\ln(c\,x^n)))}{nb}$
default	$\frac{-\frac{\left(\cot^3(a+b\ln(c\,x^n))\right)}{3}+\cot(a+b\ln(c\,x^n))-\frac{\pi}{2}+\operatorname{arccot}(\cot(a+b\ln(c\,x^n)))}{nb}$
risch	$\ln(x)+\frac{4i\left(3(x^n)^{4ib}c^{4ib}e^{2b\pi\operatorname{csgn}(ic\,x^n)^3}e^{-2b\pi\operatorname{csgn}(ic\,x^n)^2\operatorname{csgn}(ic)}e^{-2b\pi\operatorname{csgn}(ic\,x^n)^2\operatorname{csgn}(ix^n)}e^{2b\pi\operatorname{csgn}(ic\,x^n)\operatorname{csgn}(ic)\operatorname{csg}}\right.}{3bn\left((x^n)^{2ib}c^{2ib}e^{b\pi\operatorname{csgn}(ic\,x^n)^3}e^{-b\pi\operatorname{csgn}(ic\,x^n)^2\operatorname{csg}}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a+b*ln(c*x^n))^4/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(-1/3*cot(a+b*ln(c*x^n))^3+cot(a+b*ln(c*x^n))-1/2*Pi+arccot(cot(a+b*l
n(c*x^n))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2172 vs. $2(42) = 84$.
time = 0.36, size = 2172, normalized size = 49.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(b*cos(6*b*log(c))^2 + b*sin(6*b*log(c))^2)*n*cos(6*b*log(x^n) + 6*a
)^2*log(x) + 27*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*cos(4*b*log(x
^n) + 4*a)^2*log(x) + 27*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*cos(
2*b*log(x^n) + 2*a)^2*log(x) + 3*(b*cos(6*b*log(c))^2 + b*sin(6*b*log(c))^2
)*n*log(x)*sin(6*b*log(x^n) + 6*a)^2 + 27*(b*cos(4*b*log(c))^2 + b*sin(4*b*
log(c))^2)*n*log(x)*sin(4*b*log(x^n) + 4*a)^2 + 27*(b*cos(2*b*log(c))^2 + b
```

```
*sin(2*b*log(c))^2)*n*log(x)*sin(2*b*log(x^n) + 2*a)^2 + 3*b*n*log(x) - 2*(
3*b*n*cos(6*b*log(c))*log(x) + 3*(3*(b*cos(6*b*log(c))*cos(4*b*log(c)) + b*
sin(6*b*log(c))*sin(4*b*log(c)))*n*log(x) - 2*cos(4*b*log(c))*sin(6*b*log(c
)) + 2*cos(6*b*log(c))*sin(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) - 3*(3*(b*c
os(6*b*log(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n*log(x
) - 2*cos(2*b*log(c))*sin(6*b*log(c)) + 2*cos(6*b*log(c))*sin(2*b*log(c)))*
cos(2*b*log(x^n) + 2*a) + 3*(3*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6
*b*log(c))*sin(4*b*log(c)))*n*log(x) + 2*cos(6*b*log(c))*cos(4*b*log(c)) +
2*sin(6*b*log(c))*sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) - 3*(3*(b*cos(2*
b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(2*b*log(c)))*n*log(x) + 2
*cos(6*b*log(c))*cos(2*b*log(c)) + 2*sin(6*b*log(c))*sin(2*b*log(c)))*sin(2
*b*log(x^n) + 2*a) - 4*sin(6*b*log(c)))*cos(6*b*log(x^n) + 6*a) + 6*(3*b*n*
cos(4*b*log(c))*log(x) - 9*(b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*l
og(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n) + 2*a)*log(x) - 9*(b*cos(2*b*log
(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*n*log(x)*sin(2*b*
log(x^n) + 2*a) - 2*sin(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) - 6*(3*b*n*cos
(2*b*log(c))*log(x) - 2*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 2*(3*b*n
*log(x)*sin(6*b*log(c)) + 3*(3*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6
*b*log(c))*sin(4*b*log(c)))*n*log(x) + 2*cos(6*b*log(c))*cos(4*b*log(c)) +
2*sin(6*b*log(c))*sin(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) - 3*(3*(b*cos(2*
b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(2*b*log(c)))*n*log(x) + 2
*cos(6*b*log(c))*cos(2*b*log(c)) + 2*sin(6*b*log(c))*sin(2*b*log(c)))*cos(2
*b*log(x^n) + 2*a) - 3*(3*(b*cos(6*b*log(c))*cos(4*b*log(c)) + b*sin(6*b*lo
g(c))*sin(4*b*log(c)))*n*log(x) - 2*cos(4*b*log(c))*sin(6*b*log(c)) + 2*cos
(6*b*log(c))*sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) + 3*(3*(b*cos(6*b*log
(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n*log(x) - 2*cos(
2*b*log(c))*sin(6*b*log(c)) + 2*cos(6*b*log(c))*sin(2*b*log(c)))*sin(2*b*lo
g(x^n) + 2*a) + 4*cos(6*b*log(c)))*sin(6*b*log(x^n) + 6*a) + 6*(9*(b*cos(2*
b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*lo
g(x^n) + 2*a)*log(x) - 3*b*n*log(x)*sin(4*b*log(c)) - 9*(b*cos(4*b*log(c))*
cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log(c)))*n*log(x)*sin(2*b*log(x
^n) + 2*a) - 2*cos(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) + 6*(3*b*n*log(x)*s
in(2*b*log(c)) + 2*cos(2*b*log(c)))*sin(2*b*log(x^n) + 2*a))/((b*cos(6*b*lo
g(c))^2 + b*sin(6*b*log(c))^2)*n*cos(6*b*log(x^n) + 6*a)^2 + 9*(b*cos(4*b*l
og(c))^2 + b*sin(4*b*log(c))^2)*n*cos(4*b*log(x^n) + 4*a)^2 - 6*b*n*cos(2*b
*log(c))*cos(2*b*log(x^n) + 2*a) + 9*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c
))^2)*n*cos(2*b*log(x^n) + 2*a)^2 + (b*cos(6*b*log(c))^2 + b*sin(6*b*log(c)
)^2)*n*sin(6*b*log(x^n) + 6*a)^2 + 9*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c
))^2)*n*sin(4*b*log(x^n) + 4*a)^2 + 6*b*n*sin(2*b*log(c))*sin(2*b*log(x^n)
+ 2*a) + 9*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*sin(2*b*log(x^n) +
 2*a)^2 + b*n - 2*(b*n*cos(6*b*log(c)) + 3*(b*cos(6*b*log(c))*cos(4*b*log(c
)) + b*sin(6*b*log(c))*sin(4*b*log(c)))*n*cos(4*b*log(x^n) + 4*a) - 3*(b*co
s(6*b*log(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n*cos(2*
b*log(x^n) + 2*a) + 3*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c)
)*sin(4*b*log(c)))*n*sin(4*b*log(x^n) + 4*a) - 3*(b*cos(2*b*log(c))*sin(6*b
```

```
*log(c)) - b*cos(6*b*log(c))*sin(2*b*log(c)))*n*sin(2*b*log(x^n) + 2*a))*co
s(6*b*log(x^n) + 6*a) + 6*(b*n*cos(4*b*log(c)) - 3*(b*cos(4*b*log(c))*cos(2
*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n) + 2*a) -
 3*(b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*
n*sin(2*b*log(x^n) + 2*a))*cos(4*b*log(x^n) + 4*a) + 2*(3*(b*cos(4*b*log(c)
)*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b*log(c)))*n*cos(4*b*log(x^n) +
 4*a) - 3*(b*cos(2*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(2*b*lo
g(c)))*n*cos(2*b*log(x^n) + 2*a) + b*n*sin(6*b*log(c)) - 3*(b*cos(6*b*log(c
))*cos(4*b*log(c)) + b*sin(6*b*log(c))*sin(4*b*log(c)))*n*sin(4*b*log(x^n)
+ 4*a) + 3*(b*cos(6*b*log(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*l
og(c)))*n*sin(2*b*log(x^n) + 2*a))*sin(6*b*log(x^n) + 6*a) + 6*(3*(b*cos(2*
b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*lo
g(x^n) + 2*a) - b*n*sin(4*b*log(c)) - 3*(b*cos(4*b*log(c))*cos(2*b*log(c))
+ b*sin(4*b*log(c))*sin(2*b*log(c)))*n*sin(2*b*log(x^n) + 2*a))*sin(4*b*log
(x^n) + 4*a))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(42) = 84$.
time = 2.43, size = 132, normalized size = 3.00

$$\frac{4\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)^2+3\,\left(bn\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)\log\left(x\right)-bn\log\left(x\right)\right)\sin\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)+2\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)-2}{3\,\left(bn\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)-bn\right)\sin\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

```
[Out] 1/3*(4*cos(2*b*n*log(x) + 2*b*log(c) + 2*a)^2 + 3*(b*n*cos(2*b*n*log(x) + 2
*b*log(c) + 2*a)*log(x) - b*n*log(x))*sin(2*b*n*log(x) + 2*b*log(c) + 2*a)
+ 2*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) - 2)/((b*n*cos(2*b*n*log(x) + 2*b*
log(c) + 2*a) - b*n)*sin(2*b*n*log(x) + 2*b*log(c) + 2*a))
```

Sympy [A]
time = 1.79, size = 65, normalized size = 1.48

$$\begin{cases} \log\left(x\right)\cot^4\left(a\right) & \text{for}\: b=0 \wedge \left(b=0 \vee n=0\right) \\ \log\left(x\right)\cot^4\left(a+b\log\left(c\right)\right) & \text{for}\: n=0 \\ \frac{\log\left(cx^n\right)}{n}-\frac{\cot^3\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{\cot\left(a+b\log\left(cx^n\right)\right)}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Piecewise((log(x)*cot(a)**4, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*cot
(a + b*log(c))**4, Eq(n, 0)), (log(c*x**n)/n - cot(a + b*log(c*x**n))**3/(3
*b*n) + cot(a + b*log(c*x**n))/(b*n), True))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 8.10, size = 182, normalized size = 4.14

$$\ln(x)+\frac{\frac{4\mathrm{i}}{3\,b\,n}+\frac{\mathrm{e}^{a\,4\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,4\mathrm{i}}\,4\mathrm{i}}{3\,b\,n}}{3\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}-3\,\mathrm{e}^{a\,4\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,4\mathrm{i}}+\mathrm{e}^{a\,6\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,6\mathrm{i}}-1}+\frac{4\mathrm{i}}{3\,b\,n\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}-1\right)}+\frac{\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}\,4\mathrm{i}}{3\,b\,n\,\left(1+\mathrm{e}^{a\,4\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,4\mathrm{i}}-2\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*log(c*x^n))^4/x,x)
```

```
[Out] log(x) + (4i/(3*b*n) + (exp(a*4i)*(c*x^n)^(b*4i)*4i)/(3*b*n))/(3*exp(a*2i)*
(c*x^n)^(b*2i) - 3*exp(a*4i)*(c*x^n)^(b*4i) + exp(a*6i)*(c*x^n)^(b*6i) - 1)
 + 4i/(3*b*n*(exp(a*2i)*(c*x^n)^(b*2i) - 1)) + (exp(a*2i)*(c*x^n)^(b*2i)*4i
)/(3*b*n*(exp(a*4i)*(c*x^n)^(b*4i) - 2*exp(a*2i)*(c*x^n)^(b*2i) + 1))
```

3.225 $\int \frac{\cot^5(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=66

$$\frac{\cot^2\left(a+b\log\left(cx^n\right)\right)}{2bn}-\frac{\cot^4\left(a+b\log\left(cx^n\right)\right)}{4bn}+\frac{\log\left(\sin\left(a+b\log\left(cx^n\right)\right)\right)}{bn}$$

```
[Out] 1/2*cot(a+b*ln(c*x^n))^2/b/n-1/4*cot(a+b*ln(c*x^n))^4/b/n+ln(sin(a+b*ln(c*x
^n)))/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.118$, Rules used = {3554, 3556}

$$\frac{\log\left(\sin\left(a+b\log\left(cx^n\right)\right)\right)}{bn}-\frac{\cot^4\left(a+b\log\left(cx^n\right)\right)}{4bn}+\frac{\cot^2\left(a+b\log\left(cx^n\right)\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] Cot[a + b*Log[c*x^n]]^2/(2*b*n) - Cot[a + b*Log[c*x^n]]^4/(4*b*n) + Log[Sin
[a + b*Log[c*x^n]]]/(b*n)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^5\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \cot^5(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\
&= -\frac{\cot^4\left(a+b\log\left(cx^n\right)\right)}{4bn}-\frac{\text{Subst}\left(\int \cot^3(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\
&= \frac{\cot^2\left(a+b\log\left(cx^n\right)\right)}{2bn}-\frac{\cot^4\left(a+b\log\left(cx^n\right)\right)}{4bn}+\frac{\text{Subst}\left(\int \cot(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\
&= \frac{\cot^2\left(a+b\log\left(cx^n\right)\right)}{2bn}-\frac{\cot^4\left(a+b\log\left(cx^n\right)\right)}{4bn}+\frac{\log\left(\sin\left(a+b\log\left(cx^n\right)\right)\right)}{bn}
\end{aligned}$$

Mathematica [A]
time = 0.25, size = 69, normalized size = 1.05

$$\frac{2\cot^2(a+b\log(cx^n))-\cot^4(a+b\log(cx^n))+4\log(\cos(a+b\log(cx^n)))+4\log(\tan(a+b\log(cx^n)))}{4bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] (2*Cot[a + b*Log[c*x^n]]^2 - Cot[a + b*Log[c*x^n]]^4 + 4*Log[Cos[a + b*Log[
c*x^n]]] + 4*Log[Tan[a + b*Log[c*x^n]]])/(4*b*n)
```

Maple [A]
time = 0.14, size = 57, normalized size = 0.86

method	result
derivativedivides	$\frac{-\frac{\left(\cot^4(a+b\ln(c\,x^n))\right)}{4}+\frac{\left(\cot^2(a+b\ln(c\,x^n))\right)}{2}-\frac{\ln\left(\cot^2(a+b\ln(c\,x^n))+1\right)}{2}}{nb}$
default	$\frac{-\frac{\left(\cot^4(a+b\ln(c\,x^n))\right)}{4}+\frac{\left(\cot^2(a+b\ln(c\,x^n))\right)}{2}-\frac{\ln\left(\cot^2(a+b\ln(c\,x^n))+1\right)}{2}}{nb}$
risch	$i\ln(x)-\frac{\pi\mathrm{csgn}(ic\,x^n)^3}{n}+\frac{\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ic)}{n}+\frac{\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ix^n)}{n}-\frac{\pi\,\mathrm{csgn}(ic\,x^n)\mathrm{csgn}(ic)\mathrm{csgn}(ix^n)}{n}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a+b*ln(c*x^n))^5/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(-1/4*cot(a+b*ln(c*x^n))^4+1/2*cot(a+b*ln(c*x^n))^2-1/2*ln(cot(a+b*ln
(c*x^n))^2+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 5998 vs. $2(62) = 124$.
time = 0.44, size = 5998, normalized size = 90.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

```
[Out] 1/2*(32*(cos(6*b*log(c))^2 + sin(6*b*log(c))^2)*cos(6*b*log(x^n) + 6*a)^2 +
 48*(cos(4*b*log(c))^2 + sin(4*b*log(c))^2)*cos(4*b*log(x^n) + 4*a)^2 + 32*
(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*cos(2*b*log(x^n) + 2*a)^2 + 32*(cos
(6*b*log(c))^2 + sin(6*b*log(c))^2)*sin(6*b*log(x^n) + 6*a)^2 + 48*(cos(4*b
*log(c))^2 + sin(4*b*log(c))^2)*sin(4*b*log(x^n) + 4*a)^2 + 32*(cos(2*b*log
(c))^2 + sin(2*b*log(c))^2)*sin(2*b*log(x^n) + 2*a)^2 - 8*((cos(8*b*log(c))
*cos(6*b*log(c)) + sin(8*b*log(c))*sin(6*b*log(c)))*cos(6*b*log(x^n) + 6*a)
```

```
 - (cos(8*b*log(c))*cos(4*b*log(c)) + sin(8*b*log(c))*sin(4*b*log(c)))*cos(
4*b*log(x^n) + 4*a) + (cos(8*b*log(c))*cos(2*b*log(c)) + sin(8*b*log(c))*si
n(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + (cos(6*b*log(c))*sin(8*b*log(c)) -
 cos(8*b*log(c))*sin(6*b*log(c)))*sin(6*b*log(x^n) + 6*a) - (cos(4*b*log(c)
)*sin(8*b*log(c)) - cos(8*b*log(c))*sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a
) + (cos(2*b*log(c))*sin(8*b*log(c)) - cos(8*b*log(c))*sin(2*b*log(c)))*sin
(2*b*log(x^n) + 2*a))*cos(8*b*log(x^n) + 8*a) - 8*(10*(cos(6*b*log(c))*cos(
4*b*log(c)) + sin(6*b*log(c))*sin(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) - 8*
(cos(6*b*log(c))*cos(2*b*log(c)) + sin(6*b*log(c))*sin(2*b*log(c)))*cos(2*b
*log(x^n) + 2*a) + 10*(cos(4*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c))*si
n(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) - 8*(cos(2*b*log(c))*sin(6*b*log(c))
 - cos(6*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) + cos(6*b*log(c
)))*cos(6*b*log(x^n) + 6*a) - 8*(10*(cos(4*b*log(c))*cos(2*b*log(c)) + sin(
4*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 10*(cos(2*b*log(c))*
sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a)
- cos(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) - 8*cos(2*b*log(c))*cos(2*b*log(
x^n) + 2*a) + ((cos(8*b*log(c))^2 + sin(8*b*log(c))^2)*cos(8*b*log(x^n) + 8
*a)^2 + 16*(cos(6*b*log(c))^2 + sin(6*b*log(c))^2)*cos(6*b*log(x^n) + 6*a)^
2 + 36*(cos(4*b*log(c))^2 + sin(4*b*log(c))^2)*cos(4*b*log(x^n) + 4*a)^2 +
16*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*cos(2*b*log(x^n) + 2*a)^2 + (cos
(8*b*log(c))^2 + sin(8*b*log(c))^2)*sin(8*b*log(x^n) + 8*a)^2 + 16*(cos(6*b
*log(c))^2 + sin(6*b*log(c))^2)*sin(6*b*log(x^n) + 6*a)^2 + 36*(cos(4*b*log
(c))^2 + sin(4*b*log(c))^2)*sin(4*b*log(x^n) + 4*a)^2 + 16*(cos(2*b*log(c))
^2 + sin(2*b*log(c))^2)*sin(2*b*log(x^n) + 2*a)^2 - 2*(4*(cos(8*b*log(c))*c
os(6*b*log(c)) + sin(8*b*log(c))*sin(6*b*log(c)))*cos(6*b*log(x^n) + 6*a) -
 6*(cos(8*b*log(c))*cos(4*b*log(c)) + sin(8*b*log(c))*sin(4*b*log(c)))*cos(
4*b*log(x^n) + 4*a) + 4*(cos(8*b*log(c))*cos(2*b*log(c)) + sin(8*b*log(c))*
sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 4*(cos(6*b*log(c))*sin(8*b*log(c
)) - cos(8*b*log(c))*sin(6*b*log(c)))*sin(6*b*log(x^n) + 6*a) - 6*(cos(4*b*
log(c))*sin(8*b*log(c)) - cos(8*b*log(c))*sin(4*b*log(c)))*sin(4*b*log(x^n)
 + 4*a) + 4*(cos(2*b*log(c))*sin(8*b*log(c)) - cos(8*b*log(c))*sin(2*b*log(
c)))*sin(2*b*log(x^n) + 2*a) - cos(8*b*log(c)))*cos(8*b*log(x^n) + 8*a) - 8
*(6*(cos(6*b*log(c))*cos(4*b*log(c)) + sin(6*b*log(c))*sin(4*b*log(c)))*cos
(4*b*log(x^n) + 4*a) - 4*(cos(6*b*log(c))*cos(2*b*log(c)) + sin(6*b*log(c))
*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 6*(cos(4*b*log(c))*sin(6*b*log(
c)) - cos(6*b*log(c))*sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) - 4*(cos(2*b
*log(c))*sin(6*b*log(c)) - cos(6*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n
) + 2*a) + cos(6*b*log(c)))*cos(6*b*log(x^n) + 6*a) - 12*(4*(cos(4*b*log(c)
)*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a
) + 4*(cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)))*s
in(2*b*log(x^n) + 2*a) - cos(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) - 8*cos(2
*b*log(c))*cos(2*b*log(x^n) + 2*a) + 2*(4*(cos(6*b*log(c))*sin(8*b*log(c))
- cos(8*b*log(c))*sin(6*b*log(c)))*cos(6*b*log(x^n) + 6*a) - 6*(cos(4*b*log
(c))*sin(8*b*log(c)) - cos(8*b*log(c))*sin(4*b*log(c)))*cos(4*b*log(x^n) +
4*a) + 4*(cos(2*b*log(c))*sin(8*b*log(c)) - cos(8*b*log(c))*sin(2*b*log(c))
```

```
)*cos(2*b*log(x^n) + 2*a) - 4*(cos(8*b*log(c))*cos(6*b*log(c)) + sin(8*b*lo
g(c))*sin(6*b*log(c)))*sin(6*b*log(x^n) + 6*a) + 6*(cos(8*b*log(c))*cos(4*b
*log(c)) + sin(8*b*log(c))*sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) - 4*(co
s(8*b*log(c))*cos(2*b*log(c)) + sin(8*b*log(c))*sin(2*b*log(c)))*sin(2*b*lo
g(x^n) + 2*a) - sin(8*b*log(c)))*sin(8*b*log(x^n) + 8*a) + 8*(6*(cos(4*b*lo
g(c))*sin(6*b*log(c)) - cos(6*b*log(c))*sin(4*b*log(c)))*cos(4*b*log(x^n) +
 4*a) - 4*(cos(2*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c))*sin(2*b*log(c)
))*cos(2*b*log(x^n) + 2*a) - 6*(cos(6*b*log(c))*cos(4*b*log(c)) + sin(6*b*l
og(c))*sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) + 4*(cos(6*b*log(c))*cos(2*
b*log(c)) + sin(6*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) + sin(
6*b*log(c)))*sin(6*b*log(x^n) + 6*a) + 12*(4*(cos(2*b*log(c))*sin(4*b*log(c
)) - cos(4*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) - 4*(cos(4*b*
log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n)
 + 2*a) - sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) + 8*sin(2*b*log(c))*sin(
2*b*log(x^n) + 2*a) + 1)*log((cos(a)^2 + sin(a)...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(62) = 124$.
time = 3.16, size = 129, normalized size = 1.95

$$\frac{\left(\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)^{2}-2\,\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)+1\right)\log\left(-\frac{1}{2}\,\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)+\frac{1}{2}\right)-4\,\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)+2}{2\,\left(bn\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)^{2}-2\,bn\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)+bn\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

```
[Out] 1/2*((cos(2*b*n*log(x) + 2*b*log(c) + 2*a)^2 - 2*cos(2*b*n*log(x) + 2*b*log
(c) + 2*a) + 1)*log(-1/2*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1/2) - 4*co
s(2*b*n*log(x) + 2*b*log(c) + 2*a) + 2)/(b*n*cos(2*b*n*log(x) + 2*b*log(c)
+ 2*a)^2 - 2*b*n*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + b*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(53) = 106$.
time = 24.07, size = 136, normalized size = 2.06

$$\begin{cases} \tilde{\infty}\log\left(x\right) & \text{for}\: \left(a = 0 \vee a = - b\log\left(cx^{n}\right)\right) \wedge \left(a = - b\log\left(cx^{n}\right) \vee b = 0\right) \wedge \left(a = - b\log\left(cx^{n}\right) \vee n = 0\right) \\ \log\left(x\right)\cot^{5}\left(a\right) & \text{for}\: b = 0 \\ \log\left(x\right)\cot^{5}\left(a + b\log\left(c\right)\right) & \text{for}\: n = 0 \\ - \frac{\log\left(\tan^{2}\left(a + b\log\left(cx^{n}\right)\right) + 1\right)}{2bn} + \frac{\log\left(\tan\left(a + b\log\left(cx^{n}\right)\right)\right)}{bn} + \frac{1}{2bn\tan^{2}\left(a + b\log\left(cx^{n}\right)\right)} - \frac{1}{4bn\tan^{4}\left(a + b\log\left(cx^{n}\right)\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*ln(c*x**n))**5/x,x)
```

```
[Out] Piecewise((zoo*log(x), (Eq(a, 0) | Eq(a, -b*log(c*x**n))) & (Eq(b, 0) | Eq(
a, -b*log(c*x**n))) & (Eq(n, 0) | Eq(a, -b*log(c*x**n)))), (log(x)*cot(a)**
5, Eq(b, 0)), (log(x)*cot(a + b*log(c))**5, Eq(n, 0)), (-log(tan(a + b*log(
c*x**n))**2 + 1)/(2*b*n) + log(tan(a + b*log(c*x**n)))/(b*n) + 1/(2*b*n*tan
(a + b*log(c*x**n))**2) - 1/(4*b*n*tan(a + b*log(c*x**n))**4), True))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 6.60, size = 246, normalized size = 3.73

$$-\ln(x)\,1\mathrm{i}-\frac{8}{b\,n\,\left(1+\mathrm{e}^{a\,4\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,4\mathrm{i}}-2\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}\right)}-\frac{4}{b\,n\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}-1\right)}-\frac{4}{b\,n\,\left(1+6\,\mathrm{e}^{a\,4\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,4\mathrm{i}}-4\,\mathrm{e}^{a\,6\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,6\mathrm{i}}+\mathrm{e}^{a\,8\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,8\mathrm{i}}-4\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}\right)}+\frac{\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}-1\right)}{b\,n}-\frac{8}{b\,n\,\left(3\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}-3\,\mathrm{e}^{a\,4\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,4\mathrm{i}}+\mathrm{e}^{a\,6\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,6\mathrm{i}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*log(c*x^n))^5/x,x)
```

```
[Out] log(exp(a*2i)*(c*x^n)^(b*2i) - 1)/(b*n) - 8/(b*n*(exp(a*4i)*(c*x^n)^(b*4i)
- 2*exp(a*2i)*(c*x^n)^(b*2i) + 1)) - 4/(b*n*(exp(a*2i)*(c*x^n)^(b*2i) - 1))
 - 4/(b*n*(6*exp(a*4i)*(c*x^n)^(b*4i) - 4*exp(a*2i)*(c*x^n)^(b*2i) - 4*exp(
a*6i)*(c*x^n)^(b*6i) + exp(a*8i)*(c*x^n)^(b*8i) + 1)) - log(x)*1i - 8/(b*n*
(3*exp(a*2i)*(c*x^n)^(b*2i) - 3*exp(a*4i)*(c*x^n)^(b*4i) + exp(a*6i)*(c*x^n
)^(b*6i) - 1))
```

3.226 $\int (ex)^m \cot(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=100

$$\frac{i(ex)^{1+m}}{e(1+m)}-\frac{2i(ex)^{1+m}\,{}_2F_1\left(1,-\frac{i(1+m)}{2bdn};1-\frac{i(1+m)}{2bdn};e^{2iad}(cx^n)^{2ibd}\right)}{e(1+m)}$$

```
[Out] I*(e*x)^(1+m)/e/(1+m)-2*I*(e*x)^(1+m)*hypergeom([1, -1/2*I*(1+m)/b/d/n],[1-
1/2*I*(1+m)/b/d/n],exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/e/(1+m)
```

Rubi [A]
time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {4594, 4592, 470, 371}

$$\frac{i(ex)^{m+1}}{e(m+1)}-\frac{2i(ex)^{m+1}\,{}_2F_1\left(1,-\frac{i(m+1)}{2bdn};1-\frac{i(m+1)}{2bdn};e^{2iad}(cx^n)^{2ibd}\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Cot[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (I*(e*x)^(1 + m))/(e*(1 + m)) - ((2*I)*(e*x)^(1 + m)*Hypergeometric2F1[1, (
(-1/2*I)*(1 + m))/(b*d*n), 1 - ((I/2)*(1 + m))/(b*d*n), E^((2*I)*a*d)*(c*x^
n)^((2*I)*b*d)])/(e*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4594

```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int (ex)^m \cot (d(a+b\log (cx^n)))\ dx=\int (ex)^m \cot (d(a+b\log (cx^n)))\ dx$$

Mathematica [A]
time = 14.83, size = 182, normalized size = 1.82

$$-\frac{ix(ex)^m\left({}_2F_1\left(1,-\frac{i(1+m)}{2bdn};1-\frac{i(1+m)}{2bdn};e^{2id(a+b\log(cx^n))}\right)+\frac{e^{2iad(1+m)}(cx^n)^{2ibd}\,{}_2F_1\left(1,-\frac{i(1+m+2ibdn)}{2bdn};-\frac{i(1+m+4ibdn)}{2bdn};e^{2iad}(cx^n)^{2ibd}\right)}{1+m+2ibdn}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Cot[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((-I)*x*(e*x)^m*(Hypergeometric2F1[1, ((-1/2*I)*(1 + m))/(b*d*n), 1 - ((I/2
)*(1 + m))/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + (E^((2*I)*a*d)*(1 + m
)*(c*x^n)^((2*I)*b*d)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m + (2*I)*b*d*n))
/(b*d*n), ((-1/2*I)*(1 + m + (4*I)*b*d*n))/(b*d*n), E^((2*I)*a*d)*(c*x^n)^(
(2*I)*b*d)])/(1 + m + (2*I)*b*d*n)))/(1 + m)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (ex)^m \cot (d(a+b\ln (c\,x^n)))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*cot(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int((e*x)^m*cot(d*(a+b*ln(c*x^n))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate((x*e)^m*cot((b*log(c*x^n) + a)*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral((x*e)^m*cot(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \cot (ad + bd \log (cx^n))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*cot(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral((e*x)**m*cot(a*d + b*d*log(c*x**n)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(d\,(a + b\,\ln(c\,x^n)))\,(e\,x)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*(a + b*log(c*x^n)))*(e*x)^m,x)
```

```
[Out] int(cot(d*(a + b*log(c*x^n)))*(e*x)^m, x)
```

3.227 $\int (ex)^m \cot^2\left(d(a+b\log(cx^n))\right)\,dx$

Optimal. Leaf size=195

$$\frac{(i(1+m)-bdn)(ex)^{1+m}}{bde(1+m)n}+\frac{i(ex)^{1+m}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bden\left(1-e^{2iad}\left(cx^n\right)^{2ibd}\right)}-\frac{2i(ex)^{1+m}\,{}_2F_1\left(1,-\frac{i(1+m)}{2bdn};1-\frac{i(1+m)}{2bdn};e^{2iad}(cx^n)^{2ibd}\right)}{bden}$$

```
[Out] (I*(1+m)-b*d*n)*(e*x)^(1+m)/b/d/e/(1+m)/n+I*(e*x)^(1+m)*(1+exp(2*I*a*d)*(c*
x^n)^(2*I*b*d))/b/d/e/n/(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))-2*I*(e*x)^(1+m)*
hypergeom([1, -1/2*I*(1+m)/b/d/n],[1-1/2*I*(1+m)/b/d/n],exp(2*I*a*d)*(c*x^n
)^(2*I*b*d))/b/d/e/n
```

Rubi [A]
time = 0.14, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4594, 4592, 516, 470, 371}

$$-\frac{2i(ex)^{m+1}\,{}_2F_1\left(1,-\frac{i(m+1)}{2bdn};1-\frac{i(m+1)}{2bdn};e^{2iad}(cx^n)^{2ibd}\right)}{bden}+\frac{i(ex)^{m+1}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bden\left(1-e^{2iad}\left(cx^n\right)^{2ibd}\right)}+\frac{(ex)^{m+1}(-bdn+i(m+1))}{bde(m+1)n}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Cot[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] ((I*(1 + m) - b*d*n)*(e*x)^(1 + m))/(b*d*e*(1 + m)*n) + (I*(e*x)^(1 + m)*(1
 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*d*e*n*(1 - E^((2*I)*a*d)*(c*x^n)^
((2*I)*b*d))) - ((2*I)*(e*x)^(1 + m)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m)
)/(b*d*n), 1 - ((I/2)*(1 + m))/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])
/(b*d*e*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4594

```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int (ex)^m \cot^2(d(a+b\log(cx^n)))\,dx = \int (ex)^m \cot^2(d(a+b\log(cx^n)))\,dx$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 547 vs. 2(195) = 390.
time = 17.38, size = 547, normalized size = 2.81

$$-\frac{x(ex)^m}{1+m}+\frac{x(ex)^m\csc(d(a+b(-n\log(x)+\log(cx^n))))\csc(bdn\log(x)+d(a+b(-n\log(x)+\log(cx^n))))\sin(bdn\log(x))}{bdn}-\frac{(1+m)x^{-m}(ex)^m\csc(d(a+b(-n\log(x)+\log(cx^n))))\left(\frac{x^{1+m}\csc(d(a+b\log(cx^n)))\sin(bdn\log(x))}{1+m}-\frac{ie^{-\frac{(1+2m)(a+b(-n\log(x)+\log(cx^n)))}{bn}}\left(ie^{\frac{a+2am+b(1+m)n\log(x)+b(1+2m)(-n\log(x)+\log(cx^n))}{bn}}(1+m+2ibdn)\cot(d(a+b\log(cx^n)))-e^{\frac{a+2am+b(1+m)n\log(x)+b(1+2m)(-n\log(x)+\log(cx^n))}{bn}}(1+m+2ibdn)\,{}_2F_1\left(1,-\frac{i(1+m)}{2bdn};1-\frac{i(1+m)}{2bdn};e^{2id(a+b\log(cx^n))}\right)-e^{\frac{a(1+2m+2ibdn)}{bn}+(1+m+2ibdn)\log(x)+\frac{(1+2m+2ibdn)(-n\log(x)+\log(cx^n))}{n}}(1+m)\,{}_2F_1\left(1,-\frac{i(1+m+2ibdn)}{2bdn};-\frac{i(1+m+4ibdn)}{2bdn};e^{2id(a+b\log(cx^n))}\right)\right)\sin(d(a+b(-n\log(x)+\log(cx^n))))}{(1+m)(1+m+2ibdn)}\right)}{bdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Cot[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] -((x*(e*x)^m)/(1 + m)) + (x*(e*x)^m*Csc[d*(a + b*(-(n*Log[x]) + Log[c*x^n])
)]*Csc[b*d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Sin[b*d*n*Log[x
]])/(b*d*n) - ((1 + m)*(e*x)^m*Csc[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*((
x^(1 + m)*Csc[d*(a + b*Log[c*x^n])]*Sin[b*d*n*Log[x]])/(1 + m) - (I*(I*E^((
a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n]))/(b
*n))*(1 + m + (2*I)*b*d*n)*Cot[d*(a + b*Log[c*x^n])] - E^((a + 2*a*m + b*(1
 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n]))/(b*n))*(1 + m + (2
```

```
*I)*b*d*n)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m))/(b*d*n), 1 - ((I/2)*(1 +
 m))/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] - E^((a*(1 + 2*m + (2*I)*b*d*
n))/(b*n) + (1 + m + (2*I)*b*d*n)*Log[x] + ((1 + 2*m + (2*I)*b*d*n)*(-(n*Lo
g[x]) + Log[c*x^n]))/n)*(1 + m)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m + (2*
I)*b*d*n))/(b*d*n), ((-1/2*I)*(1 + m + (4*I)*b*d*n))/(b*d*n), E^((2*I)*d*(a
 + b*Log[c*x^n]))])*Sin[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))])/(E^(((1 + 2*
m)*(a + b*(-(n*Log[x]) + Log[c*x^n])))/(b*n))*(1 + m)*(1 + m + (2*I)*b*d*n)
)))/(b*d*n*x^m)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\cot^2\left(d(a+b\ln\left(c\,x^n\right))\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*cot(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int((e*x)^m*cot(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] -((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x*cos(2*b*d*log(x^n
) + 2*a*d)^2*e^(m*log(x) + m) + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*lo
g(c))^2)*n*x*e^(m*log(x) + m)*sin(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n*x*e^(m*
log(x) + m) - 2*(b*d*n*cos(2*b*d*log(c))*e^m - (m*sin(2*b*d*log(c)) + sin(2
*b*d*log(c)))*e^m)*x*x^m*cos(2*b*d*log(x^n) + 2*a*d) + 2*(b*d*n*e^m*sin(2*b
*d*log(c)) + (m*cos(2*b*d*log(c)) + cos(2*b*d*log(c)))*e^m)*x*x^m*sin(2*b*d
*log(x^n) + 2*a*d) + ((b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(
c))^2 + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*m^2 + 2
*(b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*m)*n^2*cos(2*b
*d*log(x^n) + 2*a*d)^2*e^m + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b
*d*log(c))^2 + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*
m^2 + 2*(b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*m)*n^2*
e^m*sin(2*b*d*log(x^n) + 2*a*d)^2 - 2*(b^2*d^2*m^2*cos(2*b*d*log(c)) + 2*b^
2*d^2*m*cos(2*b*d*log(c)) + b^2*d^2*cos(2*b*d*log(c)))*n^2*cos(2*b*d*log(x^
n) + 2*a*d)*e^m + 2*(b^2*d^2*m^2*sin(2*b*d*log(c)) + 2*b^2*d^2*m*sin(2*b*d*
log(c)) + b^2*d^2*sin(2*b*d*log(c)))*n^2*e^m*sin(2*b*d*log(x^n) + 2*a*d) +
(b^2*d^2*m^2 + 2*b^2*d^2*m + b^2*d^2)*n^2*e^m)*integrate((x^m*cos(b*d*log(x
^n) + a*d)*sin(b*d*log(c)) + x^m*cos(b*d*log(c))*sin(b*d*log(x^n) + a*d))/(
```

```
2*b^2*d^2*n^2*cos(b*d*log(c))*cos(b*d*log(x^n) + a*d) - 2*b^2*d^2*n^2*sin(b
*d*log(c))*sin(b*d*log(x^n) + a*d) + b^2*d^2*n^2 + (b^2*d^2*cos(b*d*log(c))
^2 + b^2*d^2*sin(b*d*log(c))^2)*n^2*cos(b*d*log(x^n) + a*d)^2 + (b^2*d^2*co
s(b*d*log(c))^2 + b^2*d^2*sin(b*d*log(c))^2)*n^2*sin(b*d*log(x^n) + a*d)^2)
, x) - ((b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2 + (b^2*d
^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*m^2 + 2*(b^2*d^2*cos(
2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*m)*n^2*cos(2*b*d*log(x^n) +
2*a*d)^2*e^m + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2 +
 (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*m^2 + 2*(b^2*d
^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*m)*n^2*e^m*sin(2*b*d*
log(x^n) + 2*a*d)^2 - 2*(b^2*d^2*m^2*cos(2*b*d*log(c)) + 2*b^2*d^2*m*cos(2*
b*d*log(c)) + b^2*d^2*cos(2*b*d*log(c)))*n^2*cos(2*b*d*log(x^n) + 2*a*d)*e^
m + 2*(b^2*d^2*m^2*sin(2*b*d*log(c)) + 2*b^2*d^2*m*sin(2*b*d*log(c)) + b^2*
d^2*sin(2*b*d*log(c)))*n^2*e^m*sin(2*b*d*log(x^n) + 2*a*d) + (b^2*d^2*m^2 +
 2*b^2*d^2*m + b^2*d^2)*n^2*e^m)*integrate(-(x^m*cos(b*d*log(x^n) + a*d)*si
n(b*d*log(c)) + x^m*cos(b*d*log(c))*sin(b*d*log(x^n) + a*d))/(2*b^2*d^2*n^2
*cos(b*d*log(c))*cos(b*d*log(x^n) + a*d) - 2*b^2*d^2*n^2*sin(b*d*log(c))*si
n(b*d*log(x^n) + a*d) - b^2*d^2*n^2 - (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*
sin(b*d*log(c))^2)*n^2*cos(b*d*log(x^n) + a*d)^2 - (b^2*d^2*cos(b*d*log(c))
^2 + b^2*d^2*sin(b*d*log(c))^2)*n^2*sin(b*d*log(x^n) + a*d)^2), x))/((b*d*c
os(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2 + (b*d*cos(2*b*d*log(c))^2 + b
*d*sin(2*b*d*log(c))^2)*m)*n*cos(2*b*d*log(x^n) + 2*a*d)^2 + (b*d*cos(2*b*d
*log(c))^2 + b*d*sin(2*b*d*log(c))^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2
*b*d*log(c))^2)*m)*n*sin(2*b*d*log(x^n) + 2*a*d)^2 - 2*(b*d*m*cos(2*b*d*log
(c)) + b*d*cos(2*b*d*log(c)))*n*cos(2*b*d*log(x^n) + 2*a*d) + 2*(b*d*m*sin(
2*b*d*log(c)) + b*d*sin(2*b*d*log(c)))*n*sin(2*b*d*log(x^n) + 2*a*d) + (b*d
*m + b*d)*n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral((x*e)^m*cot(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \cot^2 (ad + bd \log (cx^n))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*cot(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Integral((e*x)**m*cot(a*d + b*d*log(c*x**n))**2, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(d\,(a+b\,\ln\left(c\,x^n\right)))^2\,\left(e\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*(a + b*log(c*x^n)))^2*(e*x)^m,x)
```

```
[Out] int(cot(d*(a + b*log(c*x^n)))^2*(e*x)^m, x)
```

3.228 $\int (ex)^m \cot^3(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=350

$$\frac{(i(1+m)-bdn)(1+m+2ibdn)(ex)^{1+m}}{2b^2d^2e(1+m)n^2}+\frac{(ex)^{1+m}\left(1+e^{2iad}(cx^n)^{2ibd}\right)^2}{2bden\left(1-e^{2iad}\left(cx^n\right)^{2ibd}\right)^2}+\frac{ie^{-2iad}(ex)^{1+m}\left(\frac{e^{2iad}(1+m-2ibdn)}{n}+\right.}{2b^2d^2en\left(1-e^{2iad}\right.}$$

```
[Out] 1/2*(I*(1+m)-b*d*n)*(1+m+2*I*b*d*n)*(e*x)^(1+m)/b^2/d^2/e/(1+m)/n^2+1/2*(e*
x)^(1+m)*(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^2/b/d/e/n/(1-exp(2*I*a*d)*(c*x^
n)^(2*I*b*d))^2+1/2*I*(e*x)^(1+m)*(exp(2*I*a*d)*(1+m-2*I*b*d*n)/n+exp(4*I*a
*d)*(1+m+2*I*b*d*n)*(c*x^n)^(2*I*b*d)/n)/b^2/d^2/e/exp(2*I*a*d)/n/(1-exp(2*
I*a*d)*(c*x^n)^(2*I*b*d))-I*(-2*b^2*d^2*n^2+m^2+2*m+1)*(e*x)^(1+m)*hypergeo
m([1, -1/2*I*(1+m)/b/d/n],[1-1/2*I*(1+m)/b/d/n],exp(2*I*a*d)*(c*x^n)^(2*I*b
*d))/b^2/d^2/e/(1+m)/n^2
```

Rubi [A]

time = 0.33, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4594, 4592, 516, 608, 470, 371}

$$-\frac{i(ex)^{m+1}\left(-2b^2d^2n^2+m^2+2m+1\right)\,{}_2F_1\left(1,-\frac{i(m+1)}{2bdn};1-\frac{i(m+1)}{2bdn};e^{2iad}(cx^n)^{2ibd}\right)}{b^2d^2e(m+1)n^2}+\frac{ie^{-2iad}(ex)^{m+1}\left(\frac{e^{4iad}(2ibdn+m+1)(cx^n)^{2ibd}}{n}+\frac{e^{2iad}(-2ibdn+m+1)}{n}\right)}{2b^2d^2en\left(1-e^{2iad}\left(cx^n\right)^{2ibd}\right)}+\frac{(ex)^{m+1}\left(1+e^{2iad}(cx^n)^{2ibd}\right)^2}{2bden\left(1-e^{2iad}\left(cx^n\right)^{2ibd}\right)^2}+\frac{(ex)^{m+1}(-bdn+i(m+1))(2ibdn+m+1)}{2b^2d^2e(m+1)n^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Cot[d*(a + b*Log[c*x^n])]^3,x]
```

```
[Out] ((I*(1 + m) - b*d*n)*(1 + m + (2*I)*b*d*n)*(e*x)^(1 + m))/(2*b^2*d^2*e*(1 +
 m)*n^2) + ((e*x)^(1 + m)*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^2)/(2*b*d
*e*n*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^2) + ((I/2)*(e*x)^(1 + m)*((E^
((2*I)*a*d)*(1 + m - (2*I)*b*d*n))/n + (E^((4*I)*a*d)*(1 + m + (2*I)*b*d*n)
*(c*x^n)^((2*I)*b*d))/n))/(b^2*d^2*e*E^((2*I)*a*d)*n*(1 - E^((2*I)*a*d)*(c*
x^n)^((2*I)*b*d))) - (I*(1 + 2*m + m^2 - 2*b^2*d^2*n^2)*(e*x)^(1 + m)*Hyper
geometric2F1[1, ((-1/2*I)*(1 + m))/(b*d*n), 1 - ((I/2)*(1 + m))/(b*d*n), E^
((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(b^2*d^2*e*(1 + m)*n^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
```

```
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 608

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && LtQ[
p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4594

```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int (ex)^m \cot^3 (d(a+b\log (cx^n)))\, dx = \int (ex)^m \cot^3 (d(a+b\log (cx^n)))\, dx$$

Mathematica [A]
time = 17.85, size = 639, normalized size = 1.83

$$-\frac{x(ex)^m\cot(d(a+b(-n\log(x)+\log(cx^n))))}{1+m}-\frac{x(ex)^m\csc^2(bdn\log(x)+d(a+b(-n\log(x)+\log(cx^n))))}{2bdn}+\frac{(1+m)x(ex)^m\csc(d(a+b(-n\log(x)+\log(cx^n))))\csc(bdn\log(x)+d(a+b(-n\log(x)+\log(cx^n))))\sin(bdn\log(x))}{2b^2d^2n^2}+\frac{(-1-2m-m^2+2b^2d^2n^2)x^{-m}(ex)^m\csc(d(a+b(-n\log(x)+\log(cx^n))))\left(\frac{x^{1+m}\csc(d(a+b\log(cx^n)))\sin(bdn\log(x))}{1+m}-\frac{i\left(ie^{\frac{a+2am+b(1+m)n\log(x)+b(1+2m)(-n\log(x)+\log(cx^n))}{bn}}(1+m+2ibdn)\cot(d(a+b\log(cx^n)))-e^{\frac{a+2am+b(1+m)n\log(x)+b(1+2m)(-n\log(x)+\log(cx^n))}{bn}}(1+m+2ibdn)\,{}_2F_1\left(1,-\frac{i(1+m)}{2bdn};1-\frac{i(1+m)}{2bdn};e^{2id(a+b\log(cx^n))}\right)-e^{\frac{a(1+2m+2ibdn)}{bn}+(1+m+2ibdn)\log(x)+\frac{(1+2m+2ibdn)(-n\log(x)+\log(cx^n))}{n}}(1+m)\,{}_2F_1\left(1,-\frac{i(1+m+2ibdn)}{2bdn};-\frac{i(1+m+4ibdn)}{2bdn};e^{2id(a+b\log(cx^n))}\right)\right)\sin(d(a+b(-n\log(x)+\log(cx^n))))}{e^{\frac{(1+2m)(a+b(-n\log(x)+\log(cx^n)))}{bn}}(1+m)(1+m+2ibdn)}\right)}{2b^2d^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Cot[d*(a + b*Log[c*x^n])]^3,x]
```

```
[Out] -((x*(e*x)^m*Cot[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))])/(1 + m)) - (x*(e*x)
^m*Csc[b*d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]^2)/(2*b*d*n) +
((1 + m)*x*(e*x)^m*Csc[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Csc[b*d*n*Log[
x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Sin[b*d*n*Log[x]])/(2*b^2*d^2*n^
2) + ((-1 - 2*m - m^2 + 2*b^2*d^2*n^2)*(e*x)^m*Csc[d*(a + b*(-(n*Log[x]) +
Log[c*x^n]))]*((x^(1 + m)*Csc[d*(a + b*Log[c*x^n])]*Sin[b*d*n*Log[x]])/(1 +
 m) - (I*(I*E^((a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) +
 Log[c*x^n]))/(b*n))*(1 + m + (2*I)*b*d*n)*Cot[d*(a + b*Log[c*x^n])] - E^((
a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n]))/(b
*n))*(1 + m + (2*I)*b*d*n)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m))/(b*d*n),
 1 - ((I/2)*(1 + m))/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] - E^((a*(1 +
2*m + (2*I)*b*d*n))/(b*n) + (1 + m + (2*I)*b*d*n)*Log[x] + ((1 + 2*m + (2*I
)*b*d*n)*(-(n*Log[x]) + Log[c*x^n]))/n)*(1 + m)*Hypergeometric2F1[1, ((-1/2
*I)*(1 + m + (2*I)*b*d*n))/(b*d*n), ((-1/2*I)*(1 + m + (4*I)*b*d*n))/(b*d*n
), E^((2*I)*d*(a + b*Log[c*x^n]))])*Sin[d*(a + b*(-(n*Log[x]) + Log[c*x^n])
)])/(E^(((1 + 2*m)*(a + b*(-(n*Log[x]) + Log[c*x^n])))/(b*n))*(1 + m)*(1 +
m + (2*I)*b*d*n))))/(2*b^2*d^2*n^2*x^m)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\cot^3\left(d(a+b\ln(c\,x^n))\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*cot(d*(a+b*ln(c*x^n)))^3,x)
```

```
[Out] int((e*x)^m*cot(d*(a+b*ln(c*x^n)))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(d*(a+b*log(c*x^n)))^3,x, algorithm="maxima")
```

```
[Out] (4*(b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x*cos(2*b*d*log(x^
n) + 2*a*d)^2*e^(m*log(x) + m) + 4*(b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d
*log(c))^2)*n*x*e^(m*log(x) + m)*sin(2*b*d*log(x^n) + 2*a*d)^2 - (2*b*d*n*c
os(2*b*d*log(c))*e^m - (m*sin(2*b*d*log(c)) + sin(2*b*d*log(c)))*e^m)*x*x^m
*cos(2*b*d*log(x^n) + 2*a*d) + (2*b*d*n*e^m*sin(2*b*d*log(c)) + (m*cos(2*b*
d*log(c)) + cos(2*b*d*log(c)))*e^m)*x*x^m*sin(2*b*d*log(x^n) + 2*a*d) - ((2
*(b*d*cos(4*b*d*log(c))*cos(2*b*d*log(c)) + b*d*sin(4*b*d*log(c))*sin(2*b*d
*log(c)))*n*e^m - ((cos(2*b*d*log(c))*sin(4*b*d*log(c)) - cos(4*b*d*log(c))
*sin(2*b*d*log(c)))*m + cos(2*b*d*log(c))*sin(4*b*d*log(c)) - cos(4*b*d*log
(c))*sin(2*b*d*log(c)))*e^m)*x*x^m*cos(2*b*d*log(x^n) + 2*a*d) + (2*(b*d*co
s(2*b*d*log(c))*sin(4*b*d*log(c)) - b*d*cos(4*b*d*log(c))*sin(2*b*d*log(c))
)*n*e^m + ((cos(4*b*d*log(c))*cos(2*b*d*log(c)) + sin(4*b*d*log(c))*sin(2*b
*d*log(c)))*m + cos(4*b*d*log(c))*cos(2*b*d*log(c)) + sin(4*b*d*log(c))*sin
(2*b*d*log(c)))*e^m)*x*x^m*sin(2*b*d*log(x^n) + 2*a*d) + (m*sin(4*b*d*log(c
)) + sin(4*b*d*log(c)))*x*e^(m*log(x) + m))*cos(4*b*d*log(x^n) + 4*a*d) - 2
*(2*b^6*d^6*n^6*e^m - (b^4*d^4*m^2 + 2*b^4*d^4*m + b^4*d^4)*n^4*e^m + (2*(b
^6*d^6*cos(4*b*d*log(c))^2 + b^6*d^6*sin(4*b*d*log(c))^2)*n^6*e^m - (b^4*d^
4*cos(4*b*d*log(c))^2 + b^4*d^4*sin(4*b*d*log(c))^2 + (b^4*d^4*cos(4*b*d*lo
g(c))^2 + b^4*d^4*sin(4*b*d*log(c))^2)*m^2 + 2*(b^4*d^4*cos(4*b*d*log(c))^2
 + b^4*d^4*sin(4*b*d*log(c))^2)*m)*n^4*e^m)*cos(4*b*d*log(x^n) + 4*a*d)^2 +
 4*(2*(b^6*d^6*cos(2*b*d*log(c))^2 + b^6*d^6*sin(2*b*d*log(c))^2)*n^6*e^m -
 (b^4*d^4*cos(2*b*d*log(c))^2 + b^4*d^4*sin(2*b*d*log(c))^2 + (b^4*d^4*cos(
2*b*d*log(c))^2 + b^4*d^4*sin(2*b*d*log(c))^2)*m^2 + 2*(b^4*d^4*cos(2*b*d*l
og(c))^2 + b^4*d^4*sin(2*b*d*log(c))^2)*m)*n^4*e^m)*cos(2*b*d*log(x^n) + 2*
a*d)^2 + (2*(b^6*d^6*cos(4*b*d*log(c))^2 + b^6*d^6*sin(4*b*d*log(c))^2)*n^6
*e^m - (b^4*d^4*cos(4*b*d*log(c))^2 + b^4*d^4*sin(4*b*d*log(c))^2 + (b^4*d^
4*cos(4*b*d*log(c))^2 + b^4*d^4*sin(4*b*d*log(c))^2)*m^2 + 2*(b^4*d^4*cos(4
*b*d*log(c))^2 + b^4*d^4*sin(4*b*d*log(c))^2)*m)*n^4*e^m)*sin(4*b*d*log(x^n
) + 4*a*d)^2 + 4*(2*(b^6*d^6*cos(2*b*d*log(c))^2 + b^6*d^6*sin(2*b*d*log(c)
)^2)*n^6*e^m - (b^4*d^4*cos(2*b*d*log(c))^2 + b^4*d^4*sin(2*b*d*log(c))^2 +
 (b^4*d^4*cos(2*b*d*log(c))^2 + b^4*d^4*sin(2*b*d*log(c))^2)*m^2 + 2*(b^4*d
^4*cos(2*b*d*log(c))^2 + b^4*d^4*sin(2*b*d*log(c))^2)*m)*n^4*e^m)*sin(2*b*d
*log(x^n) + 2*a*d)^2 + 2*(2*b^6*d^6*n^6*cos(4*b*d*log(c))*e^m - (b^4*d^4*m^
2*cos(4*b*d*log(c)) + 2*b^4*d^4*m*cos(4*b*d*log(c)) + b^4*d^4*cos(4*b*d*log
(c)))*n^4*e^m - 2*(2*(b^6*d^6*cos(4*b*d*log(c))*cos(2*b*d*log(c)) + b^6*d^6
*sin(4*b*d*log(c))*sin(2*b*d*log(c)))*n^6*e^m - (b^4*d^4*cos(4*b*d*log(c))*
cos(2*b*d*log(c)) + b^4*d^4*sin(4*b*d*log(c))*sin(2*b*d*log(c)) + (b^4*d^4*
cos(4*b*d*log(c))*cos(2*b*d*log(c)) + b^4*d^4*sin(4*b*d*log(c))*sin(2*b*d*l
og(c)))*m^2 + 2*(b^4*d^4*cos(4*b*d*log(c))*cos(2*b*d*log(c)) + b^4*d^4*sin(
4*b*d*log(c))*sin(2*b*d*log(c)))*m)*n^4*e^m)*cos(2*b*d*log(x^n) + 2*a*d) -
2*(2*(b^6*d^6*cos(2*b*d*log(c))*sin(4*b*d*log(c)) - b^6*d^6*cos(4*b*d*log(c
))*sin(2*b*d*log(c)))*n^6*e^m - (b^4*d^4*cos(2*b*d*log(c))*sin(4*b*d*log(c)
) - b^4*d^4*cos(4*b*d*log(c))*sin(2*b*d*log(c)) + (b^4*d^4*cos(2*b*d*log(c)
)*sin(4*b*d*log(c)) - b^4*d^4*cos(4*b*d*log(c))*sin(2*b*d*log(c)))*m^2 + 2*
(b^4*d^4*cos(2*b*d*log(c))*sin(4*b*d*log(c)) - b^4*d^4*cos(4*b*d*log(c))*si
```

```
n(2*b*d*log(c)))*m)*n^4*e^m)*sin(2*b*d*log(x^n) + 2*a*d))*cos(4*b*d*log(x^n
) + 4*a*d) - 4*(2*b^6*d^6*n^6*cos(2*b*d*log(c))*e^m - (b^4*d^4*m^2*cos(2*b*
d*log(c)) + 2*b^4*d^4*m*cos(2*b*d*log(c)) + b^4*d^4*cos(2*b*d*log(c)))*n^4*
e^m)*cos(2*b*d*log(x^n) + 2*a*d) - 2*(2*b^6*d^6*n^6*e^m*sin(4*b*d*log(c)) -
 (b^4*d^4*m^2*sin(4*b*d*log(c)) + 2*b^4*d^4*m*sin(4*b*d*log(c)) + b^4*d^4*s
in(4*b*d*log(c)))*n^4*e^m - 2*(2*(b^6*d^6*cos(2*b*d*log(c))*sin(4*b*d*log(c
)) - b^6*d^6*cos(4*b*d*log(c))*sin(2*b*d*log(c)))*n^6*e^m - (b^4*d^4*cos(2*
b*d*log(c))*sin(4*b*d*log(c)) - b^4*d^4*cos(4*b*d*log(c))*sin(2*b*d*log(c))
 + (b^4*d^4*cos(2*b*d*log(c))*sin(4*b*d*log(c)) - b^4*d^4*cos(4*b*d*log(c))
*sin(2*b*d*log(c)))*m^2 + 2*(b^4*d^4*cos(2*b*d*log(c))*sin(4*b*d*log(c)) -
b^4*d^4*cos(4*b*d*log(c))*sin(2*b*d*log(c)))*m)*n^4*e^m)*cos(2*b*d*log(x^n)
 + 2*a*d) + 2*(2*(b^6*d^6*cos(4*b*d*log(c))*cos(2*b*d*log(c)) + b^6*d^6*sin
(4*b*d*log(c))*sin(2*b*d*log(c)))*n^6*e^m - (b^4*d^4*cos(4*b*d*log(c))*cos(
2*b*d*log(c)) + b^4*d^4*sin(4*b*d*log(c))*sin(2*b*d*log(c)) + (b^4*d^4*cos(
4*b*d*log(c))*cos(2*b*d*log(c)) + b^4*d^4*sin(4*b*d*log(c))*sin(2*b*d*log(c
)))*m^2 + 2*(b^4*d^4*cos(4*b*d*log(c))*cos(2*b*d*log(c)) + b^4*d^4*sin(4*b*
d*log(c))*sin(2*b*d*log(c)))*m)*n^4*e^m)*sin(2*b*d*log(x^n) + 2*a*d))*sin(4
*b*d*log(x^n) + 4*a*d) + 4*(2*b^6*d^6*n^6*e^m*sin(2*b*d*log(c)) - (b^4*d^4*
m^2*sin(2*b*d*log(c)) + 2*b^4*d^4*m*sin(2*b*d*log(c)) + b^4*d^4*sin(2*b*d*l
og(c)))*n^4*e^m)*sin(2*b*d*log(x^n) + 2*a*d))*integrate(1/4*(x^m*cos(b*d*lo
g(x^n) + a*d)*sin(b*d*log(c)) + x^m*cos(b*d*log(c))*sin(b*d*log(x^n) + a*d)
)/(2*b^4*d^4*n^4*cos(b*d*log(c))*cos(b*d*log(x^...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(d*(a+b*log(c*x^n)))^3,x, algorithm="fricas")
[Out] integral((x*e)^m*cot(b*d*log(c*x^n) + a*d)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \cot^3 (ad + bd\log (cx^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*cot(d*(a+b*ln(c*x**n)))**3,x)
[Out] Integral((e*x)**m*cot(a*d + b*d*log(c*x**n))**3, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(d*(a+b*log(c*x^n)))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(d\,(a+b\,\ln\left(c\,x^n\right)))^3\,\left(e\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*(a + b*log(c*x^n)))^3*(e*x)^m,x)
```

```
[Out] int(cot(d*(a + b*log(c*x^n)))^3*(e*x)^m, x)
```

3.229 $\int \cot^p(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=190

$$x\left(1-e^{2iad}(cx^n)^{2ibd}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(-\frac{i\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{1-e^{2iad}\left(cx^n\right)^{2ibd}}\right)^p F_1\left(-\frac{i}{2bdn};p,-p;1-\frac{i}{2bdn};e^{2iad}(c$$

```
[Out] x*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^p*(-I*(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d
))/(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d)))^p*AppellF1(-1/2*I/b/d/n,p,-p,1-1/2*I
/b/d/n,exp(2*I*a*d)*(c*x^n)^(2*I*b*d),-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/((1+
exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^p)
```

Rubi [A]
time = 0.11, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4590, 4592, 1986, 525, 524}

$$x\left(1-e^{2iad}(cx^n)^{2ibd}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(-\frac{i\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{1-e^{2iad}\left(cx^n\right)^{2ibd}}\right)^p F_1\left(-\frac{i}{2bdn};p,-p;1-\frac{i}{2bdn};e^{2iad}(cx^n)^{2ibd},-e^{2iad}(cx^n)^{2ibd}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] (x*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p*(((-I)*(1 + E^((2*I)*a*d)*(c*x
^n)^((2*I)*b*d)))/(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))^p*AppellF1[(-1/2
*I)/(b*d*n), p, -p, 1 - (I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d), -
(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])/(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d
))^p
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4590

```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Cot[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int \cot^p\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int \cot^p\left(d(a+b\log\left(cx^n\right))\right)\,dx$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 458 vs. $2(190)=380$.
time = 1.36, size = 458, normalized size = 2.41

$$\frac{(-i+2bdn)x\left(\frac{i\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{-1+e^{2iad}(cx^n)^{2ibd}}\right)^p F_1\left(-\frac{i}{2bdn};p,-p;1-\frac{i}{2bdn};e^{2iad}(cx^n)^{2ibd},-e^{2iad}(cx^n)^{2ibd}\right)}{2bde^{2iad}np\left(cx^n\right)^{2ibd}F_1\left(1-\frac{i}{2bdn};p,1-p;2-\frac{i}{2bdn};e^{2iad}\left(cx^n\right)^{2ibd},-e^{2iad}\left(cx^n\right)^{2ibd}\right)+2bde^{2iad}np\left(cx^n\right)^{2ibd}F_1\left(1-\frac{i}{2bdn};1+p,-p;2-\frac{i}{2bdn};e^{2iad}\left(cx^n\right)^{2ibd},-e^{2iad}\left(cx^n\right)^{2ibd}\right)+(-i+2bdn)F_1\left(-\frac{i}{2bdn};p,-p;1-\frac{i}{2bdn};e^{2iad}\left(cx^n\right)^{2ibd},-e^{2iad}\left(cx^n\right)^{2ibd}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] ((-I + 2*b*d*n)*x*((I*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(-1 + E^((2*
I)*a*d)*(c*x^n)^((2*I)*b*d)))^p*AppellF1[(-1/2*I)/(b*d*n), p, -p, 1 - (I/2)
/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)
*b*d))])/(2*b*d*E^((2*I)*a*d)*n*p*(c*x^n)^((2*I)*b*d)*AppellF1[1 - (I/2)/(b
*d*n), p, 1 - p, 2 - (I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d), -(E^
((2*I)*a*d)*(c*x^n)^((2*I)*b*d))] + 2*b*d*E^((2*I)*a*d)*n*p*(c*x^n)^((2*I)*
b*d)*AppellF1[1 - (I/2)/(b*d*n), 1 + p, -p, 2 - (I/2)/(b*d*n), E^((2*I)*a*d
)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))] + (-I + 2*b*d*
n)*AppellF1[(-1/2*I)/(b*d*n), p, -p, 1 - (I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^
n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \cot^{p}\left(d(a+b\ln\left(c\,x^{n}\right))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*(a+b*ln(c*x^n)))^p,x)
```

```
[Out] int(cot(d*(a+b*ln(c*x^n)))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
```

```
[Out] integrate(cot((b*log(c*x^n) + a)*d)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

```
[Out] integral(cot(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cot^{p}\left(d(a+b\log\left(cx^{n}\right))\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*ln(c*x**n)))**p,x)
```

```
[Out] Integral(cot(d*(a + b*log(c*x**n)))**p, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(d\,(a+b\,\ln\,(c\,x^n)))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*(a + b*log(c*x^n)))^p,x)
```

```
[Out] int(cot(d*(a + b*log(c*x^n)))^p, x)
```

3.230 $\int (ex)^m \cot^p(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=210

$$\frac{(ex)^{1+m}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(-\frac{i\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{1-e^{2iad}(cx^n)^{2ibd}}\right)^p F_1\left(-\frac{i(1+m)}{2bdn};p,-p;1-\frac{i(1+m)}{2bdn};e^{2ia}\right.}{e(1+m)}$$

```
[Out] (e*x)^(1+m)*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^p*(-I*(1+exp(2*I*a*d)*(c*x^n
)^(2*I*b*d))/(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d)))^p*AppellF1(-1/2*I*(1+m)/b/
d/n,p,-p,1-1/2*I*(1+m)/b/d/n,exp(2*I*a*d)*(c*x^n)^(2*I*b*d),-exp(2*I*a*d)*(
c*x^n)^(2*I*b*d))/e/(1+m)/((1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^p)
```

Rubi [A]
time = 0.13, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4594, 4592, 1986, 525, 524}

$$\frac{(ex)^{m+1}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(-\frac{i\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{1-e^{2iad}(cx^n)^{2ibd}}\right)^p F_1\left(-\frac{i(m+1)}{2bdn};p,-p;1-\frac{i(m+1)}{2bdn};e^{2iad}(cx^n)^{2ibd},-e^{2iad}(cx^n)^{2ibd}\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Cot[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] ((e*x)^(1 + m)*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p*(((-I)*(1 + E^((2*
I)*a*d)*(c*x^n)^((2*I)*b*d)))/(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))^p*Ap
pellF1[((-1/2*I)*(1 + m))/(b*d*n), p, -p, 1 - ((I/2)*(1 + m))/(b*d*n), E^((
2*I)*a*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])/(e*(1
 + m)*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4592

```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4594

```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int (ex)^m \cot^p(d(a+b\log(cx^n)))\,dx = \int (ex)^m \cot^p(d(a+b\log(cx^n)))\,dx$$

Mathematica [A]
time = 1.25, size = 205, normalized size = 0.98

$$\frac{x(ex)^m\left(1-e^{2iad}(cx^n)^{2ibd}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(\frac{i\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{-1+e^{2iad}(cx^n)^{2ibd}}\right)^p F_1\left(-\frac{i(1+m)}{2bdn};p,-p;1-\frac{i(1+m)}{2bdn};e^{2iad}(cx^n)^{2ibd},-e^{2iad}(cx^n)^{2ibd}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Cot[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] (x*(e*x)^m*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p*((I*(1 + E^((2*I)*a*d)
*(c*x^n)^((2*I)*b*d)))/(-1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))^p*AppellF1
[((-1/2*I)*(1 + m))/(b*d*n), p, -p, 1 - ((I/2)*(1 + m))/(b*d*n), E^((2*I)*a
*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])/((1 + m)*(1
 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\cot^p(d(a+b\ln(c\,x^n)))\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*cot(d*(a+b*ln(c*x^n)))^p,x)
```

```
[Out] int((e*x)^m*cot(d*(a+b*ln(c*x^n)))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e)^m*cot((b*log(c*x^n) + a)*d)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

```
[Out] integral((x*e)^m*cot(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \cot^p (ad + bd \log (cx^n))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*cot(d*(a+b*ln(c*x**n)))**p,x)
```

```
[Out] Integral((e*x)**m*cot(a*d + b*d*log(c*x**n))**p, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(d\,(a+b\,\ln\left(c\,x^n\right)))^p\,(e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*(a + b*log(c*x^n)))^p*(e*x)^m,x)
[Out] int(cot(d*(a + b*log(c*x^n)))^p*(e*x)^m, x)
```

3.231 $\int \frac{\cot^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=201

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}\right)}{\sqrt{2}\,bn}+\frac{\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}\right)}{\sqrt{2}\,bn}-\frac{2\cot^{\frac{3}{2}}\left(a+b\log\left(c\right.\right.}{3bn}$$

[Out] -2/3*cot(a+b*ln(c*x^n))^(3/2)/b/n+1/2*arctan(-1+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)+1/2*arctan(1+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)+1/4*ln(1+cot(a+b*ln(c*x^n))-2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)-1/4*ln(1+cot(a+b*ln(c*x^n))+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)

Rubi [A]
time = 0.10, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3554, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}\right)}{\sqrt{2}\,bn}+\frac{\text{ArcTan}\left(\sqrt{2}\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}+1\right)}{\sqrt{2}\,bn}-\frac{2\cot^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{\log\left(\cot\left(a+b\log\left(cx^n\right)\right)-\sqrt{2}\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}+1\right)}{2\sqrt{2}\,bn}-\frac{\log\left(\cot\left(a+b\log\left(cx^n\right)\right)+\sqrt{2}\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}+1\right)}{2\sqrt{2}\,bn}$$

Antiderivative was successfully verified.

[In] Int[Cot[a + b*Log[c*x^n]]^(5/2)/x,x]

[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n)) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) - (2*Cot[a + b*Log[c*x^n]]^(3/2))/(3*b*n) + Log[1 - Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]] + Cot[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) - Log[1 + Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]] + Cot[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n)

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{\cot^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\text{Subst}\left(\int \cot^{\frac{5}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2\cot^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} - \frac{\text{Subst}\left(\int \sqrt{\cot(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2\cot^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \cot(a+b\log(cx^n))\right)}{bn}$$

$$= -\frac{2\cot^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} + \frac{2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\cot(a+b\log(cx^n))}\right)}{bn}$$

$$= -\frac{2\cot^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} - \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\cot(a+b\log(cx^n))}\right)}{bn} + \text{Su}$$

$$= -\frac{2\cot^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{\cot(a+b\log(cx^n))}\right.}{2bn}$$

$$= -\frac{2\cot^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} + \frac{\log\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))} + \cot(a+b\,\text{lo}\right.}{2\sqrt{2}\,bn}$$

$$= -\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}\,bn} + \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{\cot(a+b\log(\right.}{\sqrt{2}\,bn}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.31, size = 50, normalized size = 0.25

$$\frac{2\cot^{\frac{3}{2}}(a+b\log(cx^n))\left(-1+{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(a+b\log(cx^n))\right)\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (2*Cot[a + b*Log[c*x^n]]^(3/2)*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[a
+ b*Log[c*x^n]]^2]))/(3*b*n)
```

Maple [A]
time = 0.23, size = 139, normalized size = 0.69

method	result

derivativedivides	$\frac{-\frac{2\left(\cot^{\frac{3}{2}}(a+b\ln(c\,x^n))\right)}{3}+\frac{\sqrt{2}\left(\ln\left(\frac{1+\cot(a+b\ln(c\,x^n))-\sqrt{2}\left(\sqrt{\cot}(a+b\ln(c\,x^n))\right)}{1+\cot(a+b\ln(c\,x^n))+\sqrt{2}\left(\sqrt{\cot}(a+b\ln(c\,x^n))\right)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\cot}\right.\right.\right.}{4}}{nb}$
default	$\frac{-\frac{2\left(\cot^{\frac{3}{2}}(a+b\ln(c\,x^n))\right)}{3}+\frac{\sqrt{2}\left(\ln\left(\frac{1+\cot(a+b\ln(c\,x^n))-\sqrt{2}\left(\sqrt{\cot}(a+b\ln(c\,x^n))\right)}{1+\cot(a+b\ln(c\,x^n))+\sqrt{2}\left(\sqrt{\cot}(a+b\ln(c\,x^n))\right)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\cot}\right.\right.\right.}{4}}{nb}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a+b*ln(c*x^n))^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(-2/3*cot(a+b*ln(c*x^n))^(3/2)+1/4*2^(1/2)*(ln((1+cot(a+b*ln(c*x^n))-
2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))/(1+cot(a+b*ln(c*x^n))+2^(1/2)*cot(a+b*ln(
c*x^n))^(1/2)))+2*arctan(1+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))+2*arctan(-1+2^
(1/2)*cot(a+b*ln(c*x^n))^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(cot(b*log(c*x^n) + a)^(5/2)/x, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*ln(c*x**n))**(5/2)/x,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 3.39, size = 79, normalized size = 0.39

$$\frac{(-1)^{1/4}\operatorname{atan}\left((-1)^{1/4}\sqrt{\cot\left(a+b\ln\left(c\,x^n\right)\right)}\right)}{b\,n}-\frac{2\cot(a+b\ln\left(c\,x^n\right))^{3/2}}{3\,b\,n}-\frac{(-1)^{1/4}\operatorname{atanh}\left((-1)^{1/4}\sqrt{\cot\left(a+b\ln\left(c\,x^n\right)\right)}\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*log(c*x^n))^(5/2)/x,x)
```

```
[Out] ((-1)^(1/4)*atan((-1)^(1/4)*cot(a + b*log(c*x^n))^(1/2)))/(b*n) - (2*cot(a
+ b*log(c*x^n))^(3/2))/(3*b*n) - ((-1)^(1/4)*atanh((-1)^(1/4)*cot(a + b*log
(c*x^n))^(1/2)))/(b*n)
```

3.232 $\int \frac{\cot^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=199

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}\,bn}+\frac{\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}\,bn}-\frac{2\sqrt{\cot(a+b\log}}{bn}$$

```
[Out] 1/2*arctan(-1+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)+1/2*arctan(1+2^
(1/2)*cot(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)-1/4*ln(1+cot(a+b*ln(c*x^n))-2^(
1/2)*cot(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)+1/4*ln(1+cot(a+b*ln(c*x^n))+2^(1
/2)*cot(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)-2*cot(a+b*ln(c*x^n))^(1/2)/b/n
```

Rubi [A]

time = 0.09, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}\,bn}+\frac{\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\sqrt{2}\,bn}-\frac{\log\left(\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{2\sqrt{2}\,bn}+\frac{\log\left(\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{2\sqrt{2}\,bn}-\frac{2\sqrt{\cot(a+b\log(cx^n))}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n)) + ArcTan[1
 + Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) - (2*Sqrt[Cot[a + b*L
og[c*x^n]]])/(b*n) - Log[1 - Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]] + Cot[a +
b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) + Log[1 + Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]
] + Cot[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \cot^{\frac{3}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{2\sqrt{\cot(a+b\log(cx^n))}}{bn} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\cot(a+bx)}}\,dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{2\sqrt{\cot(a+b\log(cx^n))}}{bn} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx, x, \cot(a+b\log(cx^n))\right)}{bn} \\
&= -\frac{2\sqrt{\cot(a+b\log(cx^n))}}{bn} + \frac{2\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{\cot(a+b\log(cx^n))}\right)}{bn} \\
&= -\frac{2\sqrt{\cot(a+b\log(cx^n))}}{bn} + \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\cot(a+b\log(cx^n))}\right)}{bn} + \\
&= -\frac{2\sqrt{\cot(a+b\log(cx^n))}}{bn} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{\cot(a+b\log(cx^n}}{2bn} \\
&= -\frac{2\sqrt{\cot(a+b\log(cx^n))}}{bn} - \frac{\log\left(1-\sqrt{2}\,\sqrt{\cot(a+b\log(cx^n))}+\cot(a+}{2\sqrt{2}\,bn} \\
&= -\frac{\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}\,bn} + \frac{\tan^{-1}\left(1+\sqrt{2}\,\sqrt{\cot(a+b\log}{\sqrt{2}\,bn}
\end{aligned}
$$

Mathematica [A]
time = 0.34, size = 175, normalized size = 0.88

$$
-\frac{2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(a+b\log(cx^n))}\right)-2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(a+b\log(cx^n))}\right)+8\sqrt{\cot(a+b\log(cx^n))}+\sqrt{2}\log\left(1-\sqrt{2}\,\sqrt{\cot(a+b\log(cx^n))}+\cot(a+b\log(cx^n))\right)-\sqrt{2}\log\left(1+\sqrt{2}\,\sqrt{\cot(a+b\log(cx^n))}+\cot(a+b\log(cx^n))\right)}{4bn}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] -1/4*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]]] - 2*Sqrt[2]
*ArcTan[1 + Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]]] + 8*Sqrt[Cot[a + b*Log[c*x
^n]]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]] + Cot[a + b*Log
[c*x^n]]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]] + Cot[a + b
*Log[c*x^n]]])/(b*n)
```

Maple [A]
time = 0.07, size = 139, normalized size = 0.70

method	result
derivativedivides	$\frac{-2\left(\sqrt{\cot}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)+\frac{\sqrt{2}\,\left(\ln\left(\frac{1+\cot\left(a+b\ln\left(c\,x^{n}\right)\right)+\sqrt{2}\,\left(\sqrt{\cot}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}{1+\cot\left(a+b\ln\left(c\,x^{n}\right)\right)-\sqrt{2}\,\left(\sqrt{\cot}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}\right)+2\arctan\left(1+\sqrt{2}\,\left(\sqrt{\mathrm{co}}\right.\right.}{4}}{nb}$
default	$\frac{-2\left(\sqrt{\cot}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)+\frac{\sqrt{2}\,\left(\ln\left(\frac{1+\cot\left(a+b\ln\left(c\,x^{n}\right)\right)+\sqrt{2}\,\left(\sqrt{\cot}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}{1+\cot\left(a+b\ln\left(c\,x^{n}\right)\right)-\sqrt{2}\,\left(\sqrt{\cot}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}\right)+2\arctan\left(1+\sqrt{2}\,\left(\sqrt{\mathrm{co}}\right.\right.}{4}}{nb}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(-2*cot(a+b*ln(c*x^n))^(1/2)+1/4*2^(1/2)*(ln((1+cot(a+b*ln(c*x^n))+2^
(1/2)*cot(a+b*ln(c*x^n))^(1/2))/(1+cot(a+b*ln(c*x^n))-2^(1/2)*cot(a+b*ln(c*
x^n))^(1/2)))+2*arctan(1+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))+2*arctan(-1+2^(1
/2)*cot(a+b*ln(c*x^n))^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(cot(b*log(c*x^n) + a)^(3/2)/x, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*ln(c*x**n))**(3/2)/x,x)
```

```
[Out] Integral(cot(a + b*log(c*x**n))**(3/2)/x, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 3.32, size = 80, normalized size = 0.40

$$-\frac{2\sqrt{\cot\left(a+b\ln\left(c\,x^n\right)\right)}}{b\,n}-\frac{{\left(-1\right)}^{1/4}\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\sqrt{\cot\left(a+b\ln\left(c\,x^n\right)\right)}\right)\,1{}\mathrm{i}}{b\,n}-\frac{{\left(-1\right)}^{1/4}\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,\sqrt{\cot\left(a+b\ln\left(c\,x^n\right)\right)}\right)\,1{}\mathrm{i}}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*log(c*x^n))^(3/2)/x,x)
```

```
[Out] - (2*cot(a + b*log(c*x^n))^(1/2))/(b*n) - ((-1)^(1/4)*atan((-1)^(1/4)*cot(a
 + b*log(c*x^n))^(1/2))*1i)/(b*n) - ((-1)^(1/4)*atanh((-1)^(1/4)*cot(a + b*
log(c*x^n))^(1/2))*1i)/(b*n)
```

3.233 $\int \frac{\sqrt{\cot(a+b\log(cx^n))}}{x}\,dx$

Optimal. Leaf size=176

$$\frac{\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}\,bn}-\frac{\operatorname{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}\,bn}-\frac{\log\left(1-\sqrt{2}\sqrt{\cot(\right.}{}$$

```
[Out] -1/2*arctan(-1+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)-1/2*arctan(1+2
^(1/2)*cot(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)-1/4*ln(1+cot(a+b*ln(c*x^n))-2^
(1/2)*cot(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)+1/4*ln(1+cot(a+b*ln(c*x^n))+2^(
1/2)*cot(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}\,bn}-\frac{\operatorname{ArcTan}\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\sqrt{2}\,bn}-\frac{\log\left(\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{2\sqrt{2}\,bn}+\frac{\log\left(\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{2\sqrt{2}\,bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] ArcTan[1 - Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) - ArcTan[1 +
Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) - Log[1 - Sqrt[2]*Sqrt[C
ot[a + b*Log[c*x^n]]] + Cot[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) + Log[1 + Sq
rt[2]*Sqrt[Cot[a + b*Log[c*x^n]]] + Cot[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{\sqrt{\cot(a+b\log(cx^n))}}{x}\,dx = \frac{\text{Subst}\left(\int \sqrt{\cot(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \cot(a+b\log(cx^n))\right)}{bn}$$
$$= -\frac{2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\cot(a+b\log(cx^n))}\right)}{bn}$$
$$= \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\cot(a+b\log(cx^n))}\right)}{bn} - \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\cot(a\right.}}{bn}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{\cot(a+b\log(cx^n))}\right)}{2bn} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x}\right.}{}$$
$$= -\frac{\log\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+\cot(a+b\log(cx^n))\right)}{2\sqrt{2}\,bn} + \frac{\log\left(1+\sqrt{\right.}}{}$$
$$= \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}\,bn} - \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{\cot(a+b\log(\right.}}{\sqrt{2}\,bn}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.11, size = 48, normalized size = 0.27

$$-\frac{2\cot^{\frac{3}{2}}(a+b\log(cx^n))\;{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(a+b\log(cx^n))\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] (-2*Cot[a + b*Log[c*x^n]]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[a + b*L
og[c*x^n]]^2])/(3*b*n)
```

Maple [A]
time = 0.07, size = 122, normalized size = 0.69

method	result
derivativedivides	$-\frac{\sqrt{2}\left(\ln\left(\frac{1+\cot(a+b\ln(c\,x^n))-\sqrt{2}\left(\sqrt{\cot}\,(a+b\ln(c\,x^n))\right)}{1+\cot(a+b\ln(c\,x^n))+\sqrt{2}\left(\sqrt{\cot}\,(a+b\ln(c\,x^n))\right)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\cot}\,(a+b\ln(c\,x^n))\right)\right)+2\,\mathrm{a}\right.}{4nb}$

default	$-\frac{\sqrt{2}\left(\ln\left(\frac{1+\cot(a+b\ln(c\,x^{n}))-\sqrt{2}\,\left(\sqrt{\cot}\,(a+b\ln(c\,x^{n}))\right)}{1+\cot(a+b\ln(c\,x^{n}))+\sqrt{2}\,\left(\sqrt{\cot}\,(a+b\ln(c\,x^{n}))\right)}\right)+2\arctan\left(1+\sqrt{2}\,\left(\sqrt{\cot}\,(a+b\ln(c\,x^{n}))\right)\right)+2}{4nb}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/n/b*2^(1/2)*(ln((1+cot(a+b*ln(c*x^n))-2^(1/2)*cot(a+b*ln(c*x^n))^(1/2)
)/(1+cot(a+b*ln(c*x^n))+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2)))+2*arctan(1+2^(1/
2)*cot(a+b*ln(c*x^n))^(1/2))+2*arctan(-1+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cot(b*log(c*x^n) + a))/x, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cot\left(a+b\log\left(cx^{n}\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*ln(c*x**n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(cot(a + b*log(c*x**n)))/x, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 2.62, size = 58, normalized size = 0.33

$$\frac{(-1)^{1/4}\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{\cot\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}{b\,n}-\frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{\cot\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*log(c*x^n))^(1/2)/x,x)
```

```
[Out] ((-1)^(1/4)*atanh((-1)^(1/4)*cot(a + b*log(c*x^n))^(1/2)))/(b*n) - ((-1)^(1
/4)*atan((-1)^(1/4)*cot(a + b*log(c*x^n))^(1/2)))/(b*n)
```

3.234 $\int \frac{1}{x\sqrt{\cot(a+b\log(cx^n))}}\,dx$

Optimal. Leaf size=176

$$\frac{\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}\,bn}-\frac{\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}\,bn}+\frac{\log\left(1-\sqrt{2}\sqrt{\cot\right.}}{}$$

```
[Out] -1/2*arctan(-1+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)-1/2*arctan(1+2
^(1/2)*cot(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)+1/4*ln(1+cot(a+b*ln(c*x^n))-2^
(1/2)*cot(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)-1/4*ln(1+cot(a+b*ln(c*x^n))+2^(
1/2)*cot(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}\,bn}-\frac{\mathrm{ArcTan}\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\sqrt{2}\,bn}+\frac{\log\left(\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{2\sqrt{2}\,bn}-\frac{\log\left(\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{2\sqrt{2}\,bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Cot[a + b*Log[c*x^n]]]),x]
```

```
[Out] ArcTan[1 - Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) - ArcTan[1 +
Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) + Log[1 - Sqrt[2]*Sqrt[C
ot[a + b*Log[c*x^n]]] + Cot[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) - Log[1 + Sq
rt[2]*Sqrt[Cot[a + b*Log[c*x^n]]] + Cot[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt{\cot(a+b\log(cx^n))}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{\cot(a+bx)}}\,dx, x, \log(cx^n)\right)}{n}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx, x, \cot(a+b\log(cx^n))\right)}{bn}\\
&= -\frac{2\operatorname{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{\cot(a+b\log(cx^n))}\right)}{bn}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\cot(a+b\log(cx^n))}\right)}{bn} - \frac{\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{c\right.}}{bn}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{\cot(a+b\log(cx^n))}\right)}{2bn} - \frac{\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}\right.}{}\\
&= \frac{\log\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+\cot(a+b\log(cx^n))\right)}{2\sqrt{2}\,bn} - \frac{\log\left(1+\right.}{}\\
&= \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}\,bn} - \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{\cot(a+b\,\mathrm{lo}\right.}}{\sqrt{2}\,bn}
\end{aligned}
$$

Mathematica [A]
time = 0.17, size = 142, normalized size = 0.81

$$
\frac{2\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)-2\operatorname{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)+\log\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+\cot(a+b\log(cx^n))\right)-\log\left(1+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+\cot(a+b\log(cx^n))\right)}{2\sqrt{2}\,bn}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[Cot[a + b*Log[c*x^n]]]),x]
```

```
[Out] (2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]]] - 2*ArcTan[1 + Sqrt[2]*S
qrt[Cot[a + b*Log[c*x^n]]]] + Log[1 - Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]] +
 Cot[a + b*Log[c*x^n]]] - Log[1 + Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]] + Cot
[a + b*Log[c*x^n]]])/(2*Sqrt[2]*b*n)
```

Maple [A]
time = 0.07, size = 122, normalized size = 0.69

method	result

derivativedivides	$-\frac{\sqrt{2}\left(\ln\left(\frac{1+\cot(a+b\ln(c\,x^n))+\sqrt{2}\left(\sqrt{\cot}\,(a+b\ln(c\,x^n))\right)}{1+\cot(a+b\ln(c\,x^n))-\sqrt{2}\left(\sqrt{\cot}\,(a+b\ln(c\,x^n))\right)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\cot}\,(a+b\ln(c\,x^n))\right)\right)+2\,\mathrm{a}\right.}{4nb}$
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{1+\cot(a+b\ln(c\,x^n))+\sqrt{2}\left(\sqrt{\cot}\,(a+b\ln(c\,x^n))\right)}{1+\cot(a+b\ln(c\,x^n))-\sqrt{2}\left(\sqrt{\cot}\,(a+b\ln(c\,x^n))\right)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\cot}\,(a+b\ln(c\,x^n))\right)\right)+2\,\mathrm{a}\right.}{4nb}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/cot(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/n/b*2^(1/2)*(ln((1+cot(a+b*ln(c*x^n))+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2)
)/(1+cot(a+b*ln(c*x^n))-2^(1/2)*cot(a+b*ln(c*x^n))^(1/2)))+2*arctan(1+2^(1/
2)*cot(a+b*ln(c*x^n))^(1/2))+2*arctan(-1+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cot(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt(cot(b*log(c*x^n) + a))), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cot(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cot(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(cot(a + b*log(c*x**n)))), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cot(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 2.94, size = 57, normalized size = 0.32

$$\frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{\cot\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)\,1\mathrm{i}}{b\,n}+\frac{(-1)^{1/4}\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{\cot\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)\,1\mathrm{i}}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cot(a + b*log(c*x^n))^(1/2)),x)
```

```
[Out] ((-1)^(1/4)*atan((-1)^(1/4)*cot(a + b*log(c*x^n))^(1/2))*1i)/(b*n) + ((-1)^
(1/4)*atanh((-1)^(1/4)*cot(a + b*log(c*x^n))^(1/2))*1i)/(b*n)
```

3.235 $\int \frac{1}{x \cot^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=199

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}\right)}{\sqrt{2}\,bn}+\frac{\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}\right)}{\sqrt{2}\,bn}+\frac{2}{bn\sqrt{\cot\left(a+b\log\right.}}$$

```
[Out] 1/2*arctan(-1+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)+1/2*arctan(1+2^
(1/2)*cot(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)+1/4*ln(1+cot(a+b*ln(c*x^n))-2^(
1/2)*cot(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)-1/4*ln(1+cot(a+b*ln(c*x^n))+2^(1
/2)*cot(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)+2/b/n/cot(a+b*ln(c*x^n))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}\right)}{\sqrt{2}\,bn}+\frac{\text{ArcTan}\left(\sqrt{2}\,\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}+1\right)}{\sqrt{2}\,bn}+\frac{\log\left(\cot\left(a+b\log\left(cx^n\right)\right)-\sqrt{2}\,\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}+1\right)}{2\sqrt{2}\,bn}-\frac{\log\left(\cot\left(a+b\log\left(cx^n\right)\right)+\sqrt{2}\,\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}+1\right)}{2\sqrt{2}\,bn}+\frac{2}{bn\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Cot[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n)) + ArcTan[1
 + Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) + 2/(b*n*Sqrt[Cot[a +
 b*Log[c*x^n]]]) + Log[1 - Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]] + Cot[a + b*
Log[c*x^n]]]/(2*Sqrt[2]*b*n) - Log[1 + Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]]
+ Cot[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{x\cot^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\cot^{\frac{3}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$
$$= \frac{2}{bn\sqrt{\cot(a+b\log(cx^n))}} - \frac{\text{Subst}\left(\int \sqrt{\cot(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$
$$= \frac{2}{bn\sqrt{\cot(a+b\log(cx^n))}} + \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \cot(a+b\log(cx^n))\right)}{bn}$$
$$= \frac{2}{bn\sqrt{\cot(a+b\log(cx^n))}} + \frac{2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\cot(a+b\log(cx^n))}\right)}{bn}$$
$$= \frac{2}{bn\sqrt{\cot(a+b\log(cx^n))}} - \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\cot(a+b\log(cx^n))}\right)}{bn} +$$
$$= \frac{2}{bn\sqrt{\cot(a+b\log(cx^n))}} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{\cot(a+b\log(cx^n}\right.}{2bn}$$
$$= \frac{2}{bn\sqrt{\cot(a+b\log(cx^n))}} + \frac{\log\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+\cot(a+\right.}{2\sqrt{2}\,bn}$$
$$= -\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}\,bn} + \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{\cot(a+b\log\right.}{\sqrt{2}\,bn}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.15, size = 46, normalized size = 0.23

$$\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(a+b\log(cx^n))\right)}{bn\sqrt{\cot(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Cot[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[a + b*Log[c*x^n]]^2])/(b*n*Sqrt[Cot
[a + b*Log[c*x^n]]])
```

Maple [A]
time = 0.07, size = 139, normalized size = 0.70

method	result
derivativedivides	$\frac{\frac{2}{\sqrt{\cot(a+b\ln(c\,x^n))}}+\frac{\sqrt{2}\,\left(\ln\left(\frac{1+\cot(a+b\ln(c\,x^n))-\sqrt{2}\,\left(\sqrt{\cot}\,(a+b\ln(c\,x^n))\right)}{1+\cot(a+b\ln(c\,x^n))+\sqrt{2}\,\left(\sqrt{\cot}\,(a+b\ln(c\,x^n))\right)}\right)+2\arctan\left(1+\sqrt{2}\,\left(\sqrt{\right.\right.}{4}}{nb}$
default	$\frac{\frac{2}{\sqrt{\cot(a+b\ln(c\,x^n))}}+\frac{\sqrt{2}\,\left(\ln\left(\frac{1+\cot(a+b\ln(c\,x^n))-\sqrt{2}\,\left(\sqrt{\cot}\,(a+b\ln(c\,x^n))\right)}{1+\cot(a+b\ln(c\,x^n))+\sqrt{2}\,\left(\sqrt{\cot}\,(a+b\ln(c\,x^n))\right)}\right)+2\arctan\left(1+\sqrt{2}\,\left(\sqrt{\right.\right.}{4}}{nb}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/cot(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(2/cot(a+b*ln(c*x^n))^(1/2)+1/4*2^(1/2)*(ln((1+cot(a+b*ln(c*x^n))-2^(
1/2)*cot(a+b*ln(c*x^n))^(1/2))/(1+cot(a+b*ln(c*x^n))+2^(1/2)*cot(a+b*ln(c*x
^n))^(1/2)))+2*arctan(1+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))+2*arctan(-1+2^(1/
2)*cot(a+b*ln(c*x^n))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cot(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*cot(b*log(c*x^n) + a)^(3/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cot(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\cot^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cot(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Integral(1/(x*cot(a + b*log(c*x**n))**(3/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cot(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 2.95, size = 79, normalized size = 0.40

$$\frac{2}{b\,n\,\sqrt{\cot\left(a+b\,\ln\left(c\,x^n\right)\right)}}+\frac{{\left(-1\right)}^{1/4}\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\sqrt{\cot\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}{b\,n}-\frac{{\left(-1\right)}^{1/4}\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,\sqrt{\cot\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cot(a + b*log(c*x^n))^(3/2)),x)
```

```
[Out] 2/(b*n*cot(a + b*log(c*x^n))^(1/2)) + ((-1)^(1/4)*atan((-1)^(1/4)*cot(a + b
*log(c*x^n))^(1/2)))/(b*n) - ((-1)^(1/4)*atanh((-1)^(1/4)*cot(a + b*log(c*x
^n))^(1/2)))/(b*n)
```

3.236 $\int \frac{1}{x \cot^{\frac{5}{2}}(a+b\log(cx^n))}\, dx$

Optimal. Leaf size=201

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}\right)}{\sqrt{2}\,bn}+\frac{\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}\right)}{\sqrt{2}\,bn}+\frac{2}{3bn\cot^{\frac{3}{2}}\left(a+b\log\right.}$$

```
[Out] 2/3/b/n/cot(a+b*ln(c*x^n))^(3/2)+1/2*arctan(-1+2^(1/2)*cot(a+b*ln(c*x^n))^(
1/2))/b/n*2^(1/2)+1/2*arctan(1+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2
)-1/4*ln(1+cot(a+b*ln(c*x^n))-2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)
+1/4*ln(1+cot(a+b*ln(c*x^n))+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))/b/n*2^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3555, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}\right)}{\sqrt{2}\,bn}+\frac{\text{ArcTan}\left(\sqrt{2}\,\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}+1\right)}{\sqrt{2}\,bn}+\frac{2}{3bn\cot^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}-\frac{\log\left(\cot\left(a+b\log\left(cx^n\right)\right)-\sqrt{2}\,\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}+1\right)}{2\sqrt{2}\,bn}+\frac{\log\left(\cot\left(a+b\log\left(cx^n\right)\right)+\sqrt{2}\,\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}+1\right)}{2\sqrt{2}\,bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Cot[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n)) + ArcTan[1
 + Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) + 2/(3*b*n*Cot[a + b*
Log[c*x^n]]^(3/2)) - Log[1 - Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]] + Cot[a +
b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) + Log[1 + Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]
] + Cot[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{x \cot^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\cot^{\frac{5}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2}{3bn\cot^{\frac{3}{2}}(a+b\log(cx^n))} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\cot(a+bx)}}\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2}{3bn\cot^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx, x, \cot(a+b\log(cx^n))\right)}{bn}$$

$$= \frac{2}{3bn\cot^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{2\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{\cot(a+b\log(cx^n))}\right)}{bn}$$

$$= \frac{2}{3bn\cot^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\cot(a+b\log(cx^n))}\right)}{bn} +$$

$$= \frac{2}{3bn\cot^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{\cot(a+b\log(cx^n))}\right.}{2bn}$$

$$= \frac{2}{3bn\cot^{\frac{3}{2}}(a+b\log(cx^n))} - \frac{\log\left(1-\sqrt{2}\,\sqrt{\cot(a+b\log(cx^n))}+\cot(a+\right.}{2\sqrt{2}\,bn}$$

$$= -\frac{\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}\,bn} + \frac{\tan^{-1}\left(1+\sqrt{2}\,\sqrt{\cot(a+b\log\right.}{\sqrt{2}\,bn}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.23, size = 48, normalized size = 0.24

$$\frac{2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\cot^2(a+b\log(cx^n))\right)}{3bn\cot^{\frac{3}{2}}(a+b\log(cx^n))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Cot[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[a + b*Log[c*x^n]]^2])/(3*b*n*Cot[a
+ b*Log[c*x^n]]^(3/2))
```

Maple [A]
time = 0.07, size = 139, normalized size = 0.69

method	result
derivativedivides	$\frac{\frac{2}{3\cot(a+b\ln(c\,x^n))^{\frac{3}{2}}}+\frac{\sqrt{2}\left(\ln\left(\frac{1+\cot(a+b\ln(c\,x^n))+\sqrt{2}\left(\sqrt{\cot}(a+b\ln(c\,x^n))\right)}{1+\cot(a+b\ln(c\,x^n))-\sqrt{2}\left(\sqrt{\cot}(a+b\ln(c\,x^n))\right)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\cot}(a+b\ln(\right.\right.}{4}}{nb}$
default	$\frac{\frac{2}{3\cot(a+b\ln(c\,x^n))^{\frac{3}{2}}}+\frac{\sqrt{2}\left(\ln\left(\frac{1+\cot(a+b\ln(c\,x^n))+\sqrt{2}\left(\sqrt{\cot}(a+b\ln(c\,x^n))\right)}{1+\cot(a+b\ln(c\,x^n))-\sqrt{2}\left(\sqrt{\cot}(a+b\ln(c\,x^n))\right)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\cot}(a+b\ln(\right.\right.}{4}}{nb}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/cot(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(2/3/cot(a+b*ln(c*x^n))^(3/2)+1/4*2^(1/2)*(ln((1+cot(a+b*ln(c*x^n))+2
^(1/2)*cot(a+b*ln(c*x^n))^(1/2))/(1+cot(a+b*ln(c*x^n))-2^(1/2)*cot(a+b*ln(c
*x^n))^(1/2)))+2*arctan(1+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))+2*arctan(-1+2^(
1/2)*cot(a+b*ln(c*x^n))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cot(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*cot(b*log(c*x^n) + a)^(5/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cot(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cot(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cot(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 4.13, size = 80, normalized size = 0.40

$$\frac{2}{3\,b\,n\cot\left(a+b\,\ln\left(c\,x^n\right)\right)^{3/2}}-\frac{{\left(-1\right)}^{1/4}\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\sqrt{\cot\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)\,1{}\mathrm{i}}{b\,n}-\frac{{\left(-1\right)}^{1/4}\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,\sqrt{\cot\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)\,1{}\mathrm{i}}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cot(a + b*log(c*x^n))^(5/2)),x)
```

```
[Out] 2/(3*b*n*cot(a + b*log(c*x^n))^(3/2)) - ((-1)^(1/4)*atan((-1)^(1/4)*cot(a +
 b*log(c*x^n))^(1/2))*1i)/(b*n) - ((-1)^(1/4)*atanh((-1)^(1/4)*cot(a + b*lo
g(c*x^n))^(1/2))*1i)/(b*n)
```

3.237 $\int x^2 \sec(a + b\log(cx^n))\, dx$

Optimal. Leaf size=87

$$\frac{2e^{ia}x^3(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}\left(1-\frac{3i}{bn}\right);\frac{3}{2}\left(1-\frac{i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{3+ibn}$$

```
[Out] 2*exp(I*a)*x^3*(c*x^n)^(I*b)*hypergeom([1, 1/2-3/2*I/b/n],[3/2-3/2*I/b/n],-
exp(2*I*a)*(c*x^n)^(2*I*b))/(3+I*b*n)
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4605, 4601, 371}

$$\frac{2e^{ia}x^3(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}\left(1-\frac{3i}{bn}\right);\frac{3}{2}\left(1-\frac{i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{3+ibn}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sec[a + b*Log[c*x^n]],x]
```

```
[Out] (2*E^(I*a)*x^3*(c*x^n)^(I*b)*Hypergeometric2F1[1, (1 - (3*I)/(b*n))/2, (3*(
1 - I/(b*n)))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(3 + I*b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4605

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^2 \sec (a+b\log (cx^n))\,dx = \frac{\left(x^3(cx^n)^{-3/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{3}{n}}\sec(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(2e^{ia}x^3(cx^n)^{-3/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+ib+\frac{3}{n}}}{1+e^{2ia}x^{2ib}}\,dx,x,cx^n\right)}{n}$$
$$= \frac{2e^{ia}x^3(cx^n)^{ib}\ {}_2F_1\left(1,\frac{1}{2}\left(1-\frac{3i}{bn}\right);\frac{3}{2}\left(1-\frac{i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{3+ibn}$$

Mathematica [A]
time = 1.11, size = 86, normalized size = 0.99

$$-\frac{2ie^{ia}x^3(cx^n)^{ib}\ {}_2F_1\left(1,\frac{1}{2}-\frac{3i}{2bn};\frac{3}{2}-\frac{3i}{2bn};-e^{2i(a+b\log(cx^n))}\right)}{-3i+bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sec[a + b*Log[c*x^n]],x]
[Out] ((-2*I)*E^(I*a)*x^3*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 - ((3*I)/2)/(b*n
), 3/2 - ((3*I)/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))])/(-3*I + b*n)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^2 \sec (a+b\ln (c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sec(a+b*ln(c*x^n)),x)
[Out] int(x^2*sec(a+b*ln(c*x^n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(x^2*sec(b*log(c*x^n) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(x^2*sec(b*log(c*x^n) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sec \left(a + b \log \left(cx^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sec(a+b*ln(c*x**n)),x)
[Out] Integral(x**2*sec(a + b*log(c*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate(x^2*sec(b*log(c*x^n) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\cos \left(a + b \ln \left(c\,x^n\right)\right)} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/cos(a + b*log(c*x^n)),x)
[Out] int(x^2/cos(a + b*log(c*x^n)), x)
```

3.238 $\int x \sec(a + b\log(cx^n))\,dx$

Optimal. Leaf size=87

$$\frac{2e^{ia}x^2(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}\left(1-\frac{2i}{bn}\right);\frac{1}{2}\left(3-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{2+ibn}$$

```
[Out] 2*exp(I*a)*x^2*(c*x^n)^(I*b)*hypergeom([1, 1/2-I/b/n],[3/2-I/b/n],-exp(2*I*
a)*(c*x^n)^(2*I*b))/(2+I*b*n)
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4605, 4601, 371}

$$\frac{2e^{ia}x^2(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}\left(1-\frac{2i}{bn}\right);\frac{1}{2}\left(3-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{2+ibn}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[a + b*Log[c*x^n]],x]
```

```
[Out] (2*E^(I*a)*x^2*(c*x^n)^(I*b)*Hypergeometric2F1[1, (1 - (2*I)/(b*n))/2, (3 -
 (2*I)/(b*n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(2 + I*b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4605

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x\sec(a+b\log(cx^n))\,dx = \frac{\left(x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int x^{-1+\frac{2}{n}}\sec(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(2e^{ia}x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int \frac{x^{-1+ib+\frac{2}{n}}}{1+e^{2ia}x^{2ib}}\,dx,x,cx^n\right)}{n}$$
$$= \frac{2e^{ia}x^2(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}\left(1-\frac{2i}{bn}\right);\frac{1}{2}\left(3-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{2+ibn}$$

Mathematica [**A**]
time = 1.05, size = 82, normalized size = 0.94

$$-\frac{2ie^{ia}x^2(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}-\frac{i}{bn};\frac{3}{2}-\frac{i}{bn};-e^{2i(a+b\log(cx^n))}\right)}{-2i+bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[a + b*Log[c*x^n]],x]
[Out] ((-2*I)*E^(I*a)*x^2*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 - I/(b*n), 3/2 -
 I/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))])/(-2*I + b*n)
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int x\sec(a+b\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(a+b*ln(c*x^n)),x)
[Out] int(x*sec(a+b*ln(c*x^n)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(x*sec(b*log(c*x^n) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(x*sec(b*log(c*x^n) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sec\left(a+b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*ln(c*x**n)),x)
[Out] Integral(x*sec(a + b*log(c*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate(x*sec(b*log(c*x^n) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\cos\left(a+b\,\ln\left(c\,x^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(a + b*log(c*x^n)),x)
[Out] int(x/cos(a + b*log(c*x^n)), x)
```

3.239 $\int \sec(a+b\log(cx^n))\,dx$

Optimal. Leaf size=85

$$\frac{2e^{ia}x(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}\left(1-\frac{i}{bn}\right);\frac{1}{2}\left(3-\frac{i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{1+ibn}$$

```
[Out] 2*exp(I*a)*x*(c*x^n)^(I*b)*hypergeom([1, 1/2-1/2*I/b/n],[3/2-1/2*I/b/n],-ex
p(2*I*a)*(c*x^n)^(2*I*b))/(1+I*b*n)
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4599, 4601, 371}

$$\frac{2e^{ia}x(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}\left(1-\frac{i}{bn}\right);\frac{1}{2}\left(3-\frac{i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{1+ibn}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*Log[c*x^n]],x]
```

```
[Out] (2*E^(I*a)*x*(c*x^n)^(I*b)*Hypergeometric2F1[1, (1 - I/(b*n))/2, (3 - I/(b*
n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(1 + I*b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4599

```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \sec\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\sec(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(2e^{ia}x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+ib+\frac{1}{n}}}{1+e^{2ia}x^{2ib}}\,dx,x,cx^n\right)}{n}$$
$$= \frac{2e^{ia}x(cx^n)^{ib}\;{}_2F_1\left(1,\frac{1}{2}\left(1-\frac{i}{bn}\right);\frac{1}{2}\left(3-\frac{i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{1+ibn}$$

Mathematica [A]
time = 0.87, size = 84, normalized size = 0.99

$$-\frac{2ie^{ia}x(cx^n)^{ib}\;{}_2F_1\left(1,\frac{1}{2}-\frac{i}{2bn};\frac{3}{2}-\frac{i}{2bn};-e^{2i(a+b\log(cx^n))}\right)}{-i+bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*Log[c*x^n]],x]
[Out] ((-2*I)*E^(I*a)*x*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 - (I/2)/(b*n), 3/2
 - (I/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))])/(-I + b*n)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \sec\left(a+b\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a+b*ln(c*x^n)),x)
[Out] int(sec(a+b*ln(c*x^n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(sec(b*log(c*x^n) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(sec(b*log(c*x^n) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sec\left(a+b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n)),x)
[Out] Integral(sec(a + b*log(c*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate(sec(b*log(c*x^n) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(a+b\,\ln\left(c\,x^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*log(c*x^n)),x)
[Out] int(1/cos(a + b*log(c*x^n)), x)
```

3.240 $\int \frac{\sec(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=19

$$\frac{\tanh^{-1}\left(\sin\left(a+b\log\left(cx^n\right)\right)\right)}{bn}$$

```
[Out] arctanh(sin(a+b*ln(c*x^n)))/b/n
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3855}

$$\frac{\tanh^{-1}\left(\sin\left(a+b\log\left(cx^n\right)\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*Log[c*x^n]]/x,x]
```

```
[Out] ArcTanh[Sin[a + b*Log[c*x^n]]]/(b*n)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \sec(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= \frac{\tanh^{-1}\left(\sin\left(a+b\log\left(cx^n\right)\right)\right)}{bn}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 19, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\sin\left(a+b\log\left(cx^n\right)\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*Log[c*x^n]]/x,x]
```

```
[Out] ArcTanh[Sin[a + b*Log[c*x^n]]]/(b*n)
```

Maple [A]
time = 0.05, size = 32, normalized size = 1.68

method	result
derivativedivides	$\frac{\ln(\sec(a+b\ln(c\,x^n))+\tan(a+b\ln(c\,x^n)))}{nb}$
default	$\frac{\ln(\sec(a+b\ln(c\,x^n))+\tan(a+b\ln(c\,x^n)))}{nb}$
risch	$-\frac{\ln\left(c^{ib}(x^n)^{ib}\mathrm{e}^{-\frac{b\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ix^n)}{2}}\mathrm{e}^{\frac{b\pi\,\mathrm{csgn}(ic\,x^n)\mathrm{csgn}(ic)\mathrm{csgn}(ix^n)}{2}}\mathrm{e}^{\frac{b\pi\mathrm{csgn}(ic\,x^n)^3}{2}}\mathrm{e}^{-\frac{b\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ic)}{2}}\mathrm{e}^{ia}-i\right)}{bn}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
[Out] 1/n/b*ln(sec(a+b*ln(c*x^n))+tan(a+b*ln(c*x^n)))
```

Maxima [A]
time = 0.27, size = 31, normalized size = 1.63

$$\frac{\log\left(\sec\left(b\log\left(cx^n\right)+a\right)+\tan\left(b\log\left(cx^n\right)+a\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))/x,x, algorithm="maxima")
[Out] log(sec(b*log(c*x^n) + a) + tan(b*log(c*x^n) + a))/(b*n)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(19) = 38.
time = 3.00, size = 43, normalized size = 2.26

$$\frac{\log\left(\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+1\right)-\log\left(-\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+1\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))/x,x, algorithm="fricas")
[Out] 1/2*(log(sin(b*n*log(x) + b*log(c) + a) + 1) - log(-sin(b*n*log(x) + b*log(
c) + a) + 1))/(b*n)
```

Sympy [A]
time = 1.32, size = 51, normalized size = 2.68

$$-\begin{cases}-\log\left(x\right)\sec\left(a\right) & \text{for}\ b=0\\ -\log\left(x\right)\sec\left(a+b\log\left(c\right)\right) & \text{for}\ n=0\\ -\frac{\log\left(\tan\left(a+b\log\left(cx^n\right)\right)+\sec\left(a+b\log\left(cx^n\right)\right)\right)}{bn} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))/x,x)
```

```
[Out] -Piecewise((-log(x)*sec(a), Eq(b, 0)), (-log(x)*sec(a + b*log(c)), Eq(n, 0)
), (-log(tan(a + b*log(c*x**n)) + sec(a + b*log(c*x**n)))/(b*n), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))/x,x, algorithm="giac")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)/x, x)
```

Mupad [B]
time = 3.90, size = 66, normalized size = 3.47

$$-\frac{\ln\left(\frac{2\,{\mathrm{e}}^{a\,1{}\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,1{}\mathrm{i}}-2{}\mathrm{i}}{x}\right)}{b\,n}+\frac{\ln\left(\frac{2\,{\mathrm{e}}^{a\,1{}\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,1{}\mathrm{i}}+2{}\mathrm{i}}{x}\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cos(a + b*log(c*x^n))),x)
```

```
[Out] log((2*exp(a*1i)*(c*x^n)^(b*1i) + 2i)/x)/(b*n) - log((2*exp(a*1i)*(c*x^n)^(
b*1i) - 2i)/x)/(b*n)
```

3.241 $\int \frac{\sec(a+b\log(cx^n))}{x^2}\,dx$

Optimal. Leaf size=87

$$-\frac{2e^{ia}(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}\left(1+\frac{i}{bn}\right);\frac{1}{2}\left(3+\frac{i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{(1-ibn)x}$$

```
[Out] -2*exp(I*a)*(c*x^n)^(I*b)*hypergeom([1, 1/2+1/2*I/b/n],[3/2+1/2*I/b/n],-exp
(2*I*a)*(c*x^n)^(2*I*b))/(1-I*b*n)/x
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4605, 4601, 371}

$$-\frac{2e^{ia}(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}\left(1+\frac{i}{bn}\right);\frac{1}{2}\left(3+\frac{i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{x(1-ibn)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*Log[c*x^n]]/x^2,x]
```

```
[Out] (-2*E^(I*a)*(c*x^n)^(I*b)*Hypergeometric2F1[1, (1 + I/(b*n))/2, (3 + I/(b*n
))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((1 - I*b*n)*x)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4605

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\sec\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx = \frac{(cx^n)^{\frac{1}{n}}\,\mathrm{Subst}\left(\int x^{-1-\frac{1}{n}}\sec(a+b\log(x))\,dx, x, cx^n\right)}{nx}$$
$$= \frac{\left(2e^{ia}(cx^n)^{\frac{1}{n}}\right)\mathrm{Subst}\left(\int \frac{x^{-1+ib-\frac{1}{n}}}{1+e^{2ia}x^{2ib}}\,dx, x, cx^n\right)}{nx}$$
$$= -\frac{2e^{ia}(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}\left(1+\frac{i}{bn}\right);\frac{1}{2}\left(3+\frac{i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{(1-ibn)x}$$

Mathematica [A]
time = 0.73, size = 85, normalized size = 0.98

$$\frac{2e^{ia}(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}+\frac{i}{2bn};\frac{3}{2}+\frac{i}{2bn};-e^{2i(a+b\log(cx^n))}\right)}{(-1+ibn)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*Log[c*x^n]]/x^2,x]
[Out] (2*E^(I*a)*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 + (I/2)/(b*n), 3/2 + (I/2
)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))])/((-1 + I*b*n)*x)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(a+b\ln\left(c\,x^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a+b*ln(c*x^n))/x^2,x)
[Out] int(sec(a+b*ln(c*x^n))/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))/x^2,x, algorithm="maxima")
[Out] integrate(sec(b*log(c*x^n) + a)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))/x^2,x, algorithm="fricas")
[Out] integral(sec(b*log(c*x^n) + a)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(a + b\log\left(cx^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))/x**2,x)
[Out] Integral(sec(a + b*log(c*x**n))/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))/x^2,x, algorithm="giac")
[Out] integrate(sec(b*log(c*x^n) + a)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*cos(a + b*log(c*x^n))),x)
[Out] int(1/(x^2*cos(a + b*log(c*x^n))), x)
```

3.242 $\int \frac{\sec(a+b\log(cx^n))}{x^3}\,dx$

Optimal. Leaf size=87

$$-\frac{2e^{ia}(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}\left(1+\frac{2i}{bn}\right);\frac{1}{2}\left(3+\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{(2-ibn)x^2}$$

```
[Out] -2*exp(I*a)*(c*x^n)^(I*b)*hypergeom([1, 1/2+I/b/n],[3/2+I/b/n],-exp(2*I*a)*
(c*x^n)^(2*I*b))/(2-I*b*n)/x^2
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4605, 4601, 371}

$$-\frac{2e^{ia}(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}\left(1+\frac{2i}{bn}\right);\frac{1}{2}\left(3+\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{x^2(2-ibn)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*Log[c*x^n]]/x^3,x]
```

```
[Out] (-2*E^(I*a)*(c*x^n)^(I*b)*Hypergeometric2F1[1, (1 + (2*I)/(b*n))/2, (3 + (2
*I)/(b*n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 - I*b*n)*x^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4605

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\sec(a+b\log(cx^n))}{x^3}\,dx = \frac{(cx^n)^{2/n}\,\text{Subst}\left(\int x^{-1-\frac{2}{n}}\sec(a+b\log(x))\,dx, x, cx^n\right)}{nx^2}$$
$$= \frac{\left(2e^{ia}(cx^n)^{2/n}\right)\text{Subst}\left(\int \frac{x^{-1+ib-\frac{2}{n}}}{1+e^{2ia}x^{2ib}}\,dx, x, cx^n\right)}{nx^2}$$
$$= -\frac{2e^{ia}(cx^n)^{ib}\,{}_2F_1\left(1, \frac{1}{2}\left(1+\frac{2i}{bn}\right); \frac{1}{2}\left(3+\frac{2i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right)}{(2-ibn)x^2}$$

Mathematica [A]
time = 0.76, size = 81, normalized size = 0.93

$$\frac{2e^{ia}(cx^n)^{ib}\,{}_2F_1\left(1, \frac{1}{2}+\frac{i}{bn}; \frac{3}{2}+\frac{i}{bn}; -e^{2i(a+b\log(cx^n))}\right)}{(-2+ibn)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*Log[c*x^n]]/x^3,x]
```

```
[Out] (2*E^(I*a)*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 + I/(b*n), 3/2 + I/(b*n),
 -E^((2*I)*(a + b*Log[c*x^n]))])/((-2 + I*b*n)*x^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sec(a+b\ln(c\,x^n))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a+b*ln(c*x^n))/x^3,x)
```

```
[Out] int(sec(a+b*ln(c*x^n))/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))/x^3,x, algorithm="fricas")
```

```
[Out] integral(sec(b*log(c*x^n) + a)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(a + b\log\left(cx^{n}\right)\right)}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))/x**3,x)
```

```
[Out] Integral(sec(a + b*log(c*x**n))/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))/x^3,x, algorithm="giac")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3}\,\cos\left(a+b\,\ln\left(c\,x^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*cos(a + b*log(c*x^n))),x)
```

```
[Out] int(1/(x^3*cos(a + b*log(c*x^n))), x)
```

3.243 $\int x^2 \sec^2(a + b\log(cx^n))\,dx$

Optimal. Leaf size=87

$$\frac{4e^{2ia}x^3(cx^n)^{2ib}\,{}_2F_1\left(2,\frac{1}{2}\left(2-\frac{3i}{bn}\right);\frac{1}{2}\left(4-\frac{3i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{3+2ibn}$$

```
[Out] 4*exp(2*I*a)*x^3*(c*x^n)^(2*I*b)*hypergeom([2, 1-3/2*I/b/n],[2-3/2*I/b/n],-
exp(2*I*a)*(c*x^n)^(2*I*b))/(3+2*I*b*n)
```

Rubi [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4605, 4601, 371}

$$\frac{4e^{2ia}x^3(cx^n)^{2ib}\,{}_2F_1\left(2,\frac{1}{2}\left(2-\frac{3i}{bn}\right);\frac{1}{2}\left(4-\frac{3i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{3+2ibn}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sec[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (4*E^((2*I)*a)*x^3*(c*x^n)^((2*I)*b)*Hypergeometric2F1[2, (2 - (3*I)/(b*n))
/2, (4 - (3*I)/(b*n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(3 + (2*I)*b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4605

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^2 \sec^2(a+b\log(cx^n))\,dx = \frac{\left(x^3(cx^n)^{-3/n}\right)\text{Subst}\left(\int x^{-1+\frac{3}{n}}\sec^2(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(4e^{2ia}x^3(cx^n)^{-3/n}\right)\text{Subst}\left(\int \frac{x^{-1+2ib+\frac{3}{n}}}{(1+e^{2ia}x^{2ib})^2}\,dx,x,cx^n\right)}{n}$$
$$= \frac{4e^{2ia}x^3(cx^n)^{2ib}\,{}_2F_1\left(2,\frac{1}{2}\left(2-\frac{3i}{bn}\right);\frac{1}{2}\left(4-\frac{3i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{3+2ibn}$$

Mathematica [A]
time = 5.97, size = 160, normalized size = 1.84

$$\frac{x^3\left(3e^{2ia}(cx^n)^{2ib}\,{}_2F_1\left(1,1-\frac{3i}{2bn};2-\frac{3i}{2bn};-e^{2i(a+b\log(cx^n))}\right)+(-3i+2bn)\left(-i\,{}_2F_1\left(1,-\frac{3i}{2bn};1-\frac{3i}{2bn};-e^{2i(a+b\log(cx^n))}\right)+\tan(a+b\log(cx^n))\right)\right)}{bn(-3i+2bn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sec[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x^3*(3*E^((2*I)*a)*(c*x^n)^((2*I)*b)*Hypergeometric2F1[1, 1 - ((3*I)/2)/(b
*n), 2 - ((3*I)/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] + (-3*I + 2*b*n)*(
(-I)*Hypergeometric2F1[1, ((-3*I)/2)/(b*n), 1 - ((3*I)/2)/(b*n), -E^((2*I)*
(a + b*Log[c*x^n]))] + Tan[a + b*Log[c*x^n]])))/(b*n*(-3*I + 2*b*n))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int x^2\left(\sec^2(a+b\ln(c\,x^n))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sec(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(x^2*sec(a+b*ln(c*x^n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 2*(x^3*cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + x^3*cos(2*b*log(c))*sin(2*
b*log(x^n) + 2*a) - 3*(2*b^2*n^2*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) -
```

```
2*b^2*n^2*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b^2*cos(2*b*log(c))^2
+ b^2*sin(2*b*log(c))^2)*n^2*cos(2*b*log(x^n) + 2*a)^2 + (b^2*cos(2*b*log(c
))^2 + b^2*sin(2*b*log(c))^2)*n^2*sin(2*b*log(x^n) + 2*a)^2 + b^2*n^2)*inte
grate((x^2*cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + x^2*cos(2*b*log(c))*si
n(2*b*log(x^n) + 2*a))/(2*b^2*n^2*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) -
 2*b^2*n^2*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b^2*cos(2*b*log(c))^2
 + b^2*sin(2*b*log(c))^2)*n^2*cos(2*b*log(x^n) + 2*a)^2 + (b^2*cos(2*b*log(
c))^2 + b^2*sin(2*b*log(c))^2)*n^2*sin(2*b*log(x^n) + 2*a)^2 + b^2*n^2), x)
)/(2*b*n*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) + (b*cos(2*b*log(c))^2 + b
*sin(2*b*log(c))^2)*n*cos(2*b*log(x^n) + 2*a)^2 - 2*b*n*sin(2*b*log(c))*sin
(2*b*log(x^n) + 2*a) + (b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*sin(2*
b*log(x^n) + 2*a)^2 + b*n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(a+b*log(c*x^n))^2,x, algorithm="fricas")
[Out] integral(x^2*sec(b*log(c*x^n) + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sec^2(a + b\log(cx^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sec(a+b*ln(c*x**n))**2,x)
[Out] Integral(x**2*sec(a + b*log(c*x**n))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(a+b*log(c*x^n))^2,x, algorithm="giac")
[Out] integrate(x^2*sec(b*log(c*x^n) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/cos(a + b*log(c*x^n))^2,x)
```

```
[Out] int(x^2/cos(a + b*log(c*x^n))^2, x)
```

3.244 $\int x \sec^2(a + b\log(cx^n))\,dx$

Optimal. Leaf size=79

$$\frac{2e^{2ia}x^2(cx^n)^{2ib}\,{}_2F_1\left(2,1-\frac{i}{bn};2-\frac{i}{bn};-e^{2ia}(cx^n)^{2ib}\right)}{1+ibn}$$

```
[Out] 2*exp(2*I*a)*x^2*(c*x^n)^(2*I*b)*hypergeom([2, 1-I/b/n],[2-I/b/n],-exp(2*I*
a)*(c*x^n)^(2*I*b))/(1+I*b*n)
```

Rubi [A]
time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4605, 4601, 371}

$$\frac{2e^{2ia}x^2(cx^n)^{2ib}\,{}_2F_1\left(2,1-\frac{i}{bn};2-\frac{i}{bn};-e^{2ia}(cx^n)^{2ib}\right)}{1+ibn}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (2*E^((2*I)*a)*x^2*(c*x^n)^((2*I)*b)*Hypergeometric2F1[2, 1 - I/(b*n), 2 -
I/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(1 + I*b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4605

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x\sec^2(a+b\log(cx^n))\,dx = \frac{\left(x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int x^{-1+\frac{2}{n}}\sec^2(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(4e^{2ia}x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int \frac{x^{-1+2ib+\frac{2}{n}}}{(1+e^{2ia}x^{2ib})^2}\,dx,x,cx^n\right)}{n}$$
$$= \frac{2e^{2ia}x^2(cx^n)^{2ib}\ {}_2F_1\left(2,1-\frac{i}{bn};2-\frac{i}{bn};-e^{2ia}(cx^n)^{2ib}\right)}{1+ibn}$$

Mathematica [A]
time = 5.63, size = 149, normalized size = 1.89

$$\frac{x^2\left(e^{2ia}(cx^n)^{2ib}\ {}_2F_1\left(1,1-\frac{i}{bn};2-\frac{i}{bn};-e^{2i(a+b\log(cx^n))}\right)+(-i+bn)\left(-i\,{}_2F_1\left(1,-\frac{i}{bn};1-\frac{i}{bn};-e^{2i(a+b\log(cx^n))}\right)+\tan(a+b\log(cx^n))\right)\right)}{bn(-i+bn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x^2*(E^((2*I)*a)*(c*x^n)^((2*I)*b)*Hypergeometric2F1[1, 1 - I/(b*n), 2 - I
/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] + (-I + b*n)*((-I)*Hypergeometric2F1
[1, (-I)/(b*n), 1 - I/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] + Tan[a + b*Log
[c*x^n]])))/(b*n*(-I + b*n))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int x\left(\sec^2(a+b\ln(c\,x^n))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(x*sec(a+b*ln(c*x^n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 2*(x^2*cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + x^2*cos(2*b*log(c))*sin(2*
b*log(x^n) + 2*a) - 2*(2*b^2*n^2*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) -
```

```
2*b^2*n^2*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b^2*cos(2*b*log(c))^2
+ b^2*sin(2*b*log(c))^2)*n^2*cos(2*b*log(x^n) + 2*a)^2 + (b^2*cos(2*b*log(c
))^2 + b^2*sin(2*b*log(c))^2)*n^2*sin(2*b*log(x^n) + 2*a)^2 + b^2*n^2)*inte
grate((x*cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + x*cos(2*b*log(c))*sin(2*
b*log(x^n) + 2*a))/(2*b^2*n^2*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) - 2*b
^2*n^2*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b^2*cos(2*b*log(c))^2 + b
^2*sin(2*b*log(c))^2)*n^2*cos(2*b*log(x^n) + 2*a)^2 + (b^2*cos(2*b*log(c))^
2 + b^2*sin(2*b*log(c))^2)*n^2*sin(2*b*log(x^n) + 2*a)^2 + b^2*n^2), x))/(2
*b*n*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) + (b*cos(2*b*log(c))^2 + b*sin
(2*b*log(c))^2)*n*cos(2*b*log(x^n) + 2*a)^2 - 2*b*n*sin(2*b*log(c))*sin(2*b
*log(x^n) + 2*a) + (b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*sin(2*b*lo
g(x^n) + 2*a)^2 + b*n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*log(c*x^n))^2,x, algorithm="fricas")
[Out] integral(x*sec(b*log(c*x^n) + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sec^2\left(a+b\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*ln(c*x**n))**2,x)
[Out] Integral(x*sec(a + b*log(c*x**n))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*log(c*x^n))^2,x, algorithm="giac")
[Out] integrate(x*sec(b*log(c*x^n) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(a + b*log(c*x^n))^2,x)
[Out] int(x/cos(a + b*log(c*x^n))^2, x)
```

3.245 $\int \sec^2(a+b\log(cx^n))\,dx$

Optimal. Leaf size=85

$$\frac{4e^{2ia}x(cx^n)^{2ib}\,{}_2F_1\left(2,\frac{1}{2}\left(2-\frac{i}{bn}\right);\frac{1}{2}\left(4-\frac{i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{1+2ibn}$$

```
[Out] 4*exp(2*I*a)*x*(c*x^n)^(2*I*b)*hypergeom([2, 1-1/2*I/b/n],[2-1/2*I/b/n],-ex
p(2*I*a)*(c*x^n)^(2*I*b))/(1+2*I*b*n)
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4599, 4601, 371}

$$\frac{4e^{2ia}x(cx^n)^{2ib}\,{}_2F_1\left(2,\frac{1}{2}\left(2-\frac{i}{bn}\right);\frac{1}{2}\left(4-\frac{i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{1+2ibn}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (4*E^((2*I)*a)*x*(c*x^n)^((2*I)*b)*Hypergeometric2F1[2, (2 - I/(b*n))/2, (4
 - I/(b*n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(1 + (2*I)*b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4599

```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \sec^2(a+b\log(cx^n))\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\sec^2(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(4e^{2ia}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+2ib+\frac{1}{n}}}{(1+e^{2ia}x^{2ib})^2}\,dx,x,cx^n\right)}{n}$$
$$= \frac{4e^{2ia}x(cx^n)^{2ib}\,{}_2F_1\left(2,\frac{1}{2}\left(2-\frac{i}{bn}\right);\frac{1}{2}\left(4-\frac{i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{1+2ibn}$$

Mathematica [A]
time = 6.75, size = 147, normalized size = 1.73

$$\frac{x\left(\frac{e^{2ia}(cx^n)^{2ib}\,{}_2F_1\left(1,1-\frac{i}{2bn};2-\frac{i}{2bn};-e^{2i(a+b\log(cx^n))}\right)}{-i+2bn}-i\,{}_2F_1\left(1,-\frac{i}{2bn};1-\frac{i}{2bn};-e^{2i(a+b\log(cx^n))}\right)+\tan(a+b\log(cx^n))\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x*((E^((2*I)*a)*(c*x^n)^((2*I)*b)*Hypergeometric2F1[1, 1 - (I/2)/(b*n), 2
- (I/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))])/(-I + 2*b*n) - I*Hypergeomet
ric2F1[1, (-1/2*I)/(b*n), 1 - (I/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] +
 Tan[a + b*Log[c*x^n]]))/(b*n)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \sec^2(a+b\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(sec(a+b*ln(c*x^n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 2*(x*cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + x*cos(2*b*log(c))*sin(2*b*lo
g(x^n) + 2*a) - (2*b^2*n^2*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) - 2*b^2*
n^2*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b^2*cos(2*b*log(c))^2 + b^2*
sin(2*b*log(c))^2)*n^2*cos(2*b*log(x^n) + 2*a)^2 + (b^2*cos(2*b*log(c))^2 +
 b^2*sin(2*b*log(c))^2)*n^2*sin(2*b*log(x^n) + 2*a)^2 + b^2*n^2)*integrate(
(cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + cos(2*b*log(c))*sin(2*b*log(x^n)
 + 2*a))/(2*b^2*n^2*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) - 2*b^2*n^2*sin
(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b^2*cos(2*b*log(c))^2 + b^2*sin(2*b
*log(c))^2)*n^2*cos(2*b*log(x^n) + 2*a)^2 + (b^2*cos(2*b*log(c))^2 + b^2*si
n(2*b*log(c))^2)*n^2*sin(2*b*log(x^n) + 2*a)^2 + b^2*n^2), x))/(2*b*n*cos(2
*b*log(c))*cos(2*b*log(x^n) + 2*a) + (b*cos(2*b*log(c))^2 + b*sin(2*b*log(c
))^2)*n*cos(2*b*log(x^n) + 2*a)^2 - 2*b*n*sin(2*b*log(c))*sin(2*b*log(x^n)
+ 2*a) + (b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*sin(2*b*log(x^n) + 2
*a)^2 + b*n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral(sec(b*log(c*x^n) + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sec^{2}\left(a + b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**2,x)
```

```
[Out] Integral(sec(a + b*log(c*x**n))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*log(c*x^n))^2,x)
```

```
[Out] int(1/cos(a + b*log(c*x^n))^2, x)
```

3.246 $\int \frac{\sec^2(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=18

$$\frac{\tan(a+b\log(cx^n))}{bn}$$

```
[Out] tan(a+b*ln(c*x^n))/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = $\{3852, 8\}$

$$\frac{\tan(a+b\log(cx^n))}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] Tan[a + b*Log[c*x^n]]/(b*n)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \sec^2(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\
&= -\frac{\text{Subst}\left(\int 1\,dx, x, -\tan(a+b\log(cx^n))\right)}{bn}\\
&= \frac{\tan(a+b\log(cx^n))}{bn}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 18, normalized size = 1.00

$$\frac{\tan(a+b\log(cx^n))}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] Tan[a + b*Log[c*x^n]]/(b*n)
```

Maple [A]
time = 0.09, size = 19, normalized size = 1.06

method	result
derivativedivides	$\frac{\tan(a+b\ln(c\,x^n))}{bn}$
default	$\frac{\tan(a+b\ln(c\,x^n))}{bn}$
risch	$\frac{2i}{bn\left((x^n)^{2ib}c^{2ib}\mathrm{e}^{b\pi\mathrm{csgn}(ic\,x^n)^3}\mathrm{e}^{-b\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ic)}\mathrm{e}^{-b\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ix^n)}\mathrm{e}^{b\pi\,\mathrm{csgn}(ic\,x^n)\mathrm{csgn}(ic)\mathrm{csgn}(ix^n)}\mathrm{e}^{2ia}+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a+b*ln(c*x^n))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] tan(a+b*ln(c*x^n))/b/n
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(18) = 36$.
time = 0.30, size = 165, normalized size = 9.17

$$\frac{2\left(\cos\left(2\,b\log\left(x^n\right)+2\,a\right)\sin\left(2\,b\log\left(c\right)\right)+\cos\left(2\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(x^n\right)+2\,a\right)\right)}{2\,bn\cos\left(2\,b\log\left(c\right)\right)\cos\left(2\,b\log\left(x^n\right)+2\,a\right)+\left(b\cos\left(2\,b\log\left(c\right)\right)^2+b\sin\left(2\,b\log\left(c\right)\right)^2\right)n\cos\left(2\,b\log\left(x^n\right)+2\,a\right)^2-2\,bn\sin\left(2\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(x^n\right)+2\,a\right)+\left(b\cos\left(2\,b\log\left(c\right)\right)^2+b\sin\left(2\,b\log\left(c\right)\right)^2\right)n\sin\left(2\,b\log\left(x^n\right)+2\,a\right)^2+bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] 2*(cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + cos(2*b*log(c))*sin(2*b*log(x^
n) + 2*a))/(2*b*n*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) + (b*cos(2*b*log(
c))^2 + b*sin(2*b*log(c))^2)*n*cos(2*b*log(x^n) + 2*a)^2 - 2*b*n*sin(2*b*lo
g(c))*sin(2*b*log(x^n) + 2*a) + (b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)
*n*sin(2*b*log(x^n) + 2*a)^2 + b*n)
```

Fricas [A]
time = 3.00, size = 33, normalized size = 1.83

$$\frac{\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{bn\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

```
[Out] sin(b*n*log(x) + b*log(c) + a)/(b*n*cos(b*n*log(x) + b*log(c) + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(a + b\log(cx^n))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**2/x,x)
```

```
[Out] Integral(sec(a + b*log(c*x**n))**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)^2/x, x)
```

Mupad [B]
time = 3.84, size = 29, normalized size = 1.61

$$\frac{2\mathrm{i}}{b\,n\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cos(a + b*log(c*x^n))^2),x)
```

```
[Out] 2i/(b*n*(exp(a*2i)*(c*x^n)^(b*2i) + 1))
```

3.247 $\int \frac{\sec^2(a+b\log(cx^n))}{x^2}\,dx$

Optimal. Leaf size=87

$$-\frac{4e^{2ia}(cx^n)^{2ib}\,{}_2F_1\left(2,\frac{1}{2}\left(2+\frac{i}{bn}\right);\frac{1}{2}\left(4+\frac{i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{(1-2ibn)x}$$

```
[Out] -4*exp(2*I*a)*(c*x^n)^(2*I*b)*hypergeom([2, 1+1/2*I/b/n],[2+1/2*I/b/n],-exp
(2*I*a)*(c*x^n)^(2*I*b))/(1-2*I*b*n)/x
```

Rubi [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4605, 4601, 371}

$$-\frac{4e^{2ia}(cx^n)^{2ib}\,{}_2F_1\left(2,\frac{1}{2}\left(2+\frac{i}{bn}\right);\frac{1}{2}\left(4+\frac{i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{x(1-2ibn)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*Log[c*x^n]]^2/x^2,x]
```

```
[Out] (-4*E^((2*I)*a)*(c*x^n)^((2*I)*b)*Hypergeometric2F1[2, (2 + I/(b*n))/2, (4
+ I/(b*n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((1 - (2*I)*b*n)*x)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4605

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\sec^2(a+b\log(cx^n))}{x^2}\,dx = \frac{(cx^n)^{\frac{1}{n}}\,\mathrm{Subst}\left(\int x^{-1-\frac{1}{n}}\sec^2(a+b\log(x))\,dx, x, cx^n\right)}{nx}$$
$$= \frac{\left(4e^{2ia}(cx^n)^{\frac{1}{n}}\right)\mathrm{Subst}\left(\int \frac{x^{-1+2ib-\frac{1}{n}}}{(1+e^{2ia}x^{2ib})^2}\,dx, x, cx^n\right)}{nx}$$
$$= -\frac{4e^{2ia}(cx^n)^{2ib}\,{}_2F_1\left(2, \frac{1}{2}\left(2+\frac{i}{bn}\right); \frac{1}{2}\left(4+\frac{i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right)}{(1-2ibn)x}$$

Mathematica [A]
time = 4.10, size = 160, normalized size = 1.84

$$\frac{-e^{2ia}(cx^n)^{2ib}\,{}_2F_1\left(1,1+\frac{i}{2bn};2+\frac{i}{2bn};-e^{2i(a+b\log(cx^n))}\right)+(1-2ibn)\left({}_2F_1\left(1,\frac{i}{2bn};1+\frac{i}{2bn};-e^{2i(a+b\log(cx^n))}\right)+i\tan\left(a+b\log\left(cx^n\right)\right)\right)}{bn(i+2bn)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*Log[c*x^n]]^2/x^2,x]
```

```
[Out] (-(E^((2*I)*a)*(c*x^n)^((2*I)*b)*Hypergeometric2F1[1, 1 + (I/2)/(b*n), 2 +
(I/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))]) + (1 - (2*I)*b*n)*(Hypergeomet
ric2F1[1, (I/2)/(b*n), 1 + (I/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] + I*
Tan[a + b*Log[c*x^n]]))/(b*n*(I + 2*b*n)*x)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\sec^2\left(a+b\ln\left(c\,x^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a+b*ln(c*x^n))^2/x^2,x)
```

```
[Out] int(sec(a+b*ln(c*x^n))^2/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] 2*((2*b^2*n^2*x*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) - 2*b^2*n^2*x*sin(2
*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*l
```

```
og(c))^2)*n^2*x*cos(2*b*log(x^n) + 2*a)^2 + (b^2*cos(2*b*log(c))^2 + b^2*si
n(2*b*log(c))^2)*n^2*x*sin(2*b*log(x^n) + 2*a)^2 + b^2*n^2*x)*integrate((co
s(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + cos(2*b*log(c))*sin(2*b*log(x^n) +
2*a))/(2*b^2*n^2*x^2*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) - 2*b^2*n^2*x^
2*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b^2*cos(2*b*log(c))^2 + b^2*si
n(2*b*log(c))^2)*n^2*x^2*cos(2*b*log(x^n) + 2*a)^2 + (b^2*cos(2*b*log(c))^2
 + b^2*sin(2*b*log(c))^2)*n^2*x^2*sin(2*b*log(x^n) + 2*a)^2 + b^2*n^2*x^2),
 x) + cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + cos(2*b*log(c))*sin(2*b*log
(x^n) + 2*a))/(2*b*n*x*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) + (b*cos(2*b
*log(c))^2 + b*sin(2*b*log(c))^2)*n*x*cos(2*b*log(x^n) + 2*a)^2 - 2*b*n*x*s
in(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b*cos(2*b*log(c))^2 + b*sin(2*b*l
og(c))^2)*n*x*sin(2*b*log(x^n) + 2*a)^2 + b*n*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^2/x^2,x, algorithm="fricas")
[Out] integral(sec(b*log(c*x^n) + a)^2/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2\left(a + b\log\left(cx^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**2/x**2,x)
[Out] Integral(sec(a + b*log(c*x**n))**2/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^2/x^2,x, algorithm="giac")
[Out] integrate(sec(b*log(c*x^n) + a)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\cos\left(a + b\,\ln\left(c\,x^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*cos(a + b*log(c*x^n))^2),x)
[Out] int(1/(x^2*cos(a + b*log(c*x^n))^2), x)
```

3.248 $\int \frac{\sec^2(a+b\log(cx^n))}{x^3}\,dx$

Optimal. Leaf size=79

$$-\frac{2e^{2ia}(cx^n)^{2ib}\,{}_2F_1\left(2,1+\frac{i}{bn};2+\frac{i}{bn};-e^{2ia}(cx^n)^{2ib}\right)}{(1-ibn)x^2}$$

```
[Out] -2*exp(2*I*a)*(c*x^n)^(2*I*b)*hypergeom([2, 1+I/b/n],[2+I/b/n],-exp(2*I*a)*
(c*x^n)^(2*I*b))/(1-I*b*n)/x^2
```

Rubi [A]
time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = $\{4605, 4601, 371\}$

$$-\frac{2e^{2ia}(cx^n)^{2ib}\,{}_2F_1\left(2,1+\frac{i}{bn};2+\frac{i}{bn};-e^{2ia}(cx^n)^{2ib}\right)}{x^2(1-ibn)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*Log[c*x^n]]^2/x^3,x]
```

```
[Out] (-2*E^((2*I)*a)*(c*x^n)^((2*I)*b)*Hypergeometric2F1[2, 1 + I/(b*n), 2 + I/(
b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((1 - I*b*n)*x^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4605

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\sec^2(a+b\log(cx^n))}{x^3}\,dx = \frac{(cx^n)^{2/n}\,\mathrm{Subst}\left(\int x^{-1-\frac{2}{n}}\sec^2(a+b\log(x))\,dx, x, cx^n\right)}{nx^2}$$
$$= \frac{\left(4e^{2ia}(cx^n)^{2/n}\right)\mathrm{Subst}\left(\int \frac{x^{-1+2ib-\frac{2}{n}}}{(1+e^{2ia}x^{2ib})^2}\,dx, x, cx^n\right)}{nx^2}$$
$$= -\frac{2e^{2ia}(cx^n)^{2ib}\;{}_2F_1\left(2, 1+\frac{i}{bn}; 2+\frac{i}{bn}; -e^{2ia}(cx^n)^{2ib}\right)}{(1-ibn)x^2}$$

Mathematica [A]
time = 3.95, size = 150, normalized size = 1.90

$$\frac{-e^{2ia}(cx^n)^{2ib}\;{}_2F_1\left(1,1+\frac{i}{bn};2+\frac{i}{bn};-e^{2i(a+b\log(cx^n))}\right)+(i+bn)\left(-i\,{}_2F_1\left(1,\frac{i}{bn};1+\frac{i}{bn};-e^{2i(a+b\log(cx^n))}\right)+\tan(a+b\log(cx^n))\right)}{bn(i+bn)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*Log[c*x^n]]^2/x^3,x]
```

```
[Out] (-(E^((2*I)*a)*(c*x^n)^((2*I)*b)*Hypergeometric2F1[1, 1 + I/(b*n), 2 + I/(b
*n), -E^((2*I)*(a + b*Log[c*x^n]))]) + (I + b*n)*((-I)*Hypergeometric2F1[1,
 I/(b*n), 1 + I/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] + Tan[a + b*Log[c*x^n
]]))/(b*n*(I + b*n)*x^2)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(a+b\ln(c\,x^n))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a+b*ln(c*x^n))^2/x^3,x)
```

```
[Out] int(sec(a+b*ln(c*x^n))^2/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] 2*(2*(2*b^2*n^2*x^2*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) - 2*b^2*n^2*x^2
*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b^2*cos(2*b*log(c))^2 + b^2*sin
```

```
(2*b*log(c))^2)*n^2*x^2*cos(2*b*log(x^n) + 2*a)^2 + (b^2*cos(2*b*log(c))^2
+ b^2*sin(2*b*log(c))^2)*n^2*x^2*sin(2*b*log(x^n) + 2*a)^2 + b^2*n^2*x^2)*i
ntegrate((cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + cos(2*b*log(c))*sin(2*b
*log(x^n) + 2*a))/(2*b^2*n^2*x^3*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) -
2*b^2*n^2*x^3*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b^2*cos(2*b*log(c)
)^2 + b^2*sin(2*b*log(c))^2)*n^2*x^3*cos(2*b*log(x^n) + 2*a)^2 + (b^2*cos(2
*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2*x^3*sin(2*b*log(x^n) + 2*a)^2 + b
^2*n^2*x^3), x) + cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + cos(2*b*log(c))
*sin(2*b*log(x^n) + 2*a))/(2*b*n*x^2*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a
) + (b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*x^2*cos(2*b*log(x^n) + 2*
a)^2 - 2*b*n*x^2*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b*cos(2*b*log(c
))^2 + b*sin(2*b*log(c))^2)*n*x^2*sin(2*b*log(x^n) + 2*a)^2 + b*n*x^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^2/x^3,x, algorithm="fricas")
[Out] integral(sec(b*log(c*x^n) + a)^2/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2\left(a + b\log\left(cx^n\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**2/x**3,x)
[Out] Integral(sec(a + b*log(c*x**n))**2/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^2/x^3,x, algorithm="giac")
[Out] integrate(sec(b*log(c*x^n) + a)^2/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\cos\left(a + b\,\ln\left(c\,x^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*cos(a + b*log(c*x^n))^2),x)
[Out] int(1/(x^3*cos(a + b*log(c*x^n))^2), x)
```

3.249 $\int x \sec^3(a+b\log(cx^n))\,dx$

Optimal. Leaf size=87

$$\frac{8e^{3ia}x^2(cx^n)^{3ib}\,{}_2F_1\left(3,\frac{1}{2}\left(3-\frac{2i}{bn}\right);\frac{1}{2}\left(5-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{2+3ibn}$$

```
[Out] 8*exp(3*I*a)*x^2*(c*x^n)^(3*I*b)*hypergeom([3, 3/2-I/b/n],[5/2-I/b/n],-exp(
2*I*a)*(c*x^n)^(2*I*b))/(2+3*I*b*n)
```

Rubi [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4605, 4601, 371}

$$\frac{8e^{3ia}x^2(cx^n)^{3ib}\,{}_2F_1\left(3,\frac{1}{2}\left(3-\frac{2i}{bn}\right);\frac{1}{2}\left(5-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{2+3ibn}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (8*E^((3*I)*a)*x^2*(c*x^n)^((3*I)*b)*Hypergeometric2F1[3, (3 - (2*I)/(b*n))
/2, (5 - (2*I)/(b*n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(2 + (3*I)*b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4605

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x\sec^3(a+b\log(cx^n))\,dx = \frac{\left(x^2(cx^n)^{-2/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{2}{n}}\sec^3(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(8e^{3ia}x^2(cx^n)^{-2/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+3ib+\frac{2}{n}}}{(1+e^{2ia}x^{2ib})^3}\,dx,x,cx^n\right)}{n}$$
$$= \frac{8e^{3ia}x^2(cx^n)^{3ib}\,{}_2F_1\left(3,\frac{1}{2}\left(3-\frac{2i}{bn}\right);\frac{1}{2}\left(5-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{2+3ibn}$$

Mathematica [A]
time = 6.07, size = 118, normalized size = 1.36

$$\frac{x^2\left(2e^{ia}(2-ibn)(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}-\frac{i}{bn};\frac{3}{2}-\frac{i}{bn};-e^{2i(a+b\log(cx^n))}\right)+\sec(a+b\log(cx^n))(-2+bn\tan(a+b\log(cx^n)))\right)}{2b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (x^2*(2*E^(I*a)*(2 - I*b*n)*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 - I/(b*n
), 3/2 - I/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] + Sec[a + b*Log[c*x^n]]*(-
2 + b*n*Tan[a + b*Log[c*x^n]])))/(2*b^2*n^2)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int x\left(\sec^3(a+b\ln(c\,x^n))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(x*sec(a+b*ln(c*x^n))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] -((b*n*sin(b*log(c)) + 2*cos(b*log(c)))*x^2*cos(b*log(x^n) + a) + (b*n*cos(
b*log(c)) - 2*sin(b*log(c)))*x^2*sin(b*log(x^n) + a) + (((b*cos(3*b*log(c))
```

```
*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(3*b*log(c)))*n + 2*cos(4*b*log(c))
*cos(3*b*log(c)) + 2*sin(4*b*log(c))*sin(3*b*log(c)))*x^2*cos(3*b*log(x^n)
+ 3*a) - ((b*cos(b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(b*log(c)
))*n - 2*cos(4*b*log(c))*cos(b*log(c)) - 2*sin(4*b*log(c))*sin(b*log(c)))*x
^2*cos(b*log(x^n) + a) - ((b*cos(4*b*log(c))*cos(3*b*log(c)) + b*sin(4*b*lo
g(c))*sin(3*b*log(c)))*n - 2*cos(3*b*log(c))*sin(4*b*log(c)) + 2*cos(4*b*lo
g(c))*sin(3*b*log(c)))*x^2*sin(3*b*log(x^n) + 3*a) + ((b*cos(4*b*log(c))*co
s(b*log(c)) + b*sin(4*b*log(c))*sin(b*log(c)))*n + 2*cos(b*log(c))*sin(4*b*
log(c)) - 2*cos(4*b*log(c))*sin(b*log(c)))*x^2*sin(b*log(x^n) + a))*cos(4*b
*log(x^n) + 4*a) - (2*((b*cos(2*b*log(c))*sin(3*b*log(c)) - b*cos(3*b*log(c
))*sin(2*b*log(c)))*n - 2*cos(3*b*log(c))*cos(2*b*log(c)) - 2*sin(3*b*log(c
))*sin(2*b*log(c)))*x^2*cos(2*b*log(x^n) + 2*a) - 2*((b*cos(3*b*log(c))*cos
(2*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*log(c)))*n + 2*cos(2*b*log(c))*sin
(3*b*log(c)) - 2*cos(3*b*log(c))*sin(2*b*log(c)))*x^2*sin(2*b*log(x^n) + 2*
a) + (b*n*sin(3*b*log(c)) - 2*cos(3*b*log(c)))*x^2)*cos(3*b*log(x^n) + 3*a)
 - 2*(((b*cos(b*log(c))*sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)))*
n - 2*cos(2*b*log(c))*cos(b*log(c)) - 2*sin(2*b*log(c))*sin(b*log(c)))*x^2*
cos(b*log(x^n) + a) - ((b*cos(2*b*log(c))*cos(b*log(c)) + b*sin(2*b*log(c))
*sin(b*log(c)))*n + 2*cos(b*log(c))*sin(2*b*log(c)) - 2*cos(2*b*log(c))*sin
(b*log(c)))*x^2*sin(b*log(x^n) + a))*cos(2*b*log(x^n) + 2*a) - (b^4*n^4*cos
(b*log(c)) + 4*b^2*n^2*cos(b*log(c)) + ((b^4*cos(4*b*log(c))^2*cos(b*log(c)
) + b^4*cos(b*log(c))*sin(4*b*log(c))^2)*n^4 + 4*(b^2*cos(4*b*log(c))^2*cos
(b*log(c)) + b^2*cos(b*log(c))*sin(4*b*log(c))^2)*n^2)*cos(4*b*log(x^n) + 4
*a)^2 + 4*((b^4*cos(2*b*log(c))^2*cos(b*log(c)) + b^4*cos(b*log(c))*sin(2*b
*log(c))^2)*n^4 + 4*(b^2*cos(2*b*log(c))^2*cos(b*log(c)) + b^2*cos(b*log(c)
)*sin(2*b*log(c))^2)*n^2)*cos(2*b*log(x^n) + 2*a)^2 + ((b^4*cos(4*b*log(c))
^2*cos(b*log(c)) + b^4*cos(b*log(c))*sin(4*b*log(c))^2)*n^4 + 4*(b^2*cos(4*
b*log(c))^2*cos(b*log(c)) + b^2*cos(b*log(c))*sin(4*b*log(c))^2)*n^2)*sin(4
*b*log(x^n) + 4*a)^2 + 4*((b^4*cos(2*b*log(c))^2*cos(b*log(c)) + b^4*cos(b*
log(c))*sin(2*b*log(c))^2)*n^4 + 4*(b^2*cos(2*b*log(c))^2*cos(b*log(c)) + b
^2*cos(b*log(c))*sin(2*b*log(c))^2)*n^2)*sin(2*b*log(x^n) + 2*a)^2 + 2*(b^4
*n^4*cos(4*b*log(c))*cos(b*log(c)) + 4*b^2*n^2*cos(4*b*log(c))*cos(b*log(c)
) + 2*((b^4*cos(4*b*log(c))*cos(2*b*log(c))*cos(b*log(c)) + b^4*cos(b*log(c
))*sin(4*b*log(c))*sin(2*b*log(c)))*n^4 + 4*(b^2*cos(4*b*log(c))*cos(2*b*lo
g(c))*cos(b*log(c)) + b^2*cos(b*log(c))*sin(4*b*log(c))*sin(2*b*log(c)))*n^
2)*cos(2*b*log(x^n) + 2*a) + 2*((b^4*cos(2*b*log(c))*cos(b*log(c))*sin(4*b*
log(c)) - b^4*cos(4*b*log(c))*cos(b*log(c))*sin(2*b*log(c)))*n^4 + 4*(b^2*c
os(2*b*log(c))*cos(b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*cos(b*lo
g(c))*sin(2*b*log(c)))*n^2)*sin(2*b*log(x^n) + 2*a))*cos(4*b*log(x^n) + 4*a
) + 4*(b^4*n^4*cos(2*b*log(c))*cos(b*log(c)) + 4*b^2*n^2*cos(2*b*log(c))*co
s(b*log(c)))*cos(2*b*log(x^n) + 2*a) - 2*(b^4*n^4*cos(b*log(c))*sin(4*b*log
(c)) + 4*b^2*n^2*cos(b*log(c))*sin(4*b*log(c)) + 2*((b^4*cos(2*b*log(c))*co
s(b*log(c))*sin(4*b*log(c)) - b^4*cos(4*b*log(c))*cos(b*log(c))*sin(2*b*log
(c)))*n^4 + 4*(b^2*cos(2*b*log(c))*cos(b*log(c))*sin(4*b*log(c)) - b^2*cos(
4*b*log(c))*cos(b*log(c))*sin(2*b*log(c)))*n^2)*cos(2*b*log(x^n) + 2*a) - 2
```

```
*((b^4*cos(4*b*log(c))*cos(2*b*log(c))*cos(b*log(c)) + b^4*cos(b*log(c))*si
n(4*b*log(c))*sin(2*b*log(c)))*n^4 + 4*(b^2*cos(4*b*log(c))*cos(2*b*log(c))
*cos(b*log(c)) + b^2*cos(b*log(c))*sin(4*b*log(c))*sin(2*b*log(c)))*n^2)*si
n(2*b*log(x^n) + 2*a))*sin(4*b*log(x^n) + 4*a) - 4*(b^4*n^4*cos(b*log(c))*s
in(2*b*log(c)) + 4*b^2*n^2*cos(b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n)
+ 2*a))*integrate(((x*cos(2*b*log(c))*cos(b*log(x^n) + a) + x*sin(2*b*log(c
))*sin(b*log(x^n) + a))*cos(2*b*log(x^n) + 2*a) + x*cos(b*log(x^n) + a) - (
x*cos(b*log(x^n) + a)*sin(2*b*log(c)) - x*cos(2*b*log(c))*sin(b*log(x^n) +
a))*sin(2*b*log(x^n) + 2*a))/(2*b^2*n^2*cos(2*b*log(c))*cos(2*b*log(x^n) +
2*a) - 2*b^2*n^2*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b^2*cos(2*b*log
(c))^2 + b^2*sin(2*b*log(c))^2)*n^2*cos(2*b*log(x^n) + 2*a)^2 + (b^2*cos(2*
b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2*sin(2*b*log(x^n) + 2*a)^2 + b^2*n^
2), x) - (b^4*n^4*sin(b*log(c)) + 4*b^2*n^2*sin(b*log(c)) + ((b^4*cos(4*b*l
og(c))^2*sin(b*log(c)) + b^4*sin(4*b*log(c))^2*sin(b*log(c)))*n^4 + 4*(b^2*
cos(4*b*log(c))^2*sin(b*log(c)) + b^2*sin(4*b*log(c))^2*sin(b*log(c)))*n^2)
*cos(4*b*log(x^n) + 4*a)^2 + 4*((b^4*cos(2*b*log(c))^2*sin(b*log(c)) + b^4*
sin(2*b*log(c))^2*sin(b*log(c)))*n^4 + 4*(b^2*cos(2*b*log(c))^2*sin(b*log(c
)) + b^2*sin(2*b*log(c))^2*sin(b*log(c)))*n^2)*cos(2*b*log(x^n) + 2*a)^2 +
((b^4*cos(4*b*log(c))^2*sin(b*log(c)) + b^4*sin(4*b*log(c))^2*sin(b*log(c))
)*n^4 + 4*(b^2*cos(4*b*log(c))^2*sin(b*log(c)) ...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] integral(x*sec(b*log(c*x^n) + a)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sec^3\left(a + b\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*ln(c*x**n))**3,x)
```

```
[Out] Integral(x*sec(a + b*log(c*x**n))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*log(c*x^n))^3,x, algorithm="giac")
[Out] integrate(x*sec(b*log(c*x^n) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(a + b*log(c*x^n))^3,x)
[Out] int(x/cos(a + b*log(c*x^n))^3, x)
```

3.250 $\int \sec^3(a+b\log(cx^n))\,dx$

Optimal. Leaf size=85

$$\frac{8e^{3ia}x(cx^n)^{3ib}\,{}_2F_1\left(3,\frac{1}{2}\left(3-\frac{i}{bn}\right);\frac{1}{2}\left(5-\frac{i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{1+3ibn}$$

```
[Out] 8*exp(3*I*a)*x*(c*x^n)^(3*I*b)*hypergeom([3, 3/2-1/2*I/b/n],[5/2-1/2*I/b/n]
,-exp(2*I*a)*(c*x^n)^(2*I*b))/(1+3*I*b*n)
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4599, 4601, 371}

$$\frac{8e^{3ia}x(cx^n)^{3ib}\,{}_2F_1\left(3,\frac{1}{2}\left(3-\frac{i}{bn}\right);\frac{1}{2}\left(5-\frac{i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{1+3ibn}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (8*E^((3*I)*a)*x*(c*x^n)^((3*I)*b)*Hypergeometric2F1[3, (3 - I/(b*n))/2, (5
 - I/(b*n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(1 + (3*I)*b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4599

```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \sec^3(a+b\log(cx^n))\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\mathrm{Subst}\left(\int x^{-1+\frac{1}{n}}\sec^3(a+b\log(x))\,dx, x, cx^n\right)}{n}$$
$$= \frac{\left(8e^{3ia}x(cx^n)^{-1/n}\right)\mathrm{Subst}\left(\int \frac{x^{-1+3ib+\frac{1}{n}}}{(1+e^{2ia}x^{2ib})^3}\,dx, x, cx^n\right)}{n}$$
$$= \frac{8e^{3ia}x(cx^n)^{3ib}\,{}_2F_1\left(3, \frac{1}{2}\left(3-\frac{i}{bn}\right); \frac{1}{2}\left(5-\frac{i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right)}{1+3ibn}$$

Mathematica [A]
time = 5.48, size = 120, normalized size = 1.41

$$\frac{x\left(2e^{ia}(1-ibn)(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}-\frac{i}{2bn};\frac{3}{2}-\frac{i}{2bn};-e^{2i(a+b\log(cx^n))}\right)+\sec(a+b\log(cx^n))(-1+bn\tan(a+b\log(cx^n)))\right)}{2b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (x*(2*E^(I*a)*(1 - I*b*n)*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 - (I/2)/(b
*n), 3/2 - (I/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] + Sec[a + b*Log[c*x^
n]]*(-1 + b*n*Tan[a + b*Log[c*x^n]])))/(2*b^2*n^2)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \sec^3(a+b\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(sec(a+b*ln(c*x^n))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] -((b*n*sin(b*log(c)) + cos(b*log(c)))*x*cos(b*log(x^n) + a) + (b*n*cos(b*lo
g(c)) - sin(b*log(c)))*x*sin(b*log(x^n) + a) + (((b*cos(3*b*log(c))*sin(4*b
```

```
*log(c)) - b*cos(4*b*log(c))*sin(3*b*log(c)))*n + cos(4*b*log(c))*cos(3*b*l
og(c)) + sin(4*b*log(c))*sin(3*b*log(c)))*x*cos(3*b*log(x^n) + 3*a) - ((b*c
os(b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(b*log(c)))*n - cos(4*b
*log(c))*cos(b*log(c)) - sin(4*b*log(c))*sin(b*log(c)))*x*cos(b*log(x^n) +
a) - ((b*cos(4*b*log(c))*cos(3*b*log(c)) + b*sin(4*b*log(c))*sin(3*b*log(c)
))*n - cos(3*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c))*sin(3*b*log(c)))*x
*sin(3*b*log(x^n) + 3*a) + ((b*cos(4*b*log(c))*cos(b*log(c)) + b*sin(4*b*lo
g(c))*sin(b*log(c)))*n + cos(b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*si
n(b*log(c)))*x*sin(b*log(x^n) + a))*cos(4*b*log(x^n) + 4*a) - (2*((b*cos(2*
b*log(c))*sin(3*b*log(c)) - b*cos(3*b*log(c))*sin(2*b*log(c)))*n - cos(3*b*
log(c))*cos(2*b*log(c)) - sin(3*b*log(c))*sin(2*b*log(c)))*x*cos(2*b*log(x^
n) + 2*a) - 2*((b*cos(3*b*log(c))*cos(2*b*log(c)) + b*sin(3*b*log(c))*sin(2
*b*log(c)))*n + cos(2*b*log(c))*sin(3*b*log(c)) - cos(3*b*log(c))*sin(2*b*l
og(c)))*x*sin(2*b*log(x^n) + 2*a) + (b*n*sin(3*b*log(c)) - cos(3*b*log(c)))
*x)*cos(3*b*log(x^n) + 3*a) - 2*(((b*cos(b*log(c))*sin(2*b*log(c)) - b*cos(
2*b*log(c))*sin(b*log(c)))*n - cos(2*b*log(c))*cos(b*log(c)) - sin(2*b*log(
c))*sin(b*log(c)))*x*cos(b*log(x^n) + a) - ((b*cos(2*b*log(c))*cos(b*log(c)
) + b*sin(2*b*log(c))*sin(b*log(c)))*n + cos(b*log(c))*sin(2*b*log(c)) - co
s(2*b*log(c))*sin(b*log(c)))*x*sin(b*log(x^n) + a))*cos(2*b*log(x^n) + 2*a)
 - (b^4*n^4*cos(b*log(c)) + b^2*n^2*cos(b*log(c)) + ((b^4*cos(4*b*log(c))^2
*cos(b*log(c)) + b^4*cos(b*log(c))*sin(4*b*log(c))^2)*n^4 + (b^2*cos(4*b*lo
g(c))^2*cos(b*log(c)) + b^2*cos(b*log(c))*sin(4*b*log(c))^2)*n^2)*cos(4*b*l
og(x^n) + 4*a)^2 + 4*((b^4*cos(2*b*log(c))^2*cos(b*log(c)) + b^4*cos(b*log(
c))*sin(2*b*log(c))^2)*n^4 + (b^2*cos(2*b*log(c))^2*cos(b*log(c)) + b^2*cos
(b*log(c))*sin(2*b*log(c))^2)*n^2)*cos(2*b*log(x^n) + 2*a)^2 + ((b^4*cos(4*
b*log(c))^2*cos(b*log(c)) + b^4*cos(b*log(c))*sin(4*b*log(c))^2)*n^4 + (b^2
*cos(4*b*log(c))^2*cos(b*log(c)) + b^2*cos(b*log(c))*sin(4*b*log(c))^2)*n^2
)*sin(4*b*log(x^n) + 4*a)^2 + 4*((b^4*cos(2*b*log(c))^2*cos(b*log(c)) + b^4
*cos(b*log(c))*sin(2*b*log(c))^2)*n^4 + (b^2*cos(2*b*log(c))^2*cos(b*log(c)
) + b^2*cos(b*log(c))*sin(2*b*log(c))^2)*n^2)*sin(2*b*log(x^n) + 2*a)^2 + 2
*(b^4*n^4*cos(4*b*log(c))*cos(b*log(c)) + b^2*n^2*cos(4*b*log(c))*cos(b*log
(c)) + 2*((b^4*cos(4*b*log(c))*cos(2*b*log(c))*cos(b*log(c)) + b^4*cos(b*lo
g(c))*sin(4*b*log(c))*sin(2*b*log(c)))*n^4 + (b^2*cos(4*b*log(c))*cos(2*b*l
og(c))*cos(b*log(c)) + b^2*cos(b*log(c))*sin(4*b*log(c))*sin(2*b*log(c)))*n
^2)*cos(2*b*log(x^n) + 2*a) + 2*((b^4*cos(2*b*log(c))*cos(b*log(c))*sin(4*b
*log(c)) - b^4*cos(4*b*log(c))*cos(b*log(c))*sin(2*b*log(c)))*n^4 + (b^2*co
s(2*b*log(c))*cos(b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*cos(b*log
(c))*sin(2*b*log(c)))*n^2)*sin(2*b*log(x^n) + 2*a))*cos(4*b*log(x^n) + 4*a)
 + 4*(b^4*n^4*cos(2*b*log(c))*cos(b*log(c)) + b^2*n^2*cos(2*b*log(c))*cos(b
*log(c)))*cos(2*b*log(x^n) + 2*a) - 2*(b^4*n^4*cos(b*log(c))*sin(4*b*log(c)
) + b^2*n^2*cos(b*log(c))*sin(4*b*log(c)) + 2*((b^4*cos(2*b*log(c))*cos(b*l
og(c))*sin(4*b*log(c)) - b^4*cos(4*b*log(c))*cos(b*log(c))*sin(2*b*log(c)))
*n^4 + (b^2*cos(2*b*log(c))*cos(b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log
(c))*cos(b*log(c))*sin(2*b*log(c)))*n^2)*cos(2*b*log(x^n) + 2*a) - 2*((b^4*
cos(4*b*log(c))*cos(2*b*log(c))*cos(b*log(c)) + b^4*cos(b*log(c))*sin(4*b*l
```

```
og(c))*sin(2*b*log(c)))*n^4 + (b^2*cos(4*b*log(c))*cos(2*b*log(c))*cos(b*lo
g(c)) + b^2*cos(b*log(c))*sin(4*b*log(c))*sin(2*b*log(c)))*n^2)*sin(2*b*log
(x^n) + 2*a))*sin(4*b*log(x^n) + 4*a) - 4*(b^4*n^4*cos(b*log(c))*sin(2*b*lo
g(c)) + b^2*n^2*cos(b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a))*int
egrate(((cos(2*b*log(c))*cos(b*log(x^n) + a) + sin(2*b*log(c))*sin(b*log(x^
n) + a))*cos(2*b*log(x^n) + 2*a) - (cos(b*log(x^n) + a)*sin(2*b*log(c)) - c
os(2*b*log(c))*sin(b*log(x^n) + a))*sin(2*b*log(x^n) + 2*a) + cos(b*log(x^n
) + a))/(2*b^2*n^2*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) - 2*b^2*n^2*sin(
2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*
log(c))^2)*n^2*cos(2*b*log(x^n) + 2*a)^2 + (b^2*cos(2*b*log(c))^2 + b^2*sin
(2*b*log(c))^2)*n^2*sin(2*b*log(x^n) + 2*a)^2 + b^2*n^2), x) - (b^4*n^4*sin
(b*log(c)) + b^2*n^2*sin(b*log(c)) + ((b^4*cos(4*b*log(c))^2*sin(b*log(c))
+ b^4*sin(4*b*log(c))^2*sin(b*log(c)))*n^4 + (b^2*cos(4*b*log(c))^2*sin(b*l
og(c)) + b^2*sin(4*b*log(c))^2*sin(b*log(c)))*n^2)*cos(4*b*log(x^n) + 4*a)^
2 + 4*((b^4*cos(2*b*log(c))^2*sin(b*log(c)) + b^4*sin(2*b*log(c))^2*sin(b*l
og(c)))*n^4 + (b^2*cos(2*b*log(c))^2*sin(b*log(c)) + b^2*sin(2*b*log(c))^2*
sin(b*log(c)))*n^2)*cos(2*b*log(x^n) + 2*a)^2 + ((b^4*cos(4*b*log(c))^2*sin
(b*log(c)) + b^4*sin(4*b*log(c))^2*sin(b*log(c)))*n^4 + (b^2*cos(4*b*log(c)
)^2*sin(b*log(c)) + b^2*sin(4*b*log(c))^2*sin(b*log(c)))*n^2)*sin(4*b*log(x
^n) + 4*a)^2 + 4*((b^4*cos(2*b*log(c))^2*sin(b*...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] integral(sec(b*log(c*x^n) + a)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sec^3(a + b\log(cx^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**3,x)
```

```
[Out] Integral(sec(a + b*log(c*x**n))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*log(c*x^n))^3,x)
```

```
[Out] int(1/cos(a + b*log(c*x^n))^3, x)
```

3.251 $\int \frac{\sec^3(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=55

$$\frac{\tanh^{-1}\left(\sin\left(a+b\log\left(cx^n\right)\right)\right)}{2bn}+\frac{\sec\left(a+b\log\left(cx^n\right)\right)\tan\left(a+b\log\left(cx^n\right)\right)}{2bn}$$

```
[Out] 1/2*arctanh(sin(a+b*ln(c*x^n)))/b/n+1/2*sec(a+b*ln(c*x^n))*tan(a+b*ln(c*x^n
))/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3853, 3855}

$$\frac{\tanh^{-1}\left(\sin\left(a+b\log\left(cx^n\right)\right)\right)}{2bn}+\frac{\tan\left(a+b\log\left(cx^n\right)\right)\sec\left(a+b\log\left(cx^n\right)\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] ArcTanh[Sin[a + b*Log[c*x^n]]]/(2*b*n) + (Sec[a + b*Log[c*x^n]]*Tan[a + b*L
og[c*x^n]])/(2*b*n)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \sec^3(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\ &= \frac{\sec\left(a+b\log\left(cx^n\right)\right)\tan\left(a+b\log\left(cx^n\right)\right)}{2bn}+\frac{\text{Subst}\left(\int \sec(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{2n}\\ &= \frac{\tanh^{-1}\left(\sin\left(a+b\log\left(cx^n\right)\right)\right)}{2bn}+\frac{\sec\left(a+b\log\left(cx^n\right)\right)\tan\left(a+b\log\left(cx^n\right)\right)}{2bn}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 55, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\sin\left(a+b\log\left(cx^n\right)\right)\right)}{2bn}+\frac{\sec\left(a+b\log\left(cx^n\right)\right)\tan\left(a+b\log\left(cx^n\right)\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] ArcTanh[Sin[a + b*Log[c*x^n]]]/(2*b*n) + (Sec[a + b*Log[c*x^n]]*Tan[a + b*L
og[c*x^n]])/(2*b*n)
```

Maple [A]
time = 0.26, size = 59, normalized size = 1.07

method	result
derivativedivides	$\frac{\frac{\sec(a+b\ln(c\,x^n))\tan(a+b\ln(c\,x^n))}{2}+\frac{\ln(\sec(a+b\ln(c\,x^n))+\tan(a+b\ln(c\,x^n)))}{2}}{nb}$
default	$\frac{\frac{\sec(a+b\ln(c\,x^n))\tan(a+b\ln(c\,x^n))}{2}+\frac{\ln(\sec(a+b\ln(c\,x^n))+\tan(a+b\ln(c\,x^n)))}{2}}{nb}$
risch	$-\frac{i c^{ib}(x^n)^{ib}\left(c^{2ib}(x^n)^{2ib}e^{\frac{3b\pi\mathrm{csgn}(ic\,x^n)^3}{2}}e^{-\frac{3b\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ic)}{2}}e^{-\frac{3b\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ix^n)}{2}}e^{\frac{3b\pi\,\mathrm{csgn}(ic\,x^n)\mathrm{csgn}(ic)\mathrm{csgn}(}{2}}\right.}{bn\left((x^n)^{2ib}c^{2ib}e^{b\pi\mathrm{csgn}(ic\,x^n)^3}e^{-b\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ic)}e^{-b\pi\mathrm{csgn}}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(1/2*sec(a+b*ln(c*x^n))*tan(a+b*ln(c*x^n))+1/2*ln(sec(a+b*ln(c*x^n))+
tan(a+b*ln(c*x^n))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] -(((cos(3*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(3*b*log(c)))*cos(
3*b*log(x^n) + 3*a) - (cos(b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(
b*log(c)))*cos(b*log(x^n) + a) - (cos(4*b*log(c))*cos(3*b*log(c)) + sin(4*b
*log(c))*sin(3*b*log(c)))*sin(3*b*log(x^n) + 3*a) + (cos(4*b*log(c))*cos(b*
log(c)) + sin(4*b*log(c))*sin(b*log(c)))*sin(b*log(x^n) + a))*cos(4*b*log(x
^n) + 4*a) - (2*(cos(2*b*log(c))*sin(3*b*log(c)) - cos(3*b*log(c))*sin(2*b*
log(c)))*cos(2*b*log(x^n) + 2*a) - 2*(cos(3*b*log(c))*cos(2*b*log(c)) + sin
```

```
(3*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) + sin(3*b*log(c)))*co
s(3*b*log(x^n) + 3*a) - 2*((cos(b*log(c))*sin(2*b*log(c)) - cos(2*b*log(c))
*sin(b*log(c)))*cos(b*log(x^n) + a) - (cos(2*b*log(c))*cos(b*log(c)) + sin(
2*b*log(c))*sin(b*log(c)))*sin(b*log(x^n) + a))*cos(2*b*log(x^n) + 2*a) - (
4*b*n*cos(2*b*log(c))*cos(b*log(c))*cos(2*b*log(x^n) + 2*a) - 4*b*n*cos(b*l
og(c))*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b*cos(4*b*log(c))^2*cos(b
*log(c)) + b*cos(b*log(c))*sin(4*b*log(c))^2)*n*cos(4*b*log(x^n) + 4*a)^2 +
 4*(b*cos(2*b*log(c))^2*cos(b*log(c)) + b*cos(b*log(c))*sin(2*b*log(c))^2)*
n*cos(2*b*log(x^n) + 2*a)^2 + (b*cos(4*b*log(c))^2*cos(b*log(c)) + b*cos(b*
log(c))*sin(4*b*log(c))^2)*n*sin(4*b*log(x^n) + 4*a)^2 + 4*(b*cos(2*b*log(c
))^2*cos(b*log(c)) + b*cos(b*log(c))*sin(2*b*log(c))^2)*n*sin(2*b*log(x^n)
 + 2*a)^2 + b*n*cos(b*log(c)) + 2*(b*n*cos(4*b*log(c))*cos(b*log(c)) + 2*(b*
cos(4*b*log(c))*cos(2*b*log(c))*cos(b*log(c)) + b*cos(b*log(c))*sin(4*b*log
(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n) + 2*a) + 2*(b*cos(2*b*log(c))*cos(
b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*cos(b*log(c))*sin(2*b*log(c))
)*n*sin(2*b*log(x^n) + 2*a))*cos(4*b*log(x^n) + 4*a) - 2*(b*n*cos(b*log(c))
*sin(4*b*log(c)) + 2*(b*cos(2*b*log(c))*cos(b*log(c))*sin(4*b*log(c)) - b*c
os(4*b*log(c))*cos(b*log(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n) + 2*a) - 2
*(b*cos(4*b*log(c))*cos(2*b*log(c))*cos(b*log(c)) + b*cos(b*log(c))*sin(4*b
*log(c))*sin(2*b*log(c)))*n*sin(2*b*log(x^n) + 2*a))*sin(4*b*log(x^n) + 4*a
))*integrate(((cos(2*b*log(c))*cos(b*log(x^n) + a) + sin(2*b*log(c))*sin(b*
log(x^n) + a))*cos(2*b*log(x^n) + 2*a) - (cos(b*log(x^n) + a)*sin(2*b*log(c
)) - cos(2*b*log(c))*sin(b*log(x^n) + a))*sin(2*b*log(x^n) + 2*a) + cos(b*l
og(x^n) + a))/((cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*x*cos(2*b*log(x^n) +
 2*a)^2 + (cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*x*sin(2*b*log(x^n) + 2*a)
^2 + 2*x*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) - 2*x*sin(2*b*log(c))*sin(
2*b*log(x^n) + 2*a) + x), x) - (4*b*n*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*
a)*sin(b*log(c)) - 4*b*n*sin(2*b*log(c))*sin(b*log(c))*sin(2*b*log(x^n) + 2
*a) + (b*cos(4*b*log(c))^2*sin(b*log(c)) + b*sin(4*b*log(c))^2*sin(b*log(c)
))*n*cos(4*b*log(x^n) + 4*a)^2 + 4*(b*cos(2*b*log(c))^2*sin(b*log(c)) + b*s
in(2*b*log(c))^2*sin(b*log(c)))*n*cos(2*b*log(x^n) + 2*a)^2 + (b*cos(4*b*lo
g(c))^2*sin(b*log(c)) + b*sin(4*b*log(c))^2*sin(b*log(c)))*n*sin(4*b*log(x^
n) + 4*a)^2 + 4*(b*cos(2*b*log(c))^2*sin(b*log(c)) + b*sin(2*b*log(c))^2*si
n(b*log(c)))*n*sin(2*b*log(x^n) + 2*a)^2 + b*n*sin(b*log(c)) + 2*(b*n*cos(4
*b*log(c))*sin(b*log(c)) + 2*(b*cos(4*b*log(c))*cos(2*b*log(c))*sin(b*log(c
)) + b*sin(4*b*log(c))*sin(2*b*log(c))*sin(b*log(c)))*n*cos(2*b*log(x^n) +
2*a) + 2*(b*cos(2*b*log(c))*sin(4*b*log(c))*sin(b*log(c)) - b*cos(4*b*log(c
))*sin(2*b*log(c))*sin(b*log(c)))*n*sin(2*b*log(x^n) + 2*a))*cos(4*b*log(x^
n) + 4*a) - 2*(b*n*sin(4*b*log(c))*sin(b*log(c)) + 2*(b*cos(2*b*log(c))*sin
(4*b*log(c))*sin(b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c))*sin(b*log(c)
))*n*cos(2*b*log(x^n) + 2*a) - 2*(b*cos(4*b*log(c))*cos(2*b*log(c))*sin(b*l
og(c)) + b*sin(4*b*log(c))*sin(2*b*log(c))*sin(b*log(c)))*n*sin(2*b*log(x^n
) + 2*a))*sin(4*b*log(x^n) + 4*a))*integrate(((cos(b*log(x^n) + a)*sin(2*b*
log(c)) - cos(2*b*log(c))*sin(b*log(x^n) + a))*cos(2*b*log(x^n) + 2*a) + (c
os(2*b*log(c))*cos(b*log(x^n) + a) + sin(2*b*log(c))*sin(b*log(x^n) + a))*s
```

```
in(2*b*log(x^n) + 2*a) - sin(b*log(x^n) + a))/((cos(2*b*log(c))^2 + sin(2*b
*log(c))^2)*x*cos(2*b*log(x^n) + 2*a)^2 + (cos(2*b*log(c))^2 + sin(2*b*log(
c))^2)*x*sin(2*b*log(x^n) + 2*a)^2 + 2*x*cos(2*b*log(c))*cos(2*b*log(x^n) +
 2*a) - 2*x*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + x), x) + cos(b*log(x^
n) + a)*sin(b*log(c)) + ((cos(4*b*log(c))*cos(3*b*log(c)) + sin(4*b*log(c))
*sin(3*b*log(c)))*cos(3*b*log(x^n) + 3*a) - (cos(4*b*log(c))*cos(b*log(c))
+ sin(4*b*log(c))*sin(b*log(c)))*cos(b*log(x^n) + a) + (cos(3*b*log(c))*sin
(4*b*log(c)) - cos(4*b*log(c))*sin(3*b*log(c)))*sin(3*b*log(x^n) + 3*a) - (
cos(b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(b*log(c)))*sin(b*log(x^
n) + a))*sin(4*b*log(x^n) + 4*a) - (2*(cos(3*b*log(c))*cos(2*b*log(c)) + si
n(3*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 2*(cos(2*b*log(c))
*sin(3*b*log(c)) - cos(3*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a)
 + cos(3*b*log(c)))*sin(3*b*log(x^n) + 3*a) - 2*((cos(2*b*log(c))*cos(b*log
(c)) + sin(2*b*log(c))*sin(b*log(c)))*cos(b*log(x^n) + a) + (cos(b*log(c))*
sin(2*b*log(c)) - cos(2*b*log(c))*sin(b*log(c))...
```

Fricas [A]
time = 2.38, size = 100, normalized size = 1.82

$$\frac{\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}\log\left(\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+1\right)-\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}\log\left(-\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+1\right)+2\,\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{4\,bn\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] 1/4*(cos(b*n*log(x) + b*log(c) + a)^2*log(sin(b*n*log(x) + b*log(c) + a) +
1) - cos(b*n*log(x) + b*log(c) + a)^2*log(-sin(b*n*log(x) + b*log(c) + a) +
 1) + 2*sin(b*n*log(x) + b*log(c) + a))/(b*n*cos(b*n*log(x) + b*log(c) + a)
^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{3}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**3/x,x)
```

```
[Out] Integral(sec(a + b*log(c*x**n))**3/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)^3/x, x)
```

Mupad [B]
time = 6.30, size = 178, normalized size = 3.24

$$\frac{\ln\left(-\frac{1\mathrm{i}}{x}-\frac{\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,1\mathrm{i}}}{x}\right)}{2\,b\,n}-\frac{\ln\left(\frac{1\mathrm{i}}{x}-\frac{\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,1\mathrm{i}}}{x}\right)}{2\,b\,n}+\frac{\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,1\mathrm{i}}\,2\mathrm{i}}{b\,n\,\left(2\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}+\mathrm{e}^{a\,4\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,4\mathrm{i}}+1\right)}-\frac{\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,1\mathrm{i}}\,1\mathrm{i}}{b\,n\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cos(a + b*log(c*x^n))^3),x)
```

```
[Out] log(- 1i/x - (exp(a*1i)*(c*x^n)^(b*1i))/x)/(2*b*n) - log(1i/x - (exp(a*1i)*
(c*x^n)^(b*1i))/x)/(2*b*n) + (exp(a*1i)*(c*x^n)^(b*1i)*2i)/(b*n*(2*exp(a*2i
)*(c*x^n)^(b*2i) + exp(a*4i)*(c*x^n)^(b*4i) + 1)) - (exp(a*1i)*(c*x^n)^(b*1
i)*1i)/(b*n*(exp(a*2i)*(c*x^n)^(b*2i) + 1))
```

3.252 $\int \frac{\sec^3(a+b\log(cx^n))}{x^2}\,dx$

Optimal. Leaf size=87

$$-\frac{8e^{3ia}(cx^n)^{3ib}\,{}_2F_1\left(3,\frac{1}{2}\left(3+\frac{i}{bn}\right);\frac{1}{2}\left(5+\frac{i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{(1-3ibn)x}$$

```
[Out] -8*exp(3*I*a)*(c*x^n)^(3*I*b)*hypergeom([3, 3/2+1/2*I/b/n],[5/2+1/2*I/b/n],
-exp(2*I*a)*(c*x^n)^(2*I*b))/(1-3*I*b*n)/x
```

Rubi [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4605, 4601, 371}

$$-\frac{8e^{3ia}(cx^n)^{3ib}\,{}_2F_1\left(3,\frac{1}{2}\left(3+\frac{i}{bn}\right);\frac{1}{2}\left(5+\frac{i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{x(1-3ibn)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*Log[c*x^n]]^3/x^2,x]
```

```
[Out] (-8*E^((3*I)*a)*(c*x^n)^((3*I)*b)*Hypergeometric2F1[3, (3 + I/(b*n))/2, (5
+ I/(b*n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((1 - (3*I)*b*n)*x)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4605

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\sec^3(a+b\log(cx^n))}{x^2}\,dx = \frac{(cx^n)^{\frac{1}{n}}\,\text{Subst}\left(\int x^{-1-\frac{1}{n}}\sec^3(a+b\log(x))\,dx,x,cx^n\right)}{nx}$$
$$= \frac{\left(8e^{3ia}(cx^n)^{\frac{1}{n}}\right)\text{Subst}\left(\int \frac{x^{-1+3ib-\frac{1}{n}}}{(1+e^{2ia}x^{2ib})^3}\,dx,x,cx^n\right)}{nx}$$
$$= -\frac{8e^{3ia}(cx^n)^{3ib}\,{}_2F_1\left(3,\frac{1}{2}\left(3+\frac{i}{bn}\right);\frac{1}{2}\left(5+\frac{i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{(1-3ibn)x}$$

Mathematica [A]
time = 5.73, size = 123, normalized size = 1.41

$$\frac{-2ie^{ia}(-i+bn)(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}+\frac{i}{2bn};\frac{3}{2}+\frac{i}{2bn};-e^{2i(a+b\log(cx^n))}\right)+\sec(a+b\log(cx^n))(1+bn\tan(a+b\log(cx^n)))}{2b^2n^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*Log[c*x^n]]^3/x^2,x]
```

```
[Out] ((-2*I)*E^(I*a)*(-I + b*n)*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 + (I/2)/(
b*n), 3/2 + (I/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] + Sec[a + b*Log[c*x
^n]]*(1 + b*n*Tan[a + b*Log[c*x^n]]))/(2*b^2*n^2*x)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(a+b\ln(c\,x^n))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a+b*ln(c*x^n))^3/x^2,x)
```

```
[Out] int(sec(a+b*ln(c*x^n))^3/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^3/x^2,x, algorithm="maxima")
```

```
[Out] -((((b*cos(3*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(3*b*log(c)))
*n - cos(4*b*log(c))*cos(3*b*log(c)) - sin(4*b*log(c))*sin(3*b*log(c)))*cos
```

(3*b*log(x^n) + 3*a) - ((b*cos(b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c)
)*sin(b*log(c)))*n + cos(4*b*log(c))*cos(b*log(c)) + sin(4*b*log(c))*sin(b*
log(c)))*cos(b*log(x^n) + a) - ((b*cos(4*b*log(c))*cos(3*b*log(c)) + b*sin(
4*b*log(c))*sin(3*b*log(c)))*n + cos(3*b*log(c))*sin(4*b*log(c)) - cos(4*b*
log(c))*sin(3*b*log(c)))*sin(3*b*log(x^n) + 3*a) + ((b*cos(4*b*log(c))*cos(
b*log(c)) + b*sin(4*b*log(c))*sin(b*log(c)))*n - cos(b*log(c))*sin(4*b*log(
c)) + cos(4*b*log(c))*sin(b*log(c)))*sin(b*log(x^n) + a))*cos(4*b*log(x^n)
+ 4*a) - (b*n*sin(3*b*log(c)) + 2*((b*cos(2*b*log(c))*sin(3*b*log(c)) - b*c
os(3*b*log(c))*sin(2*b*log(c)))*n + cos(3*b*log(c))*cos(2*b*log(c)) + sin(3
*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) - 2*((b*cos(3*b*log(c))
*cos(2*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*log(c)))*n - cos(2*b*log(c))*s
in(3*b*log(c)) + cos(3*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) +
cos(3*b*log(c)))*cos(3*b*log(x^n) + 3*a) - 2*(((b*cos(b*log(c))*sin(2*b*lo
g(c)) - b*cos(2*b*log(c))*sin(b*log(c)))*n + cos(2*b*log(c))*cos(b*log(c))
+ sin(2*b*log(c))*sin(b*log(c)))*cos(b*log(x^n) + a) - ((b*cos(2*b*log(c))*
cos(b*log(c)) + b*sin(2*b*log(c))*sin(b*log(c)))*n - cos(b*log(c))*sin(2*b*
log(c)) + cos(2*b*log(c))*sin(b*log(c)))*sin(b*log(x^n) + a))*cos(2*b*log(x
^n) + 2*a) + (b*n*sin(b*log(c)) - cos(b*log(c)))*cos(b*log(x^n) + a) - (((b
^4*cos(4*b*log(c))^2*cos(b*log(c)) + b^4*cos(b*log(c))*sin(4*b*log(c))^2)*n
^4 + (b^2*cos(4*b*log(c))^2*cos(b*log(c)) + b^2*cos(b*log(c))*sin(4*b*log(c
))^2)*n^2)*x*cos(4*b*log(x^n) + 4*a)^2 + 4*((b^4*cos(2*b*log(c))^2*cos(b*lo
g(c)) + b^4*cos(b*log(c))*sin(2*b*log(c))^2)*n^4 + (b^2*cos(2*b*log(c))^2*c
os(b*log(c)) + b^2*cos(b*log(c))*sin(2*b*log(c))^2)*n^2)*x*cos(2*b*log(x^n)
+ 2*a)^2 + ((b^4*cos(4*b*log(c))^2*cos(b*log(c)) + b^4*cos(b*log(c))*sin(4
*b*log(c))^2)*n^4 + (b^2*cos(4*b*log(c))^2*cos(b*log(c)) + b^2*cos(b*log(c)
)*sin(4*b*log(c))^2)*n^2)*x*sin(4*b*log(x^n) + 4*a)^2 + 4*((b^4*cos(2*b*log
(c))^2*cos(b*log(c)) + b^4*cos(b*log(c))*sin(2*b*log(c))^2)*n^4 + (b^2*cos(
2*b*log(c))^2*cos(b*log(c)) + b^2*cos(b*log(c))*sin(2*b*log(c))^2)*n^2)*x*s
in(2*b*log(x^n) + 2*a)^2 + 4*(b^4*n^4*cos(2*b*log(c))*cos(b*log(c)) + b^2*n
^2*cos(2*b*log(c))*cos(b*log(c)))*x*cos(2*b*log(x^n) + 2*a) - 4*(b^4*n^4*co
s(b*log(c))*sin(2*b*log(c)) + b^2*n^2*cos(b*log(c))*sin(2*b*log(c)))*x*sin(
2*b*log(x^n) + 2*a) + (b^4*n^4*cos(b*log(c)) + b^2*n^2*cos(b*log(c)))*x + 2
(2((b^4*cos(4*b*log(c))*cos(2*b*log(c))*cos(b*log(c)) + b^4*cos(b*log(c))
*sin(4*b*log(c))*sin(2*b*log(c)))*n^4 + (b^2*cos(4*b*log(c))*cos(2*b*log(c)
)*cos(b*log(c)) + b^2*cos(b*log(c))*sin(4*b*log(c))*sin(2*b*log(c)))*n^2)*x
*cos(2*b*log(x^n) + 2*a) + 2*((b^4*cos(2*b*log(c))*cos(b*log(c))*sin(4*b*lo
g(c)) - b^4*cos(4*b*log(c))*cos(b*log(c))*sin(2*b*log(c)))*n^4 + (b^2*cos(2
*b*log(c))*cos(b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*cos(b*log(c)
)*sin(2*b*log(c)))*n^2)*x*sin(2*b*log(x^n) + 2*a) + (b^4*n^4*cos(4*b*log(c)
)*cos(b*log(c)) + b^2*n^2*cos(4*b*log(c))*cos(b*log(c)))*x)*cos(4*b*log(x^n
) + 4*a) - 2*(2*((b^4*cos(2*b*log(c))*cos(b*log(c))*sin(4*b*log(c)) - b^4*c
os(4*b*log(c))*cos(b*log(c))*sin(2*b*log(c)))*n^4 + (b^2*cos(2*b*log(c))*co
s(b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*cos(b*log(c))*sin(2*b*log
(c)))*n^2)*x*cos(2*b*log(x^n) + 2*a) - 2*((b^4*cos(4*b*log(c))*cos(2*b*log(
c))*cos(b*log(c)) + b^4*cos(b*log(c))*sin(4*b*log(c))*sin(2*b*log(c)))*n^4

```
+ (b^2*cos(4*b*log(c))*cos(2*b*log(c))*cos(b*log(c)) + b^2*cos(b*log(c))*si
n(4*b*log(c))*sin(2*b*log(c)))*n^2)*x*sin(2*b*log(x^n) + 2*a) + (b^4*n^4*co
s(b*log(c))*sin(4*b*log(c)) + b^2*n^2*cos(b*log(c))*sin(4*b*log(c)))*x)*sin
(4*b*log(x^n) + 4*a))*integrate(((cos(2*b*log(c))*cos(b*log(x^n) + a) + sin
(2*b*log(c))*sin(b*log(x^n) + a))*cos(2*b*log(x^n) + 2*a) - (cos(b*log(x^n)
 + a)*sin(2*b*log(c)) - cos(2*b*log(c))*sin(b*log(x^n) + a))*sin(2*b*log(x^
n) + 2*a) + cos(b*log(x^n) + a))/(2*b^2*n^2*x^2*cos(2*b*log(c))*cos(2*b*log
(x^n) + 2*a) - 2*b^2*n^2*x^2*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b^2
*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2*x^2*cos(2*b*log(x^n) + 2*a)
^2 + (b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2*x^2*sin(2*b*log(x^
n) + 2*a)^2 + b^2*n^2*x^2), x) - (((b^4*cos(4*b*log(c))^2*sin(b*log(c)) + b
^4*sin(4*b*log(c))^2*sin(b*log(c)))*n^4 + (b^2*cos(4*b*log(c))^2*sin(b*log(
c)) + b^2*sin(4*b*log(c))^2*sin(b*log(c)))*n^2)*x*cos(4*b*log(x^n) + 4*a)^2
 + 4*((b^4*cos(2*b*log(c))^2*sin(b*log(c)) + b^4*sin(2*b*log(c))^2*sin(b*lo
g(c)))*n^4 + (b^2*cos(2*b*log(c))^2*sin(b*log(c)) + b^2*sin(2*b*log(c))^2*s
in(b*log(c)))*n^2)*x*cos(2*b*log(x^n) + 2*a)^2 + ((b^4*cos(4*b*log(c))^2*si
n(b*log(c)) + b^4*sin(4*b*log(c))^2*sin(b*log(c)))*n^4 + (b^2*cos(4*b*log(c
))^2*sin(b*log(c)) + b^2*sin(4*b*log(c))^2*sin(b*log(c)))*n^2)*x*sin(4*b*lo
g(x^n) + 4*a)^2 + 4*((b^4*cos(2*b*log(c))^2*sin(b*log(c)) + b^4*sin(2*b*log
(c))^2*sin(b*log(c)))*n^4 + (b^2*cos(2*b*log(c)...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(sec(b*log(c*x^n) + a)^3/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3\left(a + b\log\left(cx^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**3/x**2,x)
```

```
[Out] Integral(sec(a + b*log(c*x**n))**3/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)^3/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 \cos\left(a + b \ln\left(c\,x^n\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*cos(a + b*log(c*x^n))^3),x)
```

```
[Out] int(1/(x^2*cos(a + b*log(c*x^n))^3), x)
```

3.253 $\int \frac{\sec^3(a+b\log(cx^n))}{x^3}\,dx$

Optimal. Leaf size=87

$$-\frac{8e^{3ia}(cx^n)^{3ib}\,{}_2F_1\left(3,\frac{1}{2}\left(3+\frac{2i}{bn}\right);\frac{1}{2}\left(5+\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{(2-3ibn)x^2}$$

```
[Out] -8*exp(3*I*a)*(c*x^n)^(3*I*b)*hypergeom([3, 3/2+I/b/n],[5/2+I/b/n],-exp(2*I
*a)*(c*x^n)^(2*I*b))/(2-3*I*b*n)/x^2
```

Rubi [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4605, 4601, 371}

$$-\frac{8e^{3ia}(cx^n)^{3ib}\,{}_2F_1\left(3,\frac{1}{2}\left(3+\frac{2i}{bn}\right);\frac{1}{2}\left(5+\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{x^2(2-3ibn)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*Log[c*x^n]]^3/x^3,x]
```

```
[Out] (-8*E^((3*I)*a)*(c*x^n)^((3*I)*b)*Hypergeometric2F1[3, (3 + (2*I)/(b*n))/2,
 (5 + (2*I)/(b*n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 - (3*I)*b*n)*x
^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4605

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\sec^3(a+b\log(cx^n))}{x^3}\,dx = \frac{(cx^n)^{2/n}\operatorname{Subst}\left(\int x^{-1-\frac{2}{n}}\sec^3(a+b\log(x))\,dx, x, cx^n\right)}{nx^2}$$
$$= \frac{\left(8e^{3ia}(cx^n)^{2/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+3ib-\frac{2}{n}}}{(1+e^{2ia}x^{2ib})^3}\,dx, x, cx^n\right)}{nx^2}$$
$$= -\frac{8e^{3ia}(cx^n)^{3ib}\ {}_2F_1\left(3, \frac{1}{2}\left(3+\frac{2i}{bn}\right); \frac{1}{2}\left(5+\frac{2i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right)}{(2-3ibn)x^2}$$

Mathematica [**A**]
time = 5.75, size = 119, normalized size = 1.37

$$\frac{-2ie^{ia}(-2i+bn)(cx^n)^{ib}\ {}_2F_1\left(1,\frac{1}{2}+\frac{i}{bn};\frac{3}{2}+\frac{i}{bn};-e^{2i(a+b\log(cx^n))}\right)+\sec(a+b\log(cx^n))(2+bn\tan(a+b\log(cx^n)))}{2b^2n^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*Log[c*x^n]]^3/x^3,x]
```

```
[Out] ((-2*I)*E^(I*a)*(-2*I + b*n)*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 + I/(b*
n), 3/2 + I/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] + Sec[a + b*Log[c*x^n]]*(
2 + b*n*Tan[a + b*Log[c*x^n]]))/(2*b^2*n^2*x^2)
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(a+b\ln(c\,x^n))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a+b*ln(c*x^n))^3/x^3,x)
```

```
[Out] int(sec(a+b*ln(c*x^n))^3/x^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^3/x^3,x, algorithm="maxima")
```

```
[Out] -((((b*cos(3*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(3*b*log(c)))
*n - 2*cos(4*b*log(c))*cos(3*b*log(c)) - 2*sin(4*b*log(c))*sin(3*b*log(c)))
*cos(3*b*log(x^n) + 3*a) - ((b*cos(b*log(c))*sin(4*b*log(c)) - b*cos(4*b*lo
g(c))*sin(b*log(c)))*n + 2*cos(4*b*log(c))*cos(b*log(c)) + 2*sin(4*b*log(c)
)*sin(b*log(c)))*cos(b*log(x^n) + a) - ((b*cos(4*b*log(c))*cos(3*b*log(c))
+ b*sin(4*b*log(c))*sin(3*b*log(c)))*n + 2*cos(3*b*log(c))*sin(4*b*log(c))
- 2*cos(4*b*log(c))*sin(3*b*log(c)))*sin(3*b*log(x^n) + 3*a) + ((b*cos(4*b*
log(c))*cos(b*log(c)) + b*sin(4*b*log(c))*sin(b*log(c)))*n - 2*cos(b*log(c)
)*sin(4*b*log(c)) + 2*cos(4*b*log(c))*sin(b*log(c)))*sin(b*log(x^n) + a))*c
os(4*b*log(x^n) + 4*a) - (b*n*sin(3*b*log(c)) + 2*((b*cos(2*b*log(c))*sin(3
*b*log(c)) - b*cos(3*b*log(c))*sin(2*b*log(c)))*n + 2*cos(3*b*log(c))*cos(2
*b*log(c)) + 2*sin(3*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) - 2
*((b*cos(3*b*log(c))*cos(2*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*log(c)))*n
 - 2*cos(2*b*log(c))*sin(3*b*log(c)) + 2*cos(3*b*log(c))*sin(2*b*log(c)))*s
in(2*b*log(x^n) + 2*a) + 2*cos(3*b*log(c)))*cos(3*b*log(x^n) + 3*a) - 2*(((
b*cos(b*log(c))*sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)))*n + 2*co
s(2*b*log(c))*cos(b*log(c)) + 2*sin(2*b*log(c))*sin(b*log(c)))*cos(b*log(x^
n) + a) - ((b*cos(2*b*log(c))*cos(b*log(c)) + b*sin(2*b*log(c))*sin(b*log(c
)))*n - 2*cos(b*log(c))*sin(2*b*log(c)) + 2*cos(2*b*log(c))*sin(b*log(c)))*
sin(b*log(x^n) + a))*cos(2*b*log(x^n) + 2*a) + (b*n*sin(b*log(c)) - 2*cos(b
*log(c)))*cos(b*log(x^n) + a) - (((b^4*cos(4*b*log(c))^2*cos(b*log(c)) + b^
4*cos(b*log(c))*sin(4*b*log(c))^2)*n^4 + 4*(b^2*cos(4*b*log(c))^2*cos(b*log
(c)) + b^2*cos(b*log(c))*sin(4*b*log(c))^2)*n^2)*x^2*cos(4*b*log(x^n) + 4*a
)^2 + 4*((b^4*cos(2*b*log(c))^2*cos(b*log(c)) + b^4*cos(b*log(c))*sin(2*b*l
og(c))^2)*n^4 + 4*(b^2*cos(2*b*log(c))^2*cos(b*log(c)) + b^2*cos(b*log(c))*
sin(2*b*log(c))^2)*n^2)*x^2*cos(2*b*log(x^n) + 2*a)^2 + ((b^4*cos(4*b*log(c
))^2*cos(b*log(c)) + b^4*cos(b*log(c))*sin(4*b*log(c))^2)*n^4 + 4*(b^2*cos(
4*b*log(c))^2*cos(b*log(c)) + b^2*cos(b*log(c))*sin(4*b*log(c))^2)*n^2)*x^2
*sin(4*b*log(x^n) + 4*a)^2 + 4*((b^4*cos(2*b*log(c))^2*cos(b*log(c)) + b^4*
cos(b*log(c))*sin(2*b*log(c))^2)*n^4 + 4*(b^2*cos(2*b*log(c))^2*cos(b*log(c
)) + b^2*cos(b*log(c))*sin(2*b*log(c))^2)*n^2)*x^2*sin(2*b*log(x^n) + 2*a)^
2 + 4*(b^4*n^4*cos(2*b*log(c))*cos(b*log(c)) + 4*b^2*n^2*cos(2*b*log(c))*co
s(b*log(c)))*x^2*cos(2*b*log(x^n) + 2*a) - 4*(b^4*n^4*cos(b*log(c))*sin(2*b
*log(c)) + 4*b^2*n^2*cos(b*log(c))*sin(2*b*log(c)))*x^2*sin(2*b*log(x^n) +
2*a) + (b^4*n^4*cos(b*log(c)) + 4*b^2*n^2*cos(b*log(c)))*x^2 + 2*(2*((b^4*c
os(4*b*log(c))*cos(2*b*log(c))*cos(b*log(c)) + b^4*cos(b*log(c))*sin(4*b*lo
g(c))*sin(2*b*log(c)))*n^4 + 4*(b^2*cos(4*b*log(c))*cos(2*b*log(c))*cos(b*l
og(c)) + b^2*cos(b*log(c))*sin(4*b*log(c))*sin(2*b*log(c)))*n^2)*x^2*cos(2*
b*log(x^n) + 2*a) + 2*((b^4*cos(2*b*log(c))*cos(b*log(c))*sin(4*b*log(c)) -
 b^4*cos(4*b*log(c))*cos(b*log(c))*sin(2*b*log(c)))*n^4 + 4*(b^2*cos(2*b*lo
g(c))*cos(b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*cos(b*log(c))*sin
(2*b*log(c)))*n^2)*x^2*sin(2*b*log(x^n) + 2*a) + (b^4*n^4*cos(4*b*log(c))*c
os(b*log(c)) + 4*b^2*n^2*cos(4*b*log(c))*cos(b*log(c)))*x^2)*cos(4*b*log(x^
n) + 4*a) - 2*(2*((b^4*cos(2*b*log(c))*cos(b*log(c))*sin(4*b*log(c)) - b^4*
cos(4*b*log(c))*cos(b*log(c))*sin(2*b*log(c)))*n^4 + 4*(b^2*cos(2*b*log(c))
```

```
*cos(b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*cos(b*log(c))*sin(2*b*
log(c)))*n^2)*x^2*cos(2*b*log(x^n) + 2*a) - 2*((b^4*cos(4*b*log(c))*cos(2*b
*log(c))*cos(b*log(c)) + b^4*cos(b*log(c))*sin(4*b*log(c))*sin(2*b*log(c)))
*n^4 + 4*(b^2*cos(4*b*log(c))*cos(2*b*log(c))*cos(b*log(c)) + b^2*cos(b*log
(c))*sin(4*b*log(c))*sin(2*b*log(c)))*n^2)*x^2*sin(2*b*log(x^n) + 2*a) + (b
^4*n^4*cos(b*log(c))*sin(4*b*log(c)) + 4*b^2*n^2*cos(b*log(c))*sin(4*b*log(
c)))*x^2)*sin(4*b*log(x^n) + 4*a))*integrate(((cos(2*b*log(c))*cos(b*log(x^
n) + a) + sin(2*b*log(c))*sin(b*log(x^n) + a))*cos(2*b*log(x^n) + 2*a) - (c
os(b*log(x^n) + a)*sin(2*b*log(c)) - cos(2*b*log(c))*sin(b*log(x^n) + a))*s
in(2*b*log(x^n) + 2*a) + cos(b*log(x^n) + a))/(2*b^2*n^2*x^3*cos(2*b*log(c)
)*cos(2*b*log(x^n) + 2*a) - 2*b^2*n^2*x^3*sin(2*b*log(c))*sin(2*b*log(x^n)
+ 2*a) + (b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2*x^3*cos(2*b*lo
g(x^n) + 2*a)^2 + (b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2*x^3*s
in(2*b*log(x^n) + 2*a)^2 + b^2*n^2*x^3), x) - (((b^4*cos(4*b*log(c))^2*sin(
b*log(c)) + b^4*sin(4*b*log(c))^2*sin(b*log(c)))*n^4 + 4*(b^2*cos(4*b*log(c
))^2*sin(b*log(c)) + b^2*sin(4*b*log(c))^2*sin(b*log(c)))*n^2)*x^2*cos(4*b*
log(x^n) + 4*a)^2 + 4*((b^4*cos(2*b*log(c))^2*sin(b*log(c)) + b^4*sin(2*b*l
og(c))^2*sin(b*log(c)))*n^4 + 4*(b^2*cos(2*b*log(c))^2*sin(b*log(c)) + b^2*
sin(2*b*log(c))^2*sin(b*log(c)))*n^2)*x^2*cos(2*b*log(x^n) + 2*a)^2 + ((b^4
*cos(4*b*log(c))^2*sin(b*log(c)) + b^4*sin(4*b*log(c))^2*sin(b*log(c)))*n^4
 + 4*(b^2*cos(4*b*log(c))^2*sin(b*log(c)) + b^2*sin(4*b*log(c))^2*sin(b*log
(c)))*n^2)*x^2*sin(4*b*log(x^n) + 4*a)^2 + 4*((...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral(sec(b*log(c*x^n) + a)^3/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3 (a + b \log (cx^n))}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**3/x**3,x)
```

```
[Out] Integral(sec(a + b*log(c*x**n))**3/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)^3/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*cos(a + b*log(c*x^n))^3),x)
```

```
[Out] int(1/(x^3*cos(a + b*log(c*x^n))^3), x)
```

3.254 $\int x \sec^4(a+b\log(cx^n))\,dx$

Optimal. Leaf size=79

$$\frac{8e^{4ia}x^2(cx^n)^{4ib}\ {}_2F_1\left(4,2-\frac{i}{bn};3-\frac{i}{bn};-e^{2ia}(cx^n)^{2ib}\right)}{1+2ibn}$$

```
[Out] 8*exp(4*I*a)*x^2*(c*x^n)^(4*I*b)*hypergeom([4, 2-I/b/n],[3-I/b/n],-exp(2*I*
a)*(c*x^n)^(2*I*b))/(1+2*I*b*n)
```

Rubi [A]
time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4605, 4601, 371}

$$\frac{8e^{4ia}x^2(cx^n)^{4ib}\ {}_2F_1\left(4,2-\frac{i}{bn};3-\frac{i}{bn};-e^{2ia}(cx^n)^{2ib}\right)}{1+2ibn}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (8*E^((4*I)*a)*x^2*(c*x^n)^((4*I)*b)*Hypergeometric2F1[4, 2 - I/(b*n), 3 -
I/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(1 + (2*I)*b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4605

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int x \sec^4(a+b\log(cx^n))\,dx &= \frac{\left(x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int x^{-1+\frac{2}{n}}\sec^4(a+b\log(x))\,dx,x,cx^n\right)}{n}\\
&= \frac{\left(16e^{4ia}x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int \frac{x^{-1+4ib+\frac{2}{n}}}{(1+e^{2ia}x^{2ib})^4}\,dx,x,cx^n\right)}{n}\\
&= \frac{8e^{4ia}x^2(cx^n)^{4ib}\ {}_2F_1\left(4,2-\frac{i}{bn};3-\frac{i}{bn};-e^{2ia}(cx^n)^{2ib}\right)}{1+2ibn}
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 204 vs. $2(79) = 158$.
time = 13.52, size = 204, normalized size = 2.58

$$
\frac{x^2\left(2e^{2ia}(i+bn)(cx^n)^{2ib}\ {}_2F_1\left(1,1-\frac{i}{bn};2-\frac{i}{bn};-e^{2i(a+b\log(cx^n))}\right)-2i(1+b^2n^2)\ {}_2F_1\left(1,-\frac{i}{bn};1-\frac{i}{bn};-e^{2i(a+b\log(cx^n))}\right)+\sec^2(a+b\log(cx^n))\left(-bn+(1+2b^2n^2+(1+b^2n^2)\cos(2(a+b\log(cx^n))))\tan(a+b\log(cx^n))\right)\right)}{3b^3n^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x^2*(2*E^((2*I)*a)*(I + b*n)*(c*x^n)^((2*I)*b)*Hypergeometric2F1[1, 1 - I/
(b*n), 2 - I/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] - (2*I)*(1 + b^2*n^2)*Hy
pergeometric2F1[1, (-I)/(b*n), 1 - I/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))]
+ Sec[a + b*Log[c*x^n]]^2*(-(b*n) + (1 + 2*b^2*n^2 + (1 + b^2*n^2)*Cos[2*(a
 + b*Log[c*x^n])])*Tan[a + b*Log[c*x^n]])))/(3*b^3*n^3)
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$
\int x\left(\sec^4(a+b\ln(c\,x^n))\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(a+b*ln(c*x^n))^4,x)
```

```
[Out] int(x*sec(a+b*ln(c*x^n))^4,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

```
[Out] -4/3*(3*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*x^2*cos(4*b*log(x^n)
+ 4*a)^2 + 3*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*x^2*cos(2*b*log(
x^n) + 2*a)^2 + 3*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*x^2*sin(4*b
*log(x^n) + 4*a)^2 + 3*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*x^2*si
n(2*b*log(x^n) + 2*a)^2 + (b*n*cos(2*b*log(c)) - sin(2*b*log(c)))*x^2*cos(2
*b*log(x^n) + 2*a) - (b*n*sin(2*b*log(c)) + cos(2*b*log(c)))*x^2*sin(2*b*lo
g(x^n) + 2*a) + (((b*cos(6*b*log(c))*cos(4*b*log(c)) + b*sin(6*b*log(c))*si
n(4*b*log(c)))*n - cos(4*b*log(c))*sin(6*b*log(c)) + cos(6*b*log(c))*sin(4*
b*log(c)))*x^2*cos(4*b*log(x^n) + 4*a) - (3*(b^2*cos(2*b*log(c))*sin(6*b*lo
g(c)) - b^2*cos(6*b*log(c))*sin(2*b*log(c)))*n^2 - (b*cos(6*b*log(c))*cos(2
*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n + 2*cos(2*b*log(c))*sin(6
*b*log(c)) - 2*cos(6*b*log(c))*sin(2*b*log(c)))*x^2*cos(2*b*log(x^n) + 2*a)
 + ((b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b*log(c)))
*n + cos(6*b*log(c))*cos(4*b*log(c)) + sin(6*b*log(c))*sin(4*b*log(c)))*x^2
*sin(4*b*log(x^n) + 4*a) + (3*(b^2*cos(6*b*log(c))*cos(2*b*log(c)) + b^2*si
n(6*b*log(c))*sin(2*b*log(c)))*n^2 + (b*cos(2*b*log(c))*sin(6*b*log(c)) - b
*cos(6*b*log(c))*sin(2*b*log(c)))*n + 2*cos(6*b*log(c))*cos(2*b*log(c)) + 2
*sin(6*b*log(c))*sin(2*b*log(c)))*x^2*sin(2*b*log(x^n) + 2*a) - (b^2*n^2*si
n(6*b*log(c)) + sin(6*b*log(c)))*x^2)*cos(6*b*log(x^n) + 6*a) - (3*(3*(b^2*
cos(2*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(2*b*log(c)))*n^2
- 2*(b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log(c)))
*n + cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)))*x^2
*cos(2*b*log(x^n) + 2*a) - 3*(3*(b^2*cos(4*b*log(c))*cos(2*b*log(c)) + b^2*
sin(4*b*log(c))*sin(2*b*log(c)))*n^2 + 2*(b*cos(2*b*log(c))*sin(4*b*log(c))
 - b*cos(4*b*log(c))*sin(2*b*log(c)))*n + cos(4*b*log(c))*cos(2*b*log(c)) +
 sin(4*b*log(c))*sin(2*b*log(c)))*x^2*sin(2*b*log(x^n) + 2*a) + (3*b^2*n^2*
sin(4*b*log(c)) - b*n*cos(4*b*log(c)) + 2*sin(4*b*log(c)))*x^2)*cos(4*b*log
(x^n) + 4*a) + 18*(b^8*n^8 + b^6*n^6 + ((b^8*cos(6*b*log(c))^2 + b^8*sin(6*
b*log(c))^2)*n^8 + (b^6*cos(6*b*log(c))^2 + b^6*sin(6*b*log(c))^2)*n^6)*cos
(6*b*log(x^n) + 6*a)^2 + 9*((b^8*cos(4*b*log(c))^2 + b^8*sin(4*b*log(c))^2)
*n^8 + (b^6*cos(4*b*log(c))^2 + b^6*sin(4*b*log(c))^2)*n^6)*cos(4*b*log(x^n
) + 4*a)^2 + 9*((b^8*cos(2*b*log(c))^2 + b^8*sin(2*b*log(c))^2)*n^8 + (b^6*
cos(2*b*log(c))^2 + b^6*sin(2*b*log(c))^2)*n^6)*cos(2*b*log(x^n) + 2*a)^2 +
 ((b^8*cos(6*b*log(c))^2 + b^8*sin(6*b*log(c))^2)*n^8 + (b^6*cos(6*b*log(c)
)^2 + b^6*sin(6*b*log(c))^2)*n^6)*sin(6*b*log(x^n) + 6*a)^2 + 9*((b^8*cos(4
*b*log(c))^2 + b^8*sin(4*b*log(c))^2)*n^8 + (b^6*cos(4*b*log(c))^2 + b^6*si
n(4*b*log(c))^2)*n^6)*sin(4*b*log(x^n) + 4*a)^2 + 9*((b^8*cos(2*b*log(c))^2
 + b^8*sin(2*b*log(c))^2)*n^8 + (b^6*cos(2*b*log(c))^2 + b^6*sin(2*b*log(c)
)^2)*n^6)*sin(2*b*log(x^n) + 2*a)^2 + 2*(b^8*n^8*cos(6*b*log(c)) + b^6*n^6*
cos(6*b*log(c)) + 3*((b^8*cos(6*b*log(c))*cos(4*b*log(c)) + b^8*sin(6*b*log
(c))*sin(4*b*log(c)))*n^8 + (b^6*cos(6*b*log(c))*cos(4*b*log(c)) + b^6*sin(
6*b*log(c))*sin(4*b*log(c)))*n^6)*cos(4*b*log(x^n) + 4*a) + 3*((b^8*cos(6*b
*log(c))*cos(2*b*log(c)) + b^8*sin(6*b*log(c))*sin(2*b*log(c)))*n^8 + (b^6*
cos(6*b*log(c))*cos(2*b*log(c)) + b^6*sin(6*b*log(c))*sin(2*b*log(c)))*n^6)
*cos(2*b*log(x^n) + 2*a) + 3*((b^8*cos(4*b*log(c))*sin(6*b*log(c)) - b^8*co
```

```
s(6*b*log(c))*sin(4*b*log(c)))*n^8 + (b^6*cos(4*b*log(c))*sin(6*b*log(c)) -
 b^6*cos(6*b*log(c))*sin(4*b*log(c)))*n^6)*sin(4*b*log(x^n) + 4*a) + 3*((b^
8*cos(2*b*log(c))*sin(6*b*log(c)) - b^8*cos(6*b*log(c))*sin(2*b*log(c)))*n^
8 + (b^6*cos(2*b*log(c))*sin(6*b*log(c)) - b^6*cos(6*b*log(c))*sin(2*b*log(
c)))*n^6)*sin(2*b*log(x^n) + 2*a))*cos(6*b*log(x^n) + 6*a) + 6*(b^8*n^8*cos
(4*b*log(c)) + b^6*n^6*cos(4*b*log(c)) + 3*((b^8*cos(4*b*log(c))*cos(2*b*lo
g(c)) + b^8*sin(4*b*log(c))*sin(2*b*log(c)))*n^8 + (b^6*cos(4*b*log(c))*cos
(2*b*log(c)) + b^6*sin(4*b*log(c))*sin(2*b*log(c)))*n^6)*cos(2*b*log(x^n) +
 2*a) + 3*((b^8*cos(2*b*log(c))*sin(4*b*log(c)) - b^8*cos(4*b*log(c))*sin(2
*b*log(c)))*n^8 + (b^6*cos(2*b*log(c))*sin(4*b*log(c)) - b^6*cos(4*b*log(c)
)*sin(2*b*log(c)))*n^6)*sin(2*b*log(x^n) + 2*a))*cos(4*b*log(x^n) + 4*a) +
6*(b^8*n^8*cos(2*b*log(c)) + b^6*n^6*cos(2*b*log(c)))*cos(2*b*log(x^n) + 2*
a) - 2*(b^8*n^8*sin(6*b*log(c)) + b^6*n^6*sin(6*b*log(c)) + 3*((b^8*cos(4*b
*log(c))*sin(6*b*log(c)) - b^8*cos(6*b*log(c))*sin(4*b*log(c)))*n^8 + (b^6*
cos(4*b*log(c))*sin(6*b*log(c)) - b^6*cos(6*b*log(c))*sin(4*b*log(c)))*n^6)
*cos(4*b*log(x^n) + 4*a) + 3*((b^8*cos(2*b*log(c))*sin(6*b*log(c)) - b^8*co
s(6*b*log(c))*sin(2*b*log(c)))*n^8 + (b^6*cos(2*b*log(c))*sin(6*b*log(c)) -
 b^6*cos(6*b*log(c))*sin(2*b*log(c)))*n^6)*cos(2*b*log(x^n) + 2*a) - 3*((b^
8*cos(6*b*log(c))*cos(4*b*log(c)) + b^8*sin(6*b*log(c))*sin(4*b*log(c)))*n^
8 + (b^6*cos(6*b*log(c))*cos(4*b*log(c)) + b^6*sin(6*b*log(c))*sin(4*b*log(
c)))*n^6)*sin(4*b*log(x^n) + 4*a) - 3*((b^8*cos(6*b*log(c))*cos(2*b*log(c))
 + b^8*sin(6*b*log(c))*sin(2*b*log(c)))*n^8 + (...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] integral(x*sec(b*log(c*x^n) + a)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \sec^4\left(a + b\log\left(cx^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*ln(c*x**n))**4,x)
```

```
[Out] Integral(x*sec(a + b*log(c*x**n))**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] integrate(x*sec(b*log(c*x^n) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(a + b*log(c*x^n))^4,x)
```

```
[Out] int(x/cos(a + b*log(c*x^n))^4, x)
```

3.255 $\int \sec^4(a + b\log(cx^n))\,dx$

Optimal. Leaf size=85

$$\frac{16e^{4ia}x(cx^n)^{4ib}\,{}_2F_1\left(4,\frac{1}{2}\left(4-\frac{i}{bn}\right);\frac{1}{2}\left(6-\frac{i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{1+4ibn}$$

```
[Out] 16*exp(4*I*a)*x*(c*x^n)^(4*I*b)*hypergeom([4, 2-1/2*I/b/n],[3-1/2*I/b/n],-e
xp(2*I*a)*(c*x^n)^(2*I*b))/(1+4*I*b*n)
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4599, 4601, 371}

$$\frac{16e^{4ia}x(cx^n)^{4ib}\,{}_2F_1\left(4,\frac{1}{2}\left(4-\frac{i}{bn}\right);\frac{1}{2}\left(6-\frac{i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{1+4ibn}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (16*E^((4*I)*a)*x*(c*x^n)^((4*I)*b)*Hypergeometric2F1[4, (4 - I/(b*n))/2, (
6 - I/(b*n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(1 + (4*I)*b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4599

```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \sec^4(a+b\log(cx^n))\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\sec^4(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(16e^{4ia}x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+4ib+\frac{1}{n}}}{(1+e^{2ia}x^{2ib})^4}\,dx,x,cx^n\right)}{n}$$
$$= \frac{16e^{4ia}x(cx^n)^{4ib}\,{}_2F_1\left(4,\frac{1}{2}\left(4-\frac{i}{bn}\right);\frac{1}{2}\left(6-\frac{i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{1+4ibn}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 517 vs. 2(85) = 170.
time = 11.13, size = 517, normalized size = 6.08

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*Log[c*x^n]]^4,x]
```

```
[Out] ((1 + 4*b^2*n^2)*x*Sec[a + b*(-(n*Log[x]) + Log[c*x^n])]*Sec[a + b*n*Log[x]
 + b*(-(n*Log[x]) + Log[c*x^n])]*Sin[b*n*Log[x]])/(6*b^3*n^3) + (x*Sec[a +
b*(-(n*Log[x]) + Log[c*x^n])]*Sec[a + b*n*Log[x] + b*(-(n*Log[x]) + Log[c*x
^n])]^3*Sin[b*n*Log[x]])/(3*b*n) - ((1 + 4*b^2*n^2)*Sec[a + b*(-(n*Log[x])
+ Log[c*x^n])]*(-(E^((2*I + 1/(b*n))*(a + b*Log[c*x^n]))*Cos[a + b*(-(n*Log
[x]) + Log[c*x^n])]*Hypergeometric2F1[1, 1 - (I/2)/(b*n), 2 - (I/2)/(b*n),
-E^((2*I)*(a + b*Log[c*x^n]))]) + E^(a/(b*n) + (-(n*Log[x]) + Log[c*x^n])/n
)*(1 + (2*I)*b*n)*x*(Cos[a + b*(-(n*Log[x]) + Log[c*x^n])]*Hypergeometric2F
1[1, (-1/2*I)/(b*n), 1 - (I/2)/(b*n), -E^((2*I)*(a + b*n*Log[x] + b*(-(n*Lo
g[x]) + Log[c*x^n])))] + I*Sin[a + b*(-(n*Log[x]) + Log[c*x^n])])))/(6*b^3*
E^((a + b*(-(n*Log[x]) + Log[c*x^n]))/(b*n))*n^3*(-I + 2*b*n)) + (x*Sec[a +
 b*(-(n*Log[x]) + Log[c*x^n])]*Sec[a + b*n*Log[x] + b*(-(n*Log[x]) + Log[c*
x^n])]^2*(-Cos[a + b*(-(n*Log[x]) + Log[c*x^n])] + 2*b*n*Sin[a + b*(-(n*Log
[x]) + Log[c*x^n])]))/(6*b^2*n^2)
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int \sec^4(a+b\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a+b*ln(c*x^n))^4,x)
```

```
[Out] int(sec(a+b*ln(c*x^n))^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

```
[Out] -1/3*(6*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*x*cos(4*b*log(x^n) +
4*a)^2 + 6*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*x*cos(2*b*log(x^n)
 + 2*a)^2 + 6*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*x*sin(4*b*log(x
^n) + 4*a)^2 + 6*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*x*sin(2*b*lo
g(x^n) + 2*a)^2 + (2*b*n*cos(2*b*log(c)) - sin(2*b*log(c)))*x*cos(2*b*log(x
^n) + 2*a) - (2*b*n*sin(2*b*log(c)) + cos(2*b*log(c)))*x*sin(2*b*log(x^n) +
 2*a) + ((2*(b*cos(6*b*log(c))*cos(4*b*log(c)) + b*sin(6*b*log(c))*sin(4*b*
log(c)))*n - cos(4*b*log(c))*sin(6*b*log(c)) + cos(6*b*log(c))*sin(4*b*log(
c)))*x*cos(4*b*log(x^n) + 4*a) - 2*(6*(b^2*cos(2*b*log(c))*sin(6*b*log(c))
- b^2*cos(6*b*log(c))*sin(2*b*log(c)))*n^2 - (b*cos(6*b*log(c))*cos(2*b*log
(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n + cos(2*b*log(c))*sin(6*b*log(c
)) - cos(6*b*log(c))*sin(2*b*log(c)))*x*cos(2*b*log(x^n) + 2*a) + (2*(b*cos
(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b*log(c)))*n + cos(6
*b*log(c))*cos(4*b*log(c)) + sin(6*b*log(c))*sin(4*b*log(c)))*x*sin(4*b*log
(x^n) + 4*a) + 2*(6*(b^2*cos(6*b*log(c))*cos(2*b*log(c)) + b^2*sin(6*b*log(
c))*sin(2*b*log(c)))*n^2 + (b*cos(2*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*l
og(c))*sin(2*b*log(c)))*n + cos(6*b*log(c))*cos(2*b*log(c)) + sin(6*b*log(c
))*sin(2*b*log(c)))*x*sin(2*b*log(x^n) + 2*a) - (4*b^2*n^2*sin(6*b*log(c))
+ sin(6*b*log(c)))*x)*cos(6*b*log(x^n) + 6*a) - (3*(12*(b^2*cos(2*b*log(c))
*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(2*b*log(c)))*n^2 - 4*(b*cos(4*b*
log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log(c)))*n + cos(2*b*lo
g(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)))*x*cos(2*b*log(x^n)
 + 2*a) - 3*(12*(b^2*cos(4*b*log(c))*cos(2*b*log(c)) + b^2*sin(4*b*log(c))*
sin(2*b*log(c)))*n^2 + 4*(b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log
(c))*sin(2*b*log(c)))*n + cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))
*sin(2*b*log(c)))*x*sin(2*b*log(x^n) + 2*a) + 2*(6*b^2*n^2*sin(4*b*log(c))
- b*n*cos(4*b*log(c)) + sin(4*b*log(c)))*x)*cos(4*b*log(x^n) + 4*a) + 9*(4*
b^8*n^8 + b^6*n^6 + (4*(b^8*cos(6*b*log(c))^2 + b^8*sin(6*b*log(c))^2)*n^8
+ (b^6*cos(6*b*log(c))^2 + b^6*sin(6*b*log(c))^2)*n^6)*cos(6*b*log(x^n) + 6
*a)^2 + 9*(4*(b^8*cos(4*b*log(c))^2 + b^8*sin(4*b*log(c))^2)*n^8 + (b^6*cos
(4*b*log(c))^2 + b^6*sin(4*b*log(c))^2)*n^6)*cos(4*b*log(x^n) + 4*a)^2 + 9*
(4*(b^8*cos(2*b*log(c))^2 + b^8*sin(2*b*log(c))^2)*n^8 + (b^6*cos(2*b*log(c
))^2 + b^6*sin(2*b*log(c))^2)*n^6)*cos(2*b*log(x^n) + 2*a)^2 + (4*(b^8*cos(
6*b*log(c))^2 + b^8*sin(6*b*log(c))^2)*n^8 + (b^6*cos(6*b*log(c))^2 + b^6*s
in(6*b*log(c))^2)*n^6)*sin(6*b*log(x^n) + 6*a)^2 + 9*(4*(b^8*cos(4*b*log(c)
)^2 + b^8*sin(4*b*log(c))^2)*n^8 + (b^6*cos(4*b*log(c))^2 + b^6*sin(4*b*log
(c))^2)*n^6)*sin(4*b*log(x^n) + 4*a)^2 + 9*(4*(b^8*cos(2*b*log(c))^2 + b^8*
```

```
sin(2*b*log(c))^2)*n^8 + (b^6*cos(2*b*log(c))^2 + b^6*sin(2*b*log(c))^2)*n^
6)*sin(2*b*log(x^n) + 2*a)^2 + 2*(4*b^8*n^8*cos(6*b*log(c)) + b^6*n^6*cos(6
*b*log(c)) + 3*(4*(b^8*cos(6*b*log(c))*cos(4*b*log(c)) + b^8*sin(6*b*log(c)
)*sin(4*b*log(c)))*n^8 + (b^6*cos(6*b*log(c))*cos(4*b*log(c)) + b^6*sin(6*b
*log(c))*sin(4*b*log(c)))*n^6)*cos(4*b*log(x^n) + 4*a) + 3*(4*(b^8*cos(6*b*
log(c))*cos(2*b*log(c)) + b^8*sin(6*b*log(c))*sin(2*b*log(c)))*n^8 + (b^6*c
os(6*b*log(c))*cos(2*b*log(c)) + b^6*sin(6*b*log(c))*sin(2*b*log(c)))*n^6)*
cos(2*b*log(x^n) + 2*a) + 3*(4*(b^8*cos(4*b*log(c))*sin(6*b*log(c)) - b^8*c
os(6*b*log(c))*sin(4*b*log(c)))*n^8 + (b^6*cos(4*b*log(c))*sin(6*b*log(c))
- b^6*cos(6*b*log(c))*sin(4*b*log(c)))*n^6)*sin(4*b*log(x^n) + 4*a) + 3*(4*
(b^8*cos(2*b*log(c))*sin(6*b*log(c)) - b^8*cos(6*b*log(c))*sin(2*b*log(c)))
*n^8 + (b^6*cos(2*b*log(c))*sin(6*b*log(c)) - b^6*cos(6*b*log(c))*sin(2*b*l
og(c)))*n^6)*sin(2*b*log(x^n) + 2*a))*cos(6*b*log(x^n) + 6*a) + 6*(4*b^8*n^
8*cos(4*b*log(c)) + b^6*n^6*cos(4*b*log(c)) + 3*(4*(b^8*cos(4*b*log(c))*cos
(2*b*log(c)) + b^8*sin(4*b*log(c))*sin(2*b*log(c)))*n^8 + (b^6*cos(4*b*log(
c))*cos(2*b*log(c)) + b^6*sin(4*b*log(c))*sin(2*b*log(c)))*n^6)*cos(2*b*log
(x^n) + 2*a) + 3*(4*(b^8*cos(2*b*log(c))*sin(4*b*log(c)) - b^8*cos(4*b*log(
c))*sin(2*b*log(c)))*n^8 + (b^6*cos(2*b*log(c))*sin(4*b*log(c)) - b^6*cos(4
*b*log(c))*sin(2*b*log(c)))*n^6)*sin(2*b*log(x^n) + 2*a))*cos(4*b*log(x^n)
+ 4*a) + 6*(4*b^8*n^8*cos(2*b*log(c)) + b^6*n^6*cos(2*b*log(c)))*cos(2*b*lo
g(x^n) + 2*a) - 2*(4*b^8*n^8*sin(6*b*log(c)) + b^6*n^6*sin(6*b*log(c)) + 3*
(4*(b^8*cos(4*b*log(c))*sin(6*b*log(c)) - b^8*cos(6*b*log(c))*sin(4*b*log(c
)))*n^8 + (b^6*cos(4*b*log(c))*sin(6*b*log(c)) - b^6*cos(6*b*log(c))*sin(4*
b*log(c)))*n^6)*cos(4*b*log(x^n) + 4*a) + 3*(4*(b^8*cos(2*b*log(c))*sin(6*b
*log(c)) - b^8*cos(6*b*log(c))*sin(2*b*log(c)))*n^8 + (b^6*cos(2*b*log(c))*
sin(6*b*log(c)) - b^6*cos(6*b*log(c))*sin(2*b*log(c)))*n^6)*cos(2*b*log(x^n
) + 2*a) - 3*(4*(b^8*cos(6*b*log(c))*cos(4*b*log(c)) + b^8*sin(6*b*log(c))*
sin(4*b*log(c)))*n^8 + (b^6*cos(6*b*log(c))*cos(4*b*log(c)) + b^6*sin(6*b*l
og(c))*sin(4*b*log(c)))*n^6)*sin(4*b*log(x^n) + 4*a) - 3*(4*(b^8*cos(6*b*lo
g(c))*cos(2*b*log(c)) + b^8*sin(6*b*log(c))*sin...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] integral(sec(b*log(c*x^n) + a)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sec^{4}\left(a + b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**4,x)
```

```
[Out] Integral(sec(a + b*log(c*x**n))**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*log(c*x^n))^4,x)
```

```
[Out] int(1/cos(a + b*log(c*x^n))^4, x)
```

3.256 $\int \frac{\sec^4(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=42

$$\frac{\tan\left(a+b\log\left(cx^n\right)\right)}{bn}+\frac{\tan^3\left(a+b\log\left(cx^n\right)\right)}{3bn}$$

```
[Out] tan(a+b*ln(c*x^n))/b/n+1/3*tan(a+b*ln(c*x^n))^3/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {3852}

$$\frac{\tan^3\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{\tan\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] Tan[a + b*Log[c*x^n]]/(b*n) + Tan[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^4\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \sec^4(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}\\
&= -\frac{\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -\tan\left(a+b\log\left(cx^n\right)\right)\right)}{bn}\\
&= \frac{\tan\left(a+b\log\left(cx^n\right)\right)}{bn}+\frac{\tan^3\left(a+b\log\left(cx^n\right)\right)}{3bn}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 36, normalized size = 0.86

$$\frac{\tan\left(a+b\log\left(cx^n\right)\right)+\frac{1}{3}\tan^3\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b*Log[c*x^n]]^4/x,x]

[Out] (Tan[a + b*Log[c*x^n]] + Tan[a + b*Log[c*x^n]]^3/3)/(b*n)

Maple [A]
time = 0.14, size = 37, normalized size = 0.88

method	result
derivativedivides	$-\frac{\left(-\frac{2}{3}-\frac{\left(\sec^{2}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}{3}\right)\tan\left(a+b\ln\left(c\,x^{n}\right)\right)}{nb}$
default	$-\frac{\left(-\frac{2}{3}-\frac{\left(\sec^{2}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}{3}\right)\tan\left(a+b\ln\left(c\,x^{n}\right)\right)}{nb}$
risch	$\frac{4i\left(3\left(x^{n}\right)^{2ib}c^{2ib}e^{b\pi\mathrm{csgn}\left(ic\,x^{n}\right)^{3}}e^{-b\pi\mathrm{csgn}\left(ic\,x^{n}\right)^{2}\mathrm{csgn}\left(ic\right)}e^{-b\pi\mathrm{csgn}\left(ic\,x^{n}\right)^{2}\mathrm{csgn}\left(ix^{n}\right)}e^{b\pi\,\mathrm{csgn}\left(ic\,x^{n}\right)\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ix^{n}\right)}e^{2ia}+1\right)}{3bn\left(\left(x^{n}\right)^{2ib}c^{2ib}e^{b\pi\mathrm{csgn}\left(ic\,x^{n}\right)^{3}}e^{-b\pi\mathrm{csgn}\left(ic\,x^{n}\right)^{2}\mathrm{csgn}\left(ic\right)}e^{-b\pi\mathrm{csgn}\left(ic\,x^{n}\right)^{2}\mathrm{csgn}\left(ix^{n}\right)}e^{b\pi\,\mathrm{csgn}\left(ic\,x^{n}\right)\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ix^{n}\right)}e^{2ia}+1\right)^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(a+b*ln(c*x^n))^4/x,x,method=_RETURNVERBOSE)

[Out] -1/n/b*(-2/3-1/3*sec(a+b*ln(c*x^n))^2)*tan(a+b*ln(c*x^n))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1323 vs. $2(40) = 80$.
time = 0.33, size = 1323, normalized size = 31.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b*log(c*x^n))^4/x,x, algorithm="maxima")

[Out] 4/3*((3*(cos(2*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) - 3*(cos(6*b*log(c))*cos(2*b*log(c)) + sin(6*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) + sin(6*b*log(c)))*cos(6*b*log(x^n) + 6*a) + 3*(3*(cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) - 3*(cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) + sin(4*b*log(c))*cos(4*b*log(x^n) + 4*a) + (3*(cos(6*b*log(c))*cos(2*b*log(c)) + sin(6*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 3*(cos(2*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) + cos(6*b*log(c)))*sin(6*b*log(x^n) + 6*a) + 3*(3*(cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 3*(cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) + cos(4*b*log(c)))*sin(4*b*log(x^n) + 4*a))/((b*cos(6*b*log(c))^2 + b*sin(6*b*log(c))^2)*n*cos(6*b*log(x^n) + 6*a)^2 + 9*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*cos(4*b*log(x^n) + 4*a)^2 + 6*b*n*cos(2*b*log(c))*

```
cos(2*b*log(x^n) + 2*a) + 9*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*c
os(2*b*log(x^n) + 2*a)^2 + (b*cos(6*b*log(c))^2 + b*sin(6*b*log(c))^2)*n*si
n(6*b*log(x^n) + 6*a)^2 + 9*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*s
in(4*b*log(x^n) + 4*a)^2 - 6*b*n*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) +
9*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*sin(2*b*log(x^n) + 2*a)^2 +
 b*n + 2*(b*n*cos(6*b*log(c)) + 3*(b*cos(6*b*log(c))*cos(4*b*log(c)) + b*si
n(6*b*log(c))*sin(4*b*log(c)))*n*cos(4*b*log(x^n) + 4*a) + 3*(b*cos(6*b*log
(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n
) + 2*a) + 3*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b
*log(c)))*n*sin(4*b*log(x^n) + 4*a) + 3*(b*cos(2*b*log(c))*sin(6*b*log(c))
- b*cos(6*b*log(c))*sin(2*b*log(c)))*n*sin(2*b*log(x^n) + 2*a))*cos(6*b*log
(x^n) + 6*a) + 6*(b*n*cos(4*b*log(c)) + 3*(b*cos(4*b*log(c))*cos(2*b*log(c)
) + b*sin(4*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n) + 2*a) + 3*(b*cos
(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*n*sin(2*b
*log(x^n) + 2*a))*cos(4*b*log(x^n) + 4*a) - 2*(3*(b*cos(4*b*log(c))*sin(6*b
*log(c)) - b*cos(6*b*log(c))*sin(4*b*log(c)))*n*cos(4*b*log(x^n) + 4*a) + 3
*(b*cos(2*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(2*b*log(c)))*n*
cos(2*b*log(x^n) + 2*a) + b*n*sin(6*b*log(c)) - 3*(b*cos(6*b*log(c))*cos(4*
b*log(c)) + b*sin(6*b*log(c))*sin(4*b*log(c)))*n*sin(4*b*log(x^n) + 4*a) -
3*(b*cos(6*b*log(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n
*sin(2*b*log(x^n) + 2*a))*sin(6*b*log(x^n) + 6*a) - 6*(3*(b*cos(2*b*log(c))
*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n) +
2*a) + b*n*sin(4*b*log(c)) - 3*(b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4
*b*log(c))*sin(2*b*log(c)))*n*sin(2*b*log(x^n) + 2*a))*sin(4*b*log(x^n) + 4
*a))
```

Fricas [A]
time = 3.18, size = 52, normalized size = 1.24

$$\frac{\left(2 \cos \left(bn \log \left(x\right) + b \log \left(c\right) + a\right)^{2} + 1\right) \sin \left(bn \log \left(x\right) + b \log \left(c\right) + a\right)}{3 \, bn \cos \left(bn \log \left(x\right) + b \log \left(c\right) + a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(b*n*log(x) + b*log(c) + a)^2 + 1)*sin(b*n*log(x) + b*log(c) + a)
/(b*n*cos(b*n*log(x) + b*log(c) + a)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}\left(a + b \log \left(cx^{n}\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Integral(sec(a + b*log(c*x**n))**4/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)^4/x, x)
```

Mupad [B]
time = 9.06, size = 49, normalized size = 1.17

$$\frac{4\left(\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}\,3\mathrm{i}+1\mathrm{i}\right)}{3\,b\,n\,{\left(\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cos(a + b*log(c*x^n))^4),x)
```

```
[Out] (4*(exp(a*2i)*(c*x^n)^(b*2i)*3i + 1i))/(3*b*n*(exp(a*2i)*(c*x^n)^(b*2i) + 1
)^3)
```

3.257 $\int \frac{\sec^4(a+b\log(cx^n))}{x^2}\,dx$

Optimal. Leaf size=87

$$-\frac{16e^{4ia}(cx^n)^{4ib}\,{}_2F_1\left(4,\frac{1}{2}\left(4+\frac{i}{bn}\right);\frac{1}{2}\left(6+\frac{i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{(1-4ibn)x}$$

```
[Out] -16*exp(4*I*a)*(c*x^n)^(4*I*b)*hypergeom([4, 2+1/2*I/b/n],[3+1/2*I/b/n],-ex
p(2*I*a)*(c*x^n)^(2*I*b))/(1-4*I*b*n)/x
```

Rubi [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4605, 4601, 371}

$$-\frac{16e^{4ia}(cx^n)^{4ib}\,{}_2F_1\left(4,\frac{1}{2}\left(4+\frac{i}{bn}\right);\frac{1}{2}\left(6+\frac{i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{x(1-4ibn)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*Log[c*x^n]]^4/x^2,x]
```

```
[Out] (-16*E^((4*I)*a)*(c*x^n)^((4*I)*b)*Hypergeometric2F1[4, (4 + I/(b*n))/2, (6
 + I/(b*n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((1 - (4*I)*b*n)*x)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4605

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^4(a+b\log(cx^n))}{x^2}\,dx &= \frac{(cx^n)^{\frac{1}{n}}\,\text{Subst}\left(\int x^{-1-\frac{1}{n}}\sec^4(a+b\log(x))\,dx, x, cx^n\right)}{nx} \\
&= \frac{\left(16e^{4ia}(cx^n)^{\frac{1}{n}}\right)\text{Subst}\left(\int \frac{x^{-1+4ib-\frac{1}{n}}}{(1+e^{2ia}x^{2ib})^4}\,dx, x, cx^n\right)}{nx} \\
&= -\frac{16e^{4ia}(cx^n)^{4ib}\,{}_2F_1\left(4,\frac{1}{2}\left(4+\frac{i}{bn}\right);\frac{1}{2}\left(6+\frac{i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{(1-4ibn)x}
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 215 vs. 2(87) = 174.
time = 10.18, size = 215, normalized size = 2.47

$$
\frac{-2e^{2ia}(-i+2bn)(cx^n)^{2ib}\,{}_2F_1\left(1,1+\frac{i}{2bn};2+\frac{i}{2bn};-e^{2i(a+b\log(cx^n))}\right)-2i(1+4b^2n^2)\,{}_2F_1\left(1,\frac{i}{2bn};1+\frac{i}{2bn};-e^{2i(a+b\log(cx^n))}\right)+\sec^2(a+b\log(cx^n))\left(2bn+(1+8b^2n^2+(1+4b^2n^2)\cos(2(a+b\log(cx^n))))\tan(a+b\log(cx^n))\right)}{12b^3n^3x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*Log[c*x^n]]^4/x^2,x]
```

```
[Out] (-2*E^((2*I)*a)*(-I + 2*b*n)*(c*x^n)^((2*I)*b)*Hypergeometric2F1[1, 1 + (I/
2)/(b*n), 2 + (I/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] - (2*I)*(1 + 4*b^
2*n^2)*Hypergeometric2F1[1, (I/2)/(b*n), 1 + (I/2)/(b*n), -E^((2*I)*(a + b*
Log[c*x^n]))] + Sec[a + b*Log[c*x^n]]^2*(2*b*n + (1 + 8*b^2*n^2 + (1 + 4*b^
2*n^2)*Cos[2*(a + b*Log[c*x^n])])*Tan[a + b*Log[c*x^n]]))/(12*b^3*n^3*x)
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$
\int \frac{\sec^4(a+b\ln(c\,x^n))}{x^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a+b*ln(c*x^n))^4/x^2,x)
```

```
[Out] int(sec(a+b*ln(c*x^n))^4/x^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^4/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*(6*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*cos(4*b*log(x^n) + 4*a
)^2 + 6*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*cos(2*b*log(x^n) + 2*
a)^2 + 6*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*sin(4*b*log(x^n) + 4
*a)^2 + 6*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*sin(2*b*log(x^n) +
2*a)^2 + (4*b^2*n^2*sin(6*b*log(c)) + (2*(b*cos(6*b*log(c))*cos(4*b*log(c))
 + b*sin(6*b*log(c))*sin(4*b*log(c)))*n + cos(4*b*log(c))*sin(6*b*log(c)) -
 cos(6*b*log(c))*sin(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) + 2*(6*(b^2*cos(2
*b*log(c))*sin(6*b*log(c)) - b^2*cos(6*b*log(c))*sin(2*b*log(c)))*n^2 + (b*
cos(6*b*log(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n + co
s(2*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c))*sin(2*b*log(c)))*cos(2*b*lo
g(x^n) + 2*a) + (2*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*s
in(4*b*log(c)))*n - cos(6*b*log(c))*cos(4*b*log(c)) - sin(6*b*log(c))*sin(4
*b*log(c)))*sin(4*b*log(x^n) + 4*a) - 2*(6*(b^2*cos(6*b*log(c))*cos(2*b*log
(c)) + b^2*sin(6*b*log(c))*sin(2*b*log(c)))*n^2 - (b*cos(2*b*log(c))*sin(6*
b*log(c)) - b*cos(6*b*log(c))*sin(2*b*log(c)))*n + cos(6*b*log(c))*cos(2*b*
log(c)) + sin(6*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) + sin(6*
b*log(c)))*cos(6*b*log(x^n) + 6*a) + (12*b^2*n^2*sin(4*b*log(c)) + 2*b*n*co
s(4*b*log(c)) + 3*(12*(b^2*cos(2*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*lo
g(c))*sin(2*b*log(c)))*n^2 + 4*(b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4
*b*log(c))*sin(2*b*log(c)))*n + cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*l
og(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) - 3*(12*(b^2*cos(4*b*log(c)
)*cos(2*b*log(c)) + b^2*sin(4*b*log(c))*sin(2*b*log(c)))*n^2 - 4*(b*cos(2*b
*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*n + cos(4*b*l
og(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n)
+ 2*a) + 2*sin(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) + (2*b*n*cos(2*b*log(c)
) + sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 9*((4*(b^8*cos(6*b*log(c))^2
 + b^8*sin(6*b*log(c))^2)*n^8 + (b^6*cos(6*b*log(c))^2 + b^6*sin(6*b*log(c)
)^2)*n^6)*x*cos(6*b*log(x^n) + 6*a)^2 + 9*(4*(b^8*cos(4*b*log(c))^2 + b^8*s
in(4*b*log(c))^2)*n^8 + (b^6*cos(4*b*log(c))^2 + b^6*sin(4*b*log(c))^2)*n^6
)*x*cos(4*b*log(x^n) + 4*a)^2 + 9*(4*(b^8*cos(2*b*log(c))^2 + b^8*sin(2*b*l
og(c))^2)*n^8 + (b^6*cos(2*b*log(c))^2 + b^6*sin(2*b*log(c))^2)*n^6)*x*cos(
2*b*log(x^n) + 2*a)^2 + (4*(b^8*cos(6*b*log(c))^2 + b^8*sin(6*b*log(c))^2)*
n^8 + (b^6*cos(6*b*log(c))^2 + b^6*sin(6*b*log(c))^2)*n^6)*x*sin(6*b*log(x^
n) + 6*a)^2 + 9*(4*(b^8*cos(4*b*log(c))^2 + b^8*sin(4*b*log(c))^2)*n^8 + (b
^6*cos(4*b*log(c))^2 + b^6*sin(4*b*log(c))^2)*n^6)*x*sin(4*b*log(x^n) + 4*a
)^2 + 9*(4*(b^8*cos(2*b*log(c))^2 + b^8*sin(2*b*log(c))^2)*n^8 + (b^6*cos(2
*b*log(c))^2 + b^6*sin(2*b*log(c))^2)*n^6)*x*sin(2*b*log(x^n) + 2*a)^2 + 6*
(4*b^8*n^8*cos(2*b*log(c)) + b^6*n^6*cos(2*b*log(c)))*x*cos(2*b*log(x^n) +
2*a) - 6*(4*b^8*n^8*sin(2*b*log(c)) + b^6*n^6*sin(2*b*log(c)))*x*sin(2*b*lo
g(x^n) + 2*a) + (4*b^8*n^8 + b^6*n^6)*x + 2*(3*(4*(b^8*cos(6*b*log(c))*cos(
4*b*log(c)) + b^8*sin(6*b*log(c))*sin(4*b*log(c)))*n^8 + (b^6*cos(6*b*log(c
))*cos(4*b*log(c)) + b^6*sin(6*b*log(c))*sin(4*b*log(c)))*n^6)*x*cos(4*b*lo
g(x^n) + 4*a) + 3*(4*(b^8*cos(6*b*log(c))*cos(2*b*log(c)) + b^8*sin(6*b*log
(c))*sin(2*b*log(c)))*n^8 + (b^6*cos(6*b*log(c))*cos(2*b*log(c)) + b^6*sin(
6*b*log(c))*sin(2*b*log(c)))*n^6)*x*cos(2*b*log(x^n) + 2*a) + 3*(4*(b^8*cos
```

```
(4*b*log(c))*sin(6*b*log(c)) - b^8*cos(6*b*log(c))*sin(4*b*log(c)))*n^8 + (
b^6*cos(4*b*log(c))*sin(6*b*log(c)) - b^6*cos(6*b*log(c))*sin(4*b*log(c)))*
n^6)*x*sin(4*b*log(x^n) + 4*a) + 3*(4*(b^8*cos(2*b*log(c))*sin(6*b*log(c))
- b^8*cos(6*b*log(c))*sin(2*b*log(c)))*n^8 + (b^6*cos(2*b*log(c))*sin(6*b*l
og(c)) - b^6*cos(6*b*log(c))*sin(2*b*log(c)))*n^6)*x*sin(2*b*log(x^n) + 2*a
) + (4*b^8*n^8*cos(6*b*log(c)) + b^6*n^6*cos(6*b*log(c)))*x)*cos(6*b*log(x^
n) + 6*a) + 6*(3*(4*(b^8*cos(4*b*log(c))*cos(2*b*log(c)) + b^8*sin(4*b*log(
c))*sin(2*b*log(c)))*n^8 + (b^6*cos(4*b*log(c))*cos(2*b*log(c)) + b^6*sin(4
*b*log(c))*sin(2*b*log(c)))*n^6)*x*cos(2*b*log(x^n) + 2*a) + 3*(4*(b^8*cos(
2*b*log(c))*sin(4*b*log(c)) - b^8*cos(4*b*log(c))*sin(2*b*log(c)))*n^8 + (b
^6*cos(2*b*log(c))*sin(4*b*log(c)) - b^6*cos(4*b*log(c))*sin(2*b*log(c)))*n
^6)*x*sin(2*b*log(x^n) + 2*a) + (4*b^8*n^8*cos(4*b*log(c)) + b^6*n^6*cos(4*
b*log(c)))*x)*cos(4*b*log(x^n) + 4*a) - 2*(3*(4*(b^8*cos(4*b*log(c))*sin(6*
b*log(c)) - b^8*cos(6*b*log(c))*sin(4*b*log(c)))*n^8 + (b^6*cos(4*b*log(c))
*sin(6*b*log(c)) - b^6*cos(6*b*log(c))*sin(4*b*log(c)))*n^6)*x*cos(4*b*log(
x^n) + 4*a) + 3*(4*(b^8*cos(2*b*log(c))*sin(6*b*log(c)) - b^8*cos(6*b*log(c
))*sin(2*b*log(c)))*n^8 + (b^6*cos(2*b*log(c))*sin(6*b*log(c)) - b^6*cos(6*
b*log(c))*sin(2*b*log(c)))*n^6)*x*cos(2*b*log(x^n) + 2*a) - 3*(4*(b^8*cos(6
*b*log(c))*cos(4*b*log(c)) + b^8*sin(6*b*log(c))*sin(4*b*log(c)))*n^8 + (b^
6*cos(6*b*log(c))*cos(4*b*log(c)) + b^6*sin(6*b*log(c))*sin(4*b*log(c)))*n^
6)*x*sin(4*b*log(x^n) + 4*a) - 3*(4*(b^8*cos(6*b*log(c))*cos(2*b*log(c)) +
b^8*sin(6*b*log(c))*sin(2*b*log(c)))*n^8 + (b^6...
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^4/x^2,x, algorithm="fricas")
```

```
[Out] integral(sec(b*log(c*x^n) + a)^4/x^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4\left(a + b\log\left(cx^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**4/x**2,x)
```

```
[Out] Integral(sec(a + b*log(c*x**n))**4/x**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^4/x^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)^4/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\cos\left(a+b\,\ln\left(c\,x^n\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*cos(a + b*log(c*x^n))^4),x)
```

```
[Out] int(1/(x^2*cos(a + b*log(c*x^n))^4), x)
```

3.258 $\int \frac{\sec^4(a+b\log(cx^n))}{x^3}\,dx$

Optimal. Leaf size=79

$$-\frac{8e^{4ia}(cx^n)^{4ib}\,{}_2F_1\left(4,2+\frac{i}{bn};3+\frac{i}{bn};-e^{2ia}(cx^n)^{2ib}\right)}{(1-2ibn)x^2}$$

```
[Out] -8*exp(4*I*a)*(c*x^n)^(4*I*b)*hypergeom([4, 2+I/b/n],[3+I/b/n],-exp(2*I*a)*
(c*x^n)^(2*I*b))/(1-2*I*b*n)/x^2
```

Rubi [A]
time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4605, 4601, 371}

$$-\frac{8e^{4ia}(cx^n)^{4ib}\,{}_2F_1\left(4,2+\frac{i}{bn};3+\frac{i}{bn};-e^{2ia}(cx^n)^{2ib}\right)}{x^2(1-2ibn)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*Log[c*x^n]]^4/x^3,x]
```

```
[Out] (-8*E^((4*I)*a)*(c*x^n)^((4*I)*b)*Hypergeometric2F1[4, 2 + I/(b*n), 3 + I/(
b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((1 - (2*I)*b*n)*x^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4605

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\sec^4(a+b\log(cx^n))}{x^3}\,dx = \frac{(cx^n)^{2/n}\,\text{Subst}\left(\int x^{-1-\frac{2}{n}}\sec^4(a+b\log(x))\,dx, x, cx^n\right)}{nx^2}$$
$$= \frac{\left(16e^{4ia}(cx^n)^{2/n}\right)\text{Subst}\left(\int \frac{x^{-1+4ib-\frac{2}{n}}}{(1+e^{2ia}x^{2ib})^4}\,dx, x, cx^n\right)}{nx^2}$$
$$= -\frac{8e^{4ia}(cx^n)^{4ib}\,{}_2F_1\left(4, 2+\frac{i}{bn}; 3+\frac{i}{bn}; -e^{2ia}(cx^n)^{2ib}\right)}{(1-2ibn)x^2}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 203 vs. $2(79) = 158$.
time = 10.21, size = 203, normalized size = 2.57

$$\frac{-2e^{2ia}(-i+bn)(cx^n)^{2ib}\,{}_2F_1\left(1,1+\frac{i}{bn};2+\frac{i}{bn};-e^{2i(a+b\log(cx^n))}\right)-2i(1+b^2n^2)\,{}_2F_1\left(1,\frac{i}{bn};1+\frac{i}{bn};-e^{2i(a+b\log(cx^n))}\right)+\sec^2(a+b\log(cx^n))\left(bn+\left(1+2b^2n^2+(1+b^2n^2)\cos(2(a+b\log(cx^n)))\right)\tan(a+b\log(cx^n))\right)}{3b^3n^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*Log[c*x^n]]^4/x^3,x]
```

```
[Out] (-2*E^((2*I)*a)*(-I + b*n)*(c*x^n)^((2*I)*b)*Hypergeometric2F1[1, 1 + I/(b*
n), 2 + I/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] - (2*I)*(1 + b^2*n^2)*Hyper
geometric2F1[1, I/(b*n), 1 + I/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] + Sec[
a + b*Log[c*x^n]]^2*(b*n + (1 + 2*b^2*n^2 + (1 + b^2*n^2)*Cos[2*(a + b*Log[
c*x^n])])*Tan[a + b*Log[c*x^n]]))/(3*b^3*n^3*x^2)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(a+b\ln(c\,x^n))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a+b*ln(c*x^n))^4/x^3,x)
```

```
[Out] int(sec(a+b*ln(c*x^n))^4/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^4/x^3,x, algorithm="maxima")
```

```
[Out] 4/3*(3*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*cos(4*b*log(x^n) + 4*a
)^2 + 3*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*cos(2*b*log(x^n) + 2*
a)^2 + 3*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*sin(4*b*log(x^n) + 4
*a)^2 + 3*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*sin(2*b*log(x^n) +
2*a)^2 + (b^2*n^2*sin(6*b*log(c)) + ((b*cos(6*b*log(c))*cos(4*b*log(c)) + b
*sin(6*b*log(c))*sin(4*b*log(c)))*n + cos(4*b*log(c))*sin(6*b*log(c)) - cos
(6*b*log(c))*sin(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) + (3*(b^2*cos(2*b*log
(c))*sin(6*b*log(c)) - b^2*cos(6*b*log(c))*sin(2*b*log(c)))*n^2 + (b*cos(6*
b*log(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n + 2*cos(2*
b*log(c))*sin(6*b*log(c)) - 2*cos(6*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(
x^n) + 2*a) + ((b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4
*b*log(c)))*n - cos(6*b*log(c))*cos(4*b*log(c)) - sin(6*b*log(c))*sin(4*b*l
og(c)))*sin(4*b*log(x^n) + 4*a) - (3*(b^2*cos(6*b*log(c))*cos(2*b*log(c)) +
 b^2*sin(6*b*log(c))*sin(2*b*log(c)))*n^2 - (b*cos(2*b*log(c))*sin(6*b*log(
c)) - b*cos(6*b*log(c))*sin(2*b*log(c)))*n + 2*cos(6*b*log(c))*cos(2*b*log(
c)) + 2*sin(6*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) + sin(6*b*
log(c)))*cos(6*b*log(x^n) + 6*a) + (3*b^2*n^2*sin(4*b*log(c)) + b*n*cos(4*b
*log(c)) + 3*(3*(b^2*cos(2*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*
sin(2*b*log(c)))*n^2 + 2*(b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log
(c))*sin(2*b*log(c)))*n + cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))
*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) - 3*(3*(b^2*cos(4*b*log(c))*cos(2
*b*log(c)) + b^2*sin(4*b*log(c))*sin(2*b*log(c)))*n^2 - 2*(b*cos(2*b*log(c)
)*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*n + cos(4*b*log(c))*
cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a)
+ 2*sin(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) + (b*n*cos(2*b*log(c)) + sin(2
*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 18*(((b^8*cos(6*b*log(c))^2 + b^8*sin
(6*b*log(c))^2)*n^8 + (b^6*cos(6*b*log(c))^2 + b^6*sin(6*b*log(c))^2)*n^6)*
x^2*cos(6*b*log(x^n) + 6*a)^2 + 9*((b^8*cos(4*b*log(c))^2 + b^8*sin(4*b*log
(c))^2)*n^8 + (b^6*cos(4*b*log(c))^2 + b^6*sin(4*b*log(c))^2)*n^6)*x^2*cos(
4*b*log(x^n) + 4*a)^2 + 9*((b^8*cos(2*b*log(c))^2 + b^8*sin(2*b*log(c))^2)*
n^8 + (b^6*cos(2*b*log(c))^2 + b^6*sin(2*b*log(c))^2)*n^6)*x^2*cos(2*b*log(
x^n) + 2*a)^2 + ((b^8*cos(6*b*log(c))^2 + b^8*sin(6*b*log(c))^2)*n^8 + (b^6
*cos(6*b*log(c))^2 + b^6*sin(6*b*log(c))^2)*n^6)*x^2*sin(6*b*log(x^n) + 6*a
)^2 + 9*((b^8*cos(4*b*log(c))^2 + b^8*sin(4*b*log(c))^2)*n^8 + (b^6*cos(4*b
*log(c))^2 + b^6*sin(4*b*log(c))^2)*n^6)*x^2*sin(4*b*log(x^n) + 4*a)^2 + 9*
((b^8*cos(2*b*log(c))^2 + b^8*sin(2*b*log(c))^2)*n^8 + (b^6*cos(2*b*log(c))
^2 + b^6*sin(2*b*log(c))^2)*n^6)*x^2*sin(2*b*log(x^n) + 2*a)^2 + 6*(b^8*n^8
*cos(2*b*log(c)) + b^6*n^6*cos(2*b*log(c)))*x^2*cos(2*b*log(x^n) + 2*a) - 6
*(b^8*n^8*sin(2*b*log(c)) + b^6*n^6*sin(2*b*log(c)))*x^2*sin(2*b*log(x^n) +
 2*a) + (b^8*n^8 + b^6*n^6)*x^2 + 2*(3*((b^8*cos(6*b*log(c))*cos(4*b*log(c)
) + b^8*sin(6*b*log(c))*sin(4*b*log(c)))*n^8 + (b^6*cos(6*b*log(c))*cos(4*b
*log(c)) + b^6*sin(6*b*log(c))*sin(4*b*log(c)))*n^6)*x^2*cos(4*b*log(x^n) +
 4*a) + 3*((b^8*cos(6*b*log(c))*cos(2*b*log(c)) + b^8*sin(6*b*log(c))*sin(2
*b*log(c)))*n^8 + (b^6*cos(6*b*log(c))*cos(2*b*log(c)) + b^6*sin(6*b*log(c)
)*sin(2*b*log(c)))*n^6)*x^2*cos(2*b*log(x^n) + 2*a) + 3*((b^8*cos(4*b*log(c
```

```
))*sin(6*b*log(c)) - b^8*cos(6*b*log(c))*sin(4*b*log(c)))*n^8 + (b^6*cos(4*
b*log(c))*sin(6*b*log(c)) - b^6*cos(6*b*log(c))*sin(4*b*log(c)))*n^6)*x^2*s
in(4*b*log(x^n) + 4*a) + 3*((b^8*cos(2*b*log(c))*sin(6*b*log(c)) - b^8*cos(
6*b*log(c))*sin(2*b*log(c)))*n^8 + (b^6*cos(2*b*log(c))*sin(6*b*log(c)) - b
^6*cos(6*b*log(c))*sin(2*b*log(c)))*n^6)*x^2*sin(2*b*log(x^n) + 2*a) + (b^8
*n^8*cos(6*b*log(c)) + b^6*n^6*cos(6*b*log(c)))*x^2)*cos(6*b*log(x^n) + 6*a
) + 6*(3*((b^8*cos(4*b*log(c))*cos(2*b*log(c)) + b^8*sin(4*b*log(c))*sin(2*
b*log(c)))*n^8 + (b^6*cos(4*b*log(c))*cos(2*b*log(c)) + b^6*sin(4*b*log(c))
*sin(2*b*log(c)))*n^6)*x^2*cos(2*b*log(x^n) + 2*a) + 3*((b^8*cos(2*b*log(c)
)*sin(4*b*log(c)) - b^8*cos(4*b*log(c))*sin(2*b*log(c)))*n^8 + (b^6*cos(2*b
*log(c))*sin(4*b*log(c)) - b^6*cos(4*b*log(c))*sin(2*b*log(c)))*n^6)*x^2*si
n(2*b*log(x^n) + 2*a) + (b^8*n^8*cos(4*b*log(c)) + b^6*n^6*cos(4*b*log(c)))
*x^2)*cos(4*b*log(x^n) + 4*a) - 2*(3*((b^8*cos(4*b*log(c))*sin(6*b*log(c))
- b^8*cos(6*b*log(c))*sin(4*b*log(c)))*n^8 + (b^6*cos(4*b*log(c))*sin(6*b*l
og(c)) - b^6*cos(6*b*log(c))*sin(4*b*log(c)))*n^6)*x^2*cos(4*b*log(x^n) + 4
*a) + 3*((b^8*cos(2*b*log(c))*sin(6*b*log(c)) - b^8*cos(6*b*log(c))*sin(2*b
*log(c)))*n^8 + (b^6*cos(2*b*log(c))*sin(6*b*log(c)) - b^6*cos(6*b*log(c))*
sin(2*b*log(c)))*n^6)*x^2*cos(2*b*log(x^n) + 2*a) - 3*((b^8*cos(6*b*log(c))
*cos(4*b*log(c)) + b^8*sin(6*b*log(c))*sin(4*b*log(c)))*n^8 + (b^6*cos(6*b*
log(c))*cos(4*b*log(c)) + b^6*sin(6*b*log(c))*sin(4*b*log(c)))*n^6)*x^2*sin
(4*b*log(x^n) + 4*a) - 3*((b^8*cos(6*b*log(c))*cos(2*b*log(c)) + b^8*sin(6*
b*log(c))*sin(2*b*log(c)))*n^8 + (b^6*cos(6*b*l...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^4/x^3,x, algorithm="fricas")
```

```
[Out] integral(sec(b*log(c*x^n) + a)^4/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(a + b\log(cx^n))}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**4/x**3,x)
```

```
[Out] Integral(sec(a + b*log(c*x**n))**4/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^4/x^3,x, algorithm="giac")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)^4/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\cos\left(a+b\,\ln\left(c\,x^n\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*cos(a + b*log(c*x^n))^4),x)
```

```
[Out] int(1/(x^3*cos(a + b*log(c*x^n))^4), x)
```

3.259 $\int \left(-\left(\left(1+b^2n^2\right)\sec\left(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\sec^3\left(a-\right.\right.$

Optimal. Leaf size=41

$$-x\sec\left(a+b\log\left(cx^n\right)\right)+bnx\sec\left(a+b\log\left(cx^n\right)\right)\tan\left(a+b\log\left(cx^n\right)\right)$$

```
[Out] -x*sec(a+b*ln(c*x^n))+b*n*x*sec(a+b*ln(c*x^n))*tan(a+b*ln(c*x^n))
```

Rubi [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.10, antiderivative size = 175, normalized size of antiderivative = 4.27, number of steps used = 7, number of rules used = 3, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}=0.068$, Rules used = {4599, 4601, 371}

$$\frac{16e^{3ia}b^2n^2x\left(cx^n\right)^{3ib}\,{}_2F_1\left(3,\frac{1}{2}\left(3-\frac{i}{bn}\right);\frac{1}{2}\left(5-\frac{i}{bn}\right);-e^{2ia}\left(cx^n\right)^{2ib}\right)}{1+3ibn}-2e^{ia}x(1-ibn)\left(cx^n\right)^{ib}\,{}_2F_1\left(1,\frac{1}{2}\left(1-\frac{i}{bn}\right);\frac{1}{2}\left(3-\frac{i}{bn}\right);-e^{2ia}\left(cx^n\right)^{2ib}\right)$$

Antiderivative was successfully verified.

```
[In] Int[-((1 + b^2*n^2)*Sec[a + b*Log[c*x^n]]) + 2*b^2*n^2*Sec[a + b*Log[c*x^n]
]^3,x]
```

```
[Out] -2*E^(I*a)*(1 - I*b*n)*x*(c*x^n)^(I*b)*Hypergeometric2F1[1, (1 - I/(b*n))/2
, (3 - I/(b*n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))] + (16*b^2*E^((3*I)*a)*
n^2*x*(c*x^n)^((3*I)*b)*Hypergeometric2F1[3, (3 - I/(b*n))/2, (5 - I/(b*n))
/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(1 + (3*I)*b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4599

```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \left(-\left(1+b^2n^2\right) \sec \left(a+b \log \left(cx^n\right)\right)+2b^2n^2 \sec^3 \left(a+b \log \left(cx^n\right)\right)\right) \, dx = \left(2b^2n^2\right) \int \sec^3 \left(a+b \log \left(cx^n\right)\right) \, dx$$

$$= \left(2b^2nx\left(cx^n\right)^{-1/n}\right) \text{Subst}\left(\int x^{-1+}\right.$$

$$= \left(16b^2e^{3ia}nx\left(cx^n\right)^{-1/n}\right) \text{Subst}\left(\int\right.$$

$$= -2e^{ia}(1-ibn)x\left(cx^n\right)^{ib} \, {}_2F_1\left(1, \frac{1}{2}\right.$$

Mathematica [A]
time = 1.67, size = 29, normalized size = 0.71

$$x \sec \left(a+b \log \left(cx^n\right)\right) \left(-1+bn \tan \left(a+b \log \left(cx^n\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-((1 + b^2*n^2)*Sec[a + b*Log[c*x^n]]) + 2*b^2*n^2*Sec[a + b*Log[
c*x^n]]^3,x]
```

```
[Out] x*Sec[a + b*Log[c*x^n]]*(-1 + b*n*Tan[a + b*Log[c*x^n]])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.70, size = 525, normalized size = 12.80

method	result
risch	$-\frac{2i\left(x^n\right)^{ib}c^{ib}x\left(nb\,c^{2ib}\left(x^n\right)^{2ib}\mathrm{e}^{\frac{3b\pi\mathrm{csgn}\left(ic\,x^n\right)^3}{2}}\mathrm{e}^{-\frac{3b\pi\mathrm{csgn}\left(ic\,x^n\right)^2\mathrm{csgn}\left(ic\right)}{2}}\mathrm{e}^{-\frac{3b\pi\mathrm{csgn}\left(ic\,x^n\right)^2\mathrm{csgn}\left(ix^n\right)}{2}}\mathrm{e}^{\frac{3b\pi\,\mathrm{csgn}\left(ic\,x^n\right)\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ix^n\right)}{2}}\mathrm{e}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^2*n^2+1)*sec(a+b*ln(c*x^n))+2*b^2*n^2*sec(a+b*ln(c*x^n))^3,x,method
=_RETURNVERBOSE)
```

```
[Out] -2*I*(x^n)^(I*b)*c^(I*b)*x/(((x^n)^(I*b))^2*(c^(I*b))^2*exp(b*Pi*csgn(I*c*x
^n)^3)*exp(-b*Pi*csgn(I*c*x^n)^2*csgn(I*c))*exp(-b*Pi*csgn(I*c*x^n)^2*csgn(
I*x^n))*exp(b*Pi*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n))*exp(2*I*a)+1)^2*(n*b*
(c^(I*b))^2*((x^n)^(I*b))^2*exp(3/2*b*Pi*csgn(I*c*x^n)^3)*exp(-3/2*b*Pi*csg
n(I*c*x^n)^2*csgn(I*c))*exp(-3/2*b*Pi*csgn(I*c*x^n)^2*csgn(I*x^n))*exp(3/2*
b*Pi*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n))*exp(3*I*a)-b*n*exp(1/2*b*Pi*csgn(
I*c*x^n)^3)*exp(-1/2*b*Pi*csgn(I*c*x^n)^2*csgn(I*c))*exp(-1/2*b*Pi*csgn(I*c
```

```
*x^n)^2*csgn(I*x^n))*exp(1/2*b*Pi*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n))*exp(
I*a)-I*((x^n)^(I*b))^2*(c^(I*b))^2*exp(3/2*b*Pi*csgn(I*c*x^n)^3)*exp(-3/2*b
*Pi*csgn(I*c*x^n)^2*csgn(I*c))*exp(-3/2*b*Pi*csgn(I*c*x^n)^2*csgn(I*x^n))*e
xp(3/2*b*Pi*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n))*exp(3*I*a)-I*exp(1/2*b*Pi*
csgn(I*c*x^n)^3)*exp(-1/2*b*Pi*csgn(I*c*x^n)^2*csgn(I*c))*exp(-1/2*b*Pi*csg
n(I*c*x^n)^2*csgn(I*x^n))*exp(1/2*b*Pi*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n))
*exp(I*a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1696 vs. $2(41) = 82$.
time = 0.67, size = 1696, normalized size = 41.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^2*n^2+1)*sec(a+b*log(c*x^n))+2*b^2*n^2*sec(a+b*log(c*x^n))^3,
x, algorithm="maxima")
```

```
[Out] -2*((b*n*sin(b*log(c)) + cos(b*log(c)))*x*cos(b*log(x^n) + a) + (b*n*cos(b*
log(c)) - sin(b*log(c)))*x*sin(b*log(x^n) + a) + (((b*cos(3*b*log(c))*sin(4
*b*log(c)) - b*cos(4*b*log(c))*sin(3*b*log(c)))*n + cos(4*b*log(c))*cos(3*b
*log(c)) + sin(4*b*log(c))*sin(3*b*log(c)))*x*cos(3*b*log(x^n) + 3*a) - ((b
*cos(b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(b*log(c)))*n - cos(4
*b*log(c))*cos(b*log(c)) - sin(4*b*log(c))*sin(b*log(c)))*x*cos(b*log(x^n)
+ a) - ((b*cos(4*b*log(c))*cos(3*b*log(c)) + b*sin(4*b*log(c))*sin(3*b*log(
c)))*n - cos(3*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c))*sin(3*b*log(c)))
*x*sin(3*b*log(x^n) + 3*a) + ((b*cos(4*b*log(c))*cos(b*log(c)) + b*sin(4*b*
log(c))*sin(b*log(c)))*n + cos(b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*
sin(b*log(c)))*x*sin(b*log(x^n) + a))*cos(4*b*log(x^n) + 4*a) - (2*((b*cos(
2*b*log(c))*sin(3*b*log(c)) - b*cos(3*b*log(c))*sin(2*b*log(c)))*n - cos(3*
b*log(c))*cos(2*b*log(c)) - sin(3*b*log(c))*sin(2*b*log(c)))*x*cos(2*b*log(
x^n) + 2*a) - 2*((b*cos(3*b*log(c))*cos(2*b*log(c)) + b*sin(3*b*log(c))*sin
(2*b*log(c)))*n + cos(2*b*log(c))*sin(3*b*log(c)) - cos(3*b*log(c))*sin(2*b
*log(c)))*x*sin(2*b*log(x^n) + 2*a) + (b*n*sin(3*b*log(c)) - cos(3*b*log(c)
))*x)*cos(3*b*log(x^n) + 3*a) - 2*(((b*cos(b*log(c))*sin(2*b*log(c)) - b*co
s(2*b*log(c))*sin(b*log(c)))*n - cos(2*b*log(c))*cos(b*log(c)) - sin(2*b*lo
g(c))*sin(b*log(c)))*x*cos(b*log(x^n) + a) - ((b*cos(2*b*log(c))*cos(b*log(
c)) + b*sin(2*b*log(c))*sin(b*log(c)))*n + cos(b*log(c))*sin(2*b*log(c)) -
cos(2*b*log(c))*sin(b*log(c)))*x*sin(b*log(x^n) + a))*cos(2*b*log(x^n) + 2*
a) + (((b*cos(4*b*log(c))*cos(3*b*log(c)) + b*sin(4*b*log(c))*sin(3*b*log(c
)))*n - cos(3*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c))*sin(3*b*log(c)))*
x*cos(3*b*log(x^n) + 3*a) - ((b*cos(4*b*log(c))*cos(b*log(c)) + b*sin(4*b*l
og(c))*sin(b*log(c)))*n + cos(b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*s
in(b*log(c)))*x*cos(b*log(x^n) + a) + ((b*cos(3*b*log(c))*sin(4*b*log(c)) -
 b*cos(4*b*log(c))*sin(3*b*log(c)))*n + cos(4*b*log(c))*cos(3*b*log(c)) + s
```

```
in(4*b*log(c))*sin(3*b*log(c)))*x*sin(3*b*log(x^n) + 3*a) - ((b*cos(b*log(c
))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(b*log(c)))*n - cos(4*b*log(c))*c
os(b*log(c)) - sin(4*b*log(c))*sin(b*log(c)))*x*sin(b*log(x^n) + a))*sin(4*
b*log(x^n) + 4*a) - (2*((b*cos(3*b*log(c))*cos(2*b*log(c)) + b*sin(3*b*log(
c))*sin(2*b*log(c)))*n + cos(2*b*log(c))*sin(3*b*log(c)) - cos(3*b*log(c))*
sin(2*b*log(c)))*x*cos(2*b*log(x^n) + 2*a) + 2*((b*cos(2*b*log(c))*sin(3*b*
log(c)) - b*cos(3*b*log(c))*sin(2*b*log(c)))*n - cos(3*b*log(c))*cos(2*b*lo
g(c)) - sin(3*b*log(c))*sin(2*b*log(c)))*x*sin(2*b*log(x^n) + 2*a) + (b*n*c
os(3*b*log(c)) + sin(3*b*log(c)))*x)*sin(3*b*log(x^n) + 3*a) - 2*(((b*cos(2
*b*log(c))*cos(b*log(c)) + b*sin(2*b*log(c))*sin(b*log(c)))*n + cos(b*log(c
))*sin(2*b*log(c)) - cos(2*b*log(c))*sin(b*log(c)))*x*cos(b*log(x^n) + a) +
 ((b*cos(b*log(c))*sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)))*n - c
os(2*b*log(c))*cos(b*log(c)) - sin(2*b*log(c))*sin(b*log(c)))*x*sin(b*log(x
^n) + a))*sin(2*b*log(x^n) + 2*a))/((cos(4*b*log(c))^2 + sin(4*b*log(c))^2)
*cos(4*b*log(x^n) + 4*a)^2 + 4*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*cos(
2*b*log(x^n) + 2*a)^2 + (cos(4*b*log(c))^2 + sin(4*b*log(c))^2)*sin(4*b*log
(x^n) + 4*a)^2 + 4*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*sin(2*b*log(x^n)
 + 2*a)^2 + 2*(2*(cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b
*log(c)))*cos(2*b*log(x^n) + 2*a) + 2*(cos(2*b*log(c))*sin(4*b*log(c)) - co
s(4*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) + cos(4*b*log(c)))*c
os(4*b*log(x^n) + 4*a) + 4*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) - 2*(2*(
cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)))*cos(2*b*
log(x^n) + 2*a) - 2*(cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(
2*b*log(c)))*sin(2*b*log(x^n) + 2*a) + sin(4*b*log(c)))*sin(4*b*log(x^n) +
4*a) - 4*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + 1)
```

Fricas [A]
time = 3.29, size = 47, normalized size = 1.15

$$\frac{bnx\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^2*n^2+1)*sec(a+b*log(c*x^n))+2*b^2*n^2*sec(a+b*log(c*x^n))^3,
x, algorithm="fricas")
```

```
[Out] (b*n*x*sin(b*n*log(x) + b*log(c) + a) - x*cos(b*n*log(x) + b*log(c) + a))/c
os(b*n*log(x) + b*log(c) + a)^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(2b^{2}n^{2}\sec^{2}\left(a+b\log\left(cx^{n}\right)\right)-b^{2}n^{2}-1\right)\sec\left(a+b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b**2*n**2+1)*sec(a+b*ln(c*x**n))+2*b**2*n**2*sec(a+b*ln(c*x**n)
)**3,x)
```

```
[Out] Integral((2*b**2*n**2*sec(a + b*log(c*x**n))**2 - b**2*n**2 - 1)*sec(a + b*
log(c*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^2*n^2+1)*sec(a+b*log(c*x^n))+2*b^2*n^2*sec(a+b*log(c*x^n))^3,
x, algorithm="giac")
```

```
[Out] integrate(2*b^2*n^2*sec(b*log(c*x^n) + a)^3 - (b^2*n^2 + 1)*sec(b*log(c*x^n
) + a), x)
```

Mupad [B]
time = 3.42, size = 87, normalized size = 2.12

$$\frac{2\,x\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,1\mathrm{i}}\,\left(-1+b\,n\,1\mathrm{i}\right)-2\,x\,\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}\,\left(1+b\,n\,1\mathrm{i}\right)}{{\left(\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*b^2*n^2)/cos(a + b*log(c*x^n))^3 - (b^2*n^2 + 1)/cos(a + b*log(c*x^n
)),x)
```

```
[Out] (2*x*exp(a*1i)*(c*x^n)^(b*1i)*(b*n*1i - 1) - 2*x*exp(a*1i)*exp(a*2i)*(c*x^n
)^(b*1i)*(c*x^n)^(b*2i)*(b*n*1i + 1))/(exp(a*2i)*(c*x^n)^(b*2i) + 1)^2
```

3.260 $\int x^m \sec^3\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\,dx$

Optimal. Leaf size=110

$$\frac{x^{1+m}\sec\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)}{2(1+m)}+\frac{x^{1+m}\sec\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\tan\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)}{2\sqrt{-(1+m)^2}}$$

```
[Out] 1/2*x^(1+m)*sec(a+2*ln(c*x^(1/2*(-(1+m)^2)^(1/2))))/(1+m)+1/2*x^(1+m)*sec(a
+2*ln(c*x^(1/2*(-(1+m)^2)^(1/2))))*tan(a+2*ln(c*x^(1/2*(-(1+m)^2)^(1/2))))/
(-(1+m)^2)^(1/2)
```

Rubi [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.14, antiderivative size = 146, normalized size of antiderivative = 1.33, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {4605, 4601, 371}

$$\frac{8e^{3ia}x^{m+1}\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)^{6i}{}_2F_1\left(3,\frac{1}{2}\left(3-\frac{i(m+1)}{\sqrt{-(m+1)^2}}\right);\frac{1}{2}\left(5-\frac{i(m+1)}{\sqrt{-(m+1)^2}}\right);-e^{2ia}\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)^{4i}\right)}{1-i\left(-3\sqrt{-(m+1)^2}+im\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^m*Sec[a + 2*Log[c*x^(Sqrt[-(1 + m)^2]/2)]]^3,x]
```

```
[Out] (8*E^((3*I)*a)*x^(1 + m)*(c*x^(Sqrt[-(1 + m)^2]/2))^(6*I)*Hypergeometric2F1
[3, (3 - (I*(1 + m))/Sqrt[-(1 + m)^2])/2, (5 - (I*(1 + m))/Sqrt[-(1 + m)^2]
)/2, -(E^((2*I)*a)*(c*x^(Sqrt[-(1 + m)^2]/2))^(4*I))])/(1 - I*(I*m - 3*Sqrt
[-(1 + m)^2]))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4605

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
```

```
((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^m \sec^3\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)dx = \frac{\left(2x^{1+m}\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)^{-\frac{2(1+m)}{\sqrt{-(1+m)^2}}}\right)\text{Subst}\left(\int x^{-1+\frac{2(1+m)}{\sqrt{-(1+m)^2}}}\right.}{\sqrt{-(1+m)^2}}$$

$$= \frac{\left(16e^{3ia}x^{1+m}\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)^{-\frac{2(1+m)}{\sqrt{-(1+m)^2}}}\right)\text{Subst}\left(\int \frac{x^{(-1+6i)+\frac{2}{\sqrt{-}}}}{(1+e^{2ia}x}\right.}{\sqrt{-(1+m)^2}}$$

$$= \frac{8e^{3ia}x^{1+m}\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)^{6i}\,{}_2F_1\left(3,\frac{1}{2}\left(3-\frac{i(1+m)}{\sqrt{-(1+m)^2}}\right);\frac{1}{2}\left(\right.\right.}{1-i\left(im-3\sqrt{-(1+\right.}}$$

Mathematica [A]
time = 2.29, size = 198, normalized size = 1.80

$$\frac{x^{1+m}\left((1+m)\cos\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)-\sqrt{-(1+m)^2}\sin\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\right)}{2(1+m)^2\left(\cos\left(\frac{a}{2}+\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)-\sin\left(\frac{a}{2}+\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\right)^2\left(\cos\left(\frac{a}{2}+\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)+\sin\left(\frac{a}{2}+\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sec[a + 2*Log[c*x^(Sqrt[-(1 + m)^2]/2)]]^3,x]
```

```
[Out] (x^(1 + m)*((1 + m)*Cos[a + 2*Log[c*x^(Sqrt[-(1 + m)^2]/2)]] - Sqrt[-(1 + m
)^2]*Sin[a + 2*Log[c*x^(Sqrt[-(1 + m)^2]/2)]]))/(2*(1 + m)^2*(Cos[a/2 + Log
[c*x^(Sqrt[-(1 + m)^2]/2)]] - Sin[a/2 + Log[c*x^(Sqrt[-(1 + m)^2]/2)]])^2*(
Cos[a/2 + Log[c*x^(Sqrt[-(1 + m)^2]/2)]] + Sin[a/2 + Log[c*x^(Sqrt[-(1 + m)
^2]/2)]])^2)
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int x^m\left(\sec^3\left(a+2\ln\left(c\,x^{\frac{\sqrt{-(1+m)^2}}{2}}\right)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sec(a+2*ln(c*x^(1/2*(-(1+m)^2)^(1/2))))^3,x)
```

```
[Out] int(x^m*sec(a+2*ln(c*x^(1/2*(-(1+m)^2)^(1/2))))^3,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 976 vs. $2(92) = 184$.
time = 0.37, size = 976, normalized size = 8.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+2*log(c*x^(1/2*(-(1+m)^2)^(1/2))))^3,x, algorithm="maxi
ma")
```

```
[Out] 2*((cos(a)*cos(2*log(c)) - sin(a)*sin(2*log(c)))*x*e^(m*log(x) + 14*arctan2
(sin(1/2*m*log(x)), cos(1/2*m*log(x))) + 14*arctan2(sin(1/2*log(x)), cos(1/
2*log(x)))) + 2*(((cos(2*a)*cos(a) + sin(2*a)*sin(a))*cos(2*log(c)) + (cos(
a)*sin(2*a) - cos(2*a)*sin(a))*sin(2*log(c)))*cos(4*log(c)) - ((cos(a)*sin(
2*a) - cos(2*a)*sin(a))*cos(2*log(c)) - (cos(2*a)*cos(a) + sin(2*a)*sin(a))
*sin(2*log(c)))*sin(4*log(c)))*x*e^(m*log(x) + 10*arctan2(sin(1/2*m*log(x))
, cos(1/2*m*log(x))) + 10*arctan2(sin(1/2*log(x)), cos(1/2*log(x)))) + (((c
os(4*a)*cos(a) + sin(4*a)*sin(a))*cos(2*log(c)) + (cos(a)*sin(4*a) - cos(4*
a)*sin(a))*sin(2*log(c)))*cos(8*log(c)) - ((cos(a)*sin(4*a) - cos(4*a)*sin(
a))*cos(2*log(c)) - (cos(4*a)*cos(a) + sin(4*a)*sin(a))*sin(2*log(c)))*sin(
8*log(c)))*x*e^(m*log(x) + 6*arctan2(sin(1/2*m*log(x)), cos(1/2*m*log(x)))
+ 6*arctan2(sin(1/2*log(x)), cos(1/2*log(x)))))/((cos(4*a)^2 + sin(4*a)^2)*
cos(8*log(c))^2 + (cos(4*a)^2 + sin(4*a)^2)*sin(8*log(c))^2 + ((cos(4*a)^2
+ sin(4*a)^2)*cos(8*log(c))^2 + (cos(4*a)^2 + sin(4*a)^2)*sin(8*log(c))^2)*
m + (m + 1)*e^(16*arctan2(sin(1/2*m*log(x)), cos(1/2*m*log(x))) + 16*arctan
2(sin(1/2*log(x)), cos(1/2*log(x)))) + 4*((cos(2*a)*cos(4*log(c)) - sin(2*a
)*sin(4*log(c)))*m + cos(2*a)*cos(4*log(c)) - sin(2*a)*sin(4*log(c)))*e^(12
*arctan2(sin(1/2*m*log(x)), cos(1/2*m*log(x))) + 12*arctan2(sin(1/2*log(x))
, cos(1/2*log(x)))) + 2*(2*(cos(2*a)^2 + sin(2*a)^2)*cos(4*log(c))^2 + 2*(c
os(2*a)^2 + sin(2*a)^2)*sin(4*log(c))^2 + (2*(cos(2*a)^2 + sin(2*a)^2)*cos(
4*log(c))^2 + 2*(cos(2*a)^2 + sin(2*a)^2)*sin(4*log(c))^2 + cos(4*a)*cos(8*
log(c)) - sin(4*a)*sin(8*log(c)))*m + cos(4*a)*cos(8*log(c)) - sin(4*a)*sin
(8*log(c)))*e^(8*arctan2(sin(1/2*m*log(x)), cos(1/2*m*log(x))) + 8*arctan2(
sin(1/2*log(x)), cos(1/2*log(x)))) + 4*((((cos(4*a)*cos(2*a) + sin(4*a)*sin
(2*a))*cos(4*log(c)) + (cos(2*a)*sin(4*a) - cos(4*a)*sin(2*a))*sin(4*log(c)
))*cos(8*log(c)) - ((cos(2*a)*sin(4*a) - cos(4*a)*sin(2*a))*cos(4*log(c)) -
 (cos(4*a)*cos(2*a) + sin(4*a)*sin(2*a))*sin(4*log(c)))*sin(8*log(c)))*m +
((cos(4*a)*cos(2*a) + sin(4*a)*sin(2*a))*cos(4*log(c)) + (cos(2*a)*sin(4*a)
 - cos(4*a)*sin(2*a))*sin(4*log(c)))*cos(8*log(c)) - ((cos(2*a)*sin(4*a) -
cos(4*a)*sin(2*a))*cos(4*log(c)) - (cos(4*a)*cos(2*a) + sin(4*a)*sin(2*a))*
sin(4*log(c)))*sin(8*log(c)))*e^(4*arctan2(sin(1/2*m*log(x)), cos(1/2*m*log
(x))) + 4*arctan2(sin(1/2*log(x)), cos(1/2*log(x)))))
```

Fricas [C] Result contains complex when optimal does not.

time = 2.07, size = 81, normalized size = 0.74

$$-\frac{2\left(2\,x^{2}x^{2\,m}e^{\left(3i\,a+6i\,\log\left(c\right)\right)}+e^{\left(5i\,a+10i\,\log\left(c\right)\right)}\right)}{(m+1)x^{4}x^{4\,m}+2\,(m+1)x^{2}x^{2\,m}e^{\left(2i\,a+4i\,\log\left(c\right)\right)}+(m+1)e^{\left(4i\,a+8i\,\log\left(c\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+2*log(c*x^(1/2*(-(1+m)^2)^(1/2))))^3,x, algorithm="fric
as")
```

```
[Out] -2*(2*x^2*x^(2*m)*e^(3*I*a + 6*I*log(c)) + e^(5*I*a + 10*I*log(c)))/((m + 1
)*x^4*x^(4*m) + 2*(m + 1)*x^2*x^(2*m)*e^(2*I*a + 4*I*log(c)) + (m + 1)*e^(4
*I*a + 8*I*log(c)))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sec(a+2*ln(c*x**(1/2*(-(1+m)**2)**(1/2))))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [C] Result contains complex when optimal does not.
time = 5.79, size = 834, normalized size = 7.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+2*log(c*x^(1/2*(-(1+m)^2)^(1/2))))^3,x, algorithm="giac
")
```

```
[Out] c^(6*I)*m*x*x^m*x^abs(m + 1)*e^(3*I*a)/(c^(8*I)*m^2*e^(4*I*a) + 2*c^(8*I)*m
*e^(4*I*a) + c^(8*I)*e^(4*I*a) + 2*c^(4*I)*m^2*x^(2*abs(m + 1))*e^(2*I*a) +
 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) + 2*c^(4*I)*x^(2*abs(m + 1))*e^(2*I
*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*abs(m + 1))) - c^(
6*I)*x*x^m*x^abs(m + 1)*abs(m + 1)*e^(3*I*a)/(c^(8*I)*m^2*e^(4*I*a) + 2*c^(
8*I)*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) + 2*c^(4*I)*m^2*x^(2*abs(m + 1))*e^(2*
I*a) + 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) + 2*c^(4*I)*x^(2*abs(m + 1))*
e^(2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*abs(m + 1)))
 + c^(6*I)*x*x^m*x^abs(m + 1)*e^(3*I*a)/(c^(8*I)*m^2*e^(4*I*a) + 2*c^(8*I)*
m*e^(4*I*a) + c^(8*I)*e^(4*I*a) + 2*c^(4*I)*m^2*x^(2*abs(m + 1))*e^(2*I*a)
+ 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) + 2*c^(4*I)*x^(2*abs(m + 1))*e^(2*
I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*abs(m + 1))) + c^
(2*I)*m*x*x^m*x^(3*abs(m + 1))*e^(I*a)/(c^(8*I)*m^2*e^(4*I*a) + 2*c^(8*I)*m
```

```
*e^(4*I*a) + c^(8*I)*e^(4*I*a) + 2*c^(4*I)*m^2*x^(2*abs(m + 1))*e^(2*I*a) +
 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) + 2*c^(4*I)*x^(2*abs(m + 1))*e^(2*I
*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*abs(m + 1))) + c^(
2*I)*x*x^m*x^(3*abs(m + 1))*abs(m + 1)*e^(I*a)/(c^(8*I)*m^2*e^(4*I*a) + 2*c
^(8*I)*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) + 2*c^(4*I)*m^2*x^(2*abs(m + 1))*e^(
2*I*a) + 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) + 2*c^(4*I)*x^(2*abs(m + 1)
)*e^(2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*abs(m + 1)
)) + c^(2*I)*x*x^m*x^(3*abs(m + 1))*e^(I*a)/(c^(8*I)*m^2*e^(4*I*a) + 2*c^(8
*I)*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) + 2*c^(4*I)*m^2*x^(2*abs(m + 1))*e^(2*I
*a) + 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) + 2*c^(4*I)*x^(2*abs(m + 1))*e
^(2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*abs(m + 1)))
```

Mupad [B]
time = 6.96, size = 176, normalized size = 1.60

$$\frac{\frac{x^{m+1}\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^{\frac{\sqrt{-m^2-2\,m-1}}{2}}\right)}^{2\mathrm{i}}\,\left(m\,1\mathrm{i}+\sqrt{-{\left(m+1\right)}^2}+1\mathrm{i}\right)}{\sqrt{-{\left(m+1\right)}^2}}-\frac{x^{m+1}\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^{\frac{\sqrt{-m^2-2\,m-1}}{2}}\right)}^{6\mathrm{i}}\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{e}^{a\,2\mathrm{i}}\,\sqrt{-{\left(m+1\right)}^2}+m\,\mathrm{e}^{a\,2\mathrm{i}}\,1\mathrm{i}\right)}{\sqrt{-{\left(m+1\right)}^2}}}{\left(m+1\right)\,{\left(\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^{\frac{\sqrt{-m^2-2\,m-1}}{2}}\right)}^{4\mathrm{i}}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/cos(a + 2*log(c*x^((-(m + 1)^2)^(1/2)/2)))^3,x)
```

```
[Out] ((x^(m + 1)*exp(a*1i)*(c*x^((- 2*m - m^2 - 1)^(1/2)/2))^2i*(m*1i + (-(m + 1
)^2)^(1/2) + 1i))/(-(m + 1)^2)^(1/2) - (x^(m + 1)*exp(a*1i)*(c*x^((- 2*m -
m^2 - 1)^(1/2)/2))^6i*(exp(a*2i)*1i - exp(a*2i)*(-(m + 1)^2)^(1/2) + m*exp(
a*2i)*1i))/(-(m + 1)^2)^(1/2))/((m + 1)*(exp(a*2i)*(c*x^((- 2*m - m^2 - 1)^
(1/2)/2))^4i + 1)^2)
```

3.261 $\int x \sec^3\left(a + 2\log\left(cx^i\right)\right)\, dx$

Optimal. Leaf size=45

$$\frac{e^{ia}\left(cx^i\right)^{2i} x^2}{\left(1 + e^{2ia}\left(cx^i\right)^{4i}\right)^2}$$

```
[Out] exp(I*a)*(c*x^I)^(2*I)*x^2/(1+exp(2*I*a)*(c*x^I)^(4*I))^2
```

Rubi [A]

time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4605, 4601, 267}

$$\frac{e^{ia} x^2 \left(cx^i\right)^{2i}}{\left(1 + e^{2ia}\left(cx^i\right)^{4i}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[a + 2*Log[c*x^I]]^3,x]
```

```
[Out] (E^(I*a)*(c*x^I)^(2*I)*x^2)/(1 + E^((2*I)*a)*(c*x^I)^(4*I))^2
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4605

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x\sec^3\left(a+2\log\left(cx^i\right)\right)\,dx = -\left(\left(i\left(cx^i\right)^{2i}x^2\right)\text{Subst}\left(\int x^{-1-2i}\sec^3(a+2\log(x))\,dx,x,cx^i\right)\right)$$
$$= -\left(\left(8ie^{3ia}\left(cx^i\right)^{2i}x^2\right)\text{Subst}\left(\int \frac{x^{-1+4i}}{\left(1+e^{2ia}x^{4i}\right)^3}\,dx,x,cx^i\right)\right)$$
$$= \frac{e^{ia}\left(cx^i\right)^{2i}x^2}{\left(1+e^{2ia}\left(cx^i\right)^{4i}\right)^2}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 127 vs. $2(45)=90$.
time = 0.17, size = 127, normalized size = 2.82

$$-\frac{\sec^2\left(a+2\log\left(cx^i\right)\right)\left(\left(1+2x^4\right)\cos\left(a+2\log\left(cx^i\right)-2i\log(x)\right)+i\left(1-2x^4\right)\sin\left(a+2\log\left(cx^i\right)-2i\log(x)\right)\right)\left(\cos\left(2\left(a+2\log\left(cx^i\right)-2i\log(x)\right)\right)+i\sin\left(2\left(a+2\log\left(cx^i\right)-2i\log(x)\right)\right)\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[a + 2*Log[c*x^I]]^3,x]
```

```
[Out] -1/4*(Sec[a + 2*Log[c*x^I]]^2*((1 + 2*x^4)*Cos[a + 2*Log[c*x^I] - (2*I)*Log
[x]] + I*(1 - 2*x^4)*Sin[a + 2*Log[c*x^I] - (2*I)*Log[x]])*(Cos[2*(a + 2*Lo
g[c*x^I] - (2*I)*Log[x])] + I*Sin[2*(a + 2*Log[c*x^I] - (2*I)*Log[x])]))/x^
4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.26, size = 209, normalized size = 4.64

method	result	size
risch	$\frac{x^2c^{2i}\left(x^i\right)^{2i}\mathrm{e}^{\pi\mathrm{csgn}\left(icx^i\right)^3-\pi\mathrm{csgn}\left(icx^i\right)^2\mathrm{csgn}(ic)-\pi\mathrm{csgn}\left(icx^i\right)^2\mathrm{csgn}\left(ix^i\right)+\pi\,\mathrm{csgn}\left(icx^i\right)\mathrm{csgn}(ic)\mathrm{csgn}\left(ix^i\right)+ia}}{\left(\left(x^i\right)^{4i}c^{4i}\mathrm{e}^{2\pi\mathrm{csgn}\left(icx^i\right)^3}\mathrm{e}^{-2\pi\mathrm{csgn}\left(icx^i\right)^2\mathrm{csgn}(ic)}\mathrm{e}^{-2\pi\mathrm{csgn}\left(icx^i\right)^2\mathrm{csgn}\left(ix^i\right)}\mathrm{e}^{2\pi\,\mathrm{csgn}\left(icx^i\right)\mathrm{csgn}(ic)\mathrm{csgn}\left(ix^i\right)}\mathrm{e}^{2ia}+1\right)^2}$	209

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(a+2*ln(c*x^I))^3,x,method=_RETURNVERBOSE)
```

```
[Out] x^2*c^(2*I)*(x^I)^(2*I)*exp(Pi*csgn(I*c*x^I)^3-Pi*csgn(I*c*x^I)^2*csgn(I*c)
-Pi*csgn(I*c*x^I)^2*csgn(I*x^I)+Pi*csgn(I*c*x^I)*csgn(I*c)*csgn(I*x^I)+I*a)
/(((x^I)^(2*I))^2*(c^(2*I))^2*exp(2*Pi*csgn(I*c*x^I)^3)*exp(-2*Pi*csgn(I*c*
x^I)^2*csgn(I*c))*exp(-2*Pi*csgn(I*c*x^I)^2*csgn(I*x^I))*exp(2*Pi*csgn(I*c*
x^I)*csgn(I*c)*csgn(I*x^I))*exp(2*I*a)+1)^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(31)=62$.
time = 0.30, size = 139, normalized size = 3.09

$$\frac{\left(\left(\cos\left(a\right)+i\,\sin\left(a\right)\right)\cos\left(2\,\log\left(c\right)\right)-\left(-i\,\cos\left(a\right)+\sin\left(a\right)\right)\sin\left(2\,\log\left(c\right)\right)\right)x^2e^{\left(6\,\arctan\left(\sin\left(\log\left(x\right)\right),\cos\left(\log\left(x\right)\right)\right)\right)}}{\left(\cos\left(4\,a\right)+i\,\sin\left(4\,a\right)\right)\cos\left(8\,\log\left(c\right)\right)+2\left(\left(\cos\left(2\,a\right)+i\,\sin\left(2\,a\right)\right)\cos\left(4\,\log\left(c\right)\right)-\left(-i\,\cos\left(2\,a\right)+\sin\left(2\,a\right)\right)\sin\left(4\,\log\left(c\right)\right)\right)e^{\left(4\,\arctan\left(\sin\left(\log\left(x\right)\right),\cos\left(\log\left(x\right)\right)\right)\right)}+\left(i\,\cos\left(4\,a\right)-\sin\left(4\,a\right)\right)\sin\left(8\,\log\left(c\right)\right)+e^{\left(8\,\arctan\left(\sin\left(\log\left(x\right)\right),\cos\left(\log\left(x\right)\right)\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+2*log(c*x^I))^3,x, algorithm="maxima")
```

```
[Out] ((cos(a) + I*sin(a))*cos(2*log(c)) - (-I*cos(a) + sin(a))*sin(2*log(c)))*x^
2*e^(6*arctan2(sin(log(x)), cos(log(x))))/((cos(4*a) + I*sin(4*a))*cos(8*lo
g(c)) + 2*((cos(2*a) + I*sin(2*a))*cos(4*log(c)) - (-I*cos(2*a) + sin(2*a))
*sin(4*log(c)))*e^(4*arctan2(sin(log(x)), cos(log(x)))) + (I*cos(4*a) - sin
(4*a))*sin(8*log(c)) + e^(8*arctan2(sin(log(x)), cos(log(x)))))
```

Fricas [A]
time = 2.89, size = 55, normalized size = 1.22

$$-\frac{2\,x^{4}e^{\left(3i\,a+6i\,\log\left(c\right)\right)}+e^{\left(5i\,a+10i\,\log\left(c\right)\right)}}{x^{8}+2\,x^{4}e^{\left(2i\,a+4i\,\log\left(c\right)\right)}+e^{\left(4i\,a+8i\,\log\left(c\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+2*log(c*x^I))^3,x, algorithm="fricas")
```

```
[Out] -(2*x^4*e^(3*I*a + 6*I*log(c)) + e^(5*I*a + 10*I*log(c)))/(x^8 + 2*x^4*e^(2
*I*a + 4*I*log(c)) + e^(4*I*a + 8*I*log(c)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sec^{3}\left(a+2\log\left(cx^{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+2*ln(c*x**I))**3,x)
```

```
[Out] Integral(x*sec(a + 2*log(c*x**I))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+2*log(c*x^I))^3,x, algorithm="giac")
```

```
[Out] integrate(x*sec(a + 2*log(c*x^I))^3, x)
```

Mupad [B]
time = 4.43, size = 46, normalized size = 1.02

$$\frac{x^{2}\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^{1\mathrm{i}}\right)}^{2\mathrm{i}}}{2\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^{1\mathrm{i}}\right)}^{4\mathrm{i}}+\mathrm{e}^{a\,4\mathrm{i}}\,{\left(c\,x^{1\mathrm{i}}\right)}^{8\mathrm{i}}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(a + 2*log(c*x^1i))^3,x)
```

```
[Out] (x^2*exp(a*1i)*(c*x^1i)^2i)/(2*exp(a*2i)*(c*x^1i)^4i + exp(a*4i)*(c*x^1i)^8
i + 1)
```

3.262 $\int \sec^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\,dx$

Optimal. Leaf size=58

$$\frac{1}{2}x\sec\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)-\frac{1}{2}ix\sec\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\tan\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)$$

```
[Out] 1/2*x*sec(a+2*ln(c*x^(1/2*I)))-1/2*I*x*sec(a+2*ln(c*x^(1/2*I)))*tan(a+2*ln(
c*x^(1/2*I)))
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 0.83, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4599, 4601, 267}

$$\frac{2e^{ia}x\left(cx^{\frac{i}{2}}\right)^{2i}}{\left(1+e^{2ia}\left(cx^{\frac{i}{2}}\right)^{4i}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + 2*Log[c*x^(I/2)]]^3,x]
```

```
[Out] (2*E^(I*a)*(c*x^(I/2))^(2*I)*x)/(1 + E^((2*I)*a)*(c*x^(I/2))^(4*I))^2
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4599

```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \sec^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\,dx = -\left(\left(2i\left(cx^{\frac{i}{2}}\right)^{2i}x\right)\operatorname{Subst}\left(\int x^{-1-2i}\sec^3(a+2\log(x))\,dx,x,cx^{\frac{i}{2}}\right)\right)$$
$$= -\left(\left(16ie^{3ia}\left(cx^{\frac{i}{2}}\right)^{2i}x\right)\operatorname{Subst}\left(\int\frac{x^{-1+4i}}{\left(1+e^{2ia}x^{4i}\right)^3}\,dx,x,cx^{\frac{i}{2}}\right)\right)$$
$$= \frac{2e^{ia}\left(cx^{\frac{i}{2}}\right)^{2i}x}{\left(1+e^{2ia}\left(cx^{\frac{i}{2}}\right)^{4i}\right)^2}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 137 vs. 2(58) = 116.
time = 0.13, size = 137, normalized size = 2.36

$$-\frac{\sec^2\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\left((1+2x^2)\cos\left(a+2\log\left(cx^{\frac{i}{2}}\right)-i\log(x)\right)+i(1-2x^2)\sin\left(a+2\log\left(cx^{\frac{i}{2}}\right)-i\log(x)\right)\right)\left(\cos\left(2\left(a+2\log\left(cx^{\frac{i}{2}}\right)-i\log(x)\right)\right)+i\sin\left(2\left(a+2\log\left(cx^{\frac{i}{2}}\right)-i\log(x)\right)\right)\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + 2*Log[c*x^(I/2)]]^3,x]
```

```
[Out] -1/2*(Sec[a + 2*Log[c*x^(I/2)]]^2*((1 + 2*x^2)*Cos[a + 2*Log[c*x^(I/2)] - I
*Log[x]] + I*(1 - 2*x^2)*Sin[a + 2*Log[c*x^(I/2)] - I*Log[x]])*(Cos[2*(a +
2*Log[c*x^(I/2)] - I*Log[x])] + I*Sin[2*(a + 2*Log[c*x^(I/2)] - I*Log[x])])
)/x^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.24, size = 208, normalized size = 3.59

method	result	s
risch	$\frac{2x\,c^{2i}\left(x^{\frac{i}{2}}\right)^{2i}\mathrm{e}^{\operatorname{csgn}\left(ic\,x^{\frac{i}{2}}\right)^3\pi-\operatorname{csgn}\left(ic\,x^{\frac{i}{2}}\right)^2\operatorname{csgn}(ic)\pi-\operatorname{csgn}\left(ic\,x^{\frac{i}{2}}\right)^2\operatorname{csgn}\left(ix^{\frac{i}{2}}\right)\pi+\operatorname{csgn}\left(ic\,x^{\frac{i}{2}}\right)\operatorname{csgn}(ic)\operatorname{csgn}\left(ix^{\frac{i}{2}}\right)\pi+ia}}{\left(\left(x^{\frac{i}{2}}\right)^{4i}c^{4i}\mathrm{e}^{2\operatorname{csgn}\left(ic\,x^{\frac{i}{2}}\right)^3\pi}\mathrm{e}^{-2\operatorname{csgn}\left(ic\,x^{\frac{i}{2}}\right)^2\operatorname{csgn}(ic)\pi}\mathrm{e}^{-2\operatorname{csgn}\left(ic\,x^{\frac{i}{2}}\right)^2\operatorname{csgn}\left(ix^{\frac{i}{2}}\right)\pi}\mathrm{e}^{2\operatorname{csgn}\left(ic\,x^{\frac{i}{2}}\right)\operatorname{csgn}(ic)\operatorname{csgn}\left(ix^{\frac{i}{2}}\right)\pi}\mathrm{e}^{2ia}+1\right)^2}$	2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a+2*ln(c*x^(1/2*I)))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*x*c^(2*I)*(x^(1/2*I))^(2*I)*exp(csgn(I*c*x^(1/2*I))^3*Pi-csgn(I*c*x^(1/2*
I))^2*csgn(I*c)*Pi-csgn(I*c*x^(1/2*I))^2*csgn(I*x^(1/2*I))*Pi+csgn(I*c*x^(1
/2*I))*csgn(I*c)*csgn(I*x^(1/2*I))*Pi+I*a)/(((x^(1/2*I))^(2*I))^2*(c^(2*I))
^2*exp(2*csgn(I*c*x^(1/2*I))^3*Pi)*exp(-2*csgn(I*c*x^(1/2*I))^2*csgn(I*c)*P
i)*exp(-2*csgn(I*c*x^(1/2*I))^2*csgn(I*x^(1/2*I))*Pi)*exp(2*csgn(I*c*x^(1/2
*I))*csgn(I*c)*csgn(I*x^(1/2*I))*Pi)*exp(2*I*a)+1)^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(40) = 80$.
time = 0.32, size = 151, normalized size = 2.60

$$\frac{2\left(\left(\cos\left(a\right)+i\sin\left(a\right)\right)\cos\left(2\log\left(c\right)\right)+\left(i\cos\left(a\right)-\sin\left(a\right)\right)\sin\left(2\log\left(c\right)\right)\right)xe^{\left(6\arctan\left(\sin\left(\frac{1}{2}\log\left(x\right)\right),\cos\left(\frac{1}{2}\log\left(x\right)\right)\right)\right)}}{\left(\cos\left(4a\right)+i\sin\left(4a\right)\right)\cos\left(8\log\left(c\right)\right)+2\left(\left(\cos\left(2a\right)+i\sin\left(2a\right)\right)\cos\left(4\log\left(c\right)\right)-\left(-i\cos\left(2a\right)+\sin\left(2a\right)\right)\sin\left(4\log\left(c\right)\right)\right)e^{\left(4\arctan\left(\sin\left(\frac{1}{2}\log\left(x\right)\right),\cos\left(\frac{1}{2}\log\left(x\right)\right)\right)\right)}+\left(i\cos\left(4a\right)-\sin\left(4a\right)\right)\sin\left(8\log\left(c\right)\right)+e^{\left(8\arctan\left(\sin\left(\frac{1}{2}\log\left(x\right)\right),\cos\left(\frac{1}{2}\log\left(x\right)\right)\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+2*log(c*x^(1/2*I)))^3,x, algorithm="maxima")
```

```
[Out] 2*((cos(a) + I*sin(a))*cos(2*log(c)) + (I*cos(a) - sin(a))*sin(2*log(c)))*x
*e^(6*arctan2(sin(1/2*log(x)), cos(1/2*log(x))))/((cos(4*a) + I*sin(4*a))*c
os(8*log(c)) + 2*((cos(2*a) + I*sin(2*a))*cos(4*log(c)) - (-I*cos(2*a) + si
n(2*a))*sin(4*log(c)))*e^(4*arctan2(sin(1/2*log(x)), cos(1/2*log(x)))) + (I
*cos(4*a) - sin(4*a))*sin(8*log(c)) + e^(8*arctan2(sin(1/2*log(x)), cos(1/2
*log(x)))))
```

Fricas [A]
time = 2.07, size = 55, normalized size = 0.95

$$-\frac{2\left(2x^{2}e^{\left(3ia+6i\log\left(c\right)\right)}+e^{\left(5ia+10i\log\left(c\right)\right)}\right)}{x^{4}+2x^{2}e^{\left(2ia+4i\log\left(c\right)\right)}+e^{\left(4ia+8i\log\left(c\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+2*log(c*x^(1/2*I)))^3,x, algorithm="fricas")
```

```
[Out] -2*(2*x^2*e^(3*I*a + 6*I*log(c)) + e^(5*I*a + 10*I*log(c)))/(x^4 + 2*x^2*e^
(2*I*a + 4*I*log(c)) + e^(4*I*a + 8*I*log(c)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sec^{3}\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+2*ln(c*x**(1/2*I)))**3,x)
```

```
[Out] Integral(sec(a + 2*log(c*x**(I/2)))**3, x)
```

Giac [A]
time = 1.10, size = 74, normalized size = 1.28

$$-\frac{2c^{10i}e^{\left(5ia\right)}}{c^{8i}e^{\left(4ia\right)}+2c^{4i}x^{2}e^{\left(2ia\right)}+x^{4}}-\frac{4c^{6i}x^{2}e^{\left(3ia\right)}}{c^{8i}e^{\left(4ia\right)}+2c^{4i}x^{2}e^{\left(2ia\right)}+x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+2*log(c*x^(1/2*I)))^3,x, algorithm="giac")
```

```
[Out] -2*c^(10*I)*e^(5*I*a)/(c^(8*I)*e^(4*I*a) + 2*c^(4*I)*x^2*e^(2*I*a) + x^4) -
 4*c^(6*I)*x^2*e^(3*I*a)/(c^(8*I)*e^(4*I*a) + 2*c^(4*I)*x^2*e^(2*I*a) + x^4
)
```

Mupad [B]
time = 4.48, size = 56, normalized size = 0.97

$$\frac{2\,x\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^{\frac{1}{2}\mathrm{i}}\right)}^{2\mathrm{i}}}{2\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^{\frac{1}{2}\mathrm{i}}\right)}^{4\mathrm{i}}+\mathrm{e}^{a\,4\mathrm{i}}\,{\left(c\,x^{\frac{1}{2}\mathrm{i}}\right)}^{8\mathrm{i}}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + 2*log(c*x^(1i/2)))^3,x)
```

```
[Out] (2*x*exp(a*1i)*(c*x^(1i/2))^2i)/(2*exp(a*2i)*(c*x^(1i/2))^4i + exp(a*4i)*(c
*x^(1i/2))^8i + 1)
```

3.263 $\int \sec^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\,dx$

Optimal. Leaf size=48

$$\frac{2e^{3ia}\left(cx^{-\frac{i}{2}}\right)^{6i}x}{\left(1+e^{2ia}\left(cx^{-\frac{i}{2}}\right)^{4i}\right)^2}$$

```
[Out] 2*exp(3*I*a)*(c/(x^(1/2*I)))^(6*I)*x/(1+exp(2*I*a)*(c/(x^(1/2*I)))^(4*I))^2
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4599, 4601, 270}

$$\frac{2e^{3ia}x\left(cx^{-\frac{i}{2}}\right)^{6i}}{\left(1+e^{2ia}\left(cx^{-\frac{i}{2}}\right)^{4i}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + 2*Log[c/x^(I/2)]]^3,x]
[Out] (2*E^((3*I)*a)*(c/x^(I/2))^(6*I)*x)/(1 + E^((2*I)*a)*(c/x^(I/2))^(4*I))^2
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4599

```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \sec^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\,dx=\left(2i\left(cx^{-\frac{i}{2}}\right)^{-2i}x\right)\text{Subst}\left(\int x^{-1+2i}\sec^3(a+2\log(x))\,dx,x,cx^{-\frac{i}{2}}\right)$$
$$=\left(16ie^{3ia}\left(cx^{-\frac{i}{2}}\right)^{-2i}x\right)\text{Subst}\left(\int\frac{x^{-1+8i}}{\left(1+e^{2ia}x^{4i}\right)^3}\,dx,x,cx^{-\frac{i}{2}}\right)$$
$$=\frac{2e^{3ia}\left(cx^{-\frac{i}{2}}\right)^{6i}x}{\left(1+e^{2ia}\left(cx^{-\frac{i}{2}}\right)^{4i}\right)^2}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 139 vs. $2(48)=96$.
time $=0.16$, size $=139$, normalized size $=2.90$

$$\frac{\sec^2\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\left((1+2x^2)\cos\left(a+2\log\left(cx^{-\frac{i}{2}}\right)+i\log(x)\right)+i(-1+2x^2)\sin\left(a+2\log\left(cx^{-\frac{i}{2}}\right)+i\log(x)\right)\right)\left(-2\cos\left(2\left(a+2\log\left(cx^{-\frac{i}{2}}\right)+i\log(x)\right)\right)+2i\sin\left(2\left(a+2\log\left(cx^{-\frac{i}{2}}\right)+i\log(x)\right)\right)\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + 2*Log[c/x^(I/2)]]^3,x]
```

```
[Out] (Sec[a + 2*Log[c/x^(I/2)]]^2*((1 + 2*x^2)*Cos[a + 2*Log[c/x^(I/2)] + I*Log[
x]] + I*(-1 + 2*x^2)*Sin[a + 2*Log[c/x^(I/2)] + I*Log[x]])*(-2*Cos[2*(a + 2
*Log[c/x^(I/2)] + I*Log[x])] + (2*I)*Sin[2*(a + 2*Log[c/x^(I/2)] + I*Log[x]
)]))/(4*x^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time $=0.24$, size $=238$, normalized size $=4.96$

method	result
risch	$\frac{2x\,c^{6i}\left(x^{\frac{i}{2}}\right)^{-6i}\mathrm{e}^{3\pi\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)^3-3\pi\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)^2\mathrm{csgn}(ic)-3\pi\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)^2\mathrm{csgn}\left(ix^{-\frac{i}{2}}\right)+3\pi\,\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)\mathrm{csgn}(ic)\mathrm{csgn}\left(ix^{-\frac{i}{2}}\right)+}}{\left(c^{4i}\left(x^{\frac{i}{2}}\right)^{-4i}\mathrm{e}^{2\pi\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)^3}\mathrm{e}^{-2\pi\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)^2\mathrm{csgn}(ic)}\mathrm{e}^{-2\pi\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)^2\mathrm{csgn}\left(ix^{-\frac{i}{2}}\right)}\mathrm{e}^{2\pi\,\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)\mathrm{csgn}(ic)\mathrm{csgn}\left(ix^{-\frac{i}{2}}\right)}\mathrm{e}^{2ia}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a+2*ln(c/(x^(1/2*I))))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*x*(c^(2*I))^3*((x^(1/2*I))^(-2*I))^3*exp(3*Pi*csgn(I*c/(x^(1/2*I)))^3-3*P
i*csgn(I*c/(x^(1/2*I)))^2*csgn(I*c)-3*Pi*csgn(I*c/(x^(1/2*I)))^2*csgn(I/(x^
(1/2*I)))+3*Pi*csgn(I*c/(x^(1/2*I)))*csgn(I*c)*csgn(I/(x^(1/2*I)))+3*I*a)/(
(c^(2*I))^2*((x^(1/2*I))^(-2*I))^2*exp(2*Pi*csgn(I*c/(x^(1/2*I)))^3)*exp(-2
*Pi*csgn(I*c/(x^(1/2*I)))^2*csgn(I*c))*exp(-2*Pi*csgn(I*c/(x^(1/2*I)))^2*cs
```

```
gn(I/(x^(1/2*I))))*exp(2*Pi*csgn(I*c/(x^(1/2*I)))*csgn(I*c)*csgn(I/(x^(1/2*
I))))*exp(2*I*a)+1)^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(27) = 54$.
time = 0.33, size = 162, normalized size = 3.38

$$\frac{2\left(\left(\cos\left(3\,a\right)+i\,\sin\left(3\,a\right)\right)\cos\left(6\,\log\left(c\right)\right)+\left(i\,\cos\left(3\,a\right)-\sin\left(3\,a\right)\right)\sin\left(6\,\log\left(c\right)\right)\right)xe^{\left(6\,\arctan\left(\sin\left(\frac{1}{2}\,\log\left(x\right)\right),\cos\left(\frac{1}{2}\,\log\left(x\right)\right)\right)\right)}}{\left(\left(\cos\left(4\,a\right)+i\,\sin\left(4\,a\right)\right)\cos\left(8\,\log\left(c\right)\right)-\left(-i\,\cos\left(4\,a\right)+\sin\left(4\,a\right)\right)\sin\left(8\,\log\left(c\right)\right)\right)e^{\left(8\,\arctan\left(\sin\left(\frac{1}{2}\,\log\left(x\right)\right),\cos\left(\frac{1}{2}\,\log\left(x\right)\right)\right)\right)}+2\left(\left(\cos\left(2\,a\right)+i\,\sin\left(2\,a\right)\right)\cos\left(4\,\log\left(c\right)\right)+\left(i\,\cos\left(2\,a\right)-\sin\left(2\,a\right)\right)\sin\left(4\,\log\left(c\right)\right)\right)e^{\left(4\,\arctan\left(\sin\left(\frac{1}{2}\,\log\left(x\right)\right),\cos\left(\frac{1}{2}\,\log\left(x\right)\right)\right)\right)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+2*log(c/(x^(1/2*I))))^3,x, algorithm="maxima")
```

```
[Out] 2*((cos(3*a) + I*sin(3*a))*cos(6*log(c)) + (I*cos(3*a) - sin(3*a))*sin(6*lo
g(c)))*x*e^(6*arctan2(sin(1/2*log(x)), cos(1/2*log(x))))/(((cos(4*a) + I*si
n(4*a))*cos(8*log(c)) - (-I*cos(4*a) + sin(4*a))*sin(8*log(c)))*e^(8*arctan
2(sin(1/2*log(x)), cos(1/2*log(x)))) + 2*((cos(2*a) + I*sin(2*a))*cos(4*log
(c)) + (I*cos(2*a) - sin(2*a))*sin(4*log(c)))*e^(4*arctan2(sin(1/2*log(x)),
 cos(1/2*log(x)))) + 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(27) = 54$.
time = 2.52, size = 57, normalized size = 1.19

$$-\frac{2\left(2\,x^{2}e^{\left(2i\,a+4i\,\log\left(c\right)\right)}+1\right)}{x^{4}e^{\left(5i\,a+10i\,\log\left(c\right)\right)}+2\,x^{2}e^{\left(3i\,a+6i\,\log\left(c\right)\right)}+e^{\left(i\,a+2i\,\log\left(c\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+2*log(c/(x^(1/2*I))))^3,x, algorithm="fricas")
```

```
[Out] -2*(2*x^2*e^(2*I*a + 4*I*log(c)) + 1)/(x^4*e^(5*I*a + 10*I*log(c)) + 2*x^2*
e^(3*I*a + 6*I*log(c)) + e^(I*a + 2*I*log(c)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sec^{3}\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+2*ln(c/(x**(1/2*I))))**3,x)
```

```
[Out] Integral(sec(a + 2*log(c/x**(I/2)))**3, x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(27) = 54$.
time = 1.11, size = 83, normalized size = 1.73

$$-\frac{4\,c^{4i}x^{2}e^{\left(2i\,a\right)}}{c^{10i}x^{4}e^{\left(5i\,a\right)}+2\,c^{6i}x^{2}e^{\left(3i\,a\right)}+c^{2i}e^{\left(i\,a\right)}}-\frac{2}{c^{10i}x^{4}e^{\left(5i\,a\right)}+2\,c^{6i}x^{2}e^{\left(3i\,a\right)}+c^{2i}e^{\left(i\,a\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+2*log(c/(x^(1/2*I))))^3,x, algorithm="giac")

[Out] -4*c^(4*I)*x^2*e^(2*I*a)/(c^(10*I)*x^4*e^(5*I*a) + 2*c^(6*I)*x^2*e^(3*I*a) + c^(2*I)*e^(I*a)) - 2/(c^(10*I)*x^4*e^(5*I*a) + 2*c^(6*I)*x^2*e^(3*I*a) + c^(2*I)*e^(I*a))

Mupad [B]
time = 6.28, size = 39, normalized size = 0.81

$$\frac{2\,x\,\mathrm{e}^{a\,3\mathrm{i}}\,{\left(\frac{c}{x^{\frac{1}{2}\mathrm{i}}}\right)}^{6\mathrm{i}}}{{\left(\mathrm{e}^{a\,2\mathrm{i}}\,{\left(\frac{c}{x^{\frac{1}{2}\mathrm{i}}}\right)}^{4\mathrm{i}}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(a + 2*log(c/x^(1i/2)))^3,x)

[Out] (2*x*exp(a*3i)*(c/x^(1i/2))^6i)/(exp(a*2i)*(c/x^(1i/2))^4i + 1)^2

3.264 $\int \sec^p\left(a+\frac{i\log(cx^n)}{n(-2+p)}\right)\,dx$

Optimal. Leaf size=95

$$\frac{e^{-2ia}(2-p)x(cx^n)^{-\frac{2}{n(2-p)}}\left(1+e^{2ia}(cx^n)^{\frac{2}{n(2-p)}}\right)\sec^p\left(a-\frac{i\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

```
[Out] 1/2*(2-p)*x*(1+exp(2*I*a)*(c*x^n)^(2/n/(2-p)))*sec(a-I*ln(c*x^n)/n/(2-p))^p
/exp(2*I*a)/(1-p)/((c*x^n)^(2/n/(2-p)))
```

Rubi [A]
time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4599, 4603, 267}

$$\frac{e^{-2ia}(2-p)x(cx^n)^{-\frac{2}{n(2-p)}}\left(1+e^{2ia}(cx^n)^{\frac{2}{n(2-p)}}\right)\sec^p\left(a-\frac{i\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + (I*Log[c*x^n])/(n*(-2 + p))]^p,x]
```

```
[Out] ((2 - p)*x*(1 + E^((2*I)*a)*(c*x^n)^(2/(n*(2 - p))))*Sec[a - (I*Log[c*x^n])
/(n*(2 - p))]^p)/(2*E^((2*I)*a)*(1 - p)*(c*x^n)^(2/(n*(2 - p))))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4599

```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4603

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sec^p\left(a+\frac{i\log(cx^n)}{n(-2+p)}\right)dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\sec^p\left(a+\frac{i\log(x)}{n(-2+p)}\right)dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-\frac{1}{n}+\frac{p}{n(-2+p)}}\left(1+e^{2ia}(cx^n)^{-\frac{2}{n(-2+p)}}\right)^p\sec^p\left(a+\frac{i\log(cx^n)}{n(-2+p)}\right)\right)\text{Subst}\left(\int \right.}{n}$$

$$= \frac{e^{-2ia}(2-p)x(cx^n)^{-\frac{2}{n(2-p)}}\left(1+e^{2ia}(cx^n)^{\frac{2}{n(2-p)}}\right)\sec^p\left(a-\frac{i\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Mathematica [A]
time = 2.02, size = 117, normalized size = 1.23

$$\frac{2^{-1+p}e^{-ia}(-2+p)x(cx^n)^{\frac{1}{n(-2+p)}}\left(\frac{e^{\frac{ia(2+p)}{-2+p}}(cx^n)^{\frac{1}{n(-2+p)}}}{e^{\frac{2iap}{-2+p}}+e^{\frac{4ia}{-2+p}}(cx^n)^{\frac{2}{n(-2+p)}}}\right)^{-1+p}}{-1+p}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[a + (I*Log[c*x^n])/(n*(-2 + p))]^p,x]
```

```
[Out] (2^(-1 + p)*(-2 + p)*x*(c*x^n)^(1/(n*(-2 + p)))*((E^((I*a*(2 + p))/(-2 + p)
)*(c*x^n)^(1/(n*(-2 + p))))/(E^(((2*I)*a*p)/(-2 + p)) + E^(((4*I)*a)/(-2 +
p))*(c*x^n)^(2/(n*(-2 + p)))))^(-1 + p))/(E^(I*a)*(-1 + p))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \sec^p\left(a+\frac{i\ln(c\,x^n)}{n\,(-2+p)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a+I*ln(c*x^n)/n/(-2+p))^p,x)
```

```
[Out] int(sec(a+I*ln(c*x^n)/n/(-2+p))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+I*log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
```

```
[Out] integrate(sec(a + I*log(c*x^n)/(n*(p - 2)))^p, x)
```

Fricas [A]
time = 2.10, size = 149, normalized size = 1.57

$$\frac{\left((p-2)xe^{\left(\frac{2\,(i\,anp-2i\,an-n\log(x)-\log(c))}{np-2\,n}\right)}+(p-2)x\right)\left(\frac{2\,e^{\left(\frac{i\,anp-2i\,an-n\log(x)-\log(c)}{np-2\,n}\right)}}{e^{\left(\frac{2\,(i\,anp-2i\,an-n\log(x)-\log(c))}{np-2\,n}\right)}+1}\right)^{p}e^{\left(-\frac{2\,(i\,anp-2i\,an-n\log(x)-\log(c))}{np-2\,n}\right)}}{2\,(p-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+I*log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
[Out] 1/2*((p - 2)*x*e^(2*(I*a*n*p - 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n)) +
(p - 2)*x)*(2*e^((I*a*n*p - 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n))/(e^(2
*(I*a*n*p - 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n)) + 1))^p*e^(-2*(I*a*n*
p - 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n))/(p - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sec^{p}\left(a+\frac{i\log\left(cx^{n}\right)}{n\,(p-2)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+I*ln(c*x**n)/n/(-2+p))**p,x)
[Out] Integral(sec(a + I*log(c*x**n)/(n*(p - 2)))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+I*log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
[Out] integrate(sec(a + I*log(c*x^n)/(n*(p - 2)))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\cos\left(a+\frac{\ln\left(c\,x^n\right)\,1{}\mathrm{i}}{n\,\left(p-2\right)}\right)}\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(a + (log(c*x^n)*1i)/(n*(p - 2))))^p,x)
[Out] int((1/cos(a + (log(c*x^n)*1i)/(n*(p - 2))))^p, x)
```

3.265 $\int \sec^p\left(a-\frac{i\log(cx^n)}{n(-2+p)}\right)\,dx$

Optimal. Leaf size=70

$$\frac{(2-p)x\left(1+e^{2ia}(cx^n)^{-\frac{2}{n(2-p)}}\right)\sec^p\left(a+\frac{i\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

```
[Out] 1/2*(2-p)*x*(1+exp(2*I*a)/((c*x^n)^(2/n/(2-p))))*sec(a+I*ln(c*x^n)/n/(2-p))
^p/(1-p)
```

Rubi [A]
time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4599, 4603, 270}

$$\frac{(2-p)x\left(1+e^{2ia}(cx^n)^{-\frac{2}{n(2-p)}}\right)\sec^p\left(a+\frac{i\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a - (I*Log[c*x^n])/(n*(-2 + p))]^p,x]
```

```
[Out] ((2 - p)*x*(1 + E^((2*I)*a)/(c*x^n)^(2/(n*(2 - p))))*Sec[a + (I*Log[c*x^n])
/(n*(2 - p))]^p)/(2*(1 - p))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4599

```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4603

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sec^p\left(a-\frac{i\log(cx^n)}{n(-2+p)}\right)dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\sec^p\left(a-\frac{i\log(x)}{n(-2+p)}\right)dx,x,cx^n\right)}{n}$$
$$= \frac{\left(x(cx^n)^{-\frac{1}{n}-\frac{p}{n(-2+p)}}\left(1+e^{2ia}(cx^n)^{\frac{2}{n(-2+p)}}\right)^p\sec^p\left(a-\frac{i\log(cx^n)}{n(-2+p)}\right)\right)\text{Subst}\left(\int x^{-1}\right.}{n}$$
$$= \frac{(2-p)x\left(1+e^{2ia}(cx^n)^{-\frac{2}{n(2-p)}}\right)\sec^p\left(a+\frac{i\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Mathematica [A]
time = 1.96, size = 117, normalized size = 1.67

$$\frac{2^{-1+p}e^{ia}(-2+p)x(cx^n)^{\frac{1}{n(-2+p)}}\left(\frac{e^{\frac{ia(2+p)}{-2+p}}(cx^n)^{\frac{1}{n(-2+p)}}}{e^{\frac{4ia}{-2+p}}+e^{\frac{2iap}{-2+p}}(cx^n)^{\frac{2}{n(-2+p)}}}\right)^{-1+p}}{-1+p}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[a - (I*Log[c*x^n])/(n*(-2 + p))]^p,x]
[Out] (2^(-1 + p)*E^(I*a)*(-2 + p)*x*(c*x^n)^(1/(n*(-2 + p)))*((E^((I*a*(2 + p))/
(-2 + p))*(c*x^n)^(1/(n*(-2 + p))))/(E^(((4*I)*a)/(-2 + p)) + E^(((2*I)*a*p
)/(-2 + p))*(c*x^n)^(2/(n*(-2 + p)))))^(-1 + p))/(-1 + p)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \sec^p\left(a-\frac{i\ln(c\,x^n)}{n\,(-2+p)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a-I*ln(c*x^n)/n/(-2+p))^p,x)
[Out] int(sec(a-I*ln(c*x^n)/n/(-2+p))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a-I*log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
```

```
[Out] integrate(sec(-a + I*log(c*x^n)/(n*(p - 2)))^p, x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(55) = 110$.
time = 2.16, size = 149, normalized size = 2.13

$$\frac{\left((p-2)xe^{\left(\frac{2\,(-i\,anp+2i\,an-n\log(x)-\log(c))}{np-2\,n}\right)}+(p-2)x\right)\left(\frac{2\,e^{\left(\frac{-i\,anp+2i\,an-n\log(x)-\log(c)}{np-2\,n}\right)}}{e^{\left(\frac{2\,(-i\,anp+2i\,an-n\log(x)-\log(c))}{np-2\,n}\right)}+1}\right)^{p}e^{\left(-\frac{2\,(-i\,anp+2i\,an-n\log(x)-\log(c))}{np-2\,n}\right)}}{2\,(p-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a-I*log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
[Out] 1/2*((p - 2)*x*e^(2*(-I*a*n*p + 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n)) +
 (p - 2)*x)*(2*e^((-I*a*n*p + 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n))/(e^
(2*(-I*a*n*p + 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n)) + 1))^p*e^(-2*(-I*
a*n*p + 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n))/(p - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sec^{p}\left(a-\frac{i\log\left(cx^{n}\right)}{n\left(p-2\right)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a-I*ln(c*x**n)/n/(-2+p))**p,x)
[Out] Integral(sec(a - I*log(c*x**n)/(n*(p - 2)))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a-I*log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
[Out] integrate(sec(a - I*log(c*x^n)/(n*(p - 2)))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\cos\left(a-\frac{\ln\left(c\,x^n\right)\,1{}\mathrm{i}}{n\,\left(p-2\right)}\right)}\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(a - (log(c*x^n)*1i)/(n*(p - 2))))^p,x)
[Out] int((1/cos(a - (log(c*x^n)*1i)/(n*(p - 2))))^p, x)
```

3.266 $\int \sqrt{\sec(a+b\log(cx^n))}\, dx$

Optimal. Leaf size=109

$$\frac{2x\sqrt{1+e^{2ia}(cx^n)^{2ib}}\, {}_2F_1\left(\frac{1}{2},\frac{1}{4}\left(1-\frac{2i}{bn}\right);\frac{1}{4}\left(5-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)\sqrt{\sec(a+b\log(cx^n))}}{2+ibn}$$

```
[Out] 2*x*hypergeom([1/2, 1/4-1/2*I/b/n],[5/4-1/2*I/b/n],-exp(2*I*a)*(c*x^n)^(2*I
*b))*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(1/2)*sec(a+b*ln(c*x^n))^(1/2)/(2+I*b*n
)
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4599, 4603, 371}

$$\frac{2x\sqrt{1+e^{2ia}(cx^n)^{2ib}}\, {}_2F_1\left(\frac{1}{2},\frac{1}{4}\left(1-\frac{2i}{bn}\right);\frac{1}{4}\left(5-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)\sqrt{\sec(a+b\log(cx^n))}}{2+ibn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[a + b*Log[c*x^n]]],x]
```

```
[Out] (2*x*Sqrt[1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Hypergeometric2F1[1/2, (1 - (2
*I)/(b*n))/4, (5 - (2*I)/(b*n))/4, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))]*Sqrt[S
ec[a + b*Log[c*x^n]]])/(2 + I*b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4599

```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4603

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\sqrt{\sec(a+b\log(x))}\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-\frac{ib}{2}-\frac{1}{n}}\sqrt{1+e^{2ia}\left(cx^n\right)^{2ib}}\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}\right)\text{Subst}\left(\int \frac{x^{-1+}}{\sqrt{1+}}\right.}{n}$$

$$= \frac{2x\sqrt{1+e^{2ia}\left(cx^n\right)^{2ib}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}\left(1-\frac{2i}{bn}\right);\frac{1}{4}\left(5-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)\sqrt{\sec(a}}{2+ibn}$$

Mathematica [A]
time = 0.36, size = 99, normalized size = 0.91

$$-\frac{2i\left(1+e^{2i(a+b\log(cx^n))}\right)x\,{}_2F_1\left(1,\frac{3}{4}-\frac{i}{2bn};\frac{5}{4}-\frac{i}{2bn};-e^{2i(a+b\log(cx^n))}\right)\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}}{-2i+bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[a + b*Log[c*x^n]]],x]
```

```
[Out] ((-2*I)*(1 + E^((2*I)*(a + b*Log[c*x^n])))*x*Hypergeometric2F1[1, 3/4 - (I/
2)/(b*n), 5/4 - (I/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))]*Sqrt[Sec[a + b*
Log[c*x^n]]])/(-2*I + b*n)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \sqrt{\sec}\left(a+b\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a+b*ln(c*x^n))^(1/2),x)
```

```
[Out] int(sec(a+b*ln(c*x^n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(b*log(c*x^n) + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sec\left(a + b\log\left(cx^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(a + b*log(c*x**n))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(b*log(c*x^n) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{1}{\cos\left(a + b\,\ln\left(c\,x^n\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(a + b*log(c*x^n)))^(1/2),x)
```

```
[Out] int((1/cos(a + b*log(c*x^n)))^(1/2), x)
```

3.267 $\int \frac{\sqrt{\sec (a+b\log (cx^n))}}{x}\,dx$

Optimal. Leaf size=54

$$\frac{2\sqrt{\cos (a+b\log (cx^n))}\,F\left(\frac{1}{2}(a+b\log (cx^n))\middle|\,2\right)\sqrt{\sec (a+b\log (cx^n))}}{bn}$$

```
[Out] 2*(cos(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cos(1/2*a+1/2*b*ln(c*x^n))*EllipticF
(sin(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))*cos(a+b*ln(c*x^n))^(1/2)*sec(a+b*ln(c*
x^n))^(1/2)/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3856, 2720}

$$\frac{2\sqrt{\sec (a+b\log (cx^n))}\,\sqrt{\cos (a+b\log (cx^n))}\,F\left(\frac{1}{2}(a+b\log (cx^n))\middle|\,2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] (2*Sqrt[Cos[a + b*Log[c*x^n]]]*EllipticF[(a + b*Log[c*x^n])/2, 2]*Sqrt[Sec[
a + b*Log[c*x^n]]])/(b*n)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\sec (a+b\log (cx^n))}}{x}\,dx &= \frac{\text{Subst}\left(\int \sqrt{\sec(a+bx)}\,dx, x, \log (cx^n)\right)}{n}\\
&= \frac{\left(\sqrt{\cos (a+b\log (cx^n))}\,\sqrt{\sec (a+b\log (cx^n))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{\cos(a+bx)}}\right.}{n}\\
&= \frac{2\sqrt{\cos (a+b\log (cx^n))}\,F\left(\frac{1}{2}(a+b\log (cx^n))\middle|\,2\right)\sqrt{\sec (a+b\log (cx^n))}}{bn}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 54, normalized size = 1.00

$$\frac{2\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}\,F\left(\frac{1}{2}\left(a+b\log\left(cx^n\right)\right)\middle|2\right)\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] (2*Sqrt[Cos[a + b*Log[c*x^n]]]*EllipticF[(a + b*Log[c*x^n])/2, 2]*Sqrt[Sec[
a + b*Log[c*x^n]]])/(b*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(86) = 172.
time = 0.35, size = 181, normalized size = 3.35

method	result
derivativedivides	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(a+b\ln(cx^n))}{2}}\,\sqrt{-2\left(c\right.}}{n\sqrt{-2\left(\sin^4\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+\sin^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)}\,\sin\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)}$
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(a+b\ln(cx^n))}{2}}\,\sqrt{-2\left(c\right.}}{n\sqrt{-2\left(\sin^4\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+\sin^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)}\,\sin\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -2/n*((2*cos(1/2*a+1/2*b*ln(c*x^n))^2-1)*sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2
)*(sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cos(1/2*a+1/2*b*ln(c*x^n))^2+1)^
(1/2)/(-2*sin(1/2*a+1/2*b*ln(c*x^n))^4+sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*
EllipticF(cos(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/sin(1/2*a+1/2*b*ln(c*x^n))/(2
*cos(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(b*log(c*x^n) + a))/x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.65, size = 78, normalized size = 1.44

$$\frac{-i\sqrt{2}\,\text{weierstrassPInverse}(-4, 0, \cos(bn\log(x) + b\log(c) + a) + i\sin(bn\log(x) + b\log(c) + a)) + i\sqrt{2}\,\text{weierstrassPInverse}(-4, 0, \cos(bn\log(x) + b\log(c) + a) - i\sin(bn\log(x) + b\log(c) + a))}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*weierstrassPInverse(-4, 0, cos(b*n*log(x) + b*log(c) + a) + I*s
in(b*n*log(x) + b*log(c) + a)) + I*sqrt(2)*weierstrassPInverse(-4, 0, cos(b
*n*log(x) + b*log(c) + a) - I*sin(b*n*log(x) + b*log(c) + a)))/(b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(a + b\log\left(cx^{n}\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(sec(a + b*log(c*x**n)))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(b*log(c*x^n) + a))/x, x)
```

Mupad [B]
time = 2.57, size = 51, normalized size = 0.94

$$\frac{2\sqrt{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}\,\sqrt{\frac{1}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}}\,\mathrm{F}\left(\frac{a}{2}+\frac{b\,\ln\left(c\,x^n\right)}{2}\middle|2\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(a + b*log(c*x^n)))^(1/2)/x,x)
```

```
[Out] (2*cos(a + b*log(c*x^n))^(1/2)*(1/cos(a + b*log(c*x^n)))^(1/2)*ellipticF(a/
2 + (b*log(c*x^n))/2, 2))/(b*n)
```

3.268 $\int \sec^{\frac{3}{2}}(a+b\log(cx^n))\,dx$

Optimal. Leaf size=109

$$\frac{2x\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i}{bn}\right);\frac{1}{4}\left(7-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)\sec^{\frac{3}{2}}(a+b\log(cx^n))}{2+3ibn}$$

```
[Out] 2*x*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(3/2)*hypergeom([3/2, 3/4-1/2*I/b/n],[7/
4-1/2*I/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))*sec(a+b*ln(c*x^n))^(3/2)/(2+3*I*b
*n)
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4599, 4603, 371}

$$\frac{2x\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i}{bn}\right);\frac{1}{4}\left(7-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)\sec^{\frac{3}{2}}(a+b\log(cx^n))}{2+3ibn}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*Log[c*x^n]]^(3/2),x]
```

```
[Out] (2*x*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2)*Hypergeometric2F1[3/2, (3 -
(2*I)/(b*n))/4, (7 - (2*I)/(b*n))/4, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))]*Sec[
a + b*Log[c*x^n]]^(3/2))/(2 + (3*I)*b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4599

```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4603

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(a+b\log(cx^n))\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\sec^{\frac{3}{2}}(a+b\log(x))\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-\frac{3ib}{2}-\frac{1}{n}}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\sec^{\frac{3}{2}}(a+b\log(cx^n))\right)\text{Subst}\left(\int \frac{x^{-1+\frac{3ib}{2}}}{(1+e^{2ia}x^2}\right.}{n}$$

$$= \frac{2x\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2} {}_2F_1\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i}{bn}\right);\frac{1}{4}\left(7-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)\sec^{\frac{3}{2}}(a+}{2+3ibn}$$

Mathematica [A]
time = 9.75, size = 190, normalized size = 1.74

$$-\frac{2ix\left(-\sqrt{1+e^{2ia}(cx^n)^{2ib}}+\left(1+e^{2ia}(cx^n)^{2ib}\right){}_2F_1\left(\frac{1}{2},-\frac{2i+bn}{4bn};\frac{3}{4}-\frac{i}{2bn};-e^{2ia}(cx^n)^{2ib}\right)\right)}{bn\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\sqrt{\frac{e^{ia}(cx^n)^{ib}}{2+2e^{2ia}(cx^n)^{2ib}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*Log[c*x^n]]^(3/2),x]
[Out] ((-2*I)*x*(-Sqrt[1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)] + (1 + E^((2*I)*a)*(c*x
^n)^((2*I)*b))*Hypergeometric2F1[1/2, -1/4*(2*I + b*n)/(b*n), 3/4 - (I/2)/(
b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))]))/(b*n*(1 + E^((2*I)*a)*(c*x^n)^((2
*I)*b))^(3/2)*Sqrt[(E^(I*a)*(c*x^n)^(I*b))/(2 + 2*E^((2*I)*a)*(c*x^n)^((2*I
)*b))])
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \sec^{\frac{3}{2}}(a+b\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a+b*ln(c*x^n))^(3/2),x)
[Out] int(sec(a+b*ln(c*x^n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sec^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Integral(sec(a + b*log(c*x**n))**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(a + b*log(c*x^n)))^(3/2),x)
```

```
[Out] int((1/cos(a + b*log(c*x^n)))^(3/2), x)
```

3.269 $\int \frac{\sec^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=89

$$-\frac{2\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{2}\left(a+b\log\left(cx^n\right)\right)\middle|\,2\right)\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}}{bn}+\frac{2\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}\,\sin\left(a\right.}{bn}$$

```
[Out] 2*sin(a+b*ln(c*x^n))*sec(a+b*ln(c*x^n))^(1/2)/b/n-2*(cos(1/2*a+1/2*b*ln(c*x
^n))^2)^(1/2)/cos(1/2*a+1/2*b*ln(c*x^n))*EllipticE(sin(1/2*a+1/2*b*ln(c*x^n
)),2^(1/2))*cos(a+b*ln(c*x^n))^(1/2)*sec(a+b*ln(c*x^n))^(1/2)/b/n
```

Rubi [A]
time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3853, 3856, 2719}

$$\frac{2\sin\left(a+b\log\left(cx^n\right)\right)\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}}{bn}-\frac{2\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{2}\left(a+b\log\left(cx^n\right)\right)\middle|\,2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (-2*Sqrt[Cos[a + b*Log[c*x^n]]]*EllipticE[(a + b*Log[c*x^n])/2, 2]*Sqrt[Sec
[a + b*Log[c*x^n]]])/(b*n) + (2*Sqrt[Sec[a + b*Log[c*x^n]]]*Sin[a + b*Log[c
*x^n]])/(b*n)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\text{Subst}\left(\int \sec^{\frac{3}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2\sqrt{\sec(a+b\log(cx^n))}\,\sin(a+b\log(cx^n))}{bn} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx, x,\right.}{n}$$

$$= \frac{2\sqrt{\sec(a+b\log(cx^n))}\,\sin(a+b\log(cx^n))}{bn} - \frac{\left(\sqrt{\cos(a+b\log(cx^n))}\,\sqrt{\sec}\right.}{}$$

$$= -\frac{2\sqrt{\cos(a+b\log(cx^n))}\,E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)\sqrt{\sec(a+b\log(cx^n))}}{bn} + \frac{2}{}$$

Mathematica [A]
time = 0.15, size = 68, normalized size = 0.76

$$\frac{2\sqrt{\sec(a+b\log(cx^n))}\left(-\sqrt{\cos(a+b\log(cx^n))}\,E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)+\sin(a+b\log(cx^n))\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (2*Sqrt[Sec[a + b*Log[c*x^n]]]*(-(Sqrt[Cos[a + b*Log[c*x^n]]]*EllipticE[(a
+ b*Log[c*x^n])/2, 2]) + Sin[a + b*Log[c*x^n]]))/(b*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(119) = 238$.
time = 0.38, size = 250, normalized size = 2.81

method	result
derivativedivides	$-\frac{2\left(-2\cos\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+\sin^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)}\left(\sin^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+\right.}{n\sqrt{-2\left(\sin^4\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right.\right.}}$
default	$-\frac{2\left(-2\cos\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+\sin^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)}\left(\sin^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+\right.}{n\sqrt{-2\left(\sin^4\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -2/n*(-2*cos(1/2*a+1/2*b*ln(c*x^n))*(-2*sin(1/2*a+1/2*b*ln(c*x^n))^4+sin(1/
2*a+1/2*b*ln(c*x^n))^2)^(1/2)*sin(1/2*a+1/2*b*ln(c*x^n))^2+(sin(1/2*a+1/2*b
*ln(c*x^n))^2)^(1/2)*(2*sin(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)*(-2*sin(1/2*a
+1/2*b*ln(c*x^n))^4+sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*EllipticE(cos(1/2*a
+1/2*b*ln(c*x^n)),2^(1/2)))/(-2*sin(1/2*a+1/2*b*ln(c*x^n))^4+sin(1/2*a+1/2*
b*ln(c*x^n))^2)^(1/2)/sin(1/2*a+1/2*b*ln(c*x^n))/(2*cos(1/2*a+1/2*b*ln(c*x^
n))^2-1)^(1/2)/b
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)^(3/2)/x, x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.36, size = 112, normalized size = 1.26

$$\frac{-i\sqrt{2}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bn\log(x)+b\log(c)+a)+i\sin(bn\log(x)+b\log(c)+a)))+i\sqrt{2}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bn\log(x)+b\log(c)+a)-i\sin(bn\log(x)+b\log(c)+a)))+\frac{2\sin(bn\log(x)+b\log(c)+a)}{\sqrt{\cos(bn\log(x)+b\log(c)+a)}}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*n*log(x
) + b*log(c) + a) + I*sin(b*n*log(x) + b*log(c) + a))) + I*sqrt(2)*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*n*log(x) + b*log(c) + a) -
I*sin(b*n*log(x) + b*log(c) + a))) + 2*sin(b*n*log(x) + b*log(c) + a)/sqrt(
cos(b*n*log(x) + b*log(c) + a)))/(b*n)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**(3/2)/x,x)
```

```
[Out] Integral(sec(a + b*log(c*x**n))**(3/2)/x, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(a+b\,\ln(c\,x^n))}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(a + b*log(c*x^n)))^(3/2)/x,x)
```

```
[Out] int((1/cos(a + b*log(c*x^n)))^(3/2)/x, x)
```

3.270 $\int \sec^{\frac{5}{2}}(a+b\log(cx^n))\,dx$

Optimal. Leaf size=109

$$\frac{2x\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i}{bn}\right);\frac{1}{4}\left(9-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)\sec^{\frac{5}{2}}(a+b\log(cx^n))}{2+5ibn}$$

```
[Out] 2*x*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(5/2)*hypergeom([5/2, 5/4-1/2*I/b/n],[9/
4-1/2*I/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))*sec(a+b*ln(c*x^n))^(5/2)/(2+5*I*b
*n)
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4599, 4603, 371}

$$\frac{2x\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i}{bn}\right);\frac{1}{4}\left(9-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)\sec^{\frac{5}{2}}(a+b\log(cx^n))}{2+5ibn}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*Log[c*x^n]]^(5/2),x]
```

```
[Out] (2*x*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^(5/2)*Hypergeometric2F1[5/2, (5 -
(2*I)/(b*n))/4, (9 - (2*I)/(b*n))/4, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))]*Sec[
a + b*Log[c*x^n]]^(5/2))/(2 + (5*I)*b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4599

```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4603

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \sec^{\frac{5}{2}}(a+b\log(cx^n))\,dx &= \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\sec^{\frac{5}{2}}(a+b\log(x))\,dx,x,cx^n\right)}{n}\\
&= \frac{\left(x(cx^n)^{-\frac{5ib}{2}-\frac{1}{n}}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}\sec^{\frac{5}{2}}(a+b\log(cx^n))\right)\text{Subst}\left(\int \frac{x^{-1+\frac{5ib}{2}+}}{(1+e^{2ia}x^{2ib}}\right.}{n}\\
&= \frac{2x\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}{}_2F_1\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i}{bn}\right);\frac{1}{4}\left(9-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)\sec^{\frac{5}{2}}(a+}{2+5ibn}
\end{aligned}
$$

Mathematica [A]
time = 1.34, size = 124, normalized size = 1.14

$$
\frac{2x\sqrt{\sec(a+b\log(cx^n))}\left(-2+(2-ibn)\left(1+e^{2ia}(cx^n)^{2ib}\right){}_2F_1\left(1,\frac{3}{4}-\frac{i}{2bn};\frac{5}{4}-\frac{i}{2bn};-e^{2i(a+b\log(cx^n))}\right)+bn\tan(a+b\log(cx^n))\right)}{3b^2n^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*Log[c*x^n]]^(5/2),x]
```

```
[Out] (2*x*Sqrt[Sec[a + b*Log[c*x^n]]]*(-2 + (2 - I*b*n)*(1 + E^((2*I)*a)*(c*x^n)
^((2*I)*b))*Hypergeometric2F1[1, 3/4 - (I/2)/(b*n), 5/4 - (I/2)/(b*n), -E^(
(2*I)*(a + b*Log[c*x^n]))] + b*n*Tan[a + b*Log[c*x^n]]))/(3*b^2*n^2)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$
\int \sec^{\frac{5}{2}}(a+b\ln(c\,x^n))\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a+b*ln(c*x^n))^(5/2),x)
```

```
[Out] int(sec(a+b*ln(c*x^n))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(a + b*log(c*x^n)))^(5/2),x)
```

```
[Out] int((1/cos(a + b*log(c*x^n)))^(5/2), x)
```

3.271 $\int \frac{\sec^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=93

$$\frac{2\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}\,F\left(\frac{1}{2}(a+b\log\left(cx^n\right))\middle|2\right)\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}}{3bn}+\frac{2\sec^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\sin\left(a+b\log\right.}{3bn}$$

```
[Out] 2/3*sec(a+b*ln(c*x^n))^(3/2)*sin(a+b*ln(c*x^n))/b/n+2/3*(cos(1/2*a+1/2*b*ln
(c*x^n))^2)^(1/2)/cos(1/2*a+1/2*b*ln(c*x^n))*EllipticF(sin(1/2*a+1/2*b*ln(c
*x^n)),2^(1/2))*cos(a+b*ln(c*x^n))^(1/2)*sec(a+b*ln(c*x^n))^(1/2)/b/n
```

Rubi [A]
time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3853, 3856, 2720}

$$\frac{2\sin\left(a+b\log\left(cx^n\right)\right)\sec^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{2\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}\,F\left(\frac{1}{2}(a+b\log\left(cx^n\right))\middle|2\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (2*Sqrt[Cos[a + b*Log[c*x^n]]]*EllipticF[(a + b*Log[c*x^n])/2, 2]*Sqrt[Sec[
a + b*Log[c*x^n]]])/(3*b*n) + (2*Sec[a + b*Log[c*x^n]]^(3/2)*Sin[a + b*Log[
c*x^n]])/(3*b*n)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\operatorname{Subst}\left(\int \sec^{\frac{5}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2\sec^{\frac{3}{2}}(a+b\log(cx^n))\sin(a+b\log(cx^n))}{3bn} + \frac{\operatorname{Subst}\left(\int \sqrt{\sec(a+bx)}\,dx, x, \mathrm{l}\right.}{3n}$$

$$= \frac{2\sec^{\frac{3}{2}}(a+b\log(cx^n))\sin(a+b\log(cx^n))}{3bn} + \frac{\left(\sqrt{\cos(a+b\log(cx^n))}\sqrt{\sec(}\right.}{}$$

$$= \frac{2\sqrt{\cos(a+b\log(cx^n))}\,F\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)\sqrt{\sec(a+b\log(cx^n))}}{3bn} + \frac{2}{}$$

Mathematica [A]
time = 0.19, size = 69, normalized size = 0.74

$$\frac{2\sec^{\frac{3}{2}}(a+b\log(cx^n))\left(\cos^{\frac{3}{2}}(a+b\log(cx^n))\,F\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)+\sin(a+b\log(cx^n))\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (2*Sec[a + b*Log[c*x^n]]^(3/2)*(Cos[a + b*Log[c*x^n]]^(3/2)*EllipticF[(a +
b*Log[c*x^n])/2, 2] + Sin[a + b*Log[c*x^n]]))/(3*b*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(119) = 238$.
time = 0.35, size = 291, normalized size = 3.13

method	result
derivativedivides	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(a+b\ln(cx^n))}{2}}\sqrt{2\left(\sin^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right),\sqrt{2}\right)\right.}{}$
default	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(a+b\ln(cx^n))}{2}}\sqrt{2\left(\sin^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right),\sqrt{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a+b*ln(c*x^n))^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/n*(-2*(sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(2*sin(1/2*a+1/2*b*ln(c*x^n
))^2-1)^(1/2)*EllipticF(cos(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))*sin(1/2*a+1/2*b
*ln(c*x^n))^2-2*sin(1/2*a+1/2*b*ln(c*x^n))^2*cos(1/2*a+1/2*b*ln(c*x^n))+(si
n(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(2*sin(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)*
EllipticF(cos(1/2*a+1/2*b*ln(c*x^n)),2^(1/2)))*((2*cos(1/2*a+1/2*b*ln(c*x^n
))^2-1)*sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/(-2*sin(1/2*a+1/2*b*ln(c*x^n))^
4+sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/(2*cos(1/2*a+1/2*b*ln(c*x^n))^2-1)^(3
/2)/sin(1/2*a+1/2*b*ln(c*x^n))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)^(5/2)/x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.90, size = 145, normalized size = 1.56

$$\frac{-i\sqrt{2}\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+i\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)+i\sqrt{2}\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-i\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)+\frac{2\sin\left(bn\log(x)+b\log(c)+a\right)}{\sqrt{\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}}}{3\,bn\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*cos(b*n*log(x) + b*log(c) + a)*weierstrassPInverse(-4, 0, c
os(b*n*log(x) + b*log(c) + a) + I*sin(b*n*log(x) + b*log(c) + a)) + I*sqrt(
2)*cos(b*n*log(x) + b*log(c) + a)*weierstrassPInverse(-4, 0, cos(b*n*log(x)
 + b*log(c) + a) - I*sin(b*n*log(x) + b*log(c) + a)) + 2*sin(b*n*log(x) + b
*log(c) + a)/sqrt(cos(b*n*log(x) + b*log(c) + a)))/(b*n*cos(b*n*log(x) + b*
log(c) + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**(5/2)/x,x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{1}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}^{5/2}}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(a + b*log(c*x^n)))^(5/2)/x,x)
```

```
[Out] int((1/cos(a + b*log(c*x^n)))^(5/2)/x, x)
```

3.272 $\int \frac{1}{\sqrt{\sec(a+b\log(cx^n))}}\,dx$

Optimal. Leaf size=110

$$\frac{2x\,{}_2F_1\left(-\frac{1}{2},-\frac{2i+bn}{4bn};\frac{1}{4}\left(3-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{(2-ibn)\sqrt{1+e^{2ia}\,(cx^n)^{2ib}}\,\sqrt{\sec(a+b\log(cx^n))}}$$

```
[Out] 2*x*hypergeom([-1/2, 1/4*(-2*I-b*n)/b/n],[3/4-1/2*I/b/n],-exp(2*I*a)*(c*x^n
)^(2*I*b))/(2-I*b*n)/(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(1/2)/sec(a+b*ln(c*x^n)
)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4599, 4603, 371}

$$\frac{2x\,{}_2F_1\left(-\frac{1}{2},-\frac{bn+2i}{4bn};\frac{1}{4}\left(3-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{(2-ibn)\sqrt{1+e^{2ia}\,(cx^n)^{2ib}}\,\sqrt{\sec(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sec[a + b*Log[c*x^n]]],x]
```

```
[Out] (2*x*Hypergeometric2F1[-1/2, -1/4*(2*I + b*n)/(b*n), (3 - (2*I)/(b*n))/4, -
(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 - I*b*n)*Sqrt[1 + E^((2*I)*a)*(c*x^n)
^((2*I)*b)]*Sqrt[Sec[a + b*Log[c*x^n]]])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4599

```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4603

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
```

```
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}}\,dx &= \frac{\left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\sqrt{\sec(a+b\log(x))}}\,dx,x,cx^n\right)}{n}\\
&= \frac{\left(x\left(cx^n\right)^{\frac{ib}{2}-\frac{1}{n}}\right)\text{Subst}\left(\int x^{-1-\frac{ib}{2}+\frac{1}{n}}\sqrt{1+e^{2ia}x^{2ib}}\,dx,x,cx^n\right)}{n\sqrt{1+e^{2ia}\left(cx^n\right)^{2ib}}\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}}\\
&= \frac{2x\,{}_2F_1\left(-\frac{1}{2},-\frac{2i+bn}{4bn};\frac{1}{4}\left(3-\frac{2i}{bn}\right);-e^{2ia}\left(cx^n\right)^{2ib}\right)}{(2-ibn)\sqrt{1+e^{2ia}\left(cx^n\right)^{2ib}}\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}}
\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 364 vs. $2(110)=220$.
time = 4.88, size = 364, normalized size = 3.31

$$\frac{2i\sqrt{2}\,be^{-ia}nx\left(cx^n\right)^{-ib}\sqrt{\frac{e^{ia}\left(cx^n\right)^{ib}}{1+e^{2ia}\left(cx^n\right)^{2ib}}}\left((2i+bn)\left(1+e^{2ia}\left(cx^n\right)^{2ib}\right)+\sqrt{1+e^{2ia}\left(cx^n\right)^{2ib}}\left(-2i-bn+e^{2ia}(-2i+bn)x^{-2ibn}\left(cx^n\right)^{2ib}\right){}_2F_1\left(\frac{1}{2},-\frac{2i+bn}{4bn};\frac{3}{4}-\frac{i}{2bn};-e^{2ia}\left(cx^n\right)^{2ib}\right)\right)}{\left(4+b^2n^2\right)\left(-2i-bn+e^{2ia}(-2i+bn)x^{-2ibn}\left(cx^n\right)^{2ib}\right)}-\frac{2x\cos\left(a-bn\log(x)+b\log\left(cx^n\right)\right)}{\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}\left(-2\cos\left(a-bn\log(x)+b\log\left(cx^n\right)\right)+bn\sin\left(a-bn\log(x)+b\log\left(cx^n\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sec[a + b*Log[c*x^n]]],x]
```

```
[Out] ((2*I)*Sqrt[2]*b*n*x*Sqrt[(E^(I*a)*(c*x^n)^(I*b))/(1 + E^((2*I)*a)*(c*x^n)^
((2*I)*b))]*((2*I + b*n)*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)) + Sqrt[1 + E^(
(2*I)*a)*(c*x^n)^((2*I)*b)]*(-2*I - b*n + (E^((2*I)*a)*(-2*I + b*n)*(c*x^n)
^((2*I)*b))/x^((2*I)*b*n))*Hypergeometric2F1[1/2, -1/4*(2*I + b*n)/(b*n), 3
/4 - (I/2)/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))]))/(E^(I*a)*(4 + b^2*n^2
)*(c*x^n)^(I*b)*(-2*I - b*n + (E^((2*I)*a)*(-2*I + b*n)*(c*x^n)^((2*I)*b))/
x^((2*I)*b*n))) - (2*x*Cos[a - b*n*Log[x] + b*Log[c*x^n]])/(Sqrt[Sec[a + b*
Log[c*x^n]]]*(-2*Cos[a - b*n*Log[x] + b*Log[c*x^n]] + b*n*Sin[a - b*n*Log[x
] + b*Log[c*x^n]]))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sec\left(a+b\ln\left(c\,x^n\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(a+b*ln(c*x^n))^(1/2),x)
```

```
[Out] int(1/sec(a+b*ln(c*x^n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(sec(b*log(c*x^n) + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sec\left(a + b\log\left(cx^{n}\right)\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(1/sqrt(sec(a + b*log(c*x**n))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(sec(b*log(c*x^n) + a)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{1}{\cos \left(a+b \ln \left(c\, x^n\right)\right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(a + b*log(c*x^n)))^(1/2),x)
[Out] int(1/(1/cos(a + b*log(c*x^n)))^(1/2), x)
```

3.273 $\int \frac{1}{x\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}}\,dx$

Optimal. Leaf size=54

$$\frac{2\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{2}\left(a+b\log\left(cx^n\right)\right)\middle|\,2\right)\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}}{bn}$$

```
[Out] 2*(cos(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cos(1/2*a+1/2*b*ln(c*x^n))*EllipticE
(sin(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))*cos(a+b*ln(c*x^n))^(1/2)*sec(a+b*ln(c*
x^n))^(1/2)/b/n
```

Rubi [A]

time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3856, 2719}

$$\frac{2\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{2}\left(a+b\log\left(cx^n\right)\right)\middle|\,2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Sec[a + b*Log[c*x^n]]]),x]
```

```
[Out] (2*Sqrt[Cos[a + b*Log[c*x^n]]]*EllipticE[(a + b*Log[c*x^n])/2, 2]*Sqrt[Sec[
a + b*Log[c*x^n]]])/(b*n)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx, x, \log\left(cx^n\right)\right)}{n}\\
&= \frac{\left(\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}\right)\text{Subst}\left(\int \sqrt{\cos(a+bx)}\,d\right.}{n}\\
&= \frac{2\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{2}\left(a+b\log\left(cx^n\right)\right)\middle|\,2\right)\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}}{bn}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 54, normalized size = 1.00

$$\frac{2E\left(\frac{1}{2}(a+b\log\left(cx^{n}\right))\middle|2\right)}{bn\sqrt{\cos\left(a+b\log\left(cx^{n}\right)\right)}\sqrt{\sec\left(a+b\log\left(cx^{n}\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[Sec[a + b*Log[c*x^n]]]),x]
```

```
[Out] (2*EllipticE[(a + b*Log[c*x^n])/2, 2])/(b*n*Sqrt[Cos[a + b*Log[c*x^n]]]*Sqr
t[Sec[a + b*Log[c*x^n]]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(86) = 172.
time = 0.30, size = 181, normalized size = 3.35

method	result
derivativedivides	$\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{a}{2}+\frac{b\ln(c x^{n})}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{a}{2}+\frac{b\ln(c x^{n})}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(a+b\ln(c x^{n}))}{2}}\sqrt{-2\left(c\right.}}{n\sqrt{-2\left(\sin^{4}\left(\frac{a}{2}+\frac{b\ln(c x^{n})}{2}\right)\right)+\sin^{2}\left(\frac{a}{2}+\frac{b\ln(c x^{n})}{2}\right)}\sin\left(\frac{a}{2}+\frac{b\ln(c x^{n})}{2}\right)}$
default	$\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{a}{2}+\frac{b\ln(c x^{n})}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{a}{2}+\frac{b\ln(c x^{n})}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(a+b\ln(c x^{n}))}{2}}\sqrt{-2\left(c\right.}}{n\sqrt{-2\left(\sin^{4}\left(\frac{a}{2}+\frac{b\ln(c x^{n})}{2}\right)\right)+\sin^{2}\left(\frac{a}{2}+\frac{b\ln(c x^{n})}{2}\right)}\sin\left(\frac{a}{2}+\frac{b\ln(c x^{n})}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/sec(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/n*((2*cos(1/2*a+1/2*b*ln(c*x^n))^2-1)*sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)
*(sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cos(1/2*a+1/2*b*ln(c*x^n))^2+1)^(
1/2)*EllipticE(cos(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/(-2*sin(1/2*a+1/2*b*ln(c
*x^n))^4+sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/sin(1/2*a+1/2*b*ln(c*x^n))/(2*
cos(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sec(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt(sec(b*log(c*x^n) + a))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.51, size = 84, normalized size = 1.56

$$\frac{i\sqrt{2}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bn\log(x)+b\log(c)+a)+i\sin(bn\log(x)+b\log(c)+a)))-i\sqrt{2}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bn\log(x)+b\log(c)+a)-i\sin(bn\log(x)+b\log(c)+a)))}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sec(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] (I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*n*log(x)
 + b*log(c) + a) + I*sin(b*n*log(x) + b*log(c) + a))) - I*sqrt(2)*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*n*log(x) + b*log(c) + a) - I
*sin(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sec(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(sec(a + b*log(c*x**n)))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sec(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sqrt(sec(b*log(c*x^n) + a))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\sqrt{\frac{1}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1/cos(a + b*log(c*x^n)))^(1/2)),x)
```

```
[Out] int(1/(x*(1/cos(a + b*log(c*x^n)))^(1/2)), x)
```

3.274 $\int \frac{1}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=109

$$\frac{2x\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{2i}{bn}\right);\frac{1}{4}\left(1-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{(2-3ibn)\left(1+e^{2ia}\left(cx^n\right)^{2ib}\right)^{3/2}\sec^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

```
[Out] 2*x*hypergeom([-3/2, -3/4-1/2*I/b/n],[1/4-1/2*I/b/n],-exp(2*I*a)*(c*x^n)^(2
*I*b))/(2-3*I*b*n)/(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(3/2)/sec(a+b*ln(c*x^n))^
(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4599, 4603, 371}

$$\frac{2x\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{2i}{bn}\right);\frac{1}{4}\left(1-\frac{2i}{bn}\right);-e^{2ia}(cx^n)^{2ib}\right)}{(2-3ibn)\left(1+e^{2ia}\left(cx^n\right)^{2ib}\right)^{3/2}\sec^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*Log[c*x^n]]^(-3/2),x]
```

```
[Out] (2*x*Hypergeometric2F1[-3/2, (-3 - (2*I)/(b*n))/4, (1 - (2*I)/(b*n))/4, -(E
^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 - (3*I)*b*n)*(1 + E^((2*I)*a)*(c*x^n)^(
(2*I)*b))^(3/2)*Sec[a + b*Log[c*x^n]]^(3/2))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4599

```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4603

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
```

```
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\sec^{\frac{3}{2}}(a+b\log(x))}\,dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{\frac{3ib}{2}-\frac{1}{n}}\right)\operatorname{Subst}\left(\int x^{-1-\frac{3ib}{2}+\frac{1}{n}}\left(1+e^{2ia}x^{2ib}\right)^{3/2}\,dx, x, cx^n\right)}{n\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\sec^{\frac{3}{2}}(a+b\log(cx^n))}$$

$$= \frac{2x\,{}_2F_1\left(-\frac{3}{2}, \frac{1}{4}\left(-3-\frac{2i}{bn}\right); \frac{1}{4}\left(1-\frac{2i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right)}{(2-3ibn)\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\sec^{\frac{3}{2}}(a+b\log(cx^n))}$$

Mathematica [A]
time = 1.52, size = 168, normalized size = 1.54

$$\frac{2x\left(3b^2n^2\left(1+e^{2ia}(cx^n)^{2ib}\right){}_2F_1\left(1, \frac{3}{4}-\frac{i}{2bn}; \frac{5}{4}-\frac{i}{2bn}; -e^{2i(a+b\log(cx^n))}\right)\sec^2(a+b\log(cx^n))+(2+ibn)(2+3bn\tan(a+b\log(cx^n)))\right)}{(2+3ibn)(-2i+bn)(2i+3bn)\sec^{\frac{3}{2}}(a+b\log(cx^n))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*Log[c*x^n]]^(-3/2),x]
```

```
[Out] (2*x*(3*b^2*n^2*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))*Hypergeometric2F1[1, 3/
4 - (I/2)/(b*n), 5/4 - (I/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))]*Sec[a +
b*Log[c*x^n]]^2 + (2 + I*b*n)*(2 + 3*b*n*Tan[a + b*Log[c*x^n]])))/((2 + (3*
I)*b*n)*(-2*I + b*n)*(2*I + 3*b*n)*Sec[a + b*Log[c*x^n]]^(3/2))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec\left(a+b\ln\left(c\,x^n\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(a+b*ln(c*x^n))^(3/2),x)
```

```
[Out] int(1/sec(a+b*ln(c*x^n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
[Out] integrate(sec(b*log(c*x^n) + a)^(-3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{3}{2}}\left(a + b\log\left(cx^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(a+b*ln(c*x**n))**(3/2),x)
[Out] Integral(sec(a + b*log(c*x**n))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
[Out] integrate(sec(b*log(c*x^n) + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{1}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(a + b*log(c*x^n)))^(3/2),x)
[Out] int(1/(1/cos(a + b*log(c*x^n)))^(3/2), x)
```

3.275 $\int \frac{1}{x \sec^{\frac{3}{2}}(a+b\log(cx^n))} \, dx$

Optimal. Leaf size=93

$$\frac{2\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}\, F\left(\frac{1}{2}\left(a+b\log\left(cx^n\right)\right)\middle|\, 2\right)\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}}{3bn}+\frac{2\sin\left(a+b\log\left(cx^n\right)\right)}{3bn\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] 2/3*sin(a+b*ln(c*x^n))/b/n/sec(a+b*ln(c*x^n))^(1/2)+2/3*(cos(1/2*a+1/2*b*ln
(c*x^n))^2)^(1/2)/cos(1/2*a+1/2*b*ln(c*x^n))*EllipticF(sin(1/2*a+1/2*b*ln(c
*x^n)),2^(1/2))*cos(a+b*ln(c*x^n))^(1/2)*sec(a+b*ln(c*x^n))^(1/2)/b/n
```

Rubi [A]

time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3854, 3856, 2720}

$$\frac{2\sin\left(a+b\log\left(cx^n\right)\right)}{3bn\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}}+\frac{2\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}\, F\left(\frac{1}{2}\left(a+b\log\left(cx^n\right)\right)\middle|\, 2\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sec[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (2*Sqrt[Cos[a + b*Log[c*x^n]]]*EllipticF[(a + b*Log[c*x^n])/2, 2]*Sqrt[Sec[
a + b*Log[c*x^n]]])/(3*b*n) + (2*Sin[a + b*Log[c*x^n]])/(3*b*n*Sqrt[Sec[a +
 b*Log[c*x^n]]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{x\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$
$$= \frac{2\sin(a+b\log(cx^n))}{3bn\sqrt{\sec(a+b\log(cx^n))}} + \frac{\text{Subst}\left(\int \sqrt{\sec(a+bx)}\,dx, x, \log(cx^n)\right)}{3n}$$
$$= \frac{2\sin(a+b\log(cx^n))}{3bn\sqrt{\sec(a+b\log(cx^n))}} + \frac{\left(\sqrt{\cos(a+b\log(cx^n))}\sqrt{\sec(a+b\log(cx^n}}{3}$$
$$= \frac{2\sqrt{\cos(a+b\log(cx^n))}\,F\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)\sqrt{\sec(a+b\log(cx^n))}}{3bn} +$$

Mathematica [A]
time = 0.17, size = 72, normalized size = 0.77

$$\frac{\sqrt{\sec(a+b\log(cx^n))}\left(2\sqrt{\cos(a+b\log(cx^n))}\,F\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)+\sin(2(a+b\log(cx^n)))\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sec[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (Sqrt[Sec[a + b*Log[c*x^n]]]*(2*Sqrt[Cos[a + b*Log[c*x^n]]]*EllipticF[(a +
b*Log[c*x^n])/2, 2] + Sin[2*(a + b*Log[c*x^n])]))/(3*b*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(119) = 238$.
time = 0.33, size = 247, normalized size = 2.66

method	result
derivativedivides	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\left(4\cos\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\left(\sin^4\left(\frac{a}{2}+\frac{b\ln(c}{2}\right.\right.\right.}{3n\sqrt{-2\left(\sin^4\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+}}$
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\left(4\cos\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\left(\sin^4\left(\frac{a}{2}+\frac{b\ln(c}{2}\right.\right.\right.}{3n\sqrt{-2\left(\sin^4\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/sec(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)

[Out] -2/3/n*((2*cos(1/2*a+1/2*b*ln(c*x^n))^2-1)*sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(4*cos(1/2*a+1/2*b*ln(c*x^n))*sin(1/2*a+1/2*b*ln(c*x^n))^4-2*sin(1/2*a+1/2*b*ln(c*x^n))^2*cos(1/2*a+1/2*b*ln(c*x^n))+(sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(2*sin(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)*EllipticF(cos(1/2*a+1/2*b*ln(c*x^n)),2^(1/2)))/(-2*sin(1/2*a+1/2*b*ln(c*x^n))^4+sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/sin(1/2*a+1/2*b*ln(c*x^n))/(2*cos(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/sec(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")

[Out] integrate(1/(x*sec(b*log(c*x^n) + a)^(3/2)), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.51, size = 107, normalized size = 1.15

$$\frac{2 \sqrt{\cos\left(bn\log\left(x\right) + b\log\left(c\right) + a\right)} \sin\left(bn\log\left(x\right) + b\log\left(c\right) + a\right) - i\sqrt{2}\,\mathrm{weierstrassPInverse}\left(-4, 0, \cos\left(bn\log\left(x\right) + b\log\left(c\right) + a\right) + i\sin\left(bn\log\left(x\right) + b\log\left(c\right) + a\right)\right) + i\sqrt{2}\,\mathrm{weierstrassPInverse}\left(-4, 0, \cos\left(bn\log\left(x\right) + b\log\left(c\right) + a\right) - i\sin\left(bn\log\left(x\right) + b\log\left(c\right) + a\right)\right)}{3\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/sec(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")

[Out] 1/3*(2*sqrt(cos(b*n*log(x) + b*log(c) + a))*sin(b*n*log(x) + b*log(c) + a) - I*sqrt(2)*weierstrassPInverse(-4, 0, cos(b*n*log(x) + b*log(c) + a) + I*sin(b*n*log(x) + b*log(c) + a)) + I*sqrt(2)*weierstrassPInverse(-4, 0, cos(b*n*log(x) + b*log(c) + a) - I*sin(b*n*log(x) + b*log(c) + a)))/(b*n)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sec^{\frac{3}{2}}\left(a + b\log\left(cx^{n}\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/sec(a+b*ln(c*x**n))**(3/2),x)

[Out] Integral(1/(x*sec(a + b*log(c*x**n))**(3/2)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sec(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
[Out] integrate(1/(x*sec(b*log(c*x^n) + a)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(\frac{1}{\cos(a+b\,\ln(c\,x^n))}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1/cos(a + b*log(c*x^n)))^(3/2)),x)
[Out] int(1/(x*(1/cos(a + b*log(c*x^n)))^(3/2)), x)
```

3.276 $\int \frac{1}{\sec^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=110

$$\frac{2x\,{}_2F_1\left(-\frac{5}{2},\frac{1}{4}\left(-5-\frac{2i}{bn}\right);-\frac{2i+bn}{4bn};-e^{2ia}(cx^n)^{2ib}\right)}{(2-5ibn)\left(1+e^{2ia}\left(cx^n\right)^{2ib}\right)^{5/2}\sec^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

```
[Out] 2*x*hypergeom([-5/2, -5/4-1/2*I/b/n],[1/4*(-2*I-b*n)/b/n],-exp(2*I*a)*(c*x^
n)^(2*I*b))/(2-5*I*b*n)/(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(5/2)/sec(a+b*ln(c*x
^n))^(5/2)
```

Rubi [A]
time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4599, 4603, 371}

$$\frac{2x\,{}_2F_1\left(-\frac{5}{2},\frac{1}{4}\left(-5-\frac{2i}{bn}\right);-\frac{bn+2i}{4bn};-e^{2ia}(cx^n)^{2ib}\right)}{(2-5ibn)\left(1+e^{2ia}\left(cx^n\right)^{2ib}\right)^{5/2}\sec^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*Log[c*x^n]]^(-5/2),x]
```

```
[Out] (2*x*Hypergeometric2F1[-5/2, (-5 - (2*I)/(b*n))/4, -1/4*(2*I + b*n)/(b*n),
-(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 - (5*I)*b*n)*(1 + E^((2*I)*a)*(c*x^n
)^((2*I)*b))^(5/2)*Sec[a + b*Log[c*x^n]]^(5/2))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4599

```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4603

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
```

```
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\sec^{\frac{5}{2}}(a+b\log(x))}\,dx, x, cx^n\right)}{n}$$
$$= \frac{\left(x(cx^n)^{\frac{5ib}{2}-\frac{1}{n}}\right)\text{Subst}\left(\int x^{-1-\frac{5ib}{2}+\frac{1}{n}}\left(1+e^{2ia}x^{2ib}\right)^{5/2}\,dx, x, cx^n\right)}{n\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}\sec^{\frac{5}{2}}(a+b\log(cx^n))}$$
$$= \frac{2x\,{}_2F_1\left(-\frac{5}{2}, \frac{1}{4}\left(-5-\frac{2i}{bn}\right); -\frac{2i+bn}{4bn}; -e^{2ia}(cx^n)^{2ib}\right)}{(2-5ibn)\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}\sec^{\frac{5}{2}}(a+b\log(cx^n))}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 861 vs. 2(110) = 220.
time = 8.84, size = 861, normalized size = 7.83

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[a + b*Log[c*x^n]]^(-5/2),x]
```

```
[Out] ((30*I)*Sqrt[2]*b^3*n^3*x^(1 - I*b*n)*Sqrt[(E^(I*(a + b*(-(n*Log[x]) + Log[
c*x^n])))*x^(I*b*n))/(1 + E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n])))*x^((
2*I)*b*n))]*((2*I + b*n)*(1 + E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n])))*
x^((2*I)*b*n)) + (-2*I - b*n + E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n])))
*(-2*I + b*n))*Sqrt[1 + E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n])))*x^((2*
I)*b*n)]*Hypergeometric2F1[1/2, -1/4*(2*I + b*n)/(b*n), 3/4 - (I/2)/(b*n),
-(E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n])))*x^((2*I)*b*n))]))/(E^(I*(a +
 b*(-(n*Log[x]) + Log[c*x^n])))*(-2*I + 5*b*n)*(2*I + 5*b*n)*(4 + b^2*n^2)*
(-2*I - b*n + E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n])))*(-2*I + b*n))) +
 Sqrt[Sec[a + b*n*Log[x] + b*(-(n*Log[x]) + Log[c*x^n])]]*(-1/4*(x*Cos[b*n*
Log[x]]*(12 + 55*b^2*n^2 + 12*Cos[2*(a + b*(-(n*Log[x]) + Log[c*x^n]))] + 6
5*b^2*n^2*Cos[2*(a + b*(-(n*Log[x]) + Log[c*x^n]))] + 4*b*n*Sin[2*(a + b*(-
(n*Log[x]) + Log[c*x^n]))]))/((-2*I + 5*b*n)*(2*I + 5*b*n)*(-2*Cos[a + b*(-
(n*Log[x]) + Log[c*x^n])] + b*n*Sin[a + b*(-(n*Log[x]) + Log[c*x^n])])) + (
x*Sin[b*n*Log[x]]*(-16*b*n - 4*b*n*Cos[2*(a + b*(-(n*Log[x]) + Log[c*x^n]))
] + 12*Sin[2*(a + b*(-(n*Log[x]) + Log[c*x^n]))] + 65*b^2*n^2*Sin[2*(a + b*
```

```
(-(n*Log[x]) + Log[c*x^n]))]))/(4*(-2*I + 5*b*n)*(2*I + 5*b*n)*(-2*Cos[a +
b*(-(n*Log[x]) + Log[c*x^n])] + b*n*Sin[a + b*(-(n*Log[x]) + Log[c*x^n])]))
 + (x*Sin[3*b*n*Log[x]]*(5*b*n*Cos[3*(a + b*(-(n*Log[x]) + Log[c*x^n]))] -
2*Sin[3*(a + b*(-(n*Log[x]) + Log[c*x^n]))]))/(2*(-2*I + 5*b*n)*(2*I + 5*b*
n)) + (x*Cos[3*b*n*Log[x]]*(2*Cos[3*(a + b*(-(n*Log[x]) + Log[c*x^n]))] + 5
*b*n*Sin[3*(a + b*(-(n*Log[x]) + Log[c*x^n]))]))/(2*(-2*I + 5*b*n)*(2*I + 5
*b*n)))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec \left(a + b \ln \left(c\, x^{n}\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(a+b*ln(c*x^n))^(5/2),x)
```

```
[Out] int(1/sec(a+b*ln(c*x^n))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)^(-5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{1}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(a + b*log(c*x^n)))^(5/2),x)
```

```
[Out] int(1/(1/cos(a + b*log(c*x^n)))^(5/2), x)
```

3.277 $\int \frac{1}{x\sec^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=93

$$\frac{6\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{2}\left(a+b\log\left(cx^n\right)\right)\middle|2\right)\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}}{5bn}+\frac{2\sin\left(a+b\log\left(cx^n\right)\right)}{5bn\sec^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

```
[Out] 2/5*sin(a+b*ln(c*x^n))/b/n/sec(a+b*ln(c*x^n))^(3/2)+6/5*(cos(1/2*a+1/2*b*ln
(c*x^n))^2)^(1/2)/cos(1/2*a+1/2*b*ln(c*x^n))*EllipticE(sin(1/2*a+1/2*b*ln(c
*x^n)),2^(1/2))*cos(a+b*ln(c*x^n))^(1/2)*sec(a+b*ln(c*x^n))^(1/2)/b/n
```

Rubi [A]

time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3854, 3856, 2719}

$$\frac{2\sin\left(a+b\log\left(cx^n\right)\right)}{5bn\sec^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}+\frac{6\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{2}\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{5bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sec[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (6*Sqrt[Cos[a + b*Log[c*x^n]]]*EllipticE[(a + b*Log[c*x^n])/2, 2]*Sqrt[Sec[
a + b*Log[c*x^n]]])/(5*b*n) + (2*Sin[a + b*Log[c*x^n]])/(5*b*n*Sec[a + b*Lo
g[c*x^n]]^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{x \sec^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sec^{\frac{5}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2\sin(a+b\log(cx^n))}{5bn\sec^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx, x, \log(cx^n)\right)}{5n}$$

$$= \frac{2\sin(a+b\log(cx^n))}{5bn\sec^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{\left(3\sqrt{\cos(a+b\log(cx^n))}\sqrt{\sec(a+b\log(cx^n)}\right.}{5n}$$

$$= \frac{6\sqrt{\cos(a+b\log(cx^n))}\,E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)\sqrt{\sec(a+b\log(cx^n))}}{5bn} +$$

Mathematica [A]
time = 0.19, size = 83, normalized size = 0.89

$$\frac{\sqrt{\sec(a+b\log(cx^n))}\left(12\sqrt{\cos(a+b\log(cx^n))}\,E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)+\sin(a+b\log(cx^n))+\sin(3(a+b\log(cx^n)))\right)}{10bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sec[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (Sqrt[Sec[a + b*Log[c*x^n]]]*(12*Sqrt[Cos[a + b*Log[c*x^n]]]*EllipticE[(a +
 b*Log[c*x^n])/2, 2] + Sin[a + b*Log[c*x^n]] + Sin[3*(a + b*Log[c*x^n])]))/
(10*b*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(119) = 238$.
time = 0.36, size = 280, normalized size = 3.01

method	result
derivativedivides	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\left(-8\cos\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\left(\sin^6\left(\frac{a}{2}+\frac{b\ln}{}\right.\right.\right.}{5n\sqrt{-2\left(\sin^4\right.}}$
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\left(-8\cos\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\left(\sin^6\left(\frac{a}{2}+\frac{b\ln}{}\right.\right.\right.}{5n\sqrt{-2\left(\sin^4\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/sec(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/n*((2*cos(1/2*a+1/2*b*ln(c*x^n))^2-1)*sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1
/2)*(-8*cos(1/2*a+1/2*b*ln(c*x^n))*sin(1/2*a+1/2*b*ln(c*x^n))^6+8*cos(1/2*a
+1/2*b*ln(c*x^n))*sin(1/2*a+1/2*b*ln(c*x^n))^4-2*sin(1/2*a+1/2*b*ln(c*x^n))
^2*cos(1/2*a+1/2*b*ln(c*x^n))-3*(sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(2*sin
(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)*EllipticE(cos(1/2*a+1/2*b*ln(c*x^n)),2^(
1/2)))/(-2*sin(1/2*a+1/2*b*ln(c*x^n))^4+sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)
/sin(1/2*a+1/2*b*ln(c*x^n))/(2*cos(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sec(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sec(b*log(c*x^n) + a)^(5/2)), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.70, size = 113, normalized size = 1.22

$$\frac{2 \cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{\frac{3}{2}}\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+3i\sqrt{2}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+i\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\right)-3i\sqrt{2}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-i\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\right)}{5\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sec(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*cos(b*n*log(x) + b*log(c) + a)^(3/2)*sin(b*n*log(x) + b*log(c) + a)
+ 3*I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*n*log
(x) + b*log(c) + a) + I*sin(b*n*log(x) + b*log(c) + a))) - 3*I*sqrt(2)*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*n*log(x) + b*log(c) + a
) - I*sin(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sec(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sec(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
[Out] integrate(1/(x*sec(b*log(c*x^n) + a)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(\frac{1}{\cos(a+b\,\ln(c\,x^n))}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1/cos(a + b*log(c*x^n)))^(5/2)),x)
[Out] int(1/(x*(1/cos(a + b*log(c*x^n)))^(5/2)), x)
```

3.278 $\int x^m \sec^3(a + b\log(cx^n))\,dx$

Optimal. Leaf size=102

$$\frac{8e^{3ia}x^{1+m}(cx^n)^{3ib}\,{}_2F_1\left(3,-\frac{i(1+m)-3bn}{2bn};-\frac{i(1+m)-5bn}{2bn};-e^{2ia}(cx^n)^{2ib}\right)}{1+m+3ibn}$$

```
[Out] 8*exp(3*I*a)*x^(1+m)*(c*x^n)^(3*I*b)*hypergeom([3, 1/2*(-I*(1+m)+3*b*n)/b/n
],[1/2*(-I*(1+m)+5*b*n)/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))/(1+m+3*I*b*n)
```

Rubi [A]
time = 0.06, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4605, 4601, 371}

$$\frac{8e^{3ia}x^{m+1}(cx^n)^{3ib}\,{}_2F_1\left(3,-\frac{i(m+1)-3bn}{2bn};-\frac{i(m+1)-5bn}{2bn};-e^{2ia}(cx^n)^{2ib}\right)}{3ibn+m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sec[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (8*E^((3*I)*a)*x^(1 + m)*(c*x^n)^((3*I)*b)*Hypergeometric2F1[3, -1/2*(I*(1
+ m) - 3*b*n)/(b*n), -1/2*(I*(1 + m) - 5*b*n)/(b*n), -(E^((2*I)*a)*(c*x^n)^
((2*I)*b))])/(1 + m + (3*I)*b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4605

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^m \sec^3(a+b\log(cx^n))\,dx = \frac{\left(x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1+m}{n}}\sec^3(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(8e^{3ia}x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\operatorname{Subst}\left(\int \frac{x^{-1+3ib+\frac{1+m}{n}}}{(1+e^{2ia}x^{2ib})^3}\,dx,x,cx^n\right)}{n}$$
$$= \frac{8e^{3ia}x^{1+m}(cx^n)^{3ib}\ {}_2F_1\left(3,-\frac{i(1+m)-3bn}{2bn};-\frac{i(1+m)-5bn}{2bn};-e^{2ia}(cx^n)^{2ib}\right)}{1+m+3ibn}$$

Mathematica [**A**]
time = 7.48, size = 139, normalized size = 1.36

$$\frac{x^{1+m}\left(4e^{ia}(1+m-ibn)(cx^n)^{ib}\ {}_2F_1\left(1,\frac{-i-im+bn}{2bn};-\frac{i(1+m+3ibn)}{2bn};-e^{2i(a+b\log(cx^n))}\right)-2\sec(a+b\log(cx^n))(1+m-bn\tan(a+b\log(cx^n)))\right)}{4b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sec[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (x^(1 + m)*(4*E^(I*a)*(1 + m - I*b*n)*(c*x^n)^(I*b)*Hypergeometric2F1[1, (-
I - I*m + b*n)/(2*b*n), ((-1/2*I)*(1 + m + (3*I)*b*n))/(b*n), -E^((2*I)*(a
+ b*Log[c*x^n]))] - 2*Sec[a + b*Log[c*x^n]]*(1 + m - b*n*Tan[a + b*Log[c*x^
n]])))/(4*b^2*n^2)
```

Maple [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$\int x^m\left(\sec^3(a+b\ln(c\,x^n))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sec(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(x^m*sec(a+b*ln(c*x^n))^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] -((b*n*sin(b*log(c)) + m*cos(b*log(c)) + cos(b*log(c)))*x*x^m*cos(b*log(x^n
) + a) + (b*n*cos(b*log(c)) - m*sin(b*log(c)) - sin(b*log(c)))*x*x^m*sin(b*
```

```
log(x^n) + a) + (((cos(4*b*log(c))*cos(3*b*log(c)) + sin(4*b*log(c))*sin(3*
b*log(c)))*m + (b*cos(3*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(3
*b*log(c)))*n + cos(4*b*log(c))*cos(3*b*log(c)) + sin(4*b*log(c))*sin(3*b*l
og(c)))*x*x^m*cos(3*b*log(x^n) + 3*a) + ((cos(4*b*log(c))*cos(b*log(c)) + s
in(4*b*log(c))*sin(b*log(c)))*m - (b*cos(b*log(c))*sin(4*b*log(c)) - b*cos(
4*b*log(c))*sin(b*log(c)))*n + cos(4*b*log(c))*cos(b*log(c)) + sin(4*b*log(
c))*sin(b*log(c)))*x*x^m*cos(b*log(x^n) + a) + ((cos(3*b*log(c))*sin(4*b*lo
g(c)) - cos(4*b*log(c))*sin(3*b*log(c)))*m - (b*cos(4*b*log(c))*cos(3*b*log
(c)) + b*sin(4*b*log(c))*sin(3*b*log(c)))*n + cos(3*b*log(c))*sin(4*b*log(c
)) - cos(4*b*log(c))*sin(3*b*log(c)))*x*x^m*sin(3*b*log(x^n) + 3*a) + ((cos
(b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(b*log(c)))*m + (b*cos(4*b*
log(c))*cos(b*log(c)) + b*sin(4*b*log(c))*sin(b*log(c)))*n + cos(b*log(c))*
sin(4*b*log(c)) - cos(4*b*log(c))*sin(b*log(c)))*x*x^m*sin(b*log(x^n) + a))
*cos(4*b*log(x^n) + 4*a) + (2*((cos(3*b*log(c))*cos(2*b*log(c)) + sin(3*b*l
og(c))*sin(2*b*log(c)))*m - (b*cos(2*b*log(c))*sin(3*b*log(c)) - b*cos(3*b*
log(c))*sin(2*b*log(c)))*n + cos(3*b*log(c))*cos(2*b*log(c)) + sin(3*b*log(
c))*sin(2*b*log(c)))*x*x^m*cos(2*b*log(x^n) + 2*a) + 2*((cos(2*b*log(c))*si
n(3*b*log(c)) - cos(3*b*log(c))*sin(2*b*log(c)))*m + (b*cos(3*b*log(c))*cos
(2*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*log(c)))*n + cos(2*b*log(c))*sin(3
*b*log(c)) - cos(3*b*log(c))*sin(2*b*log(c)))*x*x^m*sin(2*b*log(x^n) + 2*a)
 - (b*n*sin(3*b*log(c)) - m*cos(3*b*log(c)) - cos(3*b*log(c)))*x*x^m)*cos(3
*b*log(x^n) + 3*a) + 2*(((cos(2*b*log(c))*cos(b*log(c)) + sin(2*b*log(c))*s
in(b*log(c)))*m - (b*cos(b*log(c))*sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(
b*log(c)))*n + cos(2*b*log(c))*cos(b*log(c)) + sin(2*b*log(c))*sin(b*log(c)
))*x*x^m*cos(b*log(x^n) + a) + ((cos(b*log(c))*sin(2*b*log(c)) - cos(2*b*lo
g(c))*sin(b*log(c)))*m + (b*cos(2*b*log(c))*cos(b*log(c)) + b*sin(2*b*log(c
))*sin(b*log(c)))*n + cos(b*log(c))*sin(2*b*log(c)) - cos(2*b*log(c))*sin(b
*log(c)))*x*x^m*sin(b*log(x^n) + a))*cos(2*b*log(x^n) + 2*a) - (b^4*n^4*cos
(b*log(c)) + (b^2*m^2*cos(b*log(c)) + 2*b^2*m*cos(b*log(c)) + b^2*cos(b*log
(c)))*n^2 + ((b^4*cos(4*b*log(c))^2*cos(b*log(c)) + b^4*cos(b*log(c))*sin(4
*b*log(c))^2)*n^4 + (b^2*cos(4*b*log(c))^2*cos(b*log(c)) + b^2*cos(b*log(c)
)*sin(4*b*log(c))^2 + (b^2*cos(4*b*log(c))^2*cos(b*log(c)) + b^2*cos(b*log(
c))*sin(4*b*log(c))^2)*m^2 + 2*(b^2*cos(4*b*log(c))^2*cos(b*log(c)) + b^2*c
os(b*log(c))*sin(4*b*log(c))^2)*m)*n^2)*cos(4*b*log(x^n) + 4*a)^2 + 4*((b^4
*cos(2*b*log(c))^2*cos(b*log(c)) + b^4*cos(b*log(c))*sin(2*b*log(c))^2)*n^4
 + (b^2*cos(2*b*log(c))^2*cos(b*log(c)) + b^2*cos(b*log(c))*sin(2*b*log(c))
^2 + (b^2*cos(2*b*log(c))^2*cos(b*log(c)) + b^2*cos(b*log(c))*sin(2*b*log(c
))^2)*m^2 + 2*(b^2*cos(2*b*log(c))^2*cos(b*log(c)) + b^2*cos(b*log(c))*sin(
2*b*log(c))^2)*m)*n^2)*cos(2*b*log(x^n) + 2*a)^2 + ((b^4*cos(4*b*log(c))^2*
cos(b*log(c)) + b^4*cos(b*log(c))*sin(4*b*log(c))^2)*n^4 + (b^2*cos(4*b*log
(c))^2*cos(b*log(c)) + b^2*cos(b*log(c))*sin(4*b*log(c))^2 + (b^2*cos(4*b*l
og(c))^2*cos(b*log(c)) + b^2*cos(b*log(c))*sin(4*b*log(c))^2)*m^2 + 2*(b^2*
cos(4*b*log(c))^2*cos(b*log(c)) + b^2*cos(b*log(c))*sin(4*b*log(c))^2)*m)*n
^2)*sin(4*b*log(x^n) + 4*a)^2 + 4*((b^4*cos(2*b*log(c))^2*cos(b*log(c)) + b
^4*cos(b*log(c))*sin(2*b*log(c))^2)*n^4 + (b^2*cos(2*b*log(c))^2*cos(b*log(
```

```
c)) + b^2*cos(b*log(c))*sin(2*b*log(c))^2 + (b^2*cos(2*b*log(c))^2*cos(b*lo
g(c)) + b^2*cos(b*log(c))*sin(2*b*log(c))^2)*m^2 + 2*(b^2*cos(2*b*log(c))^2
*cos(b*log(c)) + b^2*cos(b*log(c))*sin(2*b*log(c))^2)*m)*n^2)*sin(2*b*log(x
^n) + 2*a)^2 + 2*(b^4*n^4*cos(4*b*log(c))*cos(b*log(c)) + (b^2*m^2*cos(4*b*
log(c))*cos(b*log(c)) + 2*b^2*m*cos(4*b*log(c))*cos(b*log(c)) + b^2*cos(4*b
*log(c))*cos(b*log(c)))*n^2 + 2*((b^4*cos(4*b*log(c))*cos(2*b*log(c))*cos(b
*log(c)) + b^4*cos(b*log(c))*sin(4*b*log(c))*sin(2*b*log(c)))*n^4 + (b^2*co
s(4*b*log(c))*cos(2*b*log(c))*cos(b*log(c)) + b^2*cos(b*log(c))*sin(4*b*log
(c))*sin(2*b*log(c)) + (b^2*cos(4*b*log(c))*cos(2*b*log(c))*cos(b*log(c)) +
 b^2*cos(b*log(c))*sin(4*b*log(c))*sin(2*b*log(c)))*m^2 + 2*(b^2*cos(4*b*lo
g(c))*cos(2*b*log(c))*cos(b*log(c)) + b^2*cos(b*log(c))*sin(4*b*log(c))*sin
(2*b*log(c)))*m)*n^2)*cos(2*b*log(x^n) + 2*a) + 2*((b^4*cos(2*b*log(c))*cos
(b*log(c))*sin(4*b*log(c)) - b^4*cos(4*b*log(c))*cos(b*log(c))*sin(2*b*log(
c)))*n^4 + (b^2*cos(2*b*log(c))*cos(b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b
*log(c))*cos(b*log(c))*sin(2*b*log(c)) + (b^2*cos(2*b*log(c))*cos(b*log(c))
*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*cos(b*log(c))*sin(2*b*log(c)))*m^2 +
 2*(b^2*cos(2*b*log(c))*cos(b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))
*cos(b*log(c))*sin(2*b*log(c)))*m)*n^2)*sin(2*b*log(x^n) + 2*a))*cos(4*b*lo
g(x^n) + 4*a) + 4*(b^4*n^4*cos(2*b*log(c))*cos(b*log(c)) + (b^2*m^2*cos(2*b
*log(c))*cos(b*log(c)) + 2*b^2*m*cos(2*b*log(c)...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*sec(b*log(c*x^n) + a)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m}\sec^{3}\left(a+b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sec(a+b*ln(c*x**n))**3,x)
```

```
[Out] Integral(x**m*sec(a + b*log(c*x**n))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*sec(b*log(c*x^n) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/cos(a + b*log(c*x^n))^3,x)
```

```
[Out] int(x^m/cos(a + b*log(c*x^n))^3, x)
```

3.279 $\int x^m \sec^2(a + b\log(cx^n))\,dx$

Optimal. Leaf size=102

$$\frac{4e^{2ia}x^{1+m}(cx^n)^{2ib}\,{}_2F_1\left(2,-\frac{i(1+m)-2bn}{2bn};-\frac{i(1+m)-4bn}{2bn};-e^{2ia}(cx^n)^{2ib}\right)}{1+m+2ibn}$$

```
[Out] 4*exp(2*I*a)*x^(1+m)*(c*x^n)^(2*I*b)*hypergeom([2, 1/2*(-I*(1+m)+2*b*n)/b/n
],[1/2*(-I*(1+m)+4*b*n)/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))/(1+m+2*I*b*n)
```

Rubi [A]
time = 0.06, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4605, 4601, 371}

$$\frac{4e^{2ia}x^{m+1}(cx^n)^{2ib}\,{}_2F_1\left(2,-\frac{i(m+1)-2bn}{2bn};-\frac{i(m+1)-4bn}{2bn};-e^{2ia}(cx^n)^{2ib}\right)}{2ibn+m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sec[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (4*E^((2*I)*a)*x^(1 + m)*(c*x^n)^((2*I)*b)*Hypergeometric2F1[2, -1/2*(I*(1
+ m) - 2*b*n)/(b*n), -1/2*(I*(1 + m) - 4*b*n)/(b*n), -(E^((2*I)*a)*(c*x^n)^
((2*I)*b))])/(1 + m + (2*I)*b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4605

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^m \sec^2\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{\left(x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1+m}{n}}\sec^2(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(4e^{2ia}x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\operatorname{Subst}\left(\int \frac{x^{-1+2ib+\frac{1+m}{n}}}{(1+e^{2ia}x^{2ib})^2}\,dx,x,cx^n\right)}{n}$$
$$= \frac{4e^{2ia}x^{1+m}(cx^n)^{2ib}\ {}_2F_1\left(2,-\frac{i(1+m)-2bn}{2bn};-\frac{i(1+m)-4bn}{2bn};-e^{2ia}(cx^n)^{2ib}\right)}{1+m+2ibn}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 482 vs. $2(102) = 204$.
time = 17.61, size = 482, normalized size = 4.73

$$\frac{x^{1+m}\sec\left(a+b(-n\log(x)+\log\left(cx^n\right))\right)\sec\left(a+bn\log(x)+b(-n\log(x)+\log\left(cx^n\right))\right)\sin(bn\log(x))}{bn}-\frac{(1+m)\sec\left(a+b(-n\log(x)+\log\left(cx^n\right))\right)\left(\frac{x^{1+m}\sec(a+b\log(cx^n))\sin(bn\log(x))}{1+m}-\frac{ie^{-\frac{(1+2m)(a+b(-n\log(x)+\log(cx^n)))}{bn}}\cos(a+b(-n\log(x)+\log(cx^n)))\left(-e^{\frac{a+2am+b(1+m)n\log(x)+b(1+2m)(-n\log(x)+\log(cx^n))}{bn}}(1+m+2ibn)\,{}_2F_1\left(1,-\frac{i(1+m)}{2bn};1-\frac{i(1+m)}{2bn};-e^{2i(a+b\log(cx^n))}\right)+e^{\frac{a(1+2m+2ibn)}{bn}+(1+m+2ibn)\log(x)+\frac{(1+2m+2ibn)(-n\log(x)+\log(cx^n))}{n}}(1+m)\,{}_2F_1\left(1,-\frac{i(1+m+2ibn)}{2bn};-\frac{i(1+m+4ibn)}{2bn};-e^{2i(a+b\log(cx^n))}\right)-ie^{\frac{a+2am+b(1+m)n\log(x)+b(1+2m)(-n\log(x)+\log(cx^n))}{bn}}(1+m+2ibn)\tan(a+b\log(cx^n))\right)}{(1+m)(1+m+2ibn)}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sec[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x^(1 + m)*Sec[a + b*(-(n*Log[x]) + Log[c*x^n])]*Sec[a + b*n*Log[x] + b*(-(
n*Log[x]) + Log[c*x^n])]*Sin[b*n*Log[x]])/(b*n) - ((1 + m)*Sec[a + b*(-(n*L
og[x]) + Log[c*x^n])]*((x^(1 + m)*Sec[a + b*Log[c*x^n]]*Sin[b*n*Log[x]])/(1
 + m) - (I*Cos[a + b*(-(n*Log[x]) + Log[c*x^n])]*(-(E^((a + 2*a*m + b*(1 +
m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n]))/(b*n))*(1 + m + (2*I)
*b*n)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m))/(b*n), 1 - ((I/2)*(1 + m))/(b
*n), -E^((2*I)*(a + b*Log[c*x^n]))]) + E^((a*(1 + 2*m + (2*I)*b*n))/(b*n) +
 (1 + m + (2*I)*b*n)*Log[x] + ((1 + 2*m + (2*I)*b*n)*(-(n*Log[x]) + Log[c*x
^n]))/n)*(1 + m)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m + (2*I)*b*n))/(b*n),
 ((-1/2*I)*(1 + m + (4*I)*b*n))/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] - I*E
^((a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n]))
/(b*n))*(1 + m + (2*I)*b*n)*Tan[a + b*Log[c*x^n]]))/(E^(((1 + 2*m)*(a + b*(
-(n*Log[x]) + Log[c*x^n])))/(b*n))*(1 + m)*(1 + m + (2*I)*b*n))))/(b*n)
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int x^m\left(\sec^2\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sec(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(x^m*sec(a+b*ln(c*x^n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 2*(x*x^m*cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + x*x^m*cos(2*b*log(c))*si
n(2*b*log(x^n) + 2*a) - ((b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2 + (
b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*m)*n^2*cos(2*b*log(x^n) + 2*
a)^2 + (b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2 + (b^2*cos(2*b*log(c)
)^2 + b^2*sin(2*b*log(c))^2)*m)*n^2*sin(2*b*log(x^n) + 2*a)^2 + 2*(b^2*m*co
s(2*b*log(c)) + b^2*cos(2*b*log(c)))*n^2*cos(2*b*log(x^n) + 2*a) - 2*(b^2*m
*sin(2*b*log(c)) + b^2*sin(2*b*log(c)))*n^2*sin(2*b*log(x^n) + 2*a) + (b^2*
m + b^2)*n^2)*integrate((x^m*cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + x^m*
cos(2*b*log(c))*sin(2*b*log(x^n) + 2*a))/(2*b^2*n^2*cos(2*b*log(c))*cos(2*b
*log(x^n) + 2*a) - 2*b^2*n^2*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b^2
*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2*cos(2*b*log(x^n) + 2*a)^2 +
 (b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2*sin(2*b*log(x^n) + 2*a
)^2 + b^2*n^2), x))/(2*b*n*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) + (b*cos
(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*cos(2*b*log(x^n) + 2*a)^2 - 2*b*n*s
in(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b*cos(2*b*log(c))^2 + b*sin(2*b*l
og(c))^2)*n*sin(2*b*log(x^n) + 2*a)^2 + b*n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*sec(b*log(c*x^n) + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sec^2(a + b \log(cx^n))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sec(a+b*ln(c*x**n))**2,x)
```

```
[Out] Integral(x**m*sec(a + b*log(c*x**n))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*sec(b*log(c*x^n) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/cos(a + b*log(c*x^n))^2,x)
```

```
[Out] int(x^m/cos(a + b*log(c*x^n))^2, x)
```

3.280 $\int x^m \sec(a+b\log(cx^n))\,dx$

Optimal. Leaf size=103

$$\frac{2e^{ia}x^{1+m}(cx^n)^{ib}\ {}_2F_1\left(1,-\frac{i+im-bn}{2bn};-\frac{i(1+m)-3bn}{2bn};-e^{2ia}(cx^n)^{2ib}\right)}{1+m+ibn}$$

```
[Out] 2*exp(I*a)*x^(1+m)*(c*x^n)^(I*b)*hypergeom([1, 1/2*(-I-I*m+b*n)/b/n],[1/2*(
-I*(1+m)+3*b*n)/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))/(1+m+I*b*n)
```

Rubi [A]
time = 0.05, antiderivative size = 99, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4605, 4601, 371}

$$\frac{2e^{ia}x^{m+1}(cx^n)^{ib}\ {}_2F_1\left(1,\frac{1}{2}\left(1-\frac{i(m+1)}{bn}\right);-\frac{i(m+1)-3bn}{2bn};-e^{2ia}(cx^n)^{2ib}\right)}{ibn+m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sec[a + b*Log[c*x^n]],x]
```

```
[Out] (2*E^(I*a)*x^(1 + m)*(c*x^n)^(I*b)*Hypergeometric2F1[1, (1 - (I*(1 + m))/(b
*n))/2, -1/2*(I*(1 + m) - 3*b*n)/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/
(1 + m + I*b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4605

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^m \sec(a+b\log(cx^n))\,dx = \frac{\left(x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1+m}{n}}\sec(a+b\log(x))\,dx, x, cx^n\right)}{n}$$
$$= \frac{\left(2e^{ia}x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\operatorname{Subst}\left(\int \frac{x^{-1+ib+\frac{1+m}{n}}}{1+e^{2ia}x^{2ib}}\,dx, x, cx^n\right)}{n}$$
$$= \frac{2e^{ia}x^{1+m}(cx^n)^{ib}\,{}_2F_1\left(1, \frac{1}{2}\left(1-\frac{i(1+m)}{bn}\right); -\frac{i(1+m)-3bn}{2bn}; -e^{2ia}(cx^n)^{2ib}\right)}{1+m+ibn}$$

Mathematica [A]
time = 1.82, size = 99, normalized size = 0.96

$$\frac{2e^{ia}x^{1+m}(cx^n)^{ib}\,{}_2F_1\left(1, \frac{-i-im+bn}{2bn}; -\frac{i(1+m+3ibn)}{2bn}; -e^{2i(a+b\log(cx^n))}\right)}{1+m+ibn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sec[a + b*Log[c*x^n]],x]
[Out] (2*E^(I*a)*x^(1 + m)*(c*x^n)^(I*b)*Hypergeometric2F1[1, (-I - I*m + b*n)/(2
*b*n), ((-1/2*I)*(1 + m + (3*I)*b*n))/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))]
)/(1 + m + I*b*n)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^m \sec(a+b\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sec(a+b*ln(c*x^n)),x)
[Out] int(x^m*sec(a+b*ln(c*x^n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(x^m*sec(b*log(c*x^n) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(x^m*sec(b*log(c*x^n) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m}\sec\left(a+b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sec(a+b*ln(c*x**n)),x)
[Out] Integral(x**m*sec(a + b*log(c*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate(x^m*sec(b*log(c*x^n) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{m}}{\cos\left(a+b\,\ln\left(c\,x^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/cos(a + b*log(c*x^n)),x)
[Out] int(x^m/cos(a + b*log(c*x^n)), x)
```

3.281 $\int x^m \sec^{\frac{5}{2}}(a + b\log(cx^n))\, dx$

Optimal. Leaf size=130

$$\frac{2x^{1+m}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},-\frac{2i+2im-5bn}{4bn};-\frac{2i+2im-9bn}{4bn};-e^{2ia}(cx^n)^{2ib}\right)\sec^{\frac{5}{2}}(a+b\log(cx^n))}{2+2m+5ibn}$$

```
[Out] 2*x^(1+m)*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(5/2)*hypergeom([5/2, 1/4*(-2*I-2*
I*m+5*b*n)/b/n],[1/4*(-2*I-2*I*m+9*b*n)/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))*s
ec(a+b*ln(c*x^n))^(5/2)/(2+2*m+5*I*b*n)
```

Rubi [A]
time = 0.07, antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4605, 4603, 371}

$$\frac{2x^{m+1}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i(m+1)}{bn}\right);-\frac{2im-9bn+2i}{4bn};-e^{2ia}(cx^n)^{2ib}\right)\sec^{\frac{5}{2}}(a+b\log(cx^n))}{5ibn+2m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sec[a + b*Log[c*x^n]]^(5/2),x]
```

```
[Out] (2*x^(1 + m)*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^(5/2)*Hypergeometric2F1[5/
2, (5 - ((2*I)*(1 + m))/(b*n))/4, -1/4*(2*I + (2*I)*m - 9*b*n)/(b*n), -(E^(
(2*I)*a)*(c*x^n)^((2*I)*b))]*Sec[a + b*Log[c*x^n]]^(5/2))/(2 + 2*m + (5*I)*
b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4603

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rule 4605

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
```

```
((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^m \sec^{\frac{5}{2}}(a+b\log(cx^n))\,dx = \frac{\left(x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{n}}\sec^{\frac{5}{2}}(a+b\log(x))\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x^{1+m}(cx^n)^{-\frac{5ib}{2}-\frac{1+m}{n}}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}\sec^{\frac{5}{2}}(a+b\log(cx^n))\right)\text{Subst}\left(\right.}{n}$$

$$= \frac{2x^{1+m}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}{}_2F_1\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i(1+m)}{bn}\right);-\frac{2i+2im-9bn}{4bn};-e^{2ia}(cx\right.}{2+2m+5ibn}$$

Mathematica [A]
time = 7.79, size = 209, normalized size = 1.61

$$\frac{2x^{1+m}\left(\sqrt{2}\,(2+2m-ibn)\sqrt{\frac{e^{ia}(cx^n)^{ib}}{1+e^{2ia}(cx^n)^{2ib}}}\sqrt{1+e^{2ia}(cx^n)^{2ib}}\,{}_2F_1\left(\frac{1}{2},\frac{-2i-2im+bn}{4bn};-\frac{2i+2im-5bn}{4bn};-e^{2ia}(cx^n)^{2ib}\right)+\sqrt{\sec(a+b\log(cx^n))}\,(-2(1+m)+bn\tan(a+b\log(cx^n)))\right)}{3b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sec[a + b*Log[c*x^n]]^(5/2),x]
```

```
[Out] (2*x^(1 + m)*(Sqrt[2]*(2 + 2*m - I*b*n)*Sqrt[(E^(I*a)*(c*x^n)^(I*b))/(1 + E
^((2*I)*a)*(c*x^n)^((2*I)*b))]*Sqrt[1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Hype
rgeometric2F1[1/2, (-2*I - (2*I)*m + b*n)/(4*b*n), -1/4*(2*I + (2*I)*m - 5*
b*n)/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))] + Sqrt[Sec[a + b*Log[c*x^n]]]
*(-2*(1 + m) + b*n*Tan[a + b*Log[c*x^n]])))/(3*b^2*n^2)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int x^m\left(\sec^{\frac{5}{2}}(a+b\ln(c\,x^n))\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sec(a+b*ln(c*x^n))^(5/2),x)
```

```
[Out] int(x^m*sec(a+b*ln(c*x^n))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*sec(b*log(c*x^n) + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sec(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \left(\frac{1}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(1/cos(a + b*log(c*x^n)))^(5/2),x)
```

```
[Out] int(x^m*(1/cos(a + b*log(c*x^n)))^(5/2), x)
```

3.282 $\int x^m \sec^{\frac{3}{2}}(a + b\log(cx^n))\, dx$

Optimal. Leaf size=130

$$\frac{2x^{1+m}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\, {}_2F_1\left(\frac{3}{2}, -\frac{2i+2im-3bn}{4bn}; -\frac{2i+2im-7bn}{4bn}; -e^{2ia}(cx^n)^{2ib}\right)\sec^{\frac{3}{2}}(a+b\log(cx^n))}{2+2m+3ibn}$$

```
[Out] 2*x^(1+m)*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(3/2)*hypergeom([3/2, 1/4*(-2*I-2*
I*m+3*b*n)/b/n],[1/4*(-2*I-2*I*m+7*b*n)/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))*s
ec(a+b*ln(c*x^n))^(3/2)/(2+2*m+3*I*b*n)
```

Rubi [A]

time = 0.07, antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4605, 4603, 371}

$$\frac{2x^{m+1}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\, {}_2F_1\left(\frac{3}{2}, \frac{1}{4}\left(3-\frac{2i(m+1)}{bn}\right); -\frac{2im-7bn+2i}{4bn}; -e^{2ia}(cx^n)^{2ib}\right)\sec^{\frac{3}{2}}(a+b\log(cx^n))}{3ibn+2m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sec[a + b*Log[c*x^n]]^(3/2),x]
```

```
[Out] (2*x^(1 + m)*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2)*Hypergeometric2F1[3/
2, (3 - ((2*I)*(1 + m))/(b*n))/4, -1/4*(2*I + (2*I)*m - 7*b*n)/(b*n), -(E^(
(2*I)*a)*(c*x^n)^((2*I)*b))]*Sec[a + b*Log[c*x^n]]^(3/2))/(2 + 2*m + (3*I)*
b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4603

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 4605

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
```

```
((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^m \sec^{\frac{3}{2}}(a+b\log(cx^n))\,dx = \frac{\left(x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{n}}\sec^{\frac{3}{2}}(a+b\log(x))\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x^{1+m}(cx^n)^{-\frac{3ib}{2}-\frac{1+m}{n}}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\sec^{\frac{3}{2}}(a+b\log(cx^n))\right)\text{Subst}\left(\int\right.}{n}$$

$$= \frac{2x^{1+m}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}{}_2F_1\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i(1+m)}{bn}\right);-\frac{2i+2im-7bn}{4bn};-e^{2ia}(cx^n\right.}{2+2m+3ibn}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 470 vs. $2(130) = 260$.
time = 10.10, size = 470, normalized size = 3.62

$$\frac{\sqrt{2}\,x^{1+m-ibn}\left(-\left((4+8m+4m^2+b^2n^2)\,x^{2ibn}\sqrt{\frac{e^{ia}(cx^n)^{ib}}{1+e^{2ia}(cx^n)^{2ib}}}\sqrt{1+e^{2ia}(cx^n)^{2ib}}\,{}_2F_1\left(\frac{1}{2},-\frac{2i+2im-3bn}{4bn};-\frac{2i+2im-7bn}{4bn};-e^{2ia}(cx^n)^{2ib}\right)\right)+(2+2m+3ibn)\left((2+2m+ibn)\sqrt{\frac{e^{ia}(cx^n)^{ib}}{1+e^{2ia}(cx^n)^{2ib}}}\sqrt{1+e^{2ia}(cx^n)^{2ib}}\,{}_2F_1\left(\frac{1}{2},-\frac{2i+2im+bn}{4bn};-\frac{2i+2im-3bn}{4bn};-e^{2ia}(cx^n)^{2ib}\right)-i\sqrt{2}\,x^{ibn}\sqrt{\sec(a+b\log(cx^n))}\,(bn\cos(bn\log(x))-2(1+m)\sin(bn\log(x)))\right)\right)}{bn(-2i-2im+3bn)\left(-2(1+m)\cos(a-bn\log(x)+b\log(cx^n))+bn\sin(a-bn\log(x)+b\log(cx^n))\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m*Sec[a + b*Log[c*x^n]]^(3/2),x]
```

```
[Out] (Sqrt[2]*x^(1 + m - I*b*n)*(-((4 + 8*m + 4*m^2 + b^2*n^2)*x^((2*I)*b*n)*Sqr
t[(E^(I*a)*(c*x^n)^(I*b))/(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))]*Sqrt[1 + E^(
(2*I)*a)*(c*x^n)^((2*I)*b)]*Hypergeometric2F1[1/2, -1/4*(2*I + (2*I)*m - 3*
b*n)/(b*n), -1/4*(2*I + (2*I)*m - 7*b*n)/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I
)*b))]) + (2 + 2*m + (3*I)*b*n)*((2 + 2*m + I*b*n)*Sqrt[(E^(I*a)*(c*x^n)^(I
*b))/(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))]*Sqrt[1 + E^((2*I)*a)*(c*x^n)^((2*
I)*b)]*Hypergeometric2F1[1/2, -1/4*(2*I + (2*I)*m + b*n)/(b*n), -1/4*(2*I +
 (2*I)*m - 3*b*n)/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))] - I*Sqrt[2]*x^(I
*b*n)*Sqrt[Sec[a + b*Log[c*x^n]]]*(b*n*Cos[b*n*Log[x]] - 2*(1 + m)*Sin[b*n*
Log[x]]))))/(b*n*(-2*I - (2*I)*m + 3*b*n)*(-2*(1 + m)*Cos[a - b*n*Log[x] +
b*Log[c*x^n]] + b*n*Sin[a - b*n*Log[x] + b*Log[c*x^n]]))
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int x^m\left(\sec^{\frac{3}{2}}(a+b\ln(c\,x^n))\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sec(a+b*ln(c*x^n))^(3/2),x)
```

```
[Out] int(x^m*sec(a+b*ln(c*x^n))^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*sec(b*log(c*x^n) + a)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sec(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\left(\frac{1}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(1/cos(a + b*log(c*x^n)))^(3/2),x)
[Out] int(x^m*(1/cos(a + b*log(c*x^n)))^(3/2), x)
```

3.283 $\int x^m \sqrt{\sec(a+b\log(cx^n))}\, dx$

Optimal. Leaf size=130

$$\frac{2x^{1+m}\sqrt{1+e^{2ia}(cx^n)^{2ib}}\,{}_2F_1\left(\frac{1}{2},-\frac{2i+2im-bn}{4bn};-\frac{2i+2im-5bn}{4bn};-e^{2ia}(cx^n)^{2ib}\right)\sqrt{\sec(a+b\log(cx^n))}}{2+2m+ibn}$$

```
[Out] 2*x^(1+m)*hypergeom([1/2, 1/4*(-2*I-2*I*m+b*n)/b/n],[1/4*(-2*I-2*I*m+5*b*n)
/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(1/2)*sec
(a+b*ln(c*x^n))^(1/2)/(2+2*m+I*b*n)
```

Rubi [A]
time = 0.06, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4605, 4603, 371}

$$\frac{2x^{m+1}\sqrt{1+e^{2ia}(cx^n)^{2ib}}\,{}_2F_1\left(\frac{1}{2},-\frac{2im-bn+2i}{4bn};-\frac{2im-5bn+2i}{4bn};-e^{2ia}(cx^n)^{2ib}\right)\sqrt{\sec(a+b\log(cx^n))}}{ibn+2m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sqrt[Sec[a + b*Log[c*x^n]]],x]
```

```
[Out] (2*x^(1 + m)*Sqrt[1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Hypergeometric2F1[1/2,
 -1/4*(2*I + (2*I)*m - b*n)/(b*n), -1/4*(2*I + (2*I)*m - 5*b*n)/(b*n), -(E^
((2*I)*a)*(c*x^n)^((2*I)*b))]*Sqrt[Sec[a + b*Log[c*x^n]]])/(2 + 2*m + I*b*n
)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4603

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rule 4605

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
```

```
((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^m\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}\,dx = \frac{\left(x^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{n}}\sqrt{\sec(a+b\log(x))}\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x^{1+m}\left(cx^n\right)^{-\frac{ib}{2}-\frac{1+m}{n}}\sqrt{1+e^{2ia}\left(cx^n\right)^{2ib}}\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}\right)\text{Subst}\left(\int\right.}{n}$$

$$= \frac{2x^{1+m}\sqrt{1+e^{2ia}\left(cx^n\right)^{2ib}}\,{}_2F_1\left(\frac{1}{2},-\frac{2i+2im-bn}{4bn};-\frac{2i+2im-5bn}{4bn};-e^{2ia}\left(cx^n\right)^{2ib}\right)}{2+2m+ibn}$$

Mathematica [A]
time = 5.68, size = 165, normalized size = 1.27

$$\frac{2\sqrt{2}\,x^{1+m}\sqrt{\frac{e^{ia}\left(cx^n\right)^{ib}}{1+e^{2ia}\left(cx^n\right)^{2ib}}}\sqrt{1+e^{2ia}\left(cx^n\right)^{2ib}}\,{}_2F_1\left(\frac{1}{2},\frac{-2i-2im+bn}{4bn};-\frac{2i+2im-5bn}{4bn};-e^{2ia}\left(cx^n\right)^{2ib}\right)}{2+2m+ibn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sqrt[Sec[a + b*Log[c*x^n]]],x]
```

```
[Out] (2*Sqrt[2]*x^(1 + m)*Sqrt[(E^(I*a)*(c*x^n)^(I*b))/(1 + E^((2*I)*a)*(c*x^n)^
((2*I)*b))]*Sqrt[1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Hypergeometric2F1[1/2,
(-2*I - (2*I)*m + b*n)/(4*b*n), -1/4*(2*I + (2*I)*m - 5*b*n)/(b*n), -(E^((2
*I)*a)*(c*x^n)^((2*I)*b))])/(2 + 2*m + I*b*n)
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int x^m\left(\sqrt{\sec}\left(a+b\ln\left(c\,x^n\right)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sec(a+b*ln(c*x^n))^(1/2),x)
```

```
[Out] int(x^m*sec(a+b*ln(c*x^n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*sqrt(sec(b*log(c*x^n) + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sqrt{\sec \left(a + b \log \left(cx^n\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sec(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(sec(a + b*log(c*x**n))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*sqrt(sec(b*log(c*x^n) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \sqrt{\frac{1}{\cos \left(a + b\,\ln \left(c\,x^n\right)\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(1/cos(a + b*log(c*x^n)))^(1/2),x)
```

```
[Out] int(x^m*(1/cos(a + b*log(c*x^n)))^(1/2), x)
```

3.284 $\int \frac{x^m}{\sqrt{\sec(a+b\log(cx^n))}}\,dx$

Optimal. Leaf size=129

$$\frac{2x^{1+m}\,{}_2F_1\left(-\frac{1}{2},-\frac{2i+2im+bn}{4bn};-\frac{2i+2im-3bn}{4bn};-e^{2ia}(cx^n)^{2ib}\right)}{(2+2m-ibn)\sqrt{1+e^{2ia}(cx^n)^{2ib}}\sqrt{\sec(a+b\log(cx^n))}}$$

```
[Out] 2*x^(1+m)*hypergeom([-1/2, 1/4*(-2*I-2*I*m-b*n)/b/n],[1/4*(-2*I-2*I*m+3*b*n
)/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))/(2+2*m-I*b*n)/(1+exp(2*I*a)*(c*x^n)^(2*
I*b))^(1/2)/sec(a+b*ln(c*x^n))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 126, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4605, 4603, 371}

$$\frac{2x^{m+1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4}\left(-\frac{2i(m+1)}{bn}-1\right);-\frac{2im-3bn+2i}{4bn};-e^{2ia}(cx^n)^{2ib}\right)}{(-ibn+2m+2)\sqrt{1+e^{2ia}(cx^n)^{2ib}}\sqrt{\sec(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[Sec[a + b*Log[c*x^n]]],x]
```

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[-1/2, (-1 - ((2*I)*(1 + m))/(b*n))/4, -1/4*(
2*I + (2*I)*m - 3*b*n)/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 + 2*m
- I*b*n)*Sqrt[1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[Sec[a + b*Log[c*x^n]]
])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4603

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rule 4605

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^m}{\sqrt{\sec(a+b\log(cx^n))}}\,dx &= \frac{\left(x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\operatorname{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}}{\sqrt{\sec(a+b\log(x))}}\,dx, x, cx^n\right)}{n} \\
&= \frac{\left(x^{1+m}(cx^n)^{\frac{ib}{2}-\frac{1+m}{n}}\right)\operatorname{Subst}\left(\int x^{-1-\frac{ib}{2}+\frac{1+m}{n}}\sqrt{1+e^{2ia}x^{2ib}}\,dx, x, cx^n\right)}{n\sqrt{1+e^{2ia}(cx^n)^{2ib}}\sqrt{\sec(a+b\log(cx^n))}} \\
&= \frac{2x^{1+m}\,{}_2F_1\left(-\frac{1}{2}, \frac{1}{4}\left(-1-\frac{2i(1+m)}{bn}\right); -\frac{2i+2im-3bn}{4bn}; -e^{2ia}(cx^n)^{2ib}\right)}{(2+2m-ibn)\sqrt{1+e^{2ia}(cx^n)^{2ib}}\sqrt{\sec(a+b\log(cx^n))}}
\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 437 vs. $2(129) = 258$.
time = 7.37, size = 437, normalized size = 3.39

$$-\frac{2be^{2i(a-bn\log(x)+b\log(cx^n))}nx^{1+m}\left((2i+2im+bn)x^{2ibn}\,{}_2F_1\left(\frac{1}{2},-\frac{2i+2im-3bn}{4bn};-\frac{2i+2im-7bn}{4bn};-e^{2ia}(cx^n)^{2ib}\right)+(-2i-2im+3bn)\,{}_2F_1\left(\frac{1}{2},-\frac{2i+2im+bn}{4bn};-\frac{2i+2im-3bn}{4bn};-e^{2ia}(cx^n)^{2ib}\right)\right)}{(2+2m-ibn)(2+2m+3ibn)\left(2+2m-ibn+e^{2i(a-bn\log(x)+b\log(cx^n))}(2+2m+ibn)\right)\sqrt{1+e^{2ia}(cx^n)^{2ib}}\sqrt{\frac{e^{ia}(cx^n)^{ib}}{2+2e^{2ia}(cx^n)^{2ib}}}}+\frac{2x^{1+m}\cos(a-bn\log(x)+b\log(cx^n))}{\sqrt{\sec(a+b\log(cx^n))}\,(2(1+m)\cos(a-bn\log(x)+b\log(cx^n))-bn\sin(a-bn\log(x)+b\log(cx^n)))}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[Sec[a + b*Log[c*x^n]]],x]
```

```
[Out] (-2*b*E^((2*I)*(a - b*n*Log[x] + b*Log[c*x^n]))*n*x^(1 + m)*((2*I + (2*I)*m
 + b*n)*x^((2*I)*b*n)*Hypergeometric2F1[1/2, -1/4*(2*I + (2*I)*m - 3*b*n)/(
b*n), -1/4*(2*I + (2*I)*m - 7*b*n)/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))]
 + (-2*I - (2*I)*m + 3*b*n)*Hypergeometric2F1[1/2, -1/4*(2*I + (2*I)*m + b*
n)/(b*n), -1/4*(2*I + (2*I)*m - 3*b*n)/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*
b))]))/((2 + 2*m - I*b*n)*(2 + 2*m + (3*I)*b*n)*(2 + 2*m - I*b*n + E^((2*I)
*(a - b*n*Log[x] + b*Log[c*x^n]))*(2 + 2*m + I*b*n))*Sqrt[1 + E^((2*I)*a)*(
c*x^n)^((2*I)*b)]*Sqrt[(E^(I*a)*(c*x^n)^(I*b))/(2 + 2*E^((2*I)*a)*(c*x^n)^(
(2*I)*b))]) + (2*x^(1 + m)*Cos[a - b*n*Log[x] + b*Log[c*x^n]])/(Sqrt[Sec[a
+ b*Log[c*x^n]]]*(2*(1 + m)*Cos[a - b*n*Log[x] + b*Log[c*x^n]] - b*n*Sin[a
- b*n*Log[x] + b*Log[c*x^n]]))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{\sec(a+b\ln(c\,x^n))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/sec(a+b*ln(c*x^n))^(1/2),x)
```

```
[Out] int(x^m/sec(a+b*ln(c*x^n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/sec(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(sec(b*log(c*x^n) + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/sec(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{\sec \left(a + b \log \left(c x^n\right)\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/sec(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(x**m/sqrt(sec(a + b*log(c*x**n))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/sec(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(sec(b*log(c*x^n) + a)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{\sqrt{\frac{1}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(1/cos(a + b*log(c*x^n)))^(1/2),x)
[Out] int(x^m/(1/cos(a + b*log(c*x^n)))^(1/2), x)
```

3.285 $\int \frac{x^m}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=130

$$\frac{2x^{1+m}\,{}_2F_1\left(-\frac{3}{2},-\frac{2i+2im+3bn}{4bn};-\frac{2i+2im-bn}{4bn};-e^{2ia}(cx^n)^{2ib}\right)}{(2+2m-3ibn)\left(1+e^{2ia}\left(cx^n\right)^{2ib}\right)^{3/2}\sec^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

```
[Out] 2*x^(1+m)*hypergeom([-3/2, 1/4*(-2*I-2*I*m-3*b*n)/b/n],[1/4*(-2*I-2*I*m+b*n
)/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))/(2+2*m-3*I*b*n)/(1+exp(2*I*a)*(c*x^n)^(
2*I*b))^(3/2)/sec(a+b*ln(c*x^n))^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4605, 4603, 371}

$$\frac{2x^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(-\frac{2i(m+1)}{bn}-3\right);-\frac{2im-bn+2i}{4bn};-e^{2ia}(cx^n)^{2ib}\right)}{(-3ibn+2m+2)\left(1+e^{2ia}\left(cx^n\right)^{2ib}\right)^{3/2}\sec^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sec[a + b*Log[c*x^n]]^(3/2),x]
```

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[-3/2, (-3 - ((2*I)*(1 + m))/(b*n))/4, -1/4*(
2*I + (2*I)*m - b*n)/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 + 2*m -
(3*I)*b*n)*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2)*Sec[a + b*Log[c*x^n]]^
(3/2))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4603

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rule 4605

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx &= \frac{\left(x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}}{\sec^{\frac{3}{2}}(a+b\log(x))}\,dx,x,cx^n\right)}{n}\\
&= \frac{\left(x^{1+m}(cx^n)^{\frac{3ib}{2}-\frac{1+m}{n}}\right)\text{Subst}\left(\int x^{-1-\frac{3ib}{2}+\frac{1+m}{n}}\left(1+e^{2ia}x^{2ib}\right)^{3/2}\,dx,x,cx^n\right)}{n\left(1+e^{2ia}\left(cx^n\right)^{2ib}\right)^{3/2}\sec^{\frac{3}{2}}(a+b\log(cx^n))}\\
&= \frac{2x^{1+m}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{2i(1+m)}{bn}\right);-\frac{2i+2im-bn}{4bn};-e^{2ia}(cx^n)^{2ib}\right)}{(2+2m-3ibn)\left(1+e^{2ia}\left(cx^n\right)^{2ib}\right)^{3/2}\sec^{\frac{3}{2}}(a+b\log(cx^n))}\end{aligned}$$

Mathematica [A]
time = 7.98, size = 230, normalized size = 1.77

$$\frac{2x^{1+m}\left(\frac{3\sqrt{2}\,b^2n^2\sqrt{\frac{e^{ia}(cx^n)^{ib}}{1+e^{2ia}(cx^n)^{2ib}}}\sqrt{1+e^{2ia}(cx^n)^{2ib}}\,{}_2F_1\left(\frac{1}{2},\frac{-2i-2im+bn}{4bn};-\frac{2i+2im-5bn}{4bn};-e^{2ia}(cx^n)^{2ib}\right)}{2+2m+ibn}+\frac{2+2m+3bn\tan(a+b\log(cx^n))}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}\right)}{4+8m+4m^2+9b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sec[a + b*Log[c*x^n]]^(3/2),x]
```

```
[Out] (2*x^(1 + m)*((3*Sqrt[2]*b^2*n^2*Sqrt[(E^(I*a)*(c*x^n)^(I*b))/(1 + E^((2*I)
*a)*(c*x^n)^((2*I)*b))]*Sqrt[1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Hypergeomet
ric2F1[1/2, (-2*I - (2*I)*m + b*n)/(4*b*n), -1/4*(2*I + (2*I)*m - 5*b*n)/(b
*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(2 + 2*m + I*b*n) + (2 + 2*m + 3*b*
n*Tan[a + b*Log[c*x^n]])/Sec[a + b*Log[c*x^n]]^(3/2)))/(4 + 8*m + 4*m^2 + 9
*b^2*n^2)
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sec\left(a+b\ln\left(c\,x^n\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/sec(a+b*ln(c*x^n))^(3/2),x)
```

```
[Out] int(x^m/sec(a+b*ln(c*x^n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/sec(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sec(b*log(c*x^n) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/sec(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sec^{\frac{3}{2}}\left(a + b\log\left(cx^n\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/sec(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Integral(x**m/sec(a + b*log(c*x**n))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/sec(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sec(b*log(c*x^n) + a)^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{\left(\frac{1}{\cos(a+b\,\ln(c\,x^n))}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(1/cos(a + b*log(c*x^n)))^(3/2),x)
[Out] int(x^m/(1/cos(a + b*log(c*x^n)))^(3/2), x)
```

3.286 $\int (ex)^m \sec^p (d(a + b \log (cx^n))) \, dx$

Optimal. Leaf size=139

$$\frac{(ex)^{1+m} \left(1 + e^{2iad}(cx^n)^{2ibd}\right)^p {}_2F_1\left(p, -\frac{i+im-bdnp}{2bdn}; \frac{1}{2}\left(2 - \frac{i(1+m)}{bdn} + p\right); -e^{2iad}(cx^n)^{2ibd}\right) \sec^p (d(a + b \log (cx^n)))}{e(1 + m + ibdnp)}$$

```
[Out] (e*x)^(1+m)*(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^p*hypergeom([p, 1/2*(-I-I*m+
b*d*n*p)/b/d/n],[1-1/2*I*(1+m)/b/d/n+1/2*p],-exp(2*I*a*d)*(c*x^n)^(2*I*b*d)
)*sec(d*(a+b*ln(c*x^n)))^p/e/(1+m+I*b*d*n*p)
```

Rubi [A]

time = 0.09, antiderivative size = 133, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4605, 4603, 371}

$$\frac{(ex)^{m+1} \left(1 + e^{2iad}(cx^n)^{2ibd}\right)^p {}_2F_1\left(p, \frac{1}{2}\left(p - \frac{i(m+1)}{bdn}\right); \frac{1}{2}\left(-\frac{i(m+1)}{bdn} + p + 2\right); -e^{2iad}(cx^n)^{2ibd}\right) \sec^p (d(a + b \log (cx^n)))}{e(ibdnp + m + 1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sec[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] ((e*x)^(1 + m)*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p*Hypergeometric2F1[
p, (((-I)*(1 + m))/(b*d*n) + p)/2, (2 - (I*(1 + m))/(b*d*n) + p)/2, -(E^((2
*I)*a*d)*(c*x^n)^((2*I)*b*d))]*Sec[d*(a + b*Log[c*x^n])]^p)/(e*(1 + m + I*b
*d*n*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4603

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 4605

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
```

```
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int (ex)^m \sec^p\left(d(a+b\log\left(cx^n\right))\right)\,dx = \frac{\left((ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{n}}\sec^p(d(a+b\log(x)))\,dx,x,c\right.}{en}$$
$$=\frac{\left((ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}-ibdp}\left(1+e^{2iad}(cx^n)^{2ibd}\right)^p\sec^p\left(d(a+b\log\left(cx^n\right)\right)\right.}{en}$$
$$=\frac{(ex)^{1+m}\left(1+e^{2iad}(cx^n)^{2ibd}\right)^p{}_2F_1\left(p,\frac{1}{2}\left(-\frac{i(1+m)}{bdn}+p\right);\frac{1}{2}\left(2-\frac{i(1+m)}{bdn}\right.\right.}{e(1+m+ibdnp)}$$

Mathematica [A]
time = 1.54, size = 169, normalized size = 1.22

$$\frac{2^px(ex)^m\left(\frac{e^{iad}(cx^n)^{ibd}}{1+e^{2iad}(cx^n)^{2ibd}}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^p{}_2F_1\left(p,-\frac{i(1+m+ibdnp)}{2bdn};\frac{1}{2}\left(2-\frac{i(1+m)}{bdn}+p\right);-e^{2iad}(cx^n)^{2ibd}\right)}{1+m+ibdnp}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sec[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] (2^p*x*(e*x)^m*((E^(I*a*d)*(c*x^n)^(I*b*d))/(1 + E^((2*I)*a*d)*(c*x^n)^((2*
I)*b*d)))^p*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p*Hypergeometric2F1[p,
((-1/2*I)*(1 + m + I*b*d*n*p))/(b*d*n), (2 - (I*(1 + m))/(b*d*n) + p)/2, -(
E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])/(1 + m + I*b*d*n*p)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (ex)^m\left(\sec^p\left(d(a+b\ln\left(c\,x^n\right))\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sec(d*(a+b*ln(c*x^n)))^p,x)
```

```
[Out] int((e*x)^m*sec(d*(a+b*ln(c*x^n)))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sec(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
[Out] integrate((x*e)^m*sec((b*log(c*x^n) + a)*d)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sec(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
[Out] integral((x*e)^m*sec(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sec^p (ad + bd \log (cx^n))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sec(d*(a+b*ln(c*x**n)))**p,x)
[Out] Integral((e*x)**m*sec(a*d + b*d*log(c*x**n))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sec(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
[Out] integrate((e*x)^m*sec((b*log(c*x^n) + a)*d)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,x)^m \left(\frac{1}{\cos\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(1/cos(d*(a + b*log(c*x^n))))^p,x)
[Out] int((e*x)^m*(1/cos(d*(a + b*log(c*x^n))))^p, x)
```

3.287 $\int x \sec^p(a+b\log(cx^n))\,dx$

Optimal. Leaf size=106

$$\frac{x^2\left(1+e^{2ia}(cx^n)^{2ib}\right)^p {}_2F_1\left(p,\frac{1}{2}\left(-\frac{2i}{bn}+p\right);\frac{1}{2}\left(2-\frac{2i}{bn}+p\right);-e^{2ia}(cx^n)^{2ib}\right)\sec^p(a+b\log(cx^n))}{2+ibnp}$$

```
[Out] x^2*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^p*hypergeom([p, -I/b/n+1/2*p],[1-I/b/n+1
/2*p],-exp(2*I*a)*(c*x^n)^(2*I*b))*sec(a+b*ln(c*x^n))^p/(2+I*b*n*p)
```

Rubi [A]
time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4605, 4603, 371}

$$\frac{x^2\left(1+e^{2ia}(cx^n)^{2ib}\right)^p {}_2F_1\left(p,\frac{1}{2}\left(p-\frac{2i}{bn}\right);\frac{1}{2}\left(p-\frac{2i}{bn}+2\right);-e^{2ia}(cx^n)^{2ib}\right)\sec^p(a+b\log(cx^n))}{2+ibnp}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[a + b*Log[c*x^n]]^p,x]
```

```
[Out] (x^2*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^p*Hypergeometric2F1[p, ((-2*I)/(b*
n) + p)/2, (2 - (2*I)/(b*n) + p)/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))]*Sec[a
 + b*Log[c*x^n]]^p)/(2 + I*b*n*p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4603

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 4605

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x\sec^p(a+b\log(cx^n))\,dx = \frac{\left(x^2(cx^n)^{-2/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{2}{n}}\sec^p(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(x^2(cx^n)^{-\frac{2}{n}-ibp}\left(1+e^{2ia}(cx^n)^{2ib}\right)^p\sec^p(a+b\log(cx^n))\right)\operatorname{Subst}\left(\int x^{-1+\frac{2}{n}+ibp}\right.}{n}$$
$$= \frac{x^2\left(1+e^{2ia}(cx^n)^{2ib}\right)^p{}_2F_1\left(p,\frac{1}{2}\left(-\frac{2i}{bn}+p\right);\frac{1}{2}\left(2-\frac{2i}{bn}+p\right);-e^{2ia}(cx^n)^{2ib}\right)\sec^p}{2+ibnp}$$

Mathematica [A]
time = 1.08, size = 142, normalized size = 1.34

$$-\frac{i2^px^2\left(\frac{e^{ia}(cx^n)^{ib}}{1+e^{2ia}(cx^n)^{2ib}}\right)^p\left(1+e^{2ia}(cx^n)^{2ib}\right)^p{}_2F_1\left(-\frac{i}{bn}+\frac{p}{2},p;1-\frac{i}{bn}+\frac{p}{2};-e^{2ia}(cx^n)^{2ib}\right)}{-2i+bnp}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[a + b*Log[c*x^n]]^p,x]
[Out] ((-I)*2^p*x^2*((E^(I*a)*(c*x^n)^(I*b))/(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))
^p*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^p*Hypergeometric2F1[(-I)/(b*n) + p/2
, p, 1 - I/(b*n) + p/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(-2*I + b*n*p)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int x(\sec^p(a+b\ln(c\,x^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(a+b*ln(c*x^n))^p,x)
[Out] int(x*sec(a+b*ln(c*x^n))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*log(c*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate(x*sec(b*log(c*x^n) + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*log(c*x^n))^p,x, algorithm="fricas")
[Out] integral(x*sec(b*log(c*x^n) + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sec^p\left(a+b\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*ln(c*x**n))**p,x)
[Out] Integral(x*sec(a + b*log(c*x**n))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*log(c*x^n))^p,x, algorithm="giac")
[Out] integrate(x*sec(b*log(c*x^n) + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(\frac{1}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1/cos(a + b*log(c*x^n)))^p,x)
[Out] int(x*(1/cos(a + b*log(c*x^n)))^p, x)
```

3.288 $\int \sec^p(a+b\log(cx^n))\,dx$

Optimal. Leaf size=107

$$\frac{x\left(1+e^{2ia}(cx^n)^{2ib}\right)^p {}_2F_1\left(p,-\frac{i-bnp}{2bn};\frac{1}{2}\left(2-\frac{i}{bn}+p\right);-e^{2ia}(cx^n)^{2ib}\right)\sec^p(a+b\log(cx^n))}{1+ibnp}$$

```
[Out] x*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^p*hypergeom([p, 1/2*(-I+b*n*p)/b/n],[1-1/2
*I/b/n+1/2*p],-exp(2*I*a)*(c*x^n)^(2*I*b))*sec(a+b*ln(c*x^n))^p/(1+I*b*n*p)
```

Rubi [A]
time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4599, 4603, 371}

$$\frac{x\left(1+e^{2ia}(cx^n)^{2ib}\right)^p {}_2F_1\left(p,-\frac{i-bnp}{2bn};\frac{1}{2}\left(p-\frac{i}{bn}+2\right);-e^{2ia}(cx^n)^{2ib}\right)\sec^p(a+b\log(cx^n))}{1+ibnp}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*Log[c*x^n]]^p,x]
```

```
[Out] (x*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^p*Hypergeometric2F1[p, -1/2*(I - b*n
*p)/(b*n), (2 - I/(b*n) + p)/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))]*Sec[a + b
*Log[c*x^n]]^p)/(1 + I*b*n*p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4599

```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4603

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \sec^p(a+b\log(cx^n))\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\sec^p(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(x(cx^n)^{-\frac{1}{n}-ibp}\left(1+e^{2ia}(cx^n)^{2ib}\right)^p\sec^p(a+b\log(cx^n))\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}+ibp}(\right.}{n}$$
$$= \frac{x\left(1+e^{2ia}(cx^n)^{2ib}\right)^p{}_2F_1\left(p,-\frac{i-bnp}{2bn};\frac{1}{2}\left(2-\frac{i}{bn}+p\right);-e^{2ia}(cx^n)^{2ib}\right)\sec^p(a+b}{1+ibnp}$$

Mathematica [A]
time = 0.89, size = 142, normalized size = 1.33

$$-\frac{i2^px\left(\frac{e^{ia}(cx^n)^{ib}}{1+e^{2ia}(cx^n)^{2ib}}\right)^p\left(1+e^{2ia}(cx^n)^{2ib}\right)^p{}_2F_1\left(p,\frac{-i+bnp}{2bn};\frac{1}{2}\left(2-\frac{i}{bn}+p\right);-e^{2ia}(cx^n)^{2ib}\right)}{-i+bnp}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*Log[c*x^n]]^p,x]
[Out] ((-I)*2^p*x*((E^(I*a)*(c*x^n)^(I*b))/(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))^p
*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^p*Hypergeometric2F1[p, (-I + b*n*p)/(2
*b*n), (2 - I/(b*n) + p)/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(-I + b*n*p)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \sec^p(a+b\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a+b*ln(c*x^n))^p,x)
[Out] int(sec(a+b*ln(c*x^n))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^p,x, algorithm="fricas")
[Out] integral(sec(b*log(c*x^n) + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sec^{p}\left(a + b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**p,x)
[Out] Integral(sec(a + b*log(c*x**n))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^p,x, algorithm="giac")
[Out] integrate(sec(b*log(c*x^n) + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(a + b*log(c*x^n)))^p,x)
[Out] int((1/cos(a + b*log(c*x^n)))^p, x)
```

3.289 $\int x^2 \csc(a + b\log(cx^n))\ dx$

Optimal. Leaf size=86

$$\frac{2e^{ia}x^3(cx^n)^{ib}\ {}_2F_1\left(1,\frac{1}{2}\left(1-\frac{3i}{bn}\right);\frac{3}{2}\left(1-\frac{i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{3i-bn}$$

```
[Out] 2*exp(I*a)*x^3*(c*x^n)^(I*b)*hypergeom([1, 1/2-3/2*I/b/n],[3/2-3/2*I/b/n],e
xp(2*I*a)*(c*x^n)^(2*I*b))/(3*I-b*n)
```

Rubi [A]
time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4606, 4602, 371}

$$\frac{2e^{ia}x^3(cx^n)^{ib}\ {}_2F_1\left(1,\frac{1}{2}\left(1-\frac{3i}{bn}\right);\frac{3}{2}\left(1-\frac{i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{-bn+3i}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csc[a + b*Log[c*x^n]],x]
```

```
[Out] (2*E^(I*a)*x^3*(c*x^n)^(I*b)*Hypergeometric2F1[1, (1 - (3*I)/(b*n))/2, (3*(
1 - I/(b*n)))/2, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/(3*I - b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4602

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[(-2*I)^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(
2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4606

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^2 \csc(a+b\log(cx^n))\,dx = \frac{\left(x^3(cx^n)^{-3/n}\right)\text{Subst}\left(\int x^{-1+\frac{3}{n}}\csc(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= -\frac{\left(2ie^{ia}x^3(cx^n)^{-3/n}\right)\text{Subst}\left(\int \frac{x^{-1+ib+\frac{3}{n}}}{1-e^{2ia}x^{2ib}}\,dx,x,cx^n\right)}{n}$$
$$= \frac{2e^{ia}x^3(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}\left(1-\frac{3i}{bn}\right);\frac{3}{2}\left(1-\frac{i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{3i-bn}$$

Mathematica [A]
time = 1.57, size = 82, normalized size = 0.95

$$-\frac{2e^{ia}x^3(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}-\frac{3i}{2bn};\frac{3}{2}-\frac{3i}{2bn};e^{2i(a+b\log(cx^n))}\right)}{-3i+bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csc[a + b*Log[c*x^n]],x]
[Out] (-2*E^(I*a)*x^3*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 - ((3*I)/2)/(b*n), 3
/2 - ((3*I)/2)/(b*n), E^((2*I)*(a + b*Log[c*x^n]))])/(-3*I + b*n)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int x^2\csc(a+b\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csc(a+b*ln(c*x^n)),x)
[Out] int(x^2*csc(a+b*ln(c*x^n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(x^2*csc(b*log(c*x^n) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(x^2*csc(b*log(c*x^n) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\csc\left(a+b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csc(a+b*ln(c*x**n)),x)
[Out] Integral(x**2*csc(a + b*log(c*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate(x^2*csc(b*log(c*x^n) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/sin(a + b*log(c*x^n)),x)
[Out] int(x^2/sin(a + b*log(c*x^n)), x)
```

3.290 $\int x \csc(a + b\log(cx^n))\, dx$

Optimal. Leaf size=86

$$\frac{2e^{ia}x^2(cx^n)^{ib}\, {}_2F_1\left(1, \frac{1}{2}\left(1-\frac{2i}{bn}\right); \frac{1}{2}\left(3-\frac{2i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right)}{2i - bn}$$

```
[Out] 2*exp(I*a)*x^2*(c*x^n)^(I*b)*hypergeom([1, 1/2-I/b/n],[3/2-I/b/n],exp(2*I*a
)*(c*x^n)^(2*I*b))/(2*I-b*n)
```

Rubi [A]
time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4606, 4602, 371}

$$\frac{2e^{ia}x^2(cx^n)^{ib}\, {}_2F_1\left(1, \frac{1}{2}\left(1-\frac{2i}{bn}\right); \frac{1}{2}\left(3-\frac{2i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right)}{-bn + 2i}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csc[a + b*Log[c*x^n]],x]
```

```
[Out] (2*E^(I*a)*x^2*(c*x^n)^(I*b)*Hypergeometric2F1[1, (1 - (2*I)/(b*n))/2, (3 -
 (2*I)/(b*n))/2, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/(2*I - b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4602

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[(-2*I)^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(
2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4606

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x\csc\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{\left(x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int x^{-1+\frac{2}{n}}\csc(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$=-\frac{\left(2ie^{ia}x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int\frac{x^{-1+ib+\frac{2}{n}}}{1-e^{2ia}x^{2ib}}\,dx,x,cx^n\right)}{n}$$
$$=\frac{2e^{ia}x^2(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}\left(1-\frac{2i}{bn}\right);\frac{1}{2}\left(3-\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{2i-bn}$$

Mathematica [A]
time = 1.53, size = 78, normalized size = 0.91

$$-\frac{2e^{ia}x^2(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}-\frac{i}{bn};\frac{3}{2}-\frac{i}{bn};e^{2i(a+b\log(cx^n))}\right)}{-2i+bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csc[a + b*Log[c*x^n]],x]
```

```
[Out] (-2*E^(I*a)*x^2*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 - I/(b*n), 3/2 - I/(
b*n), E^((2*I)*(a + b*Log[c*x^n]))])/(-2*I + b*n)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int x\csc\left(a+b\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(a+b*ln(c*x^n)),x)
```

```
[Out] int(x*csc(a+b*ln(c*x^n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] integrate(x*csc(b*log(c*x^n) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(x*csc(b*log(c*x^n) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \csc\left(a + b \log\left(cx^{n}\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(a+b*ln(c*x**n)),x)
[Out] Integral(x*csc(a + b*log(c*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate(x*csc(b*log(c*x^n) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sin(a + b*log(c*x^n)),x)
[Out] int(x/sin(a + b*log(c*x^n)), x)
```

3.291 $\int \csc(a+b\log(cx^n))\,dx$

Optimal. Leaf size=84

$$\frac{2e^{ia}x(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}\left(1-\frac{i}{bn}\right);\frac{1}{2}\left(3-\frac{i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{i-bn}$$

```
[Out] 2*exp(I*a)*x*(c*x^n)^(I*b)*hypergeom([1, 1/2-1/2*I/b/n],[3/2-1/2*I/b/n],exp
(2*I*a)*(c*x^n)^(2*I*b))/(I-b*n)
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4600, 4602, 371}

$$\frac{2e^{ia}x(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}\left(1-\frac{i}{bn}\right);\frac{1}{2}\left(3-\frac{i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{-bn+i}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*Log[c*x^n]],x]
```

```
[Out] (2*E^(I*a)*x*(c*x^n)^(I*b)*Hypergeometric2F1[1, (1 - I/(b*n))/2, (3 - I/(b*
n))/2, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/(I - b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4600

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4602

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[(-2*I)^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(
2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \csc(a+b\log(cx^n))\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\csc(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= -\frac{\left(2ie^{ia}x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+ib+\frac{1}{n}}}{1-e^{2ia}x^{2ib}}\,dx,x,cx^n\right)}{n}$$
$$= \frac{2e^{ia}x(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}\left(1-\frac{i}{bn}\right);\frac{1}{2}\left(3-\frac{i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{i-bn}$$

Mathematica [A]
time = 1.28, size = 80, normalized size = 0.95

$$-\frac{2e^{ia}x(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}-\frac{i}{2bn};\frac{3}{2}-\frac{i}{2bn};e^{2i(a+b\log(cx^n))}\right)}{-i+bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*Log[c*x^n]],x]
[Out] (-2*E^(I*a)*x*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 - (I/2)/(b*n), 3/2 - (
I/2)/(b*n), E^((2*I)*(a + b*Log[c*x^n]))])/(-I + b*n)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \csc(a+b\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(a+b*ln(c*x^n)),x)
[Out] int(csc(a+b*ln(c*x^n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(csc(b*log(c*x^n) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(csc(b*log(c*x^n) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \csc \left(a + b \log \left(cx^{n}\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*ln(c*x**n)),x)
[Out] Integral(csc(a + b*log(c*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate(csc(b*log(c*x^n) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(a + b*log(c*x^n)),x)
[Out] int(1/sin(a + b*log(c*x^n)), x)
```

3.292 $\int \frac{\csc(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=20

$$-\frac{\tanh^{-1}(\cos(a+b\log(cx^n)))}{bn}$$

[Out] -arctanh(cos(a+b*ln(c*x^n)))/b/n

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3855}

$$-\frac{\tanh^{-1}(\cos(a+b\log(cx^n)))}{bn}$$

Antiderivative was successfully verified.

[In] Int[Csc[a + b*Log[c*x^n]]/x,x]

[Out] -(ArcTanh[Cos[a + b*Log[c*x^n]]]/(b*n))

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \csc(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\ &= -\frac{\tanh^{-1}(\cos(a+b\log(cx^n)))}{bn}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 54 vs. 2(20) = 40.
time = 0.07, size = 54, normalized size = 2.70

$$-\frac{\log\left(\cos\left(\frac{a}{2}+\frac{1}{2}b\log(cx^n)\right)\right)}{bn}+\frac{\log\left(\sin\left(\frac{a}{2}+\frac{1}{2}b\log(cx^n)\right)\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[a + b*Log[c*x^n]]/x,x]

```
[Out] -(Log[Cos[a/2 + (b*Log[c*x^n])/2]]/(b*n)) + Log[Sin[a/2 + (b*Log[c*x^n])/2]
]/(b*n)
```

Maple [A]
time = 0.12, size = 33, normalized size = 1.65

method	result
derivativedivides	$-\frac{\ln(\csc(a+b\ln(c\,x^n))+\cot(a+b\ln(c\,x^n)))}{nb}$
default	$-\frac{\ln(\csc(a+b\ln(c\,x^n))+\cot(a+b\ln(c\,x^n)))}{nb}$
risch	$\frac{\ln\left(c^{ib}(x^n)^{ib}\mathrm{e}^{-\frac{b\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ix^n)}{2}}\mathrm{e}^{\frac{b\pi\,\mathrm{csgn}(ic\,x^n)\mathrm{csgn}(ic)\mathrm{csgn}(ix^n)}{2}}\mathrm{e}^{\frac{b\pi\mathrm{csgn}(ic\,x^n)^3}{2}}\mathrm{e}^{-\frac{b\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ic)}{2}}\mathrm{e}^{ia}-1\right)}{bn}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
[Out] -1/n/b*ln(csc(a+b*ln(c*x^n))+cot(a+b*ln(c*x^n)))
```

Maxima [A]
time = 0.27, size = 32, normalized size = 1.60

$$-\frac{\log\left(\cot\left(b\log\left(cx^n\right)+a\right)+\csc\left(b\log\left(cx^n\right)+a\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))/x,x, algorithm="maxima")
[Out] -log(cot(b*log(c*x^n) + a) + csc(b*log(c*x^n) + a))/(b*n)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(20) = 40$.
time = 2.55, size = 45, normalized size = 2.25

$$-\frac{\log\left(\frac{1}{2}\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\frac{1}{2}\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))/x,x, algorithm="fricas")
[Out] -1/2*(log(1/2*cos(b*n*log(x) + b*log(c) + a) + 1/2) - log(-1/2*cos(b*n*log(
x) + b*log(c) + a) + 1/2))/(b*n)
```

Sympy [A]
time = 1.32, size = 49, normalized size = 2.45

$$-\begin{cases}-\log\left(x\right)\csc\left(a\right) & \text{for}\: b=0\\-\log\left(x\right)\csc\left(a+b\log\left(c\right)\right) & \text{for}\: n=0\\\frac{\log\left(\cot\left(a+b\log\left(cx^n\right)\right)+\csc\left(a+b\log\left(cx^n\right)\right)\right)}{bn} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*ln(c*x**n))/x,x)
```

```
[Out] -Piecewise((-log(x)*csc(a), Eq(b, 0)), (-log(x)*csc(a + b*log(c)), Eq(n, 0)
), (log(cot(a + b*log(c*x**n)) + csc(a + b*log(c*x**n)))/(b*n), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))/x,x, algorithm="giac")
```

```
[Out] integrate(csc(b*log(c*x^n) + a)/x, x)
```

Mupad [B]
time = 4.00, size = 68, normalized size = 3.40

$$\frac{\ln\left(\frac{\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,1\mathrm{i}}\,2\mathrm{i}-2\mathrm{i}}{x}\right)}{b\,n}-\frac{\ln\left(\frac{\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,1\mathrm{i}}\,2\mathrm{i}+2\mathrm{i}}{x}\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*sin(a + b*log(c*x^n))),x)
```

```
[Out] log((exp(a*1i)*(c*x^n)^(b*1i)*2i - 2i)/x)/(b*n) - log((exp(a*1i)*(c*x^n)^(b
*1i)*2i + 2i)/x)/(b*n)
```

3.293 $\int \frac{\csc(a+b\log(cx^n))}{x^2}\,dx$

Optimal. Leaf size=85

$$-\frac{2e^{ia}(cx^n)^{ib}\ {}_2F_1\left(1,\frac{1}{2}\left(1+\frac{i}{bn}\right);\frac{1}{2}\left(3+\frac{i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{(i+bn)x}$$

```
[Out] -2*exp(I*a)*(c*x^n)^(I*b)*hypergeom([1, 1/2+1/2*I/b/n],[3/2+1/2*I/b/n],exp(
2*I*a)*(c*x^n)^(2*I*b))/(I+b*n)/x
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4606, 4602, 371}

$$-\frac{2e^{ia}(cx^n)^{ib}\ {}_2F_1\left(1,\frac{1}{2}\left(1+\frac{i}{bn}\right);\frac{1}{2}\left(3+\frac{i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{x(bn+i)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*Log[c*x^n]]/x^2,x]
```

```
[Out] (-2*E^(I*a)*(c*x^n)^(I*b)*Hypergeometric2F1[1, (1 + I/(b*n))/2, (3 + I/(b*n
))/2, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((I + b*n)*x)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4602

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[(-2*I)^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(
2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4606

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\csc(a+b\log(cx^n))}{x^2}\,dx = \frac{(cx^n)^{\frac{1}{n}}\,\text{Subst}\left(\int x^{-1-\frac{1}{n}}\csc(a+b\log(x))\,dx, x, cx^n\right)}{nx}$$
$$= -\frac{\left(2ie^{ia}(cx^n)^{\frac{1}{n}}\right)\text{Subst}\left(\int \frac{x^{-1+ib-\frac{1}{n}}}{1-e^{2ia}x^{2ib}}\,dx, x, cx^n\right)}{nx}$$
$$= -\frac{2e^{ia}(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}\left(1+\frac{i}{bn}\right);\frac{1}{2}\left(3+\frac{i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{(i+bn)x}$$

Mathematica [A]
time = 1.16, size = 82, normalized size = 0.96

$$-\frac{2e^{ia}(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}+\frac{i}{2bn};\frac{3}{2}+\frac{i}{2bn};e^{2i(a+b\log(cx^n))}\right)}{(i+bn)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*Log[c*x^n]]/x^2,x]
```

```
[Out] (-2*E^(I*a)*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 + (I/2)/(b*n), 3/2 + (I/
2)/(b*n), E^((2*I)*(a + b*Log[c*x^n]))])/((I + b*n)*x)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+b\ln(c\,x^n))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(a+b*ln(c*x^n))/x^2,x)
```

```
[Out] int(csc(a+b*ln(c*x^n))/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(csc(b*log(c*x^n) + a)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))/x^2,x, algorithm="fricas")
[Out] integral(csc(b*log(c*x^n) + a)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(a+b\log\left(cx^{n}\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*ln(c*x**n))/x**2,x)
[Out] Integral(csc(a + b*log(c*x**n))/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))/x^2,x, algorithm="giac")
[Out] integrate(csc(b*log(c*x^n) + a)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{2}\,\sin\left(a+b\,\ln\left(c\,x^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*sin(a + b*log(c*x^n))),x)
[Out] int(1/(x^2*sin(a + b*log(c*x^n))), x)
```

3.294 $\int \frac{\csc(a+b\log(cx^n))}{x^3}\,dx$

Optimal. Leaf size=85

$$-\frac{2e^{ia}(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}\left(1+\frac{2i}{bn}\right);\frac{1}{2}\left(3+\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{(2i+bn)x^2}$$

```
[Out] -2*exp(I*a)*(c*x^n)^(I*b)*hypergeom([1, 1/2+I/b/n],[3/2+I/b/n],exp(2*I*a)*(
c*x^n)^(2*I*b))/(2*I+b*n)/x^2
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4606, 4602, 371}

$$-\frac{2e^{ia}(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}\left(1+\frac{2i}{bn}\right);\frac{1}{2}\left(3+\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{x^2(bn+2i)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*Log[c*x^n]]/x^3,x]
```

```
[Out] (-2*E^(I*a)*(c*x^n)^(I*b)*Hypergeometric2F1[1, (1 + (2*I)/(b*n))/2, (3 + (2
*I)/(b*n))/2, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((2*I + b*n)*x^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4602

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[(-2*I)^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(
2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4606

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\csc(a+b\log(cx^n))}{x^3}\,dx = \frac{(cx^n)^{2/n}\,\text{Subst}\left(\int x^{-1-\frac{2}{n}}\csc(a+b\log(x))\,dx, x, cx^n\right)}{nx^2}$$
$$= -\frac{\left(2ie^{ia}(cx^n)^{2/n}\right)\text{Subst}\left(\int \frac{x^{-1+ib-\frac{2}{n}}}{1-e^{2ia}x^{2ib}}\,dx, x, cx^n\right)}{nx^2}$$
$$= -\frac{2e^{ia}(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}\left(1+\frac{2i}{bn}\right);\frac{1}{2}\left(3+\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{(2i+bn)x^2}$$

Mathematica [A]
time = 1.15, size = 78, normalized size = 0.92

$$-\frac{2e^{ia}(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}+\frac{i}{bn};\frac{3}{2}+\frac{i}{bn};e^{2i(a+b\log(cx^n))}\right)}{(2i+bn)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*Log[c*x^n]]/x^3,x]
[Out] (-2*E^(I*a)*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 + I/(b*n), 3/2 + I/(b*n)
, E^((2*I)*(a + b*Log[c*x^n]))])/((2*I + b*n)*x^2)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+b\ln(c\,x^n))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(a+b*ln(c*x^n))/x^3,x)
[Out] int(csc(a+b*ln(c*x^n))/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))/x^3,x, algorithm="maxima")
[Out] integrate(csc(b*log(c*x^n) + a)/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))/x^3,x, algorithm="fricas")
```

```
[Out] integral(csc(b*log(c*x^n) + a)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(a + b\log\left(cx^{n}\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*ln(c*x**n))/x**3,x)
```

```
[Out] Integral(csc(a + b*log(c*x**n))/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))/x^3,x, algorithm="giac")
```

```
[Out] integrate(csc(b*log(c*x^n) + a)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3}\,\sin\left(a+b\,\ln\left(c\,x^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*sin(a + b*log(c*x^n))),x)
```

```
[Out] int(1/(x^3*sin(a + b*log(c*x^n))), x)
```

3.295 $\int \csc^2(a + b\log(cx^n)) \, dx$

Optimal. Leaf size=84

$$-\frac{4e^{2ia}x(cx^n)^{2ib} \, {}_2F_1\left(2, \frac{1}{2}\left(2 - \frac{i}{bn}\right); \frac{1}{2}\left(4 - \frac{i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right)}{1 + 2ibn}$$

```
[Out] -4*exp(2*I*a)*x*(c*x^n)^(2*I*b)*hypergeom([2, 1-1/2*I/b/n],[2-1/2*I/b/n],ex
p(2*I*a)*(c*x^n)^(2*I*b))/(1+2*I*b*n)
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4600, 4602, 371}

$$-\frac{4e^{2ia}x(cx^n)^{2ib} \, {}_2F_1\left(2, \frac{1}{2}\left(2 - \frac{i}{bn}\right); \frac{1}{2}\left(4 - \frac{i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right)}{1 + 2ibn}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (-4*E^((2*I)*a)*x*(c*x^n)^((2*I)*b)*Hypergeometric2F1[2, (2 - I/(b*n))/2, (
4 - I/(b*n))/2, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/(1 + (2*I)*b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4600

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4602

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[(-2*I)^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(
2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \csc^2(a+b\log(cx^n))\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\csc^2(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= -\frac{\left(4e^{2ia}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+2ib+\frac{1}{n}}}{(1-e^{2ia}x^{2ib})^2}\,dx,x,cx^n\right)}{n}$$
$$= -\frac{4e^{2ia}x(cx^n)^{2ib}\,{}_2F_1\left(2,\frac{1}{2}\left(2-\frac{i}{bn}\right);\frac{1}{2}\left(4-\frac{i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{1+2ibn}$$

Mathematica [A]
time = 5.20, size = 146, normalized size = 1.74

$$\frac{x\left(-\cot(a+b\log(cx^n))-\frac{e^{2ia}(cx^n)^{2ib}\,{}_2F_1\left(1,1-\frac{i}{2bn};2-\frac{i}{2bn};e^{2i(a+b\log(cx^n))}\right)}{-i+2bn}-i\,{}_2F_1\left(1,-\frac{i}{2bn};1-\frac{i}{2bn};e^{2i(a+b\log(cx^n))}\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x*(-Cot[a + b*Log[c*x^n]] - (E^((2*I)*a)*(c*x^n)^((2*I)*b)*Hypergeometric2
F1[1, 1 - (I/2)/(b*n), 2 - (I/2)/(b*n), E^((2*I)*(a + b*Log[c*x^n]))])/(-I
+ 2*b*n) - I*Hypergeometric2F1[1, (-1/2*I)/(b*n), 1 - (I/2)/(b*n), E^((2*I)
*(a + b*Log[c*x^n]))]))/(b*n)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \csc^2(a+b\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(csc(a+b*ln(c*x^n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] (2*x*cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + 2*x*cos(2*b*log(c))*sin(2*b*
log(x^n) + 2*a) - (2*b^2*n^2*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) - 2*b^
2*n^2*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) - (b^2*cos(2*b*log(c))^2 + b^
2*sin(2*b*log(c))^2)*n^2*cos(2*b*log(x^n) + 2*a)^2 - (b^2*cos(2*b*log(c))^2
 + b^2*sin(2*b*log(c))^2)*n^2*sin(2*b*log(x^n) + 2*a)^2 - b^2*n^2)*integrat
e((cos(b*log(x^n) + a)*sin(b*log(c)) + cos(b*log(c))*sin(b*log(x^n) + a))/(
2*b^2*n^2*cos(b*log(c))*cos(b*log(x^n) + a) - 2*b^2*n^2*sin(b*log(c))*sin(b
*log(x^n) + a) + (b^2*cos(b*log(c))^2 + b^2*sin(b*log(c))^2)*n^2*cos(b*log(
x^n) + a)^2 + (b^2*cos(b*log(c))^2 + b^2*sin(b*log(c))^2)*n^2*sin(b*log(x^n
) + a)^2 + b^2*n^2), x) + (2*b^2*n^2*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a
) - 2*b^2*n^2*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) - (b^2*cos(2*b*log(c)
)^2 + b^2*sin(2*b*log(c))^2)*n^2*cos(2*b*log(x^n) + 2*a)^2 - (b^2*cos(2*b*l
og(c))^2 + b^2*sin(2*b*log(c))^2)*n^2*sin(2*b*log(x^n) + 2*a)^2 - b^2*n^2)*
integrate(-(cos(b*log(x^n) + a)*sin(b*log(c)) + cos(b*log(c))*sin(b*log(x^n
) + a))/(2*b^2*n^2*cos(b*log(c))*cos(b*log(x^n) + a) - 2*b^2*n^2*sin(b*log(
c))*sin(b*log(x^n) + a) - (b^2*cos(b*log(c))^2 + b^2*sin(b*log(c))^2)*n^2*c
os(b*log(x^n) + a)^2 - (b^2*cos(b*log(c))^2 + b^2*sin(b*log(c))^2)*n^2*sin(
b*log(x^n) + a)^2 - b^2*n^2), x))/(2*b*n*cos(2*b*log(c))*cos(2*b*log(x^n) +
 2*a) - (b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*cos(2*b*log(x^n) + 2*
a)^2 - 2*b*n*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) - (b*cos(2*b*log(c))^2
 + b*sin(2*b*log(c))^2)*n*sin(2*b*log(x^n) + 2*a)^2 - b*n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*log(c*x^n) + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^2(a + b\log(cx^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*ln(c*x**n))**2,x)
```

```
[Out] Integral(csc(a + b*log(c*x**n))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate(csc(b*log(c*x^n) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(a + b*log(c*x^n))^2,x)
```

```
[Out] int(1/sin(a + b*log(c*x^n))^2, x)
```

3.296 $\int \frac{\csc^2(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=19

$$-\frac{\cot\left(a+b\log\left(cx^n\right)\right)}{bn}$$

```
[Out] -cot(a+b*ln(c*x^n))/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = $\{3852, 8\}$

$$-\frac{\cot\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] -(Cot[a + b*Log[c*x^n]]/(b*n))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \csc^2(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\ &= -\frac{\text{Subst}\left(\int 1\,dx,x,\cot\left(a+b\log\left(cx^n\right)\right)\right)}{bn}\\ &= -\frac{\cot\left(a+b\log\left(cx^n\right)\right)}{bn}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 19, normalized size = 1.00

$$-\frac{\cot\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] -(Cot[a + b*Log[c*x^n]]/(b*n))
```

Maple [A]
time = 0.09, size = 20, normalized size = 1.05

method	result
derivativedivides	$-\frac{\cot(a+b\ln(c\,x^{n}))}{bn}$
default	$-\frac{\cot(a+b\ln(c\,x^{n}))}{bn}$
risch	$-\frac{2i}{bn\left(\left(x^{n}\right)^{2ib}c^{2ib}\mathrm{e}^{b\pi\mathrm{csgn}(ic\,x^{n})^{3}}\mathrm{e}^{-b\pi\mathrm{csgn}(ic\,x^{n})^{2}\mathrm{csgn}(ic)}\mathrm{e}^{-b\pi\mathrm{csgn}(ic\,x^{n})^{2}\mathrm{csgn}(ix^{n})}\mathrm{e}^{b\pi\,\mathrm{csgn}(ic\,x^{n})\mathrm{csgn}(ic)\mathrm{csgn}(ix^{n})}\mathrm{e}^{2ia}-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(a+b*ln(c*x^n))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] -cot(a+b*ln(c*x^n))/b/n
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(19) = 38$.
time = 0.29, size = 168, normalized size = 8.84

$$\frac{2\left(\cos\left(2\,b\log\left(x^{n}\right)+2\,a\right)\sin\left(2\,b\log\left(c\right)\right)+\cos\left(2\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(x^{n}\right)+2\,a\right)\right)}{2\,bn\cos\left(2\,b\log\left(c\right)\right)\cos\left(2\,b\log\left(x^{n}\right)+2\,a\right)-\left(b\cos\left(2\,b\log\left(c\right)\right)^{2}+b\sin\left(2\,b\log\left(c\right)\right)^{2}\right)n\cos\left(2\,b\log\left(x^{n}\right)+2\,a\right)^{2}-2\,bn\sin\left(2\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(x^{n}\right)+2\,a\right)-\left(b\cos\left(2\,b\log\left(c\right)\right)^{2}+b\sin\left(2\,b\log\left(c\right)\right)^{2}\right)n\sin\left(2\,b\log\left(x^{n}\right)+2\,a\right)^{2}-bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] 2*(cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + cos(2*b*log(c))*sin(2*b*log(x^
n) + 2*a))/(2*b*n*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) - (b*cos(2*b*log(
c))^2 + b*sin(2*b*log(c))^2)*n*cos(2*b*log(x^n) + 2*a)^2 - 2*b*n*sin(2*b*lo
g(c))*sin(2*b*log(x^n) + 2*a) - (b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)
*n*sin(2*b*log(x^n) + 2*a)^2 - b*n)
```

Fricas [A]
time = 1.80, size = 34, normalized size = 1.79

$$-\frac{\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{bn\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

```
[Out] -cos(b*n*log(x) + b*log(c) + a)/(b*n*sin(b*n*log(x) + b*log(c) + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2\left(a+b\log\left(cx^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*ln(c*x**n))**2/x,x)
[Out] Integral(csc(a + b*log(c*x**n))**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^2/x,x, algorithm="giac")
[Out] integrate(csc(b*log(c*x^n) + a)^2/x, x)
```

Mupad [B]
time = 3.90, size = 29, normalized size = 1.53

$$-\frac{2\mathrm{i}}{b\,n\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*sin(a + b*log(c*x^n))^2),x)
[Out] -2i/(b*n*(exp(a*2i)*(c*x^n)^(b*2i) - 1))
```

3.297 $\int \csc^3(a + b\log(cx^n))\,dx$

Optimal. Leaf size=84

$$-\frac{8e^{3ia}x(cx^n)^{3ib}\,{}_2F_1\left(3,\frac{1}{2}\left(3-\frac{i}{bn}\right);\frac{1}{2}\left(5-\frac{i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{i-3bn}$$

```
[Out] -8*exp(3*I*a)*x*(c*x^n)^(3*I*b)*hypergeom([3, 3/2-1/2*I/b/n],[5/2-1/2*I/b/n
],exp(2*I*a)*(c*x^n)^(2*I*b))/(I-3*b*n)
```

Rubi [A]
time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4600, 4602, 371}

$$-\frac{8e^{3ia}x(cx^n)^{3ib}\,{}_2F_1\left(3,\frac{1}{2}\left(3-\frac{i}{bn}\right);\frac{1}{2}\left(5-\frac{i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{-3bn+i}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (-8*E^((3*I)*a)*x*(c*x^n)^((3*I)*b)*Hypergeometric2F1[3, (3 - I/(b*n))/2, (
5 - I/(b*n))/2, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/(I - 3*b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4600

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4602

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[(-2*I)^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(
2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \csc^3(a+b\log(cx^n))\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\csc^3(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(8ie^{3ia}x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+3ib+\frac{1}{n}}}{(1-e^{2ia}x^{2ib})^3}\,dx,x,cx^n\right)}{n}$$
$$= -\frac{8e^{3ia}x(cx^n)^{3ib}\,{}_2F_1\left(3,\frac{1}{2}\left(3-\frac{i}{bn}\right);\frac{1}{2}\left(5-\frac{i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{i-3bn}$$

Mathematica [A]
time = 4.52, size = 117, normalized size = 1.39

$$-\frac{x\left((1+bn\cot(a+b\log(cx^n)))\csc(a+b\log(cx^n))+2e^{ia}(i+bn)(cx^n)^{ib}\,{}_2F_1\left(1,\frac{1}{2}-\frac{i}{2bn};\frac{3}{2}-\frac{i}{2bn};e^{2i(a+b\log(cx^n))}\right)\right)}{2b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*Log[c*x^n]]^3,x]
```

```
[Out] -1/2*(x*((1 + b*n*Cot[a + b*Log[c*x^n]])*Csc[a + b*Log[c*x^n]] + 2*E^(I*a)*
(I + b*n)*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 - (I/2)/(b*n), 3/2 - (I/2)
/(b*n), E^((2*I)*(a + b*Log[c*x^n]))]))/(b^2*n^2)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \csc^3(a+b\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(csc(a+b*ln(c*x^n))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] -((b*n*cos(b*log(c)) - sin(b*log(c)))*x*cos(b*log(x^n) + a) - (b*n*sin(b*lo
g(c)) + cos(b*log(c)))*x*sin(b*log(x^n) + a) + (((b*cos(4*b*log(c))*cos(3*b
```

```
*log(c)) + b*sin(4*b*log(c))*sin(3*b*log(c)))*n - cos(3*b*log(c))*sin(4*b*l
og(c)) + cos(4*b*log(c))*sin(3*b*log(c)))*x*cos(3*b*log(x^n) + 3*a) + ((b*c
os(4*b*log(c))*cos(b*log(c)) + b*sin(4*b*log(c))*sin(b*log(c)))*n + cos(b*l
og(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(b*log(c)))*x*cos(b*log(x^n) +
a) + ((b*cos(3*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(3*b*log(c)
))*n + cos(4*b*log(c))*cos(3*b*log(c)) + sin(4*b*log(c))*sin(3*b*log(c)))*x
*sin(3*b*log(x^n) + 3*a) + ((b*cos(b*log(c))*sin(4*b*log(c)) - b*cos(4*b*lo
g(c))*sin(b*log(c)))*n - cos(4*b*log(c))*cos(b*log(c)) - sin(4*b*log(c))*si
n(b*log(c)))*x*sin(b*log(x^n) + a))*cos(4*b*log(x^n) + 4*a) - (2*((b*cos(3*
b*log(c))*cos(2*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*log(c)))*n + cos(2*b*
log(c))*sin(3*b*log(c)) - cos(3*b*log(c))*sin(2*b*log(c)))*x*cos(2*b*log(x^
n) + 2*a) + 2*((b*cos(2*b*log(c))*sin(3*b*log(c)) - b*cos(3*b*log(c))*sin(2
*b*log(c)))*n - cos(3*b*log(c))*cos(2*b*log(c)) - sin(3*b*log(c))*sin(2*b*l
og(c)))*x*sin(2*b*log(x^n) + 2*a) - (b*n*cos(3*b*log(c)) + sin(3*b*log(c)))
*x)*cos(3*b*log(x^n) + 3*a) - 2*(((b*cos(2*b*log(c))*cos(b*log(c)) + b*sin(
2*b*log(c))*sin(b*log(c)))*n + cos(b*log(c))*sin(2*b*log(c)) - cos(2*b*log(
c))*sin(b*log(c)))*x*cos(b*log(x^n) + a) + ((b*cos(b*log(c))*sin(2*b*log(c)
) - b*cos(2*b*log(c))*sin(b*log(c)))*n - cos(2*b*log(c))*cos(b*log(c)) - si
n(2*b*log(c))*sin(b*log(c)))*x*sin(b*log(x^n) + a))*cos(2*b*log(x^n) + 2*a)
 + 2*(b^6*n^6 + b^4*n^4 + ((b^6*cos(4*b*log(c))^2 + b^6*sin(4*b*log(c))^2)*
n^6 + (b^4*cos(4*b*log(c))^2 + b^4*sin(4*b*log(c))^2)*n^4)*cos(4*b*log(x^n)
 + 4*a)^2 + 4*((b^6*cos(2*b*log(c))^2 + b^6*sin(2*b*log(c))^2)*n^6 + (b^4*c
os(2*b*log(c))^2 + b^4*sin(2*b*log(c))^2)*n^4)*cos(2*b*log(x^n) + 2*a)^2 +
((b^6*cos(4*b*log(c))^2 + b^6*sin(4*b*log(c))^2)*n^6 + (b^4*cos(4*b*log(c))
^2 + b^4*sin(4*b*log(c))^2)*n^4)*sin(4*b*log(x^n) + 4*a)^2 + 4*((b^6*cos(2*
b*log(c))^2 + b^6*sin(2*b*log(c))^2)*n^6 + (b^4*cos(2*b*log(c))^2 + b^4*sin
(2*b*log(c))^2)*n^4)*sin(2*b*log(x^n) + 2*a)^2 + 2*(b^6*n^6*cos(4*b*log(c))
 + b^4*n^4*cos(4*b*log(c)) - 2*((b^6*cos(4*b*log(c))*cos(2*b*log(c)) + b^6*
sin(4*b*log(c))*sin(2*b*log(c)))*n^6 + (b^4*cos(4*b*log(c))*cos(2*b*log(c))
 + b^4*sin(4*b*log(c))*sin(2*b*log(c)))*n^4)*cos(2*b*log(x^n) + 2*a) - 2*((
b^6*cos(2*b*log(c))*sin(4*b*log(c)) - b^6*cos(4*b*log(c))*sin(2*b*log(c)))*
n^6 + (b^4*cos(2*b*log(c))*sin(4*b*log(c)) - b^4*cos(4*b*log(c))*sin(2*b*lo
g(c)))*n^4)*sin(2*b*log(x^n) + 2*a))*cos(4*b*log(x^n) + 4*a) - 4*(b^6*n^6*c
os(2*b*log(c)) + b^4*n^4*cos(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) - 2*(b^6*
n^6*sin(4*b*log(c)) + b^4*n^4*sin(4*b*log(c)) - 2*((b^6*cos(2*b*log(c))*sin
(4*b*log(c)) - b^6*cos(4*b*log(c))*sin(2*b*log(c)))*n^6 + (b^4*cos(2*b*log(
c))*sin(4*b*log(c)) - b^4*cos(4*b*log(c))*sin(2*b*log(c)))*n^4)*cos(2*b*log
(x^n) + 2*a) + 2*((b^6*cos(4*b*log(c))*cos(2*b*log(c)) + b^6*sin(4*b*log(c)
)*sin(2*b*log(c)))*n^6 + (b^4*cos(4*b*log(c))*cos(2*b*log(c)) + b^4*sin(4*b
*log(c))*sin(2*b*log(c)))*n^4)*sin(2*b*log(x^n) + 2*a))*sin(4*b*log(x^n) +
4*a) + 4*(b^6*n^6*sin(2*b*log(c)) + b^4*n^4*sin(2*b*log(c)))*sin(2*b*log(x^
n) + 2*a))*integrate(1/4*(cos(b*log(x^n) + a)*sin(b*log(c)) + cos(b*log(c))
*sin(b*log(x^n) + a))/(2*b^4*n^4*cos(b*log(c))*cos(b*log(x^n) + a) - 2*b^4*
n^4*sin(b*log(c))*sin(b*log(x^n) + a) + b^4*n^4 + (b^4*cos(b*log(c))^2 + b^
4*sin(b*log(c))^2)*n^4*cos(b*log(x^n) + a)^2 + (b^4*cos(b*log(c))^2 + b^4*s
```

```
in(b*log(c))^2)*n^4*sin(b*log(x^n) + a)^2), x) + 2*(b^6*n^6 + b^4*n^4 + ((b
^6*cos(4*b*log(c))^2 + b^6*sin(4*b*log(c))^2)*n^6 + (b^4*cos(4*b*log(c))^2
+ b^4*sin(4*b*log(c))^2)*n^4)*cos(4*b*log(x^n) + 4*a)^2 + 4*((b^6*cos(2*b*l
og(c))^2 + b^6*sin(2*b*log(c))^2)*n^6 + (b^4*cos(2*b*log(c))^2 + b^4*sin(2*
b*log(c))^2)*n^4)*cos(2*b*log(x^n) + 2*a)^2 + ((b^6*cos(4*b*log(c))^2 + b^6
*sin(4*b*log(c))^2)*n^6 + (b^4*cos(4*b*log(c))^2 + b^4*sin(4*b*log(c))^2)*n
^4)*sin(4*b*log(x^n) + 4*a)^2 + 4*((b^6*cos(2*b*log(c))^2 + b^6*sin(2*b*log
(c))^2)*n^6 + (b^4*cos(2*b*log(c))^2 + b^4*sin(2*b*log(c))^2)*n^4)*sin(2*b*
log(x^n) + 2*a)^2 + 2*(b^6*n^6*cos(4*b*log(c)) + b^4*n^4*cos(4*b*log(c)) -
2*((b^6*cos(4*b*log(c))*cos(2*b*log(c)) + b^6*sin(4*b*log(c))*sin(2*b*log(c
)))*n^6 + (b^4*cos(4*b*log(c))*cos(2*b*log(c)) + b^4*sin(4*b*log(c))*sin(2*
b*log(c)))*n^4)*cos(2*b*log(x^n) + 2*a) - 2*((b^6*cos(2*b*log(c))*sin(4*b*l
og(c)) - b^6*cos(4*b*log(c))*sin(2*b*log(c)))*n^6 + (b^4*cos(2*b*log(c))*si
n(4*b*log(c)) - b^4*cos(4*b*log(c))*sin(2*b*log(c)))*n^4)*sin(2*b*log(x^n)
+ 2*a))*cos(4*b*log(x^n) + 4*a) - 4*(b^6*n^6*cos(2*b*log(c)) + b^4*n^4*cos(
2*b*log(c)))*cos(2*b*log(x^n) + 2*a) - 2*(b^6*n^6*sin(4*b*log(c)) + b^4*n^4
*sin(4*b*log(c)) - 2*((b^6*cos(2*b*log(c))*sin(4*b*log(c)) - b^6*cos(4*b*lo
g(c))*sin(2*b*log(c)))*n^6 + (b^4*cos(2*b*log(c))*sin(4*b*log(c)) - b^4*cos
(4*b*log(c))*sin(2*b*log(c)))*n^4)*cos(2*b*log(x^n) + 2*a) + 2*((b^6*cos(4*
b*log(c))*cos(2*b*log(c)) + b^6*sin(4*b*log(c))...
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] integral(csc(b*log(c*x^n) + a)^3, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^{3}\left(a + b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*ln(c*x**n))**3,x)
```

```
[Out] Integral(csc(a + b*log(c*x**n))**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] integrate(csc(b*log(c*x^n) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(a + b*log(c*x^n))^3,x)
```

```
[Out] int(1/sin(a + b*log(c*x^n))^3, x)
```

3.298 $\int \frac{\csc^3(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=55

$$-\frac{\tanh^{-1}(\cos(a+b\log(cx^n)))}{2bn}-\frac{\cot(a+b\log(cx^n))\csc(a+b\log(cx^n))}{2bn}$$

```
[Out] -1/2*arctanh(cos(a+b*ln(c*x^n)))/b/n-1/2*cot(a+b*ln(c*x^n))*csc(a+b*ln(c*x^
n))/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3853, 3855}

$$-\frac{\tanh^{-1}(\cos(a+b\log(cx^n)))}{2bn}-\frac{\cot(a+b\log(cx^n))\csc(a+b\log(cx^n))}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] -1/2*ArcTanh[Cos[a + b*Log[c*x^n]]]/(b*n) - (Cot[a + b*Log[c*x^n]]*Csc[a +
b*Log[c*x^n]])/(2*b*n)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^3(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \csc^3(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\
&= -\frac{\cot(a+b\log(cx^n))\csc(a+b\log(cx^n))}{2bn}+\frac{\text{Subst}\left(\int \csc(a+bx)\,dx, x, \log(c\right.}{2n}\\
&= -\frac{\tanh^{-1}(\cos(a+b\log(cx^n)))}{2bn}-\frac{\cot(a+b\log(cx^n))\csc(a+b\log(cx^n))}{2bn}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 107, normalized size = 1.95

$$-\frac{\csc^2\left(\frac{1}{2}(a+b\log(cx^n))\right)}{8bn}-\frac{\log\left(\cos\left(\frac{1}{2}(a+b\log(cx^n))\right)\right)}{2bn}+\frac{\log\left(\sin\left(\frac{1}{2}(a+b\log(cx^n))\right)\right)}{2bn}+\frac{\sec^2\left(\frac{1}{2}(a+b\log(cx^n))\right)}{8bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] -1/8*Csc[(a + b*Log[c*x^n])/2]^2/(b*n) - Log[Cos[(a + b*Log[c*x^n])/2]]/(2*
b*n) + Log[Sin[(a + b*Log[c*x^n])/2]]/(2*b*n) + Sec[(a + b*Log[c*x^n])/2]^2
/(8*b*n)
```

Maple [A]
time = 0.26, size = 61, normalized size = 1.11

method	result
derivativedivides	$\frac{-\frac{\csc(a+b\ln(c\,x^n))\cot(a+b\ln(c\,x^n))}{2}+\frac{\ln(\csc(a+b\ln(c\,x^n))-\cot(a+b\ln(c\,x^n)))}{2}}{nb}$
default	$\frac{-\frac{\csc(a+b\ln(c\,x^n))\cot(a+b\ln(c\,x^n))}{2}+\frac{\ln(\csc(a+b\ln(c\,x^n))-\cot(a+b\ln(c\,x^n)))}{2}}{nb}$
risch	$\frac{c^{ib}(x^n)^{ib}\left(c^{2ib}(x^n)^{2ib}\mathrm{e}^{\frac{3b\pi\mathrm{csgn}(ic\,x^n)^3}{2}}\mathrm{e}^{-\frac{3b\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ic)}{2}}\mathrm{e}^{-\frac{3b\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ix^n)}{2}}\mathrm{e}^{\frac{3b\pi\,\mathrm{csgn}(ic\,x^n)\mathrm{csgn}(ic)\mathrm{csgn}(ix^n}{2}}\right.}{bn\left((x^n)^{2ib}c^{2ib}\mathrm{e}^{b\pi\mathrm{csgn}(ic\,x^n)^3}\mathrm{e}^{-b\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ic)}\mathrm{e}^{-b\pi\mathrm{csgn}(ic}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(-1/2*csc(a+b*ln(c*x^n))*cot(a+b*ln(c*x^n))+1/2*ln(csc(a+b*ln(c*x^n))
-cot(a+b*ln(c*x^n))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2168 vs. 2(51) = 102.
time = 0.35, size = 2168, normalized size = 39.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] 1/4*(4*((cos(4*b*log(c))*cos(3*b*log(c)) + sin(4*b*log(c))*sin(3*b*log(c)))
*cos(3*b*log(x^n) + 3*a) + (cos(4*b*log(c))*cos(b*log(c)) + sin(4*b*log(c))
*sin(b*log(c)))*cos(b*log(x^n) + a) + (cos(3*b*log(c))*sin(4*b*log(c)) - co
s(4*b*log(c))*sin(3*b*log(c)))*sin(3*b*log(x^n) + 3*a) + (cos(b*log(c))*sin
(4*b*log(c)) - cos(4*b*log(c))*sin(b*log(c)))*sin(b*log(x^n) + a))*cos(4*b*
log(x^n) + 4*a) - 4*(2*(cos(3*b*log(c))*cos(2*b*log(c)) + sin(3*b*log(c))*s
```

```
in(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 2*(cos(2*b*log(c))*sin(3*b*log(c)
) - cos(3*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) - cos(3*b*log(
c)))*cos(3*b*log(x^n) + 3*a) - 8*((cos(2*b*log(c))*cos(b*log(c)) + sin(2*b*
log(c))*sin(b*log(c)))*cos(b*log(x^n) + a) + (cos(b*log(c))*sin(2*b*log(c))
 - cos(2*b*log(c))*sin(b*log(c)))*sin(b*log(x^n) + a))*cos(2*b*log(x^n) + 2
*a) + 4*cos(b*log(c))*cos(b*log(x^n) + a) - ((cos(4*b*log(c))^2 + sin(4*b*l
og(c))^2)*cos(4*b*log(x^n) + 4*a)^2 + 4*(cos(2*b*log(c))^2 + sin(2*b*log(c)
)^2)*cos(2*b*log(x^n) + 2*a)^2 + (cos(4*b*log(c))^2 + sin(4*b*log(c))^2)*si
n(4*b*log(x^n) + 4*a)^2 + 4*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*sin(2*b
*log(x^n) + 2*a)^2 - 2*(2*(cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c)
)*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 2*(cos(2*b*log(c))*sin(4*b*log
(c)) - cos(4*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) - cos(4*b*l
og(c)))*cos(4*b*log(x^n) + 4*a) - 4*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a)
 + 2*(2*(cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)))
*cos(2*b*log(x^n) + 2*a) - 2*(cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log
(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) - sin(4*b*log(c)))*sin(4*b*lo
g(x^n) + 4*a) + 4*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + 1)*log((cos(a)^
2 + sin(a)^2)*cos(b*log(c))^2 + (cos(a)^2 + sin(a)^2)*sin(b*log(c))^2 + 2*(
cos(b*log(c))*cos(a) - sin(b*log(c))*sin(a))*cos(b*log(x^n)) + cos(b*log(x^
n))^2 - 2*(cos(a)*sin(b*log(c)) + cos(b*log(c))*sin(a))*sin(b*log(x^n)) + s
in(b*log(x^n))^2) + ((cos(4*b*log(c))^2 + sin(4*b*log(c))^2)*cos(4*b*log(x^
n) + 4*a)^2 + 4*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*cos(2*b*log(x^n) +
2*a)^2 + (cos(4*b*log(c))^2 + sin(4*b*log(c))^2)*sin(4*b*log(x^n) + 4*a)^2
+ 4*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*sin(2*b*log(x^n) + 2*a)^2 - 2*(
2*(cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*cos(2
*b*log(x^n) + 2*a) + 2*(cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*s
in(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) - cos(4*b*log(c)))*cos(4*b*log(x^n)
 + 4*a) - 4*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) + 2*(2*(cos(2*b*log(c))
*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a)
 - 2*(cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*si
n(2*b*log(x^n) + 2*a) - sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) + 4*sin(2*
b*log(c))*sin(2*b*log(x^n) + 2*a) + 1)*log((cos(a)^2 + sin(a)^2)*cos(b*log(
c))^2 + (cos(a)^2 + sin(a)^2)*sin(b*log(c))^2 - 2*(cos(b*log(c))*cos(a) - s
in(b*log(c))*sin(a))*cos(b*log(x^n)) + cos(b*log(x^n))^2 + 2*(cos(a)*sin(b*
log(c)) + cos(b*log(c))*sin(a))*sin(b*log(x^n)) + sin(b*log(x^n))^2) - 4*((
cos(3*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(3*b*log(c)))*cos(3*b*
log(x^n) + 3*a) + (cos(b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(b*lo
g(c)))*cos(b*log(x^n) + a) - (cos(4*b*log(c))*cos(3*b*log(c)) + sin(4*b*log
(c))*sin(3*b*log(c)))*sin(3*b*log(x^n) + 3*a) - (cos(4*b*log(c))*cos(b*log(
c)) + sin(4*b*log(c))*sin(b*log(c)))*sin(b*log(x^n) + a))*sin(4*b*log(x^n)
+ 4*a) + 4*(2*(cos(2*b*log(c))*sin(3*b*log(c)) - cos(3*b*log(c))*sin(2*b*lo
g(c)))*cos(2*b*log(x^n) + 2*a) - 2*(cos(3*b*log(c))*cos(2*b*log(c)) + sin(3
*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) - sin(3*b*log(c)))*sin(
3*b*log(x^n) + 3*a) + 8*((cos(b*log(c))*sin(2*b*log(c)) - cos(2*b*log(c))*s
in(b*log(c)))*cos(b*log(x^n) + a) - (cos(2*b*log(c))*cos(b*log(c)) + sin(2*
```

```
b*log(c))*sin(b*log(c)))*sin(b*log(x^n) + a))*sin(2*b*log(x^n) + 2*a) - 4*s
in(b*log(c))*sin(b*log(x^n) + a))/((b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))
^2)*n*cos(4*b*log(x^n) + 4*a)^2 - 4*b*n*cos(2*b*log(c))*cos(2*b*log(x^n) +
2*a) + 4*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*cos(2*b*log(x^n) + 2
*a)^2 + (b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*sin(4*b*log(x^n) + 4*
a)^2 + 4*b*n*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + 4*(b*cos(2*b*log(c))
^2 + b*sin(2*b*log(c))^2)*n*sin(2*b*log(x^n) + 2*a)^2 + b*n + 2*(b*n*cos(4*
b*log(c)) - 2*(b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(2*
b*log(c)))*n*cos(2*b*log(x^n) + 2*a) - 2*(b*cos(2*b*log(c))*sin(4*b*log(c))
 - b*cos(4*b*log(c))*sin(2*b*log(c)))*n*sin(2*b*log(x^n) + 2*a))*cos(4*b*lo
g(x^n) + 4*a) + 2*(2*(b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))
*sin(2*b*log(c)))*n*cos(2*b*log(x^n) + 2*a) - b*n*sin(4*b*log(c)) - 2*(b*co
s(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log(c)))*n*sin(2*
b*log(x^n) + 2*a))*sin(4*b*log(x^n) + 4*a))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(51) = 102$.
time = 2.48, size = 110, normalized size = 2.00

$$-\frac{\left(\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-1\right)\log\left(\frac{1}{2}\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\frac{1}{2}\right)-\left(\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-1\right)\log\left(-\frac{1}{2}\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\frac{1}{2}\right)-2\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{4\left(bn\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-bn\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] -1/4*((cos(b*n*log(x) + b*log(c) + a)^2 - 1)*log(1/2*cos(b*n*log(x) + b*log
(c) + a) + 1/2) - (cos(b*n*log(x) + b*log(c) + a)^2 - 1)*log(-1/2*cos(b*n*l
og(x) + b*log(c) + a) + 1/2) - 2*cos(b*n*log(x) + b*log(c) + a))/(b*n*cos(b
*n*log(x) + b*log(c) + a)^2 - b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{3}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*ln(c*x**n))**3/x,x)
```

```
[Out] Integral(csc(a + b*log(c*x**n))**3/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] integrate(csc(b*log(c*x^n) + a)^3/x, x)
```

Mupad [B]
time = 6.43, size = 177, normalized size = 3.22

$$-\frac{\ln\left(-\frac{1\mathrm{i}}{x}-\frac{\mathrm{e}^{a\,1\mathrm{i}}\,(c\,x^n)^{b\,1\mathrm{i}}\,1\mathrm{i}}{x}\right)}{2\,b\,n}+\frac{\ln\left(\frac{1\mathrm{i}}{x}-\frac{\mathrm{e}^{a\,1\mathrm{i}}\,(c\,x^n)^{b\,1\mathrm{i}}\,1\mathrm{i}}{x}\right)}{2\,b\,n}+\frac{2\,\mathrm{e}^{a\,1\mathrm{i}}\,(c\,x^n)^{b\,1\mathrm{i}}}{b\,n\,\left(1+\mathrm{e}^{a\,4\mathrm{i}}\,(c\,x^n)^{b\,4\mathrm{i}}-2\,\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}}\right)}+\frac{\mathrm{e}^{a\,1\mathrm{i}}\,(c\,x^n)^{b\,1\mathrm{i}}}{b\,n\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*sin(a + b*log(c*x^n))^3),x)
```

```
[Out] log(1i/x - (exp(a*1i)*(c*x^n)^(b*1i)*1i)/x)/(2*b*n) - log(- 1i/x - (exp(a*1
i)*(c*x^n)^(b*1i)*1i)/x)/(2*b*n) + (2*exp(a*1i)*(c*x^n)^(b*1i))/(b*n*(exp(a
*4i)*(c*x^n)^(b*4i) - 2*exp(a*2i)*(c*x^n)^(b*2i) + 1)) + (exp(a*1i)*(c*x^n)
^(b*1i))/(b*n*(exp(a*2i)*(c*x^n)^(b*2i) - 1))
```

3.299 $\int \csc^4(a+b\log(cx^n))\,dx$

Optimal. Leaf size=84

$$\frac{16e^{4ia}x(cx^n)^{4ib}\,{}_2F_1\left(4,\frac{1}{2}\left(4-\frac{i}{bn}\right);\frac{1}{2}\left(6-\frac{i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{1+4ibn}$$

```
[Out] 16*exp(4*I*a)*x*(c*x^n)^(4*I*b)*hypergeom([4, 2-1/2*I/b/n],[3-1/2*I/b/n],ex
p(2*I*a)*(c*x^n)^(2*I*b))/(1+4*I*b*n)
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4600, 4602, 371}

$$\frac{16e^{4ia}x(cx^n)^{4ib}\,{}_2F_1\left(4,\frac{1}{2}\left(4-\frac{i}{bn}\right);\frac{1}{2}\left(6-\frac{i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{1+4ibn}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (16*E^((4*I)*a)*x*(c*x^n)^((4*I)*b)*Hypergeometric2F1[4, (4 - I/(b*n))/2, (
6 - I/(b*n))/2, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/(1 + (4*I)*b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4600

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4602

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[(-2*I)^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(
2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \csc^4\left(a+b\log\left(cx^n\right)\right)\,dx &= \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\csc^4(a+b\log(x))\,dx,x,cx^n\right)}{n}\\
&= \frac{\left(16e^{4ia}x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+4ib+\frac{1}{n}}}{\left(1-e^{2ia}x^{2ib}\right)^4}\,dx,x,cx^n\right)}{n}\\
&= \frac{16e^{4ia}x(cx^n)^{4ib}\ {}_2F_1\left(4,\frac{1}{2}\left(4-\frac{i}{bn}\right);\frac{1}{2}\left(6-\frac{i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{1+4ibn}
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 221 vs. $2(84) = 168$.
time = 10.69, size = 221, normalized size = 2.63

$$
\frac{x\left(-4e^{2ia}(i+2bn)\left(cx^n\right)^{2ib}\ {}_2F_1\left(1,1-\frac{i}{2bn};2-\frac{i}{2bn};e^{2i(a+b\log(cx^n))}\right)-4i(1+4b^2n^2)\ {}_2F_1\left(1,-\frac{i}{2bn};1-\frac{i}{2bn};e^{2i(a+b\log(cx^n))}\right)+\csc^3\left(a+b\log\left(cx^n\right)\right)\left(-\left((1+12b^2n^2)\cos\left(a+b\log\left(cx^n\right)\right)\right)+(1+4b^2n^2)\cos\left(3(a+b\log\left(cx^n\right))\right)-4bn\sin\left(a+b\log\left(cx^n\right)\right)\right)\right)}{24b^3n^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x*(-4*E^((2*I)*a)*(I + 2*b*n)*(c*x^n)^((2*I)*b)*Hypergeometric2F1[1, 1 - (
I/2)/(b*n), 2 - (I/2)/(b*n), E^((2*I)*(a + b*Log[c*x^n]))] - (4*I)*(1 + 4*b
^2*n^2)*Hypergeometric2F1[1, (-1/2*I)/(b*n), 1 - (I/2)/(b*n), E^((2*I)*(a +
 b*Log[c*x^n]))] + Csc[a + b*Log[c*x^n]]^3*(-((1 + 12*b^2*n^2)*Cos[a + b*Lo
g[c*x^n]]) + (1 + 4*b^2*n^2)*Cos[3*(a + b*Log[c*x^n])] - 4*b*n*Sin[a + b*Lo
g[c*x^n]])))/(24*b^3*n^3)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$
\int \csc^4\left(a+b\ln\left(c\,x^n\right)\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(a+b*ln(c*x^n))^4,x)
```

```
[Out] int(csc(a+b*ln(c*x^n))^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^4,x, algorithm="maxima")

[Out] 1/3*(6*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*x*cos(4*b*log(x^n) + 4
*a)^2 + 6*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*x*cos(2*b*log(x^n)
+ 2*a)^2 + 6*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*x*sin(4*b*log(x^
n) + 4*a)^2 + 6*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*x*sin(2*b*log
(x^n) + 2*a)^2 - (2*b*n*cos(2*b*log(c)) - sin(2*b*log(c)))*x*cos(2*b*log(x^
n) + 2*a) + (2*b*n*sin(2*b*log(c)) + cos(2*b*log(c)))*x*sin(2*b*log(x^n) +
2*a) - ((2*(b*cos(6*b*log(c))*cos(4*b*log(c)) + b*sin(6*b*log(c))*sin(4*b*l
og(c)))*n - cos(4*b*log(c))*sin(6*b*log(c)) + cos(6*b*log(c))*sin(4*b*log(c
)))*x*cos(4*b*log(x^n) + 4*a) + 2*(6*(b^2*cos(2*b*log(c))*sin(6*b*log(c)) -
 b^2*cos(6*b*log(c))*sin(2*b*log(c)))*n^2 - (b*cos(6*b*log(c))*cos(2*b*log(
c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n + cos(2*b*log(c))*sin(6*b*log(c)
) - cos(6*b*log(c))*sin(2*b*log(c)))*x*cos(2*b*log(x^n) + 2*a) + (2*(b*cos(
4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b*log(c)))*n + cos(6*
b*log(c))*cos(4*b*log(c)) + sin(6*b*log(c))*sin(4*b*log(c)))*x*sin(4*b*log(
x^n) + 4*a) - 2*(6*(b^2*cos(6*b*log(c))*cos(2*b*log(c)) + b^2*sin(6*b*log(c
))*sin(2*b*log(c)))*n^2 + (b*cos(2*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*lo
g(c))*sin(2*b*log(c)))*n + cos(6*b*log(c))*cos(2*b*log(c)) + sin(6*b*log(c)
)*sin(2*b*log(c)))*x*sin(2*b*log(x^n) + 2*a) - (4*b^2*n^2*sin(6*b*log(c)) +
 sin(6*b*log(c)))*x)*cos(6*b*log(x^n) + 6*a) + (3*(12*(b^2*cos(2*b*log(c))*
sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(2*b*log(c)))*n^2 - 4*(b*cos(4*b*l
og(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log(c)))*n + cos(2*b*log
(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)))*x*cos(2*b*log(x^n)
+ 2*a) - 3*(12*(b^2*cos(4*b*log(c))*cos(2*b*log(c)) + b^2*sin(4*b*log(c))*s
in(2*b*log(c)))*n^2 + 4*(b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(
c))*sin(2*b*log(c)))*n + cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*
sin(2*b*log(c)))*x*sin(2*b*log(x^n) + 2*a) - 2*(6*b^2*n^2*sin(4*b*log(c)) -
 b*n*cos(4*b*log(c)) + sin(4*b*log(c)))*x)*cos(4*b*log(x^n) + 4*a) + 18*(4*
b^8*n^8 + b^6*n^6 + (4*(b^8*cos(6*b*log(c))^2 + b^8*sin(6*b*log(c))^2)*n^8
+ (b^6*cos(6*b*log(c))^2 + b^6*sin(6*b*log(c))^2)*n^6)*cos(6*b*log(x^n) + 6
*a)^2 + 9*(4*(b^8*cos(4*b*log(c))^2 + b^8*sin(4*b*log(c))^2)*n^8 + (b^6*cos
(4*b*log(c))^2 + b^6*sin(4*b*log(c))^2)*n^6)*cos(4*b*log(x^n) + 4*a)^2 + 9*
(4*(b^8*cos(2*b*log(c))^2 + b^8*sin(2*b*log(c))^2)*n^8 + (b^6*cos(2*b*log(c
))^2 + b^6*sin(2*b*log(c))^2)*n^6)*cos(2*b*log(x^n) + 2*a)^2 + (4*(b^8*cos(
6*b*log(c))^2 + b^8*sin(6*b*log(c))^2)*n^8 + (b^6*cos(6*b*log(c))^2 + b^6*s
in(6*b*log(c))^2)*n^6)*sin(6*b*log(x^n) + 6*a)^2 + 9*(4*(b^8*cos(4*b*log(c)
)^2 + b^8*sin(4*b*log(c))^2)*n^8 + (b^6*cos(4*b*log(c))^2 + b^6*sin(4*b*log
(c))^2)*n^6)*sin(4*b*log(x^n) + 4*a)^2 + 9*(4*(b^8*cos(2*b*log(c))^2 + b^8*
sin(2*b*log(c))^2)*n^8 + (b^6*cos(2*b*log(c))^2 + b^6*sin(2*b*log(c))^2)*n^
6)*sin(2*b*log(x^n) + 2*a)^2 - 2*(4*b^8*n^8*cos(6*b*log(c)) + b^6*n^6*cos(6
*b*log(c)) + 3*(4*(b^8*cos(6*b*log(c))*cos(4*b*log(c)) + b^8*sin(6*b*log(c)
)*sin(4*b*log(c)))*n^8 + (b^6*cos(6*b*log(c))*cos(4*b*log(c)) + b^6*sin(6*b
*log(c))*sin(4*b*log(c)))*n^6)*cos(4*b*log(x^n) + 4*a) - 3*(4*(b^8*cos(6*b*
log(c))*cos(2*b*log(c)) + b^8*sin(6*b*log(c))*sin(2*b*log(c)))*n^8 + (b^6*c
```

```
os(6*b*log(c))*cos(2*b*log(c)) + b^6*sin(6*b*log(c))*sin(2*b*log(c)))*n^6)*
cos(2*b*log(x^n) + 2*a) + 3*(4*(b^8*cos(4*b*log(c))*sin(6*b*log(c)) - b^8*c
os(6*b*log(c))*sin(4*b*log(c)))*n^8 + (b^6*cos(4*b*log(c))*sin(6*b*log(c))
- b^6*cos(6*b*log(c))*sin(4*b*log(c)))*n^6)*sin(4*b*log(x^n) + 4*a) - 3*(4*
(b^8*cos(2*b*log(c))*sin(6*b*log(c)) - b^8*cos(6*b*log(c))*sin(2*b*log(c)))
*n^8 + (b^6*cos(2*b*log(c))*sin(6*b*log(c)) - b^6*cos(6*b*log(c))*sin(2*b*l
og(c)))*n^6)*sin(2*b*log(x^n) + 2*a))*cos(6*b*log(x^n) + 6*a) + 6*(4*b^8*n^
8*cos(4*b*log(c)) + b^6*n^6*cos(4*b*log(c)) - 3*(4*(b^8*cos(4*b*log(c))*cos
(2*b*log(c)) + b^8*sin(4*b*log(c))*sin(2*b*log(c)))*n^8 + (b^6*cos(4*b*log(
c))*cos(2*b*log(c)) + b^6*sin(4*b*log(c))*sin(2*b*log(c)))*n^6)*cos(2*b*log
(x^n) + 2*a) - 3*(4*(b^8*cos(2*b*log(c))*sin(4*b*log(c)) - b^8*cos(4*b*log(
c))*sin(2*b*log(c)))*n^8 + (b^6*cos(2*b*log(c))*sin(4*b*log(c)) - b^6*cos(4
*b*log(c))*sin(2*b*log(c)))*n^6)*sin(2*b*log(x^n) + 2*a))*cos(4*b*log(x^n)
+ 4*a) - 6*(4*b^8*n^8*cos(2*b*log(c)) + b^6*n^6*cos(2*b*log(c)))*cos(2*b*lo
g(x^n) + 2*a) + 2*(4*b^8*n^8*sin(6*b*log(c)) + b^6*n^6*sin(6*b*log(c)) + 3*
(4*(b^8*cos(4*b*log(c))*sin(6*b*log(c)) - b^8*cos(6*b*log(c))*sin(4*b*log(c
)))*n^8 + (b^6*cos(4*b*log(c))*sin(6*b*log(c)) - b^6*cos(6*b*log(c))*sin(4*
b*log(c)))*n^6)*cos(4*b*log(x^n) + 4*a) - 3*(4*(b^8*cos(2*b*log(c))*sin(6*b
*log(c)) - b^8*cos(6*b*log(c))*sin(2*b*log(c)))*n^8 + (b^6*cos(2*b*log(c))*
sin(6*b*log(c)) - b^6*cos(6*b*log(c))*sin(2*b*log(c)))*n^6)*cos(2*b*log(x^n
) + 2*a) - 3*(4*(b^8*cos(6*b*log(c))*cos(4*b*log(c)) + b^8*sin(6*b*log(c))*
sin(4*b*log(c)))*n^8 + (b^6*cos(6*b*log(c))*cos(4*b*log(c)) + b^6*sin(6*b*l
og(c))*sin(4*b*log(c)))*n^6)*sin(4*b*log(x^n) + 4*a) + 3*(4*(b^8*cos(6*b*lo
g(c))*cos(2*b*log(c)) + b^8*sin(6*b*log(c))*sin...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^4,x, algorithm="fricas")
[Out] integral(csc(b*log(c*x^n) + a)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^4\left(a + b\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*ln(c*x**n))**4,x)
[Out] Integral(csc(a + b*log(c*x**n))**4, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] integrate(csc(b*log(c*x^n) + a)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(a + b*log(c*x^n))^4,x)
```

```
[Out] int(1/sin(a + b*log(c*x^n))^4, x)
```

3.300 $\int \frac{\csc^4(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=43

$$-\frac{\cot\left(a+b\log\left(cx^n\right)\right)}{bn}-\frac{\cot^3\left(a+b\log\left(cx^n\right)\right)}{3bn}$$

```
[Out] -cot(a+b*ln(c*x^n))/b/n-1/3*cot(a+b*ln(c*x^n))^3/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3852}

$$-\frac{\cot^3\left(a+b\log\left(cx^n\right)\right)}{3bn}-\frac{\cot\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] -(Cot[a + b*Log[c*x^n]]/(b*n)) - Cot[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^4\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \csc^4(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\
&= -\frac{\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,\cot\left(a+b\log\left(cx^n\right)\right)\right)}{bn}\\
&= -\frac{\cot\left(a+b\log\left(cx^n\right)\right)}{bn}-\frac{\cot^3\left(a+b\log\left(cx^n\right)\right)}{3bn}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 56, normalized size = 1.30

$$-\frac{2\cot\left(a+b\log\left(cx^n\right)\right)}{3bn}-\frac{\cot\left(a+b\log\left(cx^n\right)\right)\csc^2\left(a+b\log\left(cx^n\right)\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] (-2*Cot[a + b*Log[c*x^n]])/(3*b*n) - (Cot[a + b*Log[c*x^n]]*Csc[a + b*Log[c
*x^n]]^2)/(3*b*n)
```

Maple [A]
time = 0.16, size = 36, normalized size = 0.84

method	result
derivativedivides	$\frac{\left(-\frac{2}{3}-\frac{\left(\csc^{2}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}{3}\right)\cot\left(a+b\ln\left(c\,x^{n}\right)\right)}{nb}$
default	$\frac{\left(-\frac{2}{3}-\frac{\left(\csc^{2}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}{3}\right)\cot\left(a+b\ln\left(c\,x^{n}\right)\right)}{nb}$
risch	$\frac{4i\left(3\left(x^{n}\right)^{2ib}c^{2ib}\mathrm{e}^{b\pi\mathrm{csgn}\left(ic\,x^{n}\right)^{3}}\mathrm{e}^{-b\pi\mathrm{csgn}\left(ic\,x^{n}\right)^{2}\mathrm{csgn}\left(ic\right)}\mathrm{e}^{-b\pi\mathrm{csgn}\left(ic\,x^{n}\right)^{2}\mathrm{csgn}\left(ix^{n}\right)}\mathrm{e}^{b\pi\,\mathrm{csgn}\left(ic\,x^{n}\right)\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ix^{n}\right)}\mathrm{e}^{2ia}-1\right)}{3bn\left(\left(x^{n}\right)^{2ib}c^{2ib}\mathrm{e}^{b\pi\mathrm{csgn}\left(ic\,x^{n}\right)^{3}}\mathrm{e}^{-b\pi\mathrm{csgn}\left(ic\,x^{n}\right)^{2}\mathrm{csgn}\left(ic\right)}\mathrm{e}^{-b\pi\mathrm{csgn}\left(ic\,x^{n}\right)^{2}\mathrm{csgn}\left(ix^{n}\right)}\mathrm{e}^{b\pi\,\mathrm{csgn}\left(ic\,x^{n}\right)\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ix^{n}\right)}\mathrm{e}^{2ia}-1\right)^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(a+b*ln(c*x^n))^4/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(-2/3-1/3*csc(a+b*ln(c*x^n))^2)*cot(a+b*ln(c*x^n))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1332 vs. $2(41) = 82$.
time = 0.30, size = 1332, normalized size = 30.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

```
[Out] 4/3*((3*(cos(2*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c))*sin(2*b*log(c)))
*cos(2*b*log(x^n) + 2*a) - 3*(cos(6*b*log(c))*cos(2*b*log(c)) + sin(6*b*log
(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) - sin(6*b*log(c)))*cos(6*b*lo
g(x^n) + 6*a) - 3*(3*(cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin
(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) - 3*(cos(4*b*log(c))*cos(2*b*log(c))
+ sin(4*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) - sin(4*b*log(c)
))*cos(4*b*log(x^n) + 4*a) + (3*(cos(6*b*log(c))*cos(2*b*log(c)) + sin(6*b*
log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 3*(cos(2*b*log(c))*sin(6
*b*log(c)) - cos(6*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) - cos
(6*b*log(c)))*sin(6*b*log(x^n) + 6*a) - 3*(3*(cos(4*b*log(c))*cos(2*b*log(c
)) + sin(4*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 3*(cos(2*b*
log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n)
 + 2*a) - cos(4*b*log(c)))*sin(4*b*log(x^n) + 4*a))/((b*cos(6*b*log(c))^2 +
 b*sin(6*b*log(c))^2)*n*cos(6*b*log(x^n) + 6*a)^2 + 9*(b*cos(4*b*log(c))^2
```

```
+ b*sin(4*b*log(c))^2)*n*cos(4*b*log(x^n) + 4*a)^2 - 6*b*n*cos(2*b*log(c))*
cos(2*b*log(x^n) + 2*a) + 9*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*c
os(2*b*log(x^n) + 2*a)^2 + (b*cos(6*b*log(c))^2 + b*sin(6*b*log(c))^2)*n*si
n(6*b*log(x^n) + 6*a)^2 + 9*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*s
in(4*b*log(x^n) + 4*a)^2 + 6*b*n*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) +
9*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*sin(2*b*log(x^n) + 2*a)^2 +
 b*n - 2*(b*n*cos(6*b*log(c)) + 3*(b*cos(6*b*log(c))*cos(4*b*log(c)) + b*si
n(6*b*log(c))*sin(4*b*log(c)))*n*cos(4*b*log(x^n) + 4*a) - 3*(b*cos(6*b*log
(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n
) + 2*a) + 3*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b
*log(c)))*n*sin(4*b*log(x^n) + 4*a) - 3*(b*cos(2*b*log(c))*sin(6*b*log(c))
- b*cos(6*b*log(c))*sin(2*b*log(c)))*n*sin(2*b*log(x^n) + 2*a))*cos(6*b*log
(x^n) + 6*a) + 6*(b*n*cos(4*b*log(c)) - 3*(b*cos(4*b*log(c))*cos(2*b*log(c)
) + b*sin(4*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n) + 2*a) - 3*(b*cos
(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*n*sin(2*b
*log(x^n) + 2*a))*cos(4*b*log(x^n) + 4*a) + 2*(3*(b*cos(4*b*log(c))*sin(6*b
*log(c)) - b*cos(6*b*log(c))*sin(4*b*log(c)))*n*cos(4*b*log(x^n) + 4*a) - 3
*(b*cos(2*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(2*b*log(c)))*n*
cos(2*b*log(x^n) + 2*a) + b*n*sin(6*b*log(c)) - 3*(b*cos(6*b*log(c))*cos(4*
b*log(c)) + b*sin(6*b*log(c))*sin(4*b*log(c)))*n*sin(4*b*log(x^n) + 4*a) +
3*(b*cos(6*b*log(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n
*sin(2*b*log(x^n) + 2*a))*sin(6*b*log(x^n) + 6*a) + 6*(3*(b*cos(2*b*log(c))
*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n) +
2*a) - b*n*sin(4*b*log(c)) - 3*(b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4
*b*log(c))*sin(2*b*log(c)))*n*sin(2*b*log(x^n) + 2*a))*sin(4*b*log(x^n) + 4
*a))
```

Fricas [A]
time = 1.83, size = 71, normalized size = 1.65

$$-\frac{2\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}-3\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{3\left(bn\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-bn\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

```
[Out] -1/3*(2*cos(b*n*log(x) + b*log(c) + a)^3 - 3*cos(b*n*log(x) + b*log(c) + a)
)/((b*n*cos(b*n*log(x) + b*log(c) + a)^2 - b*n)*sin(b*n*log(x) + b*log(c) +
 a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^{4}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Integral(csc(a + b*log(c*x**n))**4/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] integrate(csc(b*log(c*x^n) + a)^4/x, x)
```

Mupad [B]
time = 9.23, size = 49, normalized size = 1.14

$$\frac{4\left(\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}\,3\mathrm{i}-\mathrm{i}\right)}{3\,b\,n\,{\left(\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*sin(a + b*log(c*x^n))^4),x)
```

```
[Out] (4*(exp(a*2i)*(c*x^n)^(b*2i)*3i - 1i))/(3*b*n*(exp(a*2i)*(c*x^n)^(b*2i) - 1
)^3)
```

3.301 $\int \left(-\left(\left(1+b^2n^2\right)\csc\left(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\csc^3\left(a\right.\right.$

Optimal. Leaf size=42

$$-x\csc\left(a+b\log\left(cx^n\right)\right)-bnx\cot\left(a+b\log\left(cx^n\right)\right)\csc\left(a+b\log\left(cx^n\right)\right)$$

```
[Out] -x*csc(a+b*ln(c*x^n))-b*n*x*cot(a+b*ln(c*x^n))*csc(a+b*ln(c*x^n))
```

Rubi [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.09, antiderivative size = 172, normalized size of antiderivative = 4.10, number of steps used = 7, number of rules used = 3, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}=0.068$, Rules used = {4600, 4602, 371}

$$2e^{ia}x(bn+i)\left(cx^n\right)^{ib}\,{}_2F_1\left(1,\frac{1}{2}\left(1-\frac{i}{bn}\right);\frac{1}{2}\left(3-\frac{i}{bn}\right);e^{2ia}\left(cx^n\right)^{2ib}\right)-\frac{16e^{3ia}b^2n^2x\left(cx^n\right)^{3ib}\,{}_2F_1\left(3,\frac{1}{2}\left(3-\frac{i}{bn}\right);\frac{1}{2}\left(5-\frac{i}{bn}\right);e^{2ia}\left(cx^n\right)^{2ib}\right)}{-3bn+i}$$

Antiderivative was successfully verified.

```
[In] Int[-((1 + b^2*n^2)*Csc[a + b*Log[c*x^n]]) + 2*b^2*n^2*Csc[a + b*Log[c*x^n]
]^3,x]
```

```
[Out] 2*E^(I*a)*(I + b*n)*x*(c*x^n)^(I*b)*Hypergeometric2F1[1, (1 - I/(b*n))/2, (
3 - I/(b*n))/2, E^((2*I)*a)*(c*x^n)^((2*I)*b)] - (16*b^2*E^((3*I)*a)*n^2*x*
(c*x^n)^((3*I)*b)*Hypergeometric2F1[3, (3 - I/(b*n))/2, (5 - I/(b*n))/2, E^
((2*I)*a)*(c*x^n)^((2*I)*b)])/(I - 3*b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4600

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4602

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[(-2*I)^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(
2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \left(-\left(1+b^2n^2\right)\csc\left(a+b\log\left(cx^n\right)\right)+2b^2n^2\csc^3\left(a+b\log\left(cx^n\right)\right)\right)\,dx=\left(2b^2n^2\right)\int \csc^3\left(a+b\log\left(cx^n\right)\right)\,dx$$

$$=\left(2b^2nx\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\right.$$

$$=\left(16ib^2e^{3ia}nx\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int\right.$$

$$=2e^{ia}(i+bn)x\left(cx^n\right)^{ib}\,{}_2F_1\left(1,\frac{1}{2}\left(1-\right.\right.$$

Mathematica [A]
time = 0.52, size = 30, normalized size = 0.71

$$-x(1+bn\cot\left(a+b\log\left(cx^n\right)\right))\csc\left(a+b\log\left(cx^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-((1 + b^2*n^2)*Csc[a + b*Log[c*x^n]]) + 2*b^2*n^2*Csc[a + b*Log[
c*x^n]]^3,x]
```

```
[Out] -(x*(1 + b*n*Cot[a + b*Log[c*x^n]])*Csc[a + b*Log[c*x^n]])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.75, size = 523, normalized size = 12.45

method	result
risch	$\frac{2(x^n)^{ib}c^{ib}x\left(nb\,c^{2ib}(x^n)^{2ib}\mathrm{e}^{\frac{3b\pi\mathrm{csgn}(ic\,x^n)^3}{2}}\mathrm{e}^{-\frac{3b\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ic)}{2}}\mathrm{e}^{-\frac{3b\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ix^n)}{2}}\mathrm{e}^{\frac{3b\pi\,\mathrm{csgn}(ic\,x^n)\mathrm{csgn}(ic)\mathrm{csgn}(ix^n)}{2}}\mathrm{e}^{3ia}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^2*n^2+1)*csc(a+b*ln(c*x^n))+2*b^2*n^2*csc(a+b*ln(c*x^n))^3,x,method
=_RETURNVERBOSE)
```

```
[Out] 2*(x^n)^(I*b)*c^(I*b)*x/(((x^n)^(I*b))^2*(c^(I*b))^2*exp(b*Pi*csgn(I*c*x^n)
^3)*exp(-b*Pi*csgn(I*c*x^n)^2*csgn(I*c))*exp(-b*Pi*csgn(I*c*x^n)^2*csgn(I*x
^n))*exp(b*Pi*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n))*exp(2*I*a)-1)^2*(n*b*(c^
(I*b))^2*((x^n)^(I*b))^2*exp(3/2*b*Pi*csgn(I*c*x^n)^3)*exp(-3/2*b*Pi*csgn(I
*c*x^n)^2*csgn(I*c))*exp(-3/2*b*Pi*csgn(I*c*x^n)^2*csgn(I*x^n))*exp(3/2*b*P
i*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n))*exp(3*I*a)+b*n*exp(1/2*b*Pi*csgn(I*c
*x^n)^3)*exp(-1/2*b*Pi*csgn(I*c*x^n)^2*csgn(I*c))*exp(-1/2*b*Pi*csgn(I*c*x^
```

```
n)^2*csgn(I*x^n))*exp(1/2*b*Pi*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n))*exp(I*a
)-I*((x^n)^(I*b))^2*(c^(I*b))^2*exp(3/2*b*Pi*csgn(I*c*x^n)^3)*exp(-3/2*b*Pi
*csgn(I*c*x^n)^2*csgn(I*c))*exp(-3/2*b*Pi*csgn(I*c*x^n)^2*csgn(I*x^n))*exp(
3/2*b*Pi*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n))*exp(3*I*a)+I*exp(1/2*b*Pi*csg
n(I*c*x^n)^3)*exp(-1/2*b*Pi*csgn(I*c*x^n)^2*csgn(I*c))*exp(-1/2*b*Pi*csgn(I
*c*x^n)^2*csgn(I*x^n))*exp(1/2*b*Pi*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n))*ex
p(I*a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1701 vs. $2(42) = 84$.
time = 0.50, size = 1701, normalized size = 40.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^2*n^2+1)*csc(a+b*log(c*x^n))+2*b^2*n^2*csc(a+b*log(c*x^n))^3,
x, algorithm="maxima")
```

```
[Out] 2*((b*n*cos(b*log(c)) - sin(b*log(c)))*x*cos(b*log(x^n) + a) - (b*n*sin(b*l
og(c)) + cos(b*log(c)))*x*sin(b*log(x^n) + a) + (((b*cos(4*b*log(c))*cos(3*
b*log(c)) + b*sin(4*b*log(c))*sin(3*b*log(c)))*n - cos(3*b*log(c))*sin(4*b*
log(c)) + cos(4*b*log(c))*sin(3*b*log(c)))*x*cos(3*b*log(x^n) + 3*a) + ((b*
cos(4*b*log(c))*cos(b*log(c)) + b*sin(4*b*log(c))*sin(b*log(c)))*n + cos(b*
log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(b*log(c)))*x*cos(b*log(x^n) +
 a) + ((b*cos(3*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(3*b*log(c
)))*n + cos(4*b*log(c))*cos(3*b*log(c)) + sin(4*b*log(c))*sin(3*b*log(c)))*
x*sin(3*b*log(x^n) + 3*a) + ((b*cos(b*log(c))*sin(4*b*log(c)) - b*cos(4*b*l
og(c))*sin(b*log(c)))*n - cos(4*b*log(c))*cos(b*log(c)) - sin(4*b*log(c))*s
in(b*log(c)))*x*sin(b*log(x^n) + a))*cos(4*b*log(x^n) + 4*a) - (2*((b*cos(3
*b*log(c))*cos(2*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*log(c)))*n + cos(2*b
*log(c))*sin(3*b*log(c)) - cos(3*b*log(c))*sin(2*b*log(c)))*x*cos(2*b*log(x
^n) + 2*a) + 2*((b*cos(2*b*log(c))*sin(3*b*log(c)) - b*cos(3*b*log(c))*sin(
2*b*log(c)))*n - cos(3*b*log(c))*cos(2*b*log(c)) - sin(3*b*log(c))*sin(2*b*
log(c)))*x*sin(2*b*log(x^n) + 2*a) - (b*n*cos(3*b*log(c)) + sin(3*b*log(c))
)*x)*cos(3*b*log(x^n) + 3*a) - 2*(((b*cos(2*b*log(c))*cos(b*log(c)) + b*sin
(2*b*log(c))*sin(b*log(c)))*n + cos(b*log(c))*sin(2*b*log(c)) - cos(2*b*log
(c))*sin(b*log(c)))*x*cos(b*log(x^n) + a) + ((b*cos(b*log(c))*sin(2*b*log(c
)) - b*cos(2*b*log(c))*sin(b*log(c)))*n - cos(2*b*log(c))*cos(b*log(c)) - s
in(2*b*log(c))*sin(b*log(c)))*x*sin(b*log(x^n) + a))*cos(2*b*log(x^n) + 2*a
) - (((b*cos(3*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(3*b*log(c)
))*n + cos(4*b*log(c))*cos(3*b*log(c)) + sin(4*b*log(c))*sin(3*b*log(c)))*x
*cos(3*b*log(x^n) + 3*a) + ((b*cos(b*log(c))*sin(4*b*log(c)) - b*cos(4*b*lo
g(c))*sin(b*log(c)))*n - cos(4*b*log(c))*cos(b*log(c)) - sin(4*b*log(c))*si
n(b*log(c)))*x*cos(b*log(x^n) + a) - ((b*cos(4*b*log(c))*cos(3*b*log(c)) +
b*sin(4*b*log(c))*sin(3*b*log(c)))*n - cos(3*b*log(c))*sin(4*b*log(c)) + co
```

```
s(4*b*log(c))*sin(3*b*log(c)))*x*sin(3*b*log(x^n) + 3*a) - ((b*cos(4*b*log(
c))*cos(b*log(c)) + b*sin(4*b*log(c))*sin(b*log(c)))*n + cos(b*log(c))*sin(
4*b*log(c)) - cos(4*b*log(c))*sin(b*log(c)))*x*sin(b*log(x^n) + a))*sin(4*b
*log(x^n) + 4*a) + (2*((b*cos(2*b*log(c))*sin(3*b*log(c)) - b*cos(3*b*log(c
))*sin(2*b*log(c)))*n - cos(3*b*log(c))*cos(2*b*log(c)) - sin(3*b*log(c))*s
in(2*b*log(c)))*x*cos(2*b*log(x^n) + 2*a) - 2*((b*cos(3*b*log(c))*cos(2*b*l
og(c)) + b*sin(3*b*log(c))*sin(2*b*log(c)))*n + cos(2*b*log(c))*sin(3*b*log
(c)) - cos(3*b*log(c))*sin(2*b*log(c)))*x*sin(2*b*log(x^n) + 2*a) - (b*n*si
n(3*b*log(c)) - cos(3*b*log(c)))*x)*sin(3*b*log(x^n) + 3*a) + 2*(((b*cos(b*
log(c))*sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)))*n - cos(2*b*log(
c))*cos(b*log(c)) - sin(2*b*log(c))*sin(b*log(c)))*x*cos(b*log(x^n) + a) -
((b*cos(2*b*log(c))*cos(b*log(c)) + b*sin(2*b*log(c))*sin(b*log(c)))*n + co
s(b*log(c))*sin(2*b*log(c)) - cos(2*b*log(c))*sin(b*log(c)))*x*sin(b*log(x^
n) + a))*sin(2*b*log(x^n) + 2*a))/((cos(4*b*log(c))^2 + sin(4*b*log(c))^2)*
cos(4*b*log(x^n) + 4*a)^2 + 4*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*cos(2
*b*log(x^n) + 2*a)^2 + (cos(4*b*log(c))^2 + sin(4*b*log(c))^2)*sin(4*b*log(
x^n) + 4*a)^2 + 4*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*sin(2*b*log(x^n)
+ 2*a)^2 - 2*(2*(cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*
log(c)))*cos(2*b*log(x^n) + 2*a) + 2*(cos(2*b*log(c))*sin(4*b*log(c)) - cos
(4*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) - cos(4*b*log(c)))*co
s(4*b*log(x^n) + 4*a) - 4*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) + 2*(2*(c
os(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)))*cos(2*b*l
og(x^n) + 2*a) - 2*(cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2
*b*log(c)))*sin(2*b*log(x^n) + 2*a) - sin(4*b*log(c)))*sin(4*b*log(x^n) + 4
*a) + 4*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + 1)
```

Fricas [A]
time = 6.70, size = 50, normalized size = 1.19

$$\frac{bnx\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+x\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^2*n^2+1)*csc(a+b*log(c*x^n))+2*b^2*n^2*csc(a+b*log(c*x^n))^3,
x, algorithm="fricas")
```

```
[Out] (b*n*x*cos(b*n*log(x) + b*log(c) + a) + x*sin(b*n*log(x) + b*log(c) + a))/(
cos(b*n*log(x) + b*log(c) + a)^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(2b^{2}n^{2}\csc^{2}\left(a+b\log\left(cx^{n}\right)\right)-b^{2}n^{2}-1\right)\csc\left(a+b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b**2*n**2+1)*csc(a+b*ln(c*x**n))+2*b**2*n**2*csc(a+b*ln(c*x**n)
)**3,x)
```

```
[Out] Integral((2*b**2*n**2*csc(a + b*log(c*x**n))**2 - b**2*n**2 - 1)*csc(a + b*
log(c*x**n)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^2*n^2+1)*csc(a+b*log(c*x^n))+2*b^2*n^2*csc(a+b*log(c*x^n))^3,
x, algorithm="giac")
```

```
[Out] integrate(2*b^2*n^2*csc(b*log(c*x^n) + a)^3 - (b^2*n^2 + 1)*csc(b*log(c*x^n
) + a), x)
```

Mupad [**B**]
time = 3.26, size = 85, normalized size = 2.02

$$\frac{2\,x\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,1\mathrm{i}}\,\left(b\,n+1\mathrm{i}\right)+2\,x\,\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}\,\left(b\,n-\mathrm{i}\right)}{{\left(\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*b^2*n^2)/sin(a + b*log(c*x^n))^3 - (b^2*n^2 + 1)/sin(a + b*log(c*x^n
)),x)
```

```
[Out] (2*x*exp(a*1i)*(c*x^n)^(b*1i)*(b*n + 1i) + 2*x*exp(a*1i)*exp(a*2i)*(c*x^n)^
(b*1i)*(c*x^n)^(b*2i)*(b*n - 1i))/(exp(a*2i)*(c*x^n)^(b*2i) - 1)^2
```

3.302 $\int x^m \csc^3\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\,dx$

Optimal. Leaf size=110

$$\frac{x^{1+m}\csc\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)}{2(1+m)}-\frac{x^{1+m}\cot\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\csc\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)}{2\sqrt{-(1+m)^2}}$$

```
[Out] 1/2*x^(1+m)*csc(a+2*ln(c*x^(1/2*(-(1+m)^2)^(1/2))))/(1+m)-1/2*x^(1+m)*cot(a
+2*ln(c*x^(1/2*(-(1+m)^2)^(1/2))))*csc(a+2*ln(c*x^(1/2*(-(1+m)^2)^(1/2))))/
(-(1+m)^2)^(1/2)
```

Rubi [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.13, antiderivative size = 142, normalized size of antiderivative = 1.29, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {4606, 4602, 371}

$$-\frac{8e^{3ia}x^{m+1}\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)^{6i}{}_2F_1\left(3,\frac{1}{2}\left(3-\frac{i(m+1)}{\sqrt{-(m+1)^2}}\right);\frac{1}{2}\left(5-\frac{i(m+1)}{\sqrt{-(m+1)^2}}\right);e^{2ia}\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)^{4i}\right)}{im-3\sqrt{-(m+1)^2}+i}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^m*Csc[a + 2*Log[c*x^(Sqrt[-(1 + m)^2]/2)]]^3,x]
```

```
[Out] (-8*E^((3*I)*a)*x^(1 + m)*(c*x^(Sqrt[-(1 + m)^2]/2))^(6*I)*Hypergeometric2F
1[3, (3 - (I*(1 + m))/Sqrt[-(1 + m)^2])/2, (5 - (I*(1 + m))/Sqrt[-(1 + m)^2
])/2, E^((2*I)*a)*(c*x^(Sqrt[-(1 + m)^2]/2))^(4*I)])/(I + I*m - 3*Sqrt[-(1
+ m)^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4602

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[(-2*I)^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(
2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4606

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
```

```
((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^m \csc^3\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)dx = \frac{\left(2x^{1+m}\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)^{-\frac{2(1+m)}{\sqrt{-(1+m)^2}}}\right)\text{Subst}\left(\int x^{-1+\frac{2(1+m)}{\sqrt{-(1+m)^2}}}\right.}{\sqrt{-(1+m)^2}}$$

$$= \frac{\left(16ie^{3ia}x^{1+m}\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)^{-\frac{2(1+m)}{\sqrt{-(1+m)^2}}}\right)\text{Subst}\left(\int \frac{x^{(-1+6i)+}}{(1-e^{2}}\right.}{\sqrt{-(1+m)^2}}$$

$$= -\frac{8e^{3ia}x^{1+m}\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)^{6i}{}_2F_1\left(3,\frac{1}{2}\left(3-\frac{i(1+m)}{\sqrt{-(1+m)^2}}\right)\right.}{i+im-3\sqrt{-(1}}$$

Mathematica [A]
time = 2.25, size = 79, normalized size = 0.72

$$\frac{x^{1+m}\left(1+m+\sqrt{-(1+m)^2}\cot\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\right)\csc\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)}{2(1+m)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Csc[a + 2*Log[c*x^(Sqrt[-(1 + m)^2]/2)]]^3,x]
[Out] (x^(1 + m)*(1 + m + Sqrt[-(1 + m)^2]*Cot[a + 2*Log[c*x^(Sqrt[-(1 + m)^2]/2)
]])*Csc[a + 2*Log[c*x^(Sqrt[-(1 + m)^2]/2)]])/(2*(1 + m)^2)
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int x^m\left(\csc^3\left(a+2\ln\left(c\,x^{\frac{\sqrt{-(1+m)^2}}{2}}\right)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*csc(a+2*ln(c*x^(1/2*(-(1+m)^2)^(1/2))))^3,x)
[Out] int(x^m*csc(a+2*ln(c*x^(1/2*(-(1+m)^2)^(1/2))))^3,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 974 vs. $2(92) = 184$.
time = 0.38, size = 974, normalized size = 8.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(a+2*log(c*x^(1/2*(-(1+m)^2)^(1/2))))^3,x, algorithm="maxi
ma")
```

```
[Out] 2*((cos(2*log(c))*sin(a) + cos(a)*sin(2*log(c)))*x*e^(m*log(x) + 14*arctan2
(sin(1/2*m*log(x)), cos(1/2*m*log(x))) + 14*arctan2(sin(1/2*log(x)), cos(1/
2*log(x)))) + 2*(((cos(a)*sin(2*a) - cos(2*a)*sin(a))*cos(2*log(c)) - (cos(
2*a)*cos(a) + sin(2*a)*sin(a))*sin(2*log(c)))*cos(4*log(c)) + ((cos(2*a)*co
s(a) + sin(2*a)*sin(a))*cos(2*log(c)) + (cos(a)*sin(2*a) - cos(2*a)*sin(a))
*sin(2*log(c)))*sin(4*log(c)))*x*e^(m*log(x) + 10*arctan2(sin(1/2*m*log(x))
, cos(1/2*m*log(x))) + 10*arctan2(sin(1/2*log(x)), cos(1/2*log(x)))) - (((c
os(a)*sin(4*a) - cos(4*a)*sin(a))*cos(2*log(c)) - (cos(4*a)*cos(a) + sin(4*
a)*sin(a))*sin(2*log(c)))*cos(8*log(c)) + ((cos(4*a)*cos(a) + sin(4*a)*sin(
a))*cos(2*log(c)) + (cos(a)*sin(4*a) - cos(4*a)*sin(a))*sin(2*log(c)))*sin(
8*log(c)))*x*e^(m*log(x) + 6*arctan2(sin(1/2*m*log(x)), cos(1/2*m*log(x)))
+ 6*arctan2(sin(1/2*log(x)), cos(1/2*log(x)))))/((cos(4*a)^2 + sin(4*a)^2)*
cos(8*log(c))^2 + (cos(4*a)^2 + sin(4*a)^2)*sin(8*log(c))^2 + ((cos(4*a)^2
+ sin(4*a)^2)*cos(8*log(c))^2 + (cos(4*a)^2 + sin(4*a)^2)*sin(8*log(c))^2)*
m + (m + 1)*e^(16*arctan2(sin(1/2*m*log(x)), cos(1/2*m*log(x))) + 16*arctan
2(sin(1/2*log(x)), cos(1/2*log(x)))) - 4*((cos(2*a)*cos(4*log(c)) - sin(2*a
)*sin(4*log(c)))*m + cos(2*a)*cos(4*log(c)) - sin(2*a)*sin(4*log(c)))*e^(12
*arctan2(sin(1/2*m*log(x)), cos(1/2*m*log(x))) + 12*arctan2(sin(1/2*log(x))
, cos(1/2*log(x)))) + 2*(2*(cos(2*a)^2 + sin(2*a)^2)*cos(4*log(c))^2 + 2*(c
os(2*a)^2 + sin(2*a)^2)*sin(4*log(c))^2 + (2*(cos(2*a)^2 + sin(2*a)^2)*cos(
4*log(c))^2 + 2*(cos(2*a)^2 + sin(2*a)^2)*sin(4*log(c))^2 + cos(4*a)*cos(8*
log(c)) - sin(4*a)*sin(8*log(c)))*m + cos(4*a)*cos(8*log(c)) - sin(4*a)*sin
(8*log(c)))*e^(8*arctan2(sin(1/2*m*log(x)), cos(1/2*m*log(x))) + 8*arctan2(
sin(1/2*log(x)), cos(1/2*log(x)))) - 4*((((cos(4*a)*cos(2*a) + sin(4*a)*sin
(2*a))*cos(4*log(c)) + (cos(2*a)*sin(4*a) - cos(4*a)*sin(2*a))*sin(4*log(c)
))*cos(8*log(c)) - ((cos(2*a)*sin(4*a) - cos(4*a)*sin(2*a))*cos(4*log(c)) -
 (cos(4*a)*cos(2*a) + sin(4*a)*sin(2*a))*sin(4*log(c)))*sin(8*log(c)))*m +
((cos(4*a)*cos(2*a) + sin(4*a)*sin(2*a))*cos(4*log(c)) + (cos(2*a)*sin(4*a)
 - cos(4*a)*sin(2*a))*sin(4*log(c)))*cos(8*log(c)) - ((cos(2*a)*sin(4*a) -
cos(4*a)*sin(2*a))*cos(4*log(c)) - (cos(4*a)*cos(2*a) + sin(4*a)*sin(2*a))*
sin(4*log(c)))*sin(8*log(c)))*e^(4*arctan2(sin(1/2*m*log(x)), cos(1/2*m*log
(x))) + 4*arctan2(sin(1/2*log(x)), cos(1/2*log(x)))))
```

Fricas [C] Result contains complex when optimal does not.

time = 1.08, size = 83, normalized size = 0.75

$$-\frac{2\left(2i\,x^{2}x^{2\,m}e^{(3i\,a+6i\,\log(c))}-i\,e^{(5i\,a+10i\,\log(c))}\right)}{(m+1)x^{4}x^{4\,m}-2\,(m+1)x^{2}x^{2\,m}e^{(2i\,a+4i\,\log(c))}+(m+1)e^{(4i\,a+8i\,\log(c))}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(a+2*log(c*x^(1/2*(-(1+m)^2)^(1/2))))^3,x, algorithm="fric
as")
```

```
[Out] -2*(2*I*x^2*x^(2*m)*e^(3*I*a + 6*I*log(c)) - I*e^(5*I*a + 10*I*log(c)))/((m
 + 1)*x^4*x^(4*m) - 2*(m + 1)*x^2*x^(2*m)*e^(2*I*a + 4*I*log(c)) + (m + 1)*
e^(4*I*a + 8*I*log(c)))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csc(a+2*ln(c*x**(1/2*(-(1+m)**2)**(1/2))))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5006 deep
```

Giac [**C**] Result contains complex when optimal does not.
time = 5.82, size = 839, normalized size = 7.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(a+2*log(c*x^(1/2*(-(1+m)^2)^(1/2))))^3,x, algorithm="giac
")
```

```
[Out] I*c^(6*I)*m*x*x^m*x^abs(m + 1)*e^(3*I*a)/(c^(8*I)*m^2*e^(4*I*a) + 2*c^(8*I)
*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) - 2*c^(4*I)*m^2*x^(2*abs(m + 1))*e^(2*I*a)
 - 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) - 2*c^(4*I)*x^(2*abs(m + 1))*e^(2
*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*abs(m + 1))) - I
*c^(6*I)*x*x^m*x^abs(m + 1)*abs(m + 1)*e^(3*I*a)/(c^(8*I)*m^2*e^(4*I*a) + 2
*c^(8*I)*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) - 2*c^(4*I)*m^2*x^(2*abs(m + 1))*e
^(2*I*a) - 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) - 2*c^(4*I)*x^(2*abs(m +
1))*e^(2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*abs(m +
1))) + I*c^(6*I)*x*x^m*x^abs(m + 1)*e^(3*I*a)/(c^(8*I)*m^2*e^(4*I*a) + 2*c^
(8*I)*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) - 2*c^(4*I)*m^2*x^(2*abs(m + 1))*e^(2
*I*a) - 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) - 2*c^(4*I)*x^(2*abs(m + 1))
*e^(2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*abs(m + 1))
) - I*c^(2*I)*m*x*x^m*x^(3*abs(m + 1))*e^(I*a)/(c^(8*I)*m^2*e^(4*I*a) + 2*c
```

```
^(8*I)*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) - 2*c^(4*I)*m^2*x^(2*abs(m + 1))*e^(
2*I*a) - 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) - 2*c^(4*I)*x^(2*abs(m + 1)
)*e^(2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*abs(m + 1)
)) - I*c^(2*I)*x*x^m*x^(3*abs(m + 1))*abs(m + 1)*e^(I*a)/(c^(8*I)*m^2*e^(4*
I*a) + 2*c^(8*I)*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) - 2*c^(4*I)*m^2*x^(2*abs(m
 + 1))*e^(2*I*a) - 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) - 2*c^(4*I)*x^(2*
abs(m + 1))*e^(2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*
abs(m + 1))) - I*c^(2*I)*x*x^m*x^(3*abs(m + 1))*e^(I*a)/(c^(8*I)*m^2*e^(4*I
*a) + 2*c^(8*I)*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) - 2*c^(4*I)*m^2*x^(2*abs(m
+ 1))*e^(2*I*a) - 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) - 2*c^(4*I)*x^(2*a
bs(m + 1))*e^(2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*a
bs(m + 1)))
```

Mupad [B]
time = 6.96, size = 171, normalized size = 1.55

$$\frac{\frac{x^{m+1}\,{\mathrm{e}}^{a\,1{}\mathrm{i}}\,{\left(c\,x^{\frac{\sqrt{-m^2-2\,m-1}}{2}}\right)}^{6{}\mathrm{i}}\,\left({\mathrm{e}}^{a\,2{}\mathrm{i}}+{\mathrm{e}}^{a\,2{}\mathrm{i}}\,\sqrt{-{\left(m+1\right)}^2}\,1{}\mathrm{i}+m\,{\mathrm{e}}^{a\,2{}\mathrm{i}}\right)}{\sqrt{-{\left(m+1\right)}^2}}+\frac{x^{m+1}\,{\mathrm{e}}^{a\,1{}\mathrm{i}}\,{\left(c\,x^{\frac{\sqrt{-m^2-2\,m-1}}{2}}\right)}^{2{}\mathrm{i}}\,\left(m+1-\sqrt{-{\left(m+1\right)}^2}\,1{}\mathrm{i}\right)}{\sqrt{-{\left(m+1\right)}^2}}}{\left(m+1\right)\,{\left({\mathrm{e}}^{a\,2{}\mathrm{i}}\,{\left(c\,x^{\frac{\sqrt{-m^2-2\,m-1}}{2}}\right)}^{4{}\mathrm{i}}-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/sin(a + 2*log(c*x^((-(m + 1)^2)^(1/2)/2)))^3,x)
```

```
[Out] ((x^(m + 1)*exp(a*1i)*(c*x^((- 2*m - m^2 - 1)^(1/2)/2))^6i*(exp(a*2i) + exp
(a*2i)*(-(m + 1)^2)^(1/2)*1i + m*exp(a*2i)))/(-(m + 1)^2)^(1/2) + (x^(m + 1
)*exp(a*1i)*(c*x^((- 2*m - m^2 - 1)^(1/2)/2))^2i*(m - (-(m + 1)^2)^(1/2)*1i
 + 1))/(-(m + 1)^2)^(1/2))/((m + 1)*(exp(a*2i)*(c*x^((- 2*m - m^2 - 1)^(1/2
)/2))^4i - 1)^2)
```

3.303 $\int x \csc^3\left(a+2\log\left(cx^i\right)\right)\,dx$

Optimal. Leaf size=49

$$-\frac{ie^{ia}\left(cx^i\right)^{2i}x^2}{\left(1-e^{2ia}\left(cx^i\right)^{4i}\right)^2}$$

```
[Out] -I*exp(I*a)*(c*x^I)^(2*I)*x^2/(1-exp(2*I*a)*(c*x^I)^(4*I))^2
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4606, 4602, 267}

$$-\frac{ie^{ia}x^2\left(cx^i\right)^{2i}}{\left(1-e^{2ia}\left(cx^i\right)^{4i}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csc[a + 2*Log[c*x^I]]^3,x]
```

```
[Out] ((-I)*E^(I*a)*(c*x^I)^(2*I)*x^2)/(1 - E^((2*I)*a)*(c*x^I)^(4*I))^2
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4602

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[(-2*I)^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(
2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4606

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x\csc^3\left(a+2\log\left(cx^i\right)\right)\,dx &= -\left(\left(i\left(cx^i\right)^{2i}x^2\right)\operatorname{Subst}\left(\int x^{-1-2i}\csc^3(a+2\log(x))\,dx,x,cx^i\right)\right)\\ &= \left(8e^{3ia}\left(cx^i\right)^{2i}x^2\right)\operatorname{Subst}\left(\int\frac{x^{-1+4i}}{(1-e^{2ia}x^{4i})^3}\,dx,x,cx^i\right)\\ &= -\frac{ie^{ia}\left(cx^i\right)^{2i}x^2}{\left(1-e^{2ia}\left(cx^i\right)^{4i}\right)^2}\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 127 vs. $2(49) = 98$.
time = 0.20, size = 127, normalized size = 2.59

$$\frac{\csc^2\left(a+2\log\left(cx^i\right)\right)\left(i(-1+2x^4)\cos\left(a+2\log\left(cx^i\right)-2i\log(x)\right)+(1+2x^4)\sin\left(a+2\log\left(cx^i\right)-2i\log(x)\right)\right)\left(\cos\left(2(a+2\log\left(cx^i\right)-2i\log(x))\right)+i\sin\left(2(a+2\log\left(cx^i\right)-2i\log(x))\right)\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csc[a + 2*Log[c*x^I]]^3,x]
```

```
[Out] (Csc[a + 2*Log[c*x^I]]^2*(I*(-1 + 2*x^4)*Cos[a + 2*Log[c*x^I] - (2*I)*Log[x
]] + (1 + 2*x^4)*Sin[a + 2*Log[c*x^I] - (2*I)*Log[x]])*(Cos[2*(a + 2*Log[c*
x^I] - (2*I)*Log[x])] + I*Sin[2*(a + 2*Log[c*x^I] - (2*I)*Log[x])]))/(4*x^4
)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.23, size = 211, normalized size = 4.31

method	result	size
risch	$-\frac{ix^2\left(x^i\right)^{2i}c^{2i}e^{\pi\operatorname{csgn}\left(icx^i\right)^3-\pi\operatorname{csgn}\left(icx^i\right)^2\operatorname{csgn}(ic)-\pi\operatorname{csgn}\left(icx^i\right)^2\operatorname{csgn}\left(ix^i\right)+\pi\operatorname{csgn}\left(icx^i\right)\operatorname{csgn}(ic)\operatorname{csgn}\left(ix^i\right)+ia}}{\left(\left(x^i\right)^{4i}c^{4i}e^{2\pi\operatorname{csgn}\left(icx^i\right)^3}e^{-2\pi\operatorname{csgn}\left(icx^i\right)^2\operatorname{csgn}(ic)}e^{-2\pi\operatorname{csgn}\left(icx^i\right)^2\operatorname{csgn}\left(ix^i\right)}e^{2\pi\operatorname{csgn}\left(icx^i\right)\operatorname{csgn}(ic)\operatorname{csgn}\left(ix^i\right)}e^{2ia}-1\right)^2}$	211

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(a+2*ln(c*x^I))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -I*x^2*(x^I)^(2*I)*c^(2*I)*exp(Pi*csgn(I*c*x^I)^3-Pi*csgn(I*c*x^I)^2*csgn(I
*c)-Pi*csgn(I*c*x^I)^2*csgn(I*x^I)+Pi*csgn(I*c*x^I)*csgn(I*c)*csgn(I*x^I)+I
*a)/((c^(2*I))^2*((x^I)^(2*I))^2*exp(2*Pi*csgn(I*c*x^I)^3)*exp(-2*Pi*csgn(I
*c*x^I)^2*csgn(I*c))*exp(-2*Pi*csgn(I*c*x^I)^2*csgn(I*x^I))*exp(2*Pi*csgn(I
*c*x^I)*csgn(I*c)*csgn(I*x^I))*exp(2*I*a)-1)^2
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(32) = 64$.
time = 0.31, size = 139, normalized size = 2.84

$$\frac{\left((-i\cos(a)+\sin(a))\cos(2\log(c))+(\cos(a)+i\sin(a))\sin(2\log(c))\right)x^2e^{(6\arctan(\sin(\log(x)),\cos(\log(x))))}}{(\cos(4a)+i\sin(4a))\cos(8\log(c))-2\left((\cos(2a)+i\sin(2a))\cos(4\log(c))+(i\cos(2a)-\sin(2a))\sin(4\log(c))\right)e^{(4\arctan(\sin(\log(x)),\cos(\log(x))))}+(i\cos(4a)-\sin(4a))\sin(8\log(c))+e^{(8\arctan(\sin(\log(x)),\cos(\log(x))))}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(a+2*log(c*x^I))^3,x, algorithm="maxima")
```

```
[Out] ((-I*cos(a) + sin(a))*cos(2*log(c)) + (cos(a) + I*sin(a))*sin(2*log(c)))*x^
2*e^(6*arctan2(sin(log(x)), cos(log(x))))/((cos(4*a) + I*sin(4*a))*cos(8*lo
g(c)) - 2*((cos(2*a) + I*sin(2*a))*cos(4*log(c)) + (I*cos(2*a) - sin(2*a))*
sin(4*log(c)))*e^(4*arctan2(sin(log(x)), cos(log(x)))) + (I*cos(4*a) - sin(
4*a))*sin(8*log(c)) + e^(8*arctan2(sin(log(x)), cos(log(x)))))
```

Fricas [A]
time = 2.83, size = 56, normalized size = 1.14

$$\frac{-2i\,x^4e^{(3i\,a+6i\,\log(c))}+i\,e^{(5i\,a+10i\,\log(c))}}{x^8-2\,x^4e^{(2i\,a+4i\,\log(c))}+e^{(4i\,a+8i\,\log(c))}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(a+2*log(c*x^I))^3,x, algorithm="fricas")
```

```
[Out] (-2*I*x^4*e^(3*I*a + 6*I*log(c)) + I*e^(5*I*a + 10*I*log(c)))/(x^8 - 2*x^4*
e^(2*I*a + 4*I*log(c)) + e^(4*I*a + 8*I*log(c)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\csc^3\left(a+2\log\left(cx^i\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(a+2*ln(c*x**I))**3,x)
```

```
[Out] Integral(x*csc(a + 2*log(c*x**I))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(a+2*log(c*x^I))^3,x, algorithm="giac")
```

```
[Out] integrate(x*csc(a + 2*log(c*x^I))^3, x)
```

Mupad [B]
time = 4.41, size = 45, normalized size = 0.92

$$-\frac{x^2\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^{1\mathrm{i}}\right)}^{2\mathrm{i}}\,1\mathrm{i}}{1+\mathrm{e}^{a\,4\mathrm{i}}\,{\left(c\,x^{1\mathrm{i}}\right)}^{8\mathrm{i}}-2\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^{1\mathrm{i}}\right)}^{4\mathrm{i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sin(a + 2*log(c*x^1i))^3,x)
```

```
[Out] -(x^2*exp(a*1i)*(c*x^1i)^2i*1i)/(exp(a*4i)*(c*x^1i)^8i - 2*exp(a*2i)*(c*x^1
i)^4i + 1)
```

3.304 $\int \csc^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\,dx$

Optimal. Leaf size=58

$$\frac{1}{2}x\csc\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)+\frac{1}{2}ix\cot\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\csc\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)$$

```
[Out] 1/2*x*csc(a+2*ln(c*x^(1/2*I)))+1/2*I*x*cot(a+2*ln(c*x^(1/2*I)))*csc(a+2*ln(
c*x^(1/2*I)))
```

Rubi [A]
time = 0.02, antiderivative size = 51, normalized size of antiderivative = 0.88, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4600, 4602, 267}

$$-\frac{2ie^{ia}x\left(cx^{\frac{i}{2}}\right)^{2i}}{\left(1-e^{2ia}\left(cx^{\frac{i}{2}}\right)^{4i}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + 2*Log[c*x^(I/2)]]^3,x]
```

```
[Out] ((-2*I)*E^(I*a)*(c*x^(I/2))^(2*I)*x)/(1 - E^((2*I)*a)*(c*x^(I/2))^(4*I))^2
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4600

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4602

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[(-2*I)^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(
2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \csc^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\,dx &= -\left(\left(2i\left(cx^{\frac{i}{2}}\right)^{2i}x\right)\text{Subst}\left(\int x^{-1-2i}\csc^3(a+2\log(x))\,dx,x,cx^{\frac{i}{2}}\right)\right)\\
&= \left(16e^{3ia}\left(cx^{\frac{i}{2}}\right)^{2i}x\right)\text{Subst}\left(\int \frac{x^{-1+4i}}{\left(1-e^{2ia}x^{4i}\right)^3}\,dx,x,cx^{\frac{i}{2}}\right)\\
&= -\frac{2ie^{ia}\left(cx^{\frac{i}{2}}\right)^{2i}x}{\left(1-e^{2ia}\left(cx^{\frac{i}{2}}\right)^{4i}\right)^2}
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 137 vs. $2(58) = 116$.
time $= 0.16$, size $= 137$, normalized size $= 2.36$

$$
\frac{\csc^2\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\left(i(-1+2x^2)\cos\left(a+2\log\left(cx^{\frac{i}{2}}\right)-i\log(x)\right)+(1+2x^2)\sin\left(a+2\log\left(cx^{\frac{i}{2}}\right)-i\log(x)\right)\right)\left(\cos\left(2\left(a+2\log\left(cx^{\frac{i}{2}}\right)-i\log(x)\right)\right)+i\sin\left(2\left(a+2\log\left(cx^{\frac{i}{2}}\right)-i\log(x)\right)\right)\right)}{2x^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + 2*Log[c*x^(I/2)]]^3,x]
```

```
[Out] (Csc[a + 2*Log[c*x^(I/2)]]^2*(I*(-1 + 2*x^2)*Cos[a + 2*Log[c*x^(I/2)] - I*L
og[x]] + (1 + 2*x^2)*Sin[a + 2*Log[c*x^(I/2)] - I*Log[x]])*(Cos[2*(a + 2*Lo
g[c*x^(I/2)] - I*Log[x])] + I*Sin[2*(a + 2*Log[c*x^(I/2)] - I*Log[x])]))/(2
*x^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time $= 0.24$, size $= 209$, normalized size $= 3.60$

method	result	s
risch	$-\frac{2ix\left(x^{\frac{i}{2}}\right)^{2i}c^{2i}\mathrm{e}^{\mathrm{csgn}\left(icx^{\frac{i}{2}}\right)^3\pi-\mathrm{csgn}\left(icx^{\frac{i}{2}}\right)^2\mathrm{csgn}(ic)\pi-\mathrm{csgn}\left(icx^{\frac{i}{2}}\right)^2\mathrm{csgn}\left(ix^{\frac{i}{2}}\right)\pi+\mathrm{csgn}\left(icx^{\frac{i}{2}}\right)\mathrm{csgn}(ic)\mathrm{csgn}\left(ix^{\frac{i}{2}}\right)\pi+ia}}{\left(\left(x^{\frac{i}{2}}\right)^{4i}c^{4i}\mathrm{e}^{2\mathrm{csgn}\left(icx^{\frac{i}{2}}\right)^3\pi}\mathrm{e}^{-2\mathrm{csgn}\left(icx^{\frac{i}{2}}\right)^2\mathrm{csgn}(ic)\pi}\mathrm{e}^{-2\mathrm{csgn}\left(icx^{\frac{i}{2}}\right)^2\mathrm{csgn}\left(ix^{\frac{i}{2}}\right)\pi}\mathrm{e}^{2\,\mathrm{csgn}\left(icx^{\frac{i}{2}}\right)\mathrm{csgn}(ic)\mathrm{csgn}\left(ix^{\frac{i}{2}}\right)\pi}\mathrm{e}^{2ia}-1\right)^2}$	2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(a+2*ln(c*x^(1/2*I)))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*x*(x^(1/2*I))^(2*I)*c^(2*I)*exp(csgn(I*c*x^(1/2*I))^3*Pi-csgn(I*c*x^(1
/2*I))^2*csgn(I*c)*Pi-csgn(I*c*x^(1/2*I))^2*csgn(I*x^(1/2*I))*Pi+csgn(I*c*x
^(1/2*I))*csgn(I*c)*csgn(I*x^(1/2*I))*Pi+I*a)/((c^(2*I))^2*((x^(1/2*I))^(2*
I))^2*exp(2*csgn(I*c*x^(1/2*I))^3*Pi)*exp(-2*csgn(I*c*x^(1/2*I))^2*csgn(I*c
)*Pi)*exp(-2*csgn(I*c*x^(1/2*I))^2*csgn(I*x^(1/2*I))*Pi)*exp(2*csgn(I*c*x^(
1/2*I))*csgn(I*c)*csgn(I*x^(1/2*I))*Pi)*exp(2*I*a)-1)^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(40) = 80$.
time = 0.31, size = 153, normalized size = 2.64

$$-\frac{2\left(\left(i\cos\left(a\right)-\sin\left(a\right)\right)\cos\left(2\log\left(c\right)\right)-\left(\cos\left(a\right)+i\sin\left(a\right)\right)\sin\left(2\log\left(c\right)\right)\right)xe^{\left(6\arctan\left(\sin\left(\frac{1}{2}\log\left(x\right)\right),\cos\left(\frac{1}{2}\log\left(x\right)\right)\right)\right)}}{\left(\cos\left(4a\right)+i\sin\left(4a\right)\right)\cos\left(8\log\left(c\right)\right)-2\left(\left(\cos\left(2a\right)+i\sin\left(2a\right)\right)\cos\left(4\log\left(c\right)\right)+\left(i\cos\left(2a\right)-\sin\left(2a\right)\right)\sin\left(4\log\left(c\right)\right)\right)e^{\left(4\arctan\left(\sin\left(\frac{1}{2}\log\left(x\right)\right),\cos\left(\frac{1}{2}\log\left(x\right)\right)\right)\right)}+\left(i\cos\left(4a\right)-\sin\left(4a\right)\right)\sin\left(8\log\left(c\right)\right)+e^{\left(8\arctan\left(\sin\left(\frac{1}{2}\log\left(x\right)\right),\cos\left(\frac{1}{2}\log\left(x\right)\right)\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+2*log(c*x^(1/2*I)))^3,x, algorithm="maxima")
```

```
[Out] -2*((I*cos(a) - sin(a))*cos(2*log(c)) - (cos(a) + I*sin(a))*sin(2*log(c)))*
x*e^(6*arctan2(sin(1/2*log(x)), cos(1/2*log(x))))/((cos(4*a) + I*sin(4*a))*
cos(8*log(c)) - 2*((cos(2*a) + I*sin(2*a))*cos(4*log(c)) + (I*cos(2*a) - si
n(2*a))*sin(4*log(c)))*e^(4*arctan2(sin(1/2*log(x)), cos(1/2*log(x)))) + (I
*cos(4*a) - sin(4*a))*sin(8*log(c)) + e^(8*arctan2(sin(1/2*log(x)), cos(1/2
*log(x)))))
```

Fricas [A]
time = 1.94, size = 57, normalized size = 0.98

$$-\frac{2\left(2i\,x^{2}e^{\left(3i\,a+6i\,\log\left(c\right)\right)}-i\,e^{\left(5i\,a+10i\,\log\left(c\right)\right)}\right)}{x^{4}-2\,x^{2}e^{\left(2i\,a+4i\,\log\left(c\right)\right)}+e^{\left(4i\,a+8i\,\log\left(c\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+2*log(c*x^(1/2*I)))^3,x, algorithm="fricas")
```

```
[Out] -2*(2*I*x^2*e^(3*I*a + 6*I*log(c)) - I*e^(5*I*a + 10*I*log(c)))/(x^4 - 2*x^
2*e^(2*I*a + 4*I*log(c)) + e^(4*I*a + 8*I*log(c)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\csc^{3}\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+2*ln(c*x**(1/2*I)))**3,x)
```

```
[Out] Integral(csc(a + 2*log(c*x**(I/2)))**3, x)
```

Giac [A]
time = 1.11, size = 74, normalized size = 1.28

$$\frac{2i\,c^{10i}e^{\left(5i\,a\right)}}{c^{8i}e^{\left(4i\,a\right)}-2\,c^{4i}x^{2}e^{\left(2i\,a\right)}+x^{4}}-\frac{4i\,c^{6i}x^{2}e^{\left(3i\,a\right)}}{c^{8i}e^{\left(4i\,a\right)}-2\,c^{4i}x^{2}e^{\left(2i\,a\right)}+x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+2*log(c*x^(1/2*I)))^3,x, algorithm="giac")
```

```
[Out] 2*I*c^(10*I)*e^(5*I*a)/(c^(8*I)*e^(4*I*a) - 2*c^(4*I)*x^2*e^(2*I*a) + x^4)
- 4*I*c^(6*I)*x^2*e^(3*I*a)/(c^(8*I)*e^(4*I*a) - 2*c^(4*I)*x^2*e^(2*I*a) +
x^4)
```

Mupad [B]
time = 4.54, size = 55, normalized size = 0.95

$$-\frac{x\,{\mathrm{e}}^{a\,1{}\mathrm{i}}\,{\left(c\,x^{\frac{1}{2}{}\mathrm{i}}\right)}^{2{}\mathrm{i}}\,2{}\mathrm{i}}{1+{\mathrm{e}}^{a\,4{}\mathrm{i}}\,{\left(c\,x^{\frac{1}{2}{}\mathrm{i}}\right)}^{8{}\mathrm{i}}-2\,{\mathrm{e}}^{a\,2{}\mathrm{i}}\,{\left(c\,x^{\frac{1}{2}{}\mathrm{i}}\right)}^{4{}\mathrm{i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(a + 2*log(c*x^(1i/2)))^3,x)
```

```
[Out] -(x*exp(a*1i)*(c*x^(1i/2))^2i*2i)/(exp(a*4i)*(c*x^(1i/2))^8i - 2*exp(a*2i)*
(c*x^(1i/2))^4i + 1)
```

3.305 $\int \csc^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\,dx$

Optimal. Leaf size=51

$$\frac{2ie^{3ia}\left(cx^{-\frac{i}{2}}\right)^{6i}x}{\left(1-e^{2ia}\left(cx^{-\frac{i}{2}}\right)^{4i}\right)^2}$$

```
[Out] 2*I*exp(3*I*a)*(c/(x^(1/2*I)))^(6*I)*x/(1-exp(2*I*a)*(c/(x^(1/2*I)))^(4*I))
^2
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4600, 4602, 270}

$$\frac{2ie^{3ia}x\left(cx^{-\frac{i}{2}}\right)^{6i}}{\left(1-e^{2ia}\left(cx^{-\frac{i}{2}}\right)^{4i}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + 2*Log[c/x^(I/2)]]^3,x]
```

```
[Out] ((2*I)*E^((3*I)*a)*(c/x^(I/2))^(6*I)*x)/(1 - E^((2*I)*a)*(c/x^(I/2))^(4*I))
^2
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4600

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4602

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[(-2*I)^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(
2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \csc^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\,dx &= \left(2i\left(cx^{-\frac{i}{2}}\right)^{-2i}x\right)\text{Subst}\left(\int x^{-1+2i}\csc^3(a+2\log(x))\,dx,x,cx^{-\frac{i}{2}}\right)\\
&= -\left(\left(16e^{3ia}\left(cx^{-\frac{i}{2}}\right)^{-2i}x\right)\text{Subst}\left(\int \frac{x^{-1+8i}}{\left(1-e^{2ia}x^{4i}\right)^3}\,dx,x,cx^{-\frac{i}{2}}\right)\right)\\
&= \frac{2ie^{3ia}\left(cx^{-\frac{i}{2}}\right)^{6i}x}{\left(1-e^{2ia}\left(cx^{-\frac{i}{2}}\right)^{4i}\right)^2}
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 137 vs. $2(51) = 102$.
time = 0.17, size = 137, normalized size = 2.69

$$
-\frac{\csc^2\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\left((-1+2x^2)\cos\left(a+2\log\left(cx^{-\frac{i}{2}}\right)+i\log(x)\right)+i(1+2x^2)\sin\left(a+2\log\left(cx^{-\frac{i}{2}}\right)+i\log(x)\right)\right)\left(i\cos\left(2\left(a+2\log\left(cx^{-\frac{i}{2}}\right)+i\log(x)\right)\right)+\sin\left(2\left(a+2\log\left(cx^{-\frac{i}{2}}\right)+i\log(x)\right)\right)\right)}{2x^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + 2*Log[c/x^(I/2)]]^3,x]
```

```
[Out] -1/2*(Csc[a + 2*Log[c/x^(I/2)]]^2*((-1 + 2*x^2)*Cos[a + 2*Log[c/x^(I/2)] +
I*Log[x]] + I*(1 + 2*x^2)*Sin[a + 2*Log[c/x^(I/2)] + I*Log[x]])*(I*Cos[2*(a
 + 2*Log[c/x^(I/2)] + I*Log[x])] + Sin[2*(a + 2*Log[c/x^(I/2)] + I*Log[x])]
))/x^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.23, size = 239, normalized size = 4.69

method	result
risch	$\frac{2ix\,c^{6i}\left(x^{\frac{i}{2}}\right)^{-6i}\mathrm{e}^{3\pi\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)^3-3\pi\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)^2\mathrm{csgn}(ic)-3\pi\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)^2\mathrm{csgn}\left(ix^{-\frac{i}{2}}\right)+3\pi\,\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)\mathrm{csgn}(ic)\mathrm{csgn}\left(ix^{-\frac{i}{2}}\right)+3i}}{\left(c^{4i}\left(x^{\frac{i}{2}}\right)^{-4i}\mathrm{e}^{2\pi\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)^3}\mathrm{e}^{-2\pi\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)^2\mathrm{csgn}(ic)}\mathrm{e}^{-2\pi\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)^2\mathrm{csgn}\left(ix^{-\frac{i}{2}}\right)}\mathrm{e}^{2\pi\,\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)\mathrm{csgn}(ic)\mathrm{csgn}\left(ix^{-\frac{i}{2}}\right)}\mathrm{e}^{2ia}-\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(a+2*ln(c/(x^(1/2*I))))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*I*x*(c^(2*I))^3*((x^(1/2*I))^(-2*I))^3*exp(3*Pi*csgn(I*c/(x^(1/2*I)))^3-3
*Pi*csgn(I*c/(x^(1/2*I)))^2*csgn(I*c)-3*Pi*csgn(I*c/(x^(1/2*I)))^2*csgn(I/(
x^(1/2*I)))+3*Pi*csgn(I*c/(x^(1/2*I)))*csgn(I*c)*csgn(I/(x^(1/2*I)))+3*I*a)
/((c^(2*I))^2*((x^(1/2*I))^(-2*I))^2*exp(2*Pi*csgn(I*c/(x^(1/2*I)))^3)*exp(
-2*Pi*csgn(I*c/(x^(1/2*I)))^2*csgn(I*c))*exp(-2*Pi*csgn(I*c/(x^(1/2*I)))^2*
```

```
csgn(I/(x^(1/2*I))))*exp(2*Pi*csgn(I*c/(x^(1/2*I)))*csgn(I*c)*csgn(I/(x^(1/
2*I))))*exp(2*I*a)-1)^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(27) = 54$.
time = 0.31, size = 162, normalized size = 3.18

$$\frac{2\left(\left(i\cos\left(3a\right)-\sin\left(3a\right)\right)\cos\left(6\log\left(c\right)\right)-\left(\cos\left(3a\right)+i\sin\left(3a\right)\right)\sin\left(6\log\left(c\right)\right)\right)xe^{\left(6\arctan\left(\sin\left(\frac{1}{2}\log\left(x\right)\right),\cos\left(\frac{1}{2}\log\left(x\right)\right)\right)\right)}}{\left(\left(\cos\left(4a\right)+i\sin\left(4a\right)\right)\cos\left(8\log\left(c\right)\right)-\left(-i\cos\left(4a\right)+\sin\left(4a\right)\right)\sin\left(8\log\left(c\right)\right)\right)e^{\left(8\arctan\left(\sin\left(\frac{1}{2}\log\left(x\right)\right),\cos\left(\frac{1}{2}\log\left(x\right)\right)\right)\right)}-2\left(\left(\cos\left(2a\right)+i\sin\left(2a\right)\right)\cos\left(4\log\left(c\right)\right)-\left(-i\cos\left(2a\right)+\sin\left(2a\right)\right)\sin\left(4\log\left(c\right)\right)\right)e^{\left(4\arctan\left(\sin\left(\frac{1}{2}\log\left(x\right)\right),\cos\left(\frac{1}{2}\log\left(x\right)\right)\right)\right)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+2*log(c/(x^(1/2*I))))^3,x, algorithm="maxima")
```

```
[Out] 2*((I*cos(3*a) - sin(3*a))*cos(6*log(c)) - (cos(3*a) + I*sin(3*a))*sin(6*lo
g(c)))*x*e^(6*arctan2(sin(1/2*log(x)), cos(1/2*log(x))))/(((cos(4*a) + I*si
n(4*a))*cos(8*log(c)) - (-I*cos(4*a) + sin(4*a))*sin(8*log(c)))*e^(8*arctan
2(sin(1/2*log(x)), cos(1/2*log(x)))) - 2*((cos(2*a) + I*sin(2*a))*cos(4*log
(c)) - (-I*cos(2*a) + sin(2*a))*sin(4*log(c)))*e^(4*arctan2(sin(1/2*log(x))
, cos(1/2*log(x)))) + 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(27) = 54$.
time = 1.63, size = 57, normalized size = 1.12

$$-\frac{2\left(-2i\,x^{2}e^{\left(2i\,a+4i\,\log\left(c\right)\right)}+i\right)}{x^{4}e^{\left(5i\,a+10i\,\log\left(c\right)\right)}-2\,x^{2}e^{\left(3i\,a+6i\,\log\left(c\right)\right)}+e^{\left(i\,a+2i\,\log\left(c\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+2*log(c/(x^(1/2*I))))^3,x, algorithm="fricas")
```

```
[Out] -2*(-2*I*x^2*e^(2*I*a + 4*I*log(c)) + I)/(x^4*e^(5*I*a + 10*I*log(c)) - 2*x
^2*e^(3*I*a + 6*I*log(c)) + e^(I*a + 2*I*log(c)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\csc^{3}\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+2*ln(c/(x**(1/2*I))))**3,x)
```

```
[Out] Integral(csc(a + 2*log(c/x**(I/2)))**3, x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(27) = 54$.
time = 1.11, size = 83, normalized size = 1.63

$$\frac{4i\,c^{4i}x^{2}e^{\left(2i\,a\right)}}{c^{10i}x^{4}e^{\left(5i\,a\right)}-2\,c^{6i}x^{2}e^{\left(3i\,a\right)}+c^{2i}e^{\left(i\,a\right)}}-\frac{2i}{c^{10i}x^{4}e^{\left(5i\,a\right)}-2\,c^{6i}x^{2}e^{\left(3i\,a\right)}+c^{2i}e^{\left(i\,a\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+2*log(c/(x^(1/2*I))))^3,x, algorithm="giac")
```

```
[Out] 4*I*c^(4*I)*x^2*e^(2*I*a)/(c^(10*I)*x^4*e^(5*I*a) - 2*c^(6*I)*x^2*e^(3*I*a)
 + c^(2*I)*e^(I*a)) - 2*I/(c^(10*I)*x^4*e^(5*I*a) - 2*c^(6*I)*x^2*e^(3*I*a)
 + c^(2*I)*e^(I*a))
```

Mupad [B]
time = 6.32, size = 38, normalized size = 0.75

$$\frac{x\,{\mathrm{e}}^{a\,3{}\mathrm{i}}\,{\left(\frac{c}{x^{\frac{1}{2}{}\mathrm{i}}}\right)}^{6{}\mathrm{i}}\,2{}\mathrm{i}}{{\left({\mathrm{e}}^{a\,2{}\mathrm{i}}\,{\left(\frac{c}{x^{\frac{1}{2}{}\mathrm{i}}}\right)}^{4{}\mathrm{i}}-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(a + 2*log(c/x^(1i/2)))^3,x)
```

```
[Out] (x*exp(a*3i)*(c/x^(1i/2))^6i*2i)/(exp(a*2i)*(c/x^(1i/2))^4i - 1)^2
```

3.306 $\int \csc^p\left(a+\frac{i\log(cx^n)}{n(-2+p)}\right)\,dx$

Optimal. Leaf size=96

$$-\frac{e^{-2ia}(2-p)x(cx^n)^{-\frac{2}{n(2-p)}}\left(1-e^{2ia}(cx^n)^{\frac{2}{n(2-p)}}\right)\csc^p\left(a-\frac{i\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

```
[Out] -1/2*(2-p)*x*(1-exp(2*I*a)*(c*x^n)^(2/n/(2-p)))*csc(a-I*ln(c*x^n)/n/(2-p))^
p/exp(2*I*a)/(1-p)/((c*x^n)^(2/n/(2-p)))
```

Rubi [A]
time = 0.06, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4600, 4604, 267}

$$-\frac{e^{-2ia}(2-p)x(cx^n)^{-\frac{2}{n(2-p)}}\left(1-e^{2ia}(cx^n)^{\frac{2}{n(2-p)}}\right)\csc^p\left(a-\frac{i\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + (I*Log[c*x^n])/(n*(-2 + p))]^p,x]
```

```
[Out] -1/2*((2 - p)*x*(1 - E^((2*I)*a)*(c*x^n)^(2/(n*(2 - p))))*Csc[a - (I*Log[c*
x^n])/(n*(2 - p))]^p)/(E^((2*I)*a)*(1 - p)*(c*x^n)^(2/(n*(2 - p))))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4600

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4604

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \csc^p\left(a+\frac{i\log(cx^n)}{n(-2+p)}\right)\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\csc^p\left(a+\frac{i\log(x)}{n(-2+p)}\right)\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-\frac{1}{n}+\frac{p}{n(-2+p)}}\left(1-e^{2ia}(cx^n)^{-\frac{2}{n(-2+p)}}\right)^p\csc^p\left(a+\frac{i\log(cx^n)}{n(-2+p)}\right)\right)\text{Subst}\left(\int x^{\cdot}\right.}{n}$$

$$= -\frac{e^{-2ia}(2-p)x(cx^n)^{-\frac{2}{n(2-p)}}\left(1-e^{2ia}(cx^n)^{\frac{2}{n(2-p)}}\right)\csc^p\left(a-\frac{i\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Mathematica [A]
time = 1.98, size = 155, normalized size = 1.61

$$\frac{2^{-1+p}e^{-\frac{2iap}{-2+p}}(-2+p)x\left(e^{\frac{2iap}{-2+p}}-e^{\frac{4ia}{-2+p}}(cx^n)^{\frac{2}{n(-2+p)}}\right)\left(-\frac{ie^{\frac{ia(2+p)}{-2+p}}(cx^n)^{\frac{1}{n(-2+p)}}}{-e^{\frac{2iap}{-2+p}}+e^{\frac{4ia}{-2+p}}(cx^n)^{\frac{2}{n(-2+p)}}}\right)^p}{-1+p}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[a + (I*Log[c*x^n])/(n*(-2 + p))]^p,x]
```

```
[Out] (2^(-1 + p)*(-2 + p)*x*(E^(((2*I)*a*p)/(-2 + p)) - E^(((4*I)*a)/(-2 + p))*(
c*x^n)^(2/(n*(-2 + p))))*(((-I)*E^((I*a*(2 + p))/(-2 + p))*(c*x^n)^(1/(n*(-
2 + p))))/(-E^(((2*I)*a*p)/(-2 + p)) + E^(((4*I)*a)/(-2 + p))*(c*x^n)^(2/(n
*(-2 + p)))))^p)/(E^(((2*I)*a*p)/(-2 + p))*(-1 + p))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \csc^p\left(a+\frac{i\ln(c\,x^n)}{n\,(-2+p)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(a+I*ln(c*x^n)/n/(-2+p))^p,x)
```

```
[Out] int(csc(a+I*ln(c*x^n)/n/(-2+p))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+I*log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
```

[Out] integrate(csc(a + I*log(c*x^n)/(n*(p - 2)))^p, x)

Fricas [A]
time = 4.01, size = 150, normalized size = 1.56

$$\frac{\left((p-2)xe^{\left(\frac{2\,(i\,anp-2i\,an-n\log(x)-\log(c))}{np-2\,n}\right)}-(p-2)x\right)\left(\frac{2i\,e^{\left(\frac{i\,anp-2i\,an-n\log(x)-\log(c)}{np-2\,n}\right)}}{e^{\left(\frac{2\,(i\,anp-2i\,an-n\log(x)-\log(c))}{np-2\,n}\right)}-1}\right)^{p}e^{\left(-\frac{2\,(i\,anp-2i\,an-n\log(x)-\log(c))}{np-2\,n}\right)}}{2\,(p-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+I*log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
```

```
[Out] 1/2*((p - 2)*x*e^(2*(I*a*n*p - 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n)) -
(p - 2)*x)*(2*I*e^((I*a*n*p - 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n))/(e^
(2*(I*a*n*p - 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n)) - 1))^p*e^(-2*(I*a*
n*p - 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n))/(p - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^{p}\left(a+\frac{i\log\left(cx^{n}\right)}{n\left(p-2\right)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+I*ln(c*x**n)/n/(-2+p))**p,x)
```

```
[Out] Integral(csc(a + I*log(c*x**n)/(n*(p - 2)))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+I*log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
```

```
[Out] integrate(csc(a + I*log(c*x^n)/(n*(p - 2)))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\sin\left(a+\frac{\ln\left(c\,x^n\right)\,1{}\mathrm{i}}{n\,\left(p-2\right)}\right)}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(a + (log(c*x^n)*1i)/(n*(p - 2))))^p,x)
```

```
[Out] int((1/sin(a + (log(c*x^n)*1i)/(n*(p - 2))))^p, x)
```

3.307 $\int \csc^p\left(a-\frac{i\log(cx^n)}{n(-2+p)}\right)\,dx$

Optimal. Leaf size=71

$$\frac{(2-p)x\left(1-e^{2ia}(cx^n)^{-\frac{2}{n(2-p)}}\right)\csc^p\left(a+\frac{i\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

```
[Out] 1/2*(2-p)*x*(1-exp(2*I*a)/((c*x^n)^(2/n/(2-p))))*csc(a+I*ln(c*x^n)/n/(2-p))
^p/(1-p)
```

Rubi [A]
time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4600, 4604, 270}

$$\frac{(2-p)x\left(1-e^{2ia}(cx^n)^{-\frac{2}{n(2-p)}}\right)\csc^p\left(a+\frac{i\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a - (I*Log[c*x^n])/(n*(-2 + p))]^p,x]
```

```
[Out] ((2 - p)*x*(1 - E^((2*I)*a)/(c*x^n)^(2/(n*(2 - p))))*Csc[a + (I*Log[c*x^n])
/(n*(2 - p))]^p)/(2*(1 - p))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4600

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4604

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \csc^p\left(a-\frac{i\log\left(cx^n\right)}{n(-2+p)}\right)\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\mathrm{Subst}\left(\int x^{-1+\frac{1}{n}}\csc^p\left(a-\frac{i\log(x)}{n(-2+p)}\right)\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-\frac{1}{n}-\frac{p}{n(-2+p)}}\left(1-e^{2ia}(cx^n)^{\frac{2}{n(-2+p)}}\right)^p\csc^p\left(a-\frac{i\log(cx^n)}{n(-2+p)}\right)\right)\mathrm{Subst}\left(\int x\right.}{n}$$

$$= \frac{(2-p)x\left(1-e^{2ia}(cx^n)^{-\frac{2}{n(2-p)}}\right)\csc^p\left(a+\frac{i\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Mathematica [A]
time = 2.98, size = 128, normalized size = 1.80

$$\frac{2^{-1+p}(-2+p)x\left(\frac{ie^{ia}(cx^n)^{\frac{1}{n(-2+p)}}}{-1+e^{2ia}(cx^n)^{\frac{2}{n(-2+p)}}}\right)^p\left(1+e^{2ia}(cx^n)^{\frac{2}{n(-2+p)}}\left(-1+\left(1-e^{-2ia}(cx^n)^{-\frac{2}{n(-2+p)}}\right)^p\right)\right)}{-1+p}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a - (I*Log[c*x^n])/(n*(-2 + p))]^p,x]
```

```
[Out] (2^(-1 + p)*(-2 + p)*x*((I*E^(I*a)*(c*x^n)^(1/(n*(-2 + p))))/(-1 + E^((2*I)
*a)*(c*x^n)^(2/(n*(-2 + p)))))^p*(1 + E^((2*I)*a)*(c*x^n)^(2/(n*(-2 + p)))*
(-1 + (1 - 1/(E^((2*I)*a)*(c*x^n)^(2/(n*(-2 + p)))))^p)))/(-1 + p)
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \csc^p\left(a-\frac{i\ln\left(c\,x^n\right)}{n\left(-2+p\right)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(a-I*ln(c*x^n)/n/(-2+p))^p,x)
```

```
[Out] int(csc(a-I*ln(c*x^n)/n/(-2+p))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a-I*log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
```

```
[Out] integrate((-csc(-a + I*log(c*x^n)/(n*(p - 2))))^p, x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(55) = 110$.
time = 1.34, size = 150, normalized size = 2.11

$$\frac{\left((p-2)xe^{\left(\frac{2\,(-i\,anp+2i\,an-n\log(x)-\log(c))}{np-2\,n}\right)}-(p-2)x\right)\left(-\frac{2i\,e^{\left(\frac{-i\,anp+2i\,an-n\log(x)-\log(c)}{np-2\,n}\right)}}{e^{\left(\frac{2\,(-i\,anp+2i\,an-n\log(x)-\log(c))}{np-2\,n}\right)}-1}\right)^{p}e^{\left(-\frac{2\,(-i\,anp+2i\,an-n\log(x)-\log(c))}{np-2\,n}\right)}}{2\,(p-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a-I*log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
```

```
[Out] 1/2*((p - 2)*x*e^(2*(-I*a*n*p + 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n)) -
 (p - 2)*x)*(-2*I*e^((-I*a*n*p + 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n))/
(e^(2*(-I*a*n*p + 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n)) - 1))^p*e^(-2*(
-I*a*n*p + 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n))/(p - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^{p}\left(a-\frac{i\log\left(cx^{n}\right)}{n\left(p-2\right)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a-I*ln(c*x**n)/n/(-2+p))**p,x)
```

```
[Out] Integral(csc(a - I*log(c*x**n)/(n*(p - 2)))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a-I*log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
```

```
[Out] integrate(csc(a - I*log(c*x^n)/(n*(p - 2)))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\sin\left(a-\frac{\ln\left(c\,x^n\right)\,1{}\mathrm{i}}{n\,\left(p-2\right)}\right)}\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(a - (log(c*x^n)*1i)/(n*(p - 2))))^p,x)
```

```
[Out] int((1/sin(a - (log(c*x^n)*1i)/(n*(p - 2))))^p, x)
```

3.308 $\int \sqrt{\csc(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=109

$$\frac{2x\sqrt{1-e^{2ia}(cx^n)^{2ib}}\,\sqrt{\csc(a+b\log(cx^n))}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}\left(1-\frac{2i}{bn}\right);\frac{1}{4}\left(5-\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{2+ibn}$$

```
[Out] 2*x*hypergeom([1/2, 1/4-1/2*I/b/n],[5/4-1/2*I/b/n],exp(2*I*a)*(c*x^n)^(2*I*
b))*(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(1/2)*csc(a+b*ln(c*x^n))^(1/2)/(2+I*b*n)
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4600, 4604, 371}

$$\frac{2x\sqrt{1-e^{2ia}(cx^n)^{2ib}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}\left(1-\frac{2i}{bn}\right);\frac{1}{4}\left(5-\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)\sqrt{\csc(a+b\log(cx^n))}}{2+ibn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csc[a + b*Log[c*x^n]]],x]
```

```
[Out] (2*x*Sqrt[1 - E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[Csc[a + b*Log[c*x^n]]]*Hy
pergeometric2F1[1/2, (1 - (2*I)/(b*n))/4, (5 - (2*I)/(b*n))/4, E^((2*I)*a)*
(c*x^n)^((2*I)*b)])/(2 + I*b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4600

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4604

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{\csc (a+b\log (cx^n))}\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\sqrt{\csc(a+b\log(x))}\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-\frac{ib}{2}-\frac{1}{n}}\sqrt{1-e^{2ia}(cx^n)^{2ib}}\sqrt{\csc (a+b\log (cx^n))}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{i}{2}}}{\sqrt{1-e}}\right.}{n}$$

$$= \frac{2x\sqrt{1-e^{2ia}(cx^n)^{2ib}}\sqrt{\csc (a+b\log (cx^n))}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}\left(1-\frac{2i}{bn}\right);\frac{1}{4}\left(5-\frac{2i}{bn}\right);e^{2}\right.}{2+ibn}$$

Mathematica [A]
time = 0.46, size = 115, normalized size = 1.06

$$\frac{2ie^{-2ia}\left(-1+e^{2i(a+b\log(cx^n))}\right)x(cx^n)^{-2ib}\sqrt{\csc (a+b\log (cx^n))}\,{}_2F_1\left(1,\frac{3}{4}+\frac{i}{2bn};\frac{5}{4}+\frac{i}{2bn};e^{-2i(a+b\log(cx^n))}\right)}{2i+bn}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Csc[a + b*Log[c*x^n]]],x]
[Out] ((2*I)*(-1 + E^((2*I)*(a + b*Log[c*x^n])))*x*Sqrt[Csc[a + b*Log[c*x^n]]]*Hy
pergeometric2F1[1, 3/4 + (I/2)/(b*n), 5/4 + (I/2)/(b*n), E^((-2*I)*(a + b*L
og[c*x^n]))])/(E^((2*I)*a)*(2*I + b*n)*(c*x^n)^((2*I)*b))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \sqrt{\csc}\left(a+b\ln \left(c\,x^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(a+b*ln(c*x^n))^(1/2),x)
[Out] int(csc(a+b*ln(c*x^n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(csc(b*log(c*x^n) + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\csc\left(a+b\log\left(cx^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(sqrt(csc(a + b*log(c*x**n))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(csc(b*log(c*x^n) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{1}{\sin\left(a+b\,\ln\left(c\,x^{n}\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(a + b*log(c*x^n)))^(1/2),x)
```

```
[Out] int((1/sin(a + b*log(c*x^n)))^(1/2), x)
```

3.309 $\int \frac{\sqrt{\csc(a+b\log(cx^n))}}{x}\,dx$

Optimal. Leaf size=59

$$\frac{2\sqrt{\csc(a+b\log(cx^n))}\,F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b\log(cx^n)\right)\middle|2\right)\sqrt{\sin(a+b\log(cx^n))}}{bn}$$

```
[Out] -2*(sin(1/2*a+1/4*Pi+1/2*b*ln(c*x^n))^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*ln(c*
x^n))*EllipticF(cos(1/2*a+1/4*Pi+1/2*b*ln(c*x^n)),2^(1/2))*csc(a+b*ln(c*x^n
))^(1/2)*sin(a+b*ln(c*x^n))^(1/2)/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3856, 2720}

$$\frac{2\sqrt{\sin(a+b\log(cx^n))}\sqrt{\csc(a+b\log(cx^n))}\,F\left(\frac{1}{2}\left(a+b\log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csc[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] (2*Sqrt[Csc[a + b*Log[c*x^n]]]*EllipticF[(a - Pi/2 + b*Log[c*x^n])/2, 2]*Sq
rt[Sin[a + b*Log[c*x^n]]])/(b*n)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\csc(a+b\log(cx^n))}}{x}\,dx &= \frac{\text{Subst}\left(\int \sqrt{\csc(a+bx)}\,dx, x, \log(cx^n)\right)}{n}\\ &= \frac{\left(\sqrt{\csc(a+b\log(cx^n))}\sqrt{\sin(a+b\log(cx^n))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx\right.}{n}\\ &= \frac{2\sqrt{\csc(a+b\log(cx^n))}\,F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b\log(cx^n)\right)\middle|2\right)\sqrt{\sin(a+b\log(cx^n))}}{bn}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 58, normalized size = 0.98

$$-\frac{2\sqrt{\csc(a+b\log(cx^n))}\,F\left(\frac{1}{4}(-2a+\pi-2b\log(cx^n))\middle|2\right)\sqrt{\sin(a+b\log(cx^n))}}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csc[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] (-2*Sqrt[Csc[a + b*Log[c*x^n]]]*EllipticF[(-2*a + Pi - 2*b*Log[c*x^n])/4, 2
]*Sqrt[Sin[a + b*Log[c*x^n]]])/(b*n)
```

Maple [A]
time = 0.28, size = 102, normalized size = 1.73

method	result
derivativedivides	$\frac{\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+2}\,\sqrt{-\sin(a+b\ln(c\,x^n))}\ \mathrm{Ellipti}}{n\cos(a+b\ln(c\,x^n))\sqrt{\sin(a+b\ln(c\,x^n))}\,b}$
default	$\frac{\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+2}\,\sqrt{-\sin(a+b\ln(c\,x^n))}\ \mathrm{Ellipti}}{n\cos(a+b\ln(c\,x^n))\sqrt{\sin(a+b\ln(c\,x^n))}\,b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(sin(a+b*ln(c*x^n))+1)^(1/2)*(-2*sin(a+b*ln(c*x^n))+2)^(1/2)*(-sin(a+b*
ln(c*x^n)))^(1/2)*EllipticF((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2))/cos(a
+b*ln(c*x^n))/sin(a+b*ln(c*x^n))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(csc(b*log(c*x^n) + a))/x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.49, size = 78, normalized size = 1.32

$$\frac{-i\sqrt{2i}\,\mathrm{weierstrassPInverse}(4,0,\cos(bn\log(x)+b\log(c)+a)+i\sin(bn\log(x)+b\log(c)+a))+i\sqrt{-2i}\,\mathrm{weierstrassPInverse}(4,0,\cos(bn\log(x)+b\log(c)+a)-i\sin(bn\log(x)+b\log(c)+a))}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2*I)*weierstrassPInverse(4, 0, cos(b*n*log(x) + b*log(c) + a) + I*
sin(b*n*log(x) + b*log(c) + a)) + I*sqrt(-2*I)*weierstrassPInverse(4, 0, co
s(b*n*log(x) + b*log(c) + a) - I*sin(b*n*log(x) + b*log(c) + a)))/(b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\csc\left(a + b\log\left(cx^{n}\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*ln(c*x**n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(csc(a + b*log(c*x**n)))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(csc(b*log(c*x^n) + a))/x, x)
```

Mupad [B]
time = 2.62, size = 89, normalized size = 1.51

$$-\frac{2\sqrt{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}\,\mathrm{F}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}}{2}\right)\middle|2\right)\,\sqrt{{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}^2}\,\sqrt{\frac{1}{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}}}{b\,n\,\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(a + b*log(c*x^n)))^(1/2)/x,x)
```

```
[Out] -(2*sin(a + b*log(c*x^n))^(1/2)*ellipticF(asin((2^(1/2)*(1 - sin(a + b*log(
c*x^n)))^(1/2))/2), 2)*(cos(a + b*log(c*x^n))^2)^(1/2)*(1/sin(a + b*log(c*x
^n)))^(1/2))/(b*n*cos(a + b*log(c*x^n)))
```

3.310 $\int \csc^{\frac{3}{2}}(a+b\log(cx^n))\,dx$

Optimal. Leaf size=109

$$\frac{2x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}\csc^{\frac{3}{2}}(a+b\log(cx^n))\ {}_2F_1\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i}{bn}\right);\frac{1}{4}\left(7-\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{2+3ibn}$$

```
[Out] 2*x*(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(3/2)*csc(a+b*ln(c*x^n))^(3/2)*hypergeom
([3/2, 3/4-1/2*I/b/n],[7/4-1/2*I/b/n],exp(2*I*a)*(c*x^n)^(2*I*b))/(2+3*I*b*
n)
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4600, 4604, 371}

$$\frac{2x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}\ {}_2F_1\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i}{bn}\right);\frac{1}{4}\left(7-\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)\csc^{\frac{3}{2}}(a+b\log(cx^n))}{2+3ibn}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*Log[c*x^n]]^(3/2),x]
```

```
[Out] (2*x*(1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2)*Csc[a + b*Log[c*x^n]]^(3/2)*
Hypergeometric2F1[3/2, (3 - (2*I)/(b*n))/4, (7 - (2*I)/(b*n))/4, E^((2*I)*a
)*(c*x^n)^((2*I)*b)])/(2 + (3*I)*b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4600

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4604

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \csc^{\frac{3}{2}}(a+b\log(cx^n))\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\csc^{\frac{3}{2}}(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(x(cx^n)^{-\frac{3ib}{2}-\frac{1}{n}}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}\csc^{\frac{3}{2}}(a+b\log(cx^n))\right)\text{Subst}\left(\int \frac{x^{-1+\frac{3ib}{2}+}}{(1-e^{2ia}x^{2ib}}\right.}{n}$$
$$= \frac{2x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}\csc^{\frac{3}{2}}(a+b\log(cx^n))\,{}_2F_1\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i}{bn}\right);\frac{1}{4}\left(7-\frac{2i}{bn}\right);e^{2i\alpha}\right.}{2+3ibn}$$

Mathematica [**A**]
time = 7.43, size = 127, normalized size = 1.17

$$\frac{2e^{-ia}x(cx^n)^{-ib}\sqrt{\csc(a+b\log(cx^n))}\left(-1+\sqrt{1-e^{2ia}(cx^n)^{2ib}}\,{}_2F_1\left(\frac{1}{2},-\frac{2i+bn}{4bn};\frac{3}{4}-\frac{i}{2bn};e^{2ia}(cx^n)^{2ib}\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*Log[c*x^n]]^(3/2),x]
```

```
[Out] (2*x*Sqrt[Csc[a + b*Log[c*x^n]]]*(-1 + Sqrt[1 - E^((2*I)*a)*(c*x^n)^((2*I)*
b)]*Hypergeometric2F1[1/2, -1/4*(2*I + b*n)/(b*n), 3/4 - (I/2)/(b*n), E^((2
*I)*a)*(c*x^n)^((2*I)*b)]))/(b*E^(I*a)*n*(c*x^n)^(I*b))
```

Maple [**F**]
time = 0.12, size = 0, normalized size = 0.00

$$\int \csc^{\frac{3}{2}}(a+b\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(a+b*ln(c*x^n))^(3/2),x)
```

```
[Out] int(csc(a+b*ln(c*x^n))^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*log(c*x^n) + a)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^{\frac{3}{2}}\left(a + b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Integral(csc(a + b*log(c*x**n))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(a + b*log(c*x^n)))^(3/2),x)
```

```
[Out] int((1/sin(a + b*log(c*x^n)))^(3/2), x)
```

3.311 $\int \frac{\csc^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=94

$$-\frac{2\cos\left(a+b\log\left(cx^n\right)\right)\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}}{bn}-\frac{2\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b\log\left(cx^n\right)\right)\middle|2\right)\sqrt{\sin}}{bn}$$

```
[Out] -2*cos(a+b*ln(c*x^n))*csc(a+b*ln(c*x^n))^(1/2)/b/n+2*(sin(1/2*a+1/4*Pi+1/2*
b*ln(c*x^n))^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*ln(c*x^n))*EllipticE(cos(1/2*a
+1/4*Pi+1/2*b*ln(c*x^n)),2^(1/2))*csc(a+b*ln(c*x^n))^(1/2)*sin(a+b*ln(c*x^n
))^(1/2)/b/n
```

Rubi [A]
time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3853, 3856, 2719}

$$-\frac{2\cos\left(a+b\log\left(cx^n\right)\right)\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}}{bn}-\frac{2\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{2}\left(a+b\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (-2*Cos[a + b*Log[c*x^n]]*Sqrt[Csc[a + b*Log[c*x^n]]])/(b*n) - (2*Sqrt[Csc[
a + b*Log[c*x^n]]]*EllipticE[(a - Pi/2 + b*Log[c*x^n])/2, 2]*Sqrt[Sin[a + b
*Log[c*x^n]]])/(b*n)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\csc^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\text{Subst}\left(\int \csc^{\frac{3}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2\cos(a+b\log(cx^n))\sqrt{\csc(a+b\log(cx^n))}}{bn} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\csc(a+bx)}}\,dx\right.}{n}$$

$$= -\frac{2\cos(a+b\log(cx^n))\sqrt{\csc(a+b\log(cx^n))}}{bn} - \frac{\left(\sqrt{\csc(a+b\log(cx^n))}\,\sqrt{\right.}{}$$

$$= -\frac{2\cos(a+b\log(cx^n))\sqrt{\csc(a+b\log(cx^n))}}{bn} - \frac{2\sqrt{\csc(a+b\log(cx^n))}\,E(}{}$$

Mathematica [A]
time = 0.17, size = 72, normalized size = 0.77

$$-\frac{2\sqrt{\csc(a+b\log(cx^n))}\left(\cos(a+b\log(cx^n)) - E\left(\frac{1}{4}(-2a+\pi-2b\log(cx^n))\middle|2\right)\sqrt{\sin(a+b\log(cx^n))}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (-2*Sqrt[Csc[a + b*Log[c*x^n]]]*(Cos[a + b*Log[c*x^n]] - EllipticE[(-2*a +
Pi - 2*b*Log[c*x^n])/4, 2]*Sqrt[Sin[a + b*Log[c*x^n]]]))/(b*n)
```

Maple [A]
time = 0.35, size = 190, normalized size = 2.02

method	result
derivativedivides	$\frac{2\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+2}\,\sqrt{-\sin(a+b\ln(c\,x^n))}\,\text{Ellipt}}{}$
default	$\frac{2\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+2}\,\sqrt{-\sin(a+b\ln(c\,x^n))}\,\text{Ellipt}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(2*(sin(a+b*ln(c*x^n))+1)^(1/2)*(-2*sin(a+b*ln(c*x^n))+2)^(1/2)*(-sin(a
+b*ln(c*x^n)))^(1/2)*EllipticE((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2))-(s
in(a+b*ln(c*x^n))+1)^(1/2)*(-2*sin(a+b*ln(c*x^n))+2)^(1/2)*(-sin(a+b*ln(c*x
```

```
^n)))^(1/2)*EllipticF((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2))-2*cos(a+b*l
n(c*x^n))^2)/cos(a+b*ln(c*x^n))/sin(a+b*ln(c*x^n))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(csc(b*log(c*x^n) + a)^(3/2)/x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.22, size = 111, normalized size = 1.18

$$-\frac{\sqrt{2i}\ \mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+i\ \sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)))+\sqrt{-2i}\ \mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-i\ \sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)))+\frac{2\cos\left(bn\log(x)+b\log(c)+a\right)}{\sqrt{\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -(sqrt(2*I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*n*log(x)
+ b*log(c) + a) + I*sin(b*n*log(x) + b*log(c) + a))) + sqrt(-2*I)*weierstra
ssZeta(4, 0, weierstrassPInverse(4, 0, cos(b*n*log(x) + b*log(c) + a) - I*s
in(b*n*log(x) + b*log(c) + a))) + 2*cos(b*n*log(x) + b*log(c) + a)/sqrt(sin
(b*n*log(x) + b*log(c) + a)))/(b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*ln(c*x**n))**(3/2)/x,x)
```

```
[Out] Integral(csc(a + b*log(c*x**n))**(3/2)/x, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\sin(a+b\,\ln(c\,x^n))}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(a + b*log(c*x^n)))^(3/2)/x,x)
[Out] int((1/sin(a + b*log(c*x^n)))^(3/2)/x, x)
```

3.312 $\int \csc^{\frac{5}{2}}(a+b\log(cx^n))\,dx$

Optimal. Leaf size=109

$$\frac{2x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{5/2}\csc^{\frac{5}{2}}(a+b\log(cx^n))\,{}_2F_1\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i}{bn}\right);\frac{1}{4}\left(9-\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{2+5ibn}$$

```
[Out] 2*x*(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(5/2)*csc(a+b*ln(c*x^n))^(5/2)*hypergeom
([5/2, 5/4-1/2*I/b/n],[9/4-1/2*I/b/n],exp(2*I*a)*(c*x^n)^(2*I*b))/(2+5*I*b*
n)
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4600, 4604, 371}

$$\frac{2x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{5/2}{}_2F_1\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i}{bn}\right);\frac{1}{4}\left(9-\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)\csc^{\frac{5}{2}}(a+b\log(cx^n))}{2+5ibn}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*Log[c*x^n]]^(5/2),x]
```

```
[Out] (2*x*(1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^(5/2)*Csc[a + b*Log[c*x^n]]^(5/2)*
Hypergeometric2F1[5/2, (5 - (2*I)/(b*n))/4, (9 - (2*I)/(b*n))/4, E^((2*I)*a
)*(c*x^n)^((2*I)*b)])/(2 + (5*I)*b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4600

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4604

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \csc^{\frac{5}{2}}(a+b\log(cx^n))\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\csc^{\frac{5}{2}}(a+b\log(x))\,dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-\frac{5ib}{2}-\frac{1}{n}}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{5/2}\csc^{\frac{5}{2}}(a+b\log(cx^n))\right)\text{Subst}\left(\int \frac{x^{-1+\frac{5ib}{2}}}{(1-e^{2ia}x^{2}}\right.}{n}$$

$$= \frac{2x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{5/2}\csc^{\frac{5}{2}}(a+b\log(cx^n))\,{}_2F_1\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i}{bn}\right);\frac{1}{4}\left(9-\frac{2i}{bn}\right);e^{2}\right.}{2+5ibn}$$

Mathematica [A]
time = 1.67, size = 174, normalized size = 1.60

$$\frac{2e^{-2i(a-bn\log(x)+b\log(cx^n))}x^{1-2ibn}\sqrt{\csc(a+b\log(cx^n))}\left(-e^{2ia}(cx^n)^{2ib}(2+bn\cot(a+b\log(cx^n)))+(2+ibn)\left(-1+e^{2ia}(cx^n)^{2ib}\right){}_2F_1\left(1,\frac{3}{4}+\frac{i}{2bn};\frac{5}{4}+\frac{i}{2bn};e^{-2i(a+b\log(cx^n))}\right)\right)}{3b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*Log[c*x^n]]^(5/2),x]
```

```
[Out] (2*x^(1 - (2*I)*b*n)*Sqrt[Csc[a + b*Log[c*x^n]]]*(-(E^((2*I)*a)*(c*x^n)^((2
*I)*b)*(2 + b*n*Cot[a + b*Log[c*x^n]])) + (2 + I*b*n)*(-1 + E^((2*I)*a)*(c*
x^n)^((2*I)*b))*Hypergeometric2F1[1, 3/4 + (I/2)/(b*n), 5/4 + (I/2)/(b*n),
E^((-2*I)*(a + b*Log[c*x^n]))]))/(3*b^2*E^((2*I)*(a - b*n*Log[x] + b*Log[c*
x^n]))*n^2)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \csc^{\frac{5}{2}}(a+b\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(a+b*ln(c*x^n))^(5/2),x)
```

```
[Out] int(csc(a+b*ln(c*x^n))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*log(c*x^n) + a)^(5/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(a + b*log(c*x^n)))^(5/2),x)
```

```
[Out] int((1/sin(a + b*log(c*x^n)))^(5/2), x)
```

3.313 $\int \frac{\csc^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=98

$$-\frac{2\cos\left(a+b\log\left(cx^n\right)\right)\csc^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{2\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}\,F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b\log\left(cx^n\right)\right)\middle|2\right)\sqrt{\sin(}}{3bn}$$

```
[Out] -2/3*cos(a+b*ln(c*x^n))*csc(a+b*ln(c*x^n))^(3/2)/b/n-2/3*(sin(1/2*a+1/4*Pi+
1/2*b*ln(c*x^n))^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*ln(c*x^n))*EllipticF(cos(1
/2*a+1/4*Pi+1/2*b*ln(c*x^n)),2^(1/2))*csc(a+b*ln(c*x^n))^(1/2)*sin(a+b*ln(c
*x^n))^(1/2)/b/n
```

Rubi [A]
time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3853, 3856, 2720}

$$\frac{2\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}\,F\left(\frac{1}{2}\left(a+b\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{3bn}-\frac{2\cos\left(a+b\log\left(cx^n\right)\right)\csc^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (-2*Cos[a + b*Log[c*x^n]]*Csc[a + b*Log[c*x^n]]^(3/2))/(3*b*n) + (2*Sqrt[Cs
c[a + b*Log[c*x^n]]]*EllipticF[(a - Pi/2 + b*Log[c*x^n])/2, 2]*Sqrt[Sin[a +
 b*Log[c*x^n]]])/(3*b*n)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\csc^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\text{Subst}\left(\int \csc^{\frac{5}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2\cos(a+b\log(cx^n))\csc^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int \sqrt{\csc(a+bx)}\,dx, x,\right.}{3n}$$

$$= -\frac{2\cos(a+b\log(cx^n))\csc^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} + \frac{\left(\sqrt{\csc(a+b\log(cx^n))}\sqrt{\sin}\right.}{}$$

$$= -\frac{2\cos(a+b\log(cx^n))\csc^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} + \frac{2\sqrt{\csc(a+b\log(cx^n))}\,F\left(\frac{1}{2}(\right.}{}$$

Mathematica [A]
time = 0.22, size = 73, normalized size = 0.74

$$-\frac{2\csc^{\frac{3}{2}}(a+b\log(cx^n))\left(\cos(a+b\log(cx^n))+F\left(\frac{1}{4}(-2a+\pi-2b\log(cx^n))\middle|2\right)\sin^{\frac{3}{2}}(a+b\log(cx^n))\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (-2*Csc[a + b*Log[c*x^n]]^(3/2)*(Cos[a + b*Log[c*x^n]] + EllipticF[(-2*a + 
Pi - 2*b*Log[c*x^n])/4, 2]*Sin[a + b*Log[c*x^n]]^(3/2)))/(3*b*n)
```

Maple [A]
time = 0.30, size = 131, normalized size = 1.34

method	result
derivativedivides	$\frac{\sqrt{\sin(a+b\ln(cx^n))+1}\,\sqrt{-2\sin(a+b\ln(cx^n))+2}\,\sqrt{-\sin(a+b\ln(cx^n))}\,\text{EllipticF}}{3n\sin(a+b\ln(cx^n))^{\frac{3}{2}}\cos(a+b\ln(cx}$
default	$\frac{\sqrt{\sin(a+b\ln(cx^n))+1}\,\sqrt{-2\sin(a+b\ln(cx^n))+2}\,\sqrt{-\sin(a+b\ln(cx^n))}\,\text{EllipticF}}{3n\sin(a+b\ln(cx^n))^{\frac{3}{2}}\cos(a+b\ln(cx}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(a+b*ln(c*x^n))^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/n/sin(a+b*ln(c*x^n))^(3/2)*((sin(a+b*ln(c*x^n))+1)^(1/2)*(-2*sin(a+b*ln
(c*x^n))+2)^(1/2)*(-sin(a+b*ln(c*x^n)))^(1/2)*EllipticF((sin(a+b*ln(c*x^n))
+1)^(1/2),1/2*2^(1/2))*sin(a+b*ln(c*x^n))-2*cos(a+b*ln(c*x^n))^2)/cos(a+b*l
n(c*x^n))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(csc(b*log(c*x^n) + a)^(5/2)/x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.52, size = 145, normalized size = 1.48

$$\frac{-i\sqrt{2i}\,\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\mathrm{weierstrassPInverse}\left(4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+i\,\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)+i\sqrt{-2i}\,\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\mathrm{weierstrassPInverse}\left(4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-i\,\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)-\frac{2\cos\left(bn\log(x)+b\log(c)+a\right)}{\sqrt{\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}}}{3\,bn\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2*I)*sin(b*n*log(x) + b*log(c) + a)*weierstrassPInverse(4, 0,
cos(b*n*log(x) + b*log(c) + a) + I*sin(b*n*log(x) + b*log(c) + a)) + I*sqrt
(-2*I)*sin(b*n*log(x) + b*log(c) + a)*weierstrassPInverse(4, 0, cos(b*n*log
(x) + b*log(c) + a) - I*sin(b*n*log(x) + b*log(c) + a)) - 2*cos(b*n*log(x)
+ b*log(c) + a)/sqrt(sin(b*n*log(x) + b*log(c) + a)))/(b*n*sin(b*n*log(x) +
 b*log(c) + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*ln(c*x**n))**(5/2)/x,x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\sin(a+b\,\ln(c\,x^n))}\right)^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(a + b*log(c*x^n)))^(5/2)/x,x)
[Out] int((1/sin(a + b*log(c*x^n)))^(5/2)/x, x)
```

3.314 $\int \frac{1}{\sqrt{\csc(a+b\log(cx^n))}}\,dx$

Optimal. Leaf size=110

$$\frac{2x\,{}_2F_1\left(-\frac{1}{2},-\frac{2i+bn}{4bn};\frac{1}{4}\left(3-\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{(2-ibn)\sqrt{1-e^{2ia}\left(cx^n\right)^{2ib}}\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}}$$

[Out] 2*x*hypergeom([-1/2, 1/4*(-2*I-b*n)/b/n],[3/4-1/2*I/b/n],exp(2*I*a)*(c*x^n)^(2*I*b))/(2-I*b*n)/(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(1/2)/csc(a+b*ln(c*x^n))^(1/2)

Rubi [A]
time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4600, 4604, 371}

$$\frac{2x\,{}_2F_1\left(-\frac{1}{2},-\frac{bn+2i}{4bn};\frac{1}{4}\left(3-\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{(2-ibn)\sqrt{1-e^{2ia}\left(cx^n\right)^{2ib}}\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[Csc[a + b*Log[c*x^n]]],x]

[Out] (2*x*Hypergeometric2F1[-1/2, -1/4*(2*I + b*n)/(b*n), (3 - (2*I)/(b*n))/4, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((2 - I*b*n)*Sqrt[1 - E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[Csc[a + b*Log[c*x^n]]])

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4600

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4604

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
```

```
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}}\,dx = \frac{\left(x\left(cx^n\right)^{-1/n}\right)\mathrm{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\sqrt{\csc(a+b\log(x))}}\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x\left(cx^n\right)^{\frac{ib}{2}-\frac{1}{n}}\right)\mathrm{Subst}\left(\int x^{-1-\frac{ib}{2}+\frac{1}{n}}\sqrt{1-e^{2ia}x^{2ib}}\,dx,x,cx^n\right)}{n\sqrt{1-e^{2ia}\left(cx^n\right)^{2ib}}\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}}$$

$$= \frac{2x\,{}_2F_1\left(-\frac{1}{2},-\frac{2i+bn}{4bn};\frac{1}{4}\left(3-\frac{2i}{bn}\right);e^{2ia}\left(cx^n\right)^{2ib}\right)}{(2-ibn)\sqrt{1-e^{2ia}\left(cx^n\right)^{2ib}}\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 367 vs. 2(110) = 220.
time = 5.35, size = 367, normalized size = 3.34

$$2x\left(-\frac{i\sqrt{2}\,be^{-ia}n\left(cx^n\right)^{-ib}\sqrt{\frac{ie^{ia}\left(cx^n\right)^{ib}}{-1+e^{2ia}\left(cx^n\right)^{2ib}}}\left((2i+bn)\left(-1+e^{2ia}\left(cx^n\right)^{2ib}\right)+\sqrt{1-e^{2ia}\left(cx^n\right)^{2ib}}\left(2i+bn+e^{2ia}(-2i+bn)x^{-2ibn}\left(cx^n\right)^{2ib}\right){}_2F_1\left(\frac{1}{2},-\frac{2i+bn}{4bn};\frac{3}{4}-\frac{i}{2bn};e^{2ia}\left(cx^n\right)^{2ib}\right)\right)}{\left(4+b^2n^2\right)\left(-2+ibn+e^{2ia}(2+ibn)x^{-2ibn}\left(cx^n\right)^{2ib}\right)}+\frac{\sin\left(a-bn\log(x)+b\log\left(cx^n\right)\right)}{\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}\left(bn\cos\left(a-bn\log(x)+b\log\left(cx^n\right)\right)+2\sin\left(a-bn\log(x)+b\log\left(cx^n\right)\right)\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Csc[a + b*Log[c*x^n]]],x]
```

```
[Out] 2*x*(((-I)*Sqrt[2]*b*n*Sqrt[(I*E^(I*a)*(c*x^n)^(I*b))/(-1 + E^((2*I)*a)*(c*
x^n)^((2*I)*b))]*((2*I + b*n)*(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)) + Sqrt[1
 - E^((2*I)*a)*(c*x^n)^((2*I)*b)]*(2*I + b*n + (E^((2*I)*a)*(-2*I + b*n)*(c
*x^n)^((2*I)*b))/x^((2*I)*b*n))*Hypergeometric2F1[1/2, -1/4*(2*I + b*n)/(b*
n), 3/4 - (I/2)/(b*n), E^((2*I)*a)*(c*x^n)^((2*I)*b)]))/(E^(I*a)*(4 + b^2*n
^2)*(c*x^n)^(I*b)*(-2 + I*b*n + (E^((2*I)*a)*(2 + I*b*n)*(c*x^n)^((2*I)*b))
/x^((2*I)*b*n))) + Sin[a - b*n*Log[x] + b*Log[c*x^n]]/(Sqrt[Csc[a + b*Log[c
*x^n]]]*(b*n*Cos[a - b*n*Log[x] + b*Log[c*x^n]] + 2*Sin[a - b*n*Log[x] + b*
Log[c*x^n]])))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\csc\left(a+b\ln\left(c\,x^{n}\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/csc(a+b*ln(c*x^n))^(1/2),x)
```

```
[Out] int(1/csc(a+b*ln(c*x^n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csc(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(csc(b*log(c*x^n) + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csc(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\csc \left(a + b \log \left(c x^{n}\right)\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csc(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(1/sqrt(csc(a + b*log(c*x**n))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csc(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(csc(b*log(c*x^n) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{1}{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/sin(a + b*log(c*x^n)))^(1/2),x)
[Out] int(1/(1/sin(a + b*log(c*x^n)))^(1/2), x)
```

3.315 $\int \frac{1}{x\sqrt{\csc(a+b\log(cx^n))}}\,dx$

Optimal. Leaf size=59

$$\frac{2\sqrt{\csc(a+b\log(cx^n))}\,E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b\log(cx^n)\right)\middle|2\right)\sqrt{\sin(a+b\log(cx^n))}}{bn}$$

```
[Out] -2*(sin(1/2*a+1/4*Pi+1/2*b*ln(c*x^n))^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*ln(c*
x^n))*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*ln(c*x^n)),2^(1/2))*csc(a+b*ln(c*x^n
))^(1/2)*sin(a+b*ln(c*x^n))^(1/2)/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3856, 2719}

$$\frac{2\sqrt{\sin(a+b\log(cx^n))}\sqrt{\csc(a+b\log(cx^n))}\,E\left(\frac{1}{2}\left(a+b\log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Csc[a + b*Log[c*x^n]]]),x]
```

```
[Out] (2*Sqrt[Csc[a + b*Log[c*x^n]]]*EllipticE[(a - Pi/2 + b*Log[c*x^n])/2, 2]*Sq
rt[Sin[a + b*Log[c*x^n]]])/(b*n)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{\csc(a+b\log(cx^n))}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\csc(a+bx)}}\,dx, x, \log(cx^n)\right)}{n}\\
&= \frac{\left(\sqrt{\csc(a+b\log(cx^n))}\sqrt{\sin(a+b\log(cx^n))}\right)\text{Subst}\left(\int \sqrt{\sin(a+bx)}\right.}{n}\\
&= \frac{2\sqrt{\csc(a+b\log(cx^n))}\,E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b\log(cx^n)\right)\middle|2\right)\sqrt{\sin(a+b\log(cx^n}}{bn}
\end{aligned}$$

Mathematica [**A**]
time = 0.12, size = 58, normalized size = 0.98

$$-\frac{2\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{4}\left(-2a+\pi-2b\log\left(cx^n\right)\right)\middle|2\right)\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[Csc[a + b*Log[c*x^n]]]),x]
```

```
[Out] (-2*Sqrt[Csc[a + b*Log[c*x^n]]]*EllipticE[(-2*a + Pi - 2*b*Log[c*x^n])/4, 2
]*Sqrt[Sin[a + b*Log[c*x^n]]])/(b*n)
```

Maple [**A**]
time = 0.28, size = 129, normalized size = 2.19

method	result
derivativedivides	$-\frac{\sqrt{\sin\left(a+b\ln\left(c\,x^{n}\right)\right)+1}\,\sqrt{-2\sin\left(a+b\ln\left(c\,x^{n}\right)\right)+2}\,\sqrt{-\sin\left(a+b\ln\left(c\,x^{n}\right)\right)}\,\left(2\,\mathrm{Ell}\right.}{n\cos\left(a+b\ln\left(c\,x^{n}\right)\right)\sqrt{\sin\left(\right.}}$
default	$-\frac{\sqrt{\sin\left(a+b\ln\left(c\,x^{n}\right)\right)+1}\,\sqrt{-2\sin\left(a+b\ln\left(c\,x^{n}\right)\right)+2}\,\sqrt{-\sin\left(a+b\ln\left(c\,x^{n}\right)\right)}\,\left(2\,\mathrm{Ell}\right.}{n\cos\left(a+b\ln\left(c\,x^{n}\right)\right)\sqrt{\sin\left(\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/csc(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/n*(sin(a+b*ln(c*x^n))+1)^(1/2)*(-2*sin(a+b*ln(c*x^n))+2)^(1/2)*(-sin(a+b
*ln(c*x^n)))^(1/2)*(2*EllipticE((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2))-E
llipticF((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2)))/cos(a+b*ln(c*x^n))/sin(
a+b*ln(c*x^n))^(1/2)/b
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csc(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt(csc(b*log(c*x^n) + a))), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.52, size = 82, normalized size = 1.39

$$\frac{\sqrt{2i}\ \mathrm{weierstrassZeta}\left(4,0,\mathrm{weierstrassPInverse}\left(4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+i\ \sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\right)+\sqrt{-2i}\ \mathrm{weierstrassZeta}\left(4,0,\mathrm{weierstrassPInverse}\left(4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-i\ \sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csc(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2*I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*n*log(x) +
 b*log(c) + a) + I*sin(b*n*log(x) + b*log(c) + a))) + sqrt(-2*I)*weierstras
sZeta(4, 0, weierstrassPInverse(4, 0, cos(b*n*log(x) + b*log(c) + a) - I*si
n(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\csc\left(a+b\log\left(cx^{n}\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csc(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(csc(a + b*log(c*x**n)))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csc(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sqrt(csc(b*log(c*x^n) + a))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\sqrt{\frac{1}{\sin\left(a+b\,\ln\left(c\,x^{n}\right)\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1/sin(a + b*log(c*x^n)))^(1/2)),x)
```

```
[Out] int(1/(x*(1/sin(a + b*log(c*x^n)))^(1/2)), x)
```

3.316 $\int \frac{1}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=109

$$\frac{2x\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{2i}{bn}\right);\frac{1}{4}\left(1-\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{(2-3ibn)\left(1-e^{2ia}\left(cx^n\right)^{2ib}\right)^{3/2}\csc^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

```
[Out] 2*x*hypergeom([-3/2, -3/4-1/2*I/b/n],[1/4-1/2*I/b/n],exp(2*I*a)*(c*x^n)^(2*
I*b))/(2-3*I*b*n)/(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(3/2)/csc(a+b*ln(c*x^n))^(
3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4600, 4604, 371}

$$\frac{2x\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{2i}{bn}\right);\frac{1}{4}\left(1-\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{(2-3ibn)\left(1-e^{2ia}\left(cx^n\right)^{2ib}\right)^{3/2}\csc^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*Log[c*x^n]]^(-3/2),x]
```

```
[Out] (2*x*Hypergeometric2F1[-3/2, (-3 - (2*I)/(b*n))/4, (1 - (2*I)/(b*n))/4, E^(
(2*I)*a)*(c*x^n)^((2*I)*b)])/((2 - (3*I)*b*n)*(1 - E^((2*I)*a)*(c*x^n)^((2*
I)*b))^(3/2)*Csc[a + b*Log[c*x^n]]^(3/2))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4600

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4604

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
```

```
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\csc^{\frac{3}{2}}(a+b\log(x))}\,dx, x, cx^n\right)}{n}$$
$$= \frac{\left(x(cx^n)^{\frac{3ib}{2}-\frac{1}{n}}\right)\text{Subst}\left(\int x^{-1-\frac{3ib}{2}+\frac{1}{n}}\left(1-e^{2ia}x^{2ib}\right)^{3/2}\,dx, x, cx^n\right)}{n\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}\csc^{\frac{3}{2}}(a+b\log(cx^n))}$$
$$= \frac{2x\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{2i}{bn}\right);\frac{1}{4}\left(1-\frac{2i}{bn}\right);e^{2ia}(cx^n)^{2ib}\right)}{(2-3ibn)\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}\csc^{\frac{3}{2}}(a+b\log(cx^n))}$$

Mathematica [A]
time = 2.16, size = 186, normalized size = 1.71

$$\frac{2ix\left((2-ibn)(-2+3bn\cot(a+b\log(cx^n)))-3b^2e^{-2ia}n^2(cx^n)^{-2ib}\left(-1+e^{2ia}(cx^n)^{2ib}\right)\csc^2(a+b\log(cx^n))\,{}_2F_1\left(1,\frac{3}{4}+\frac{i}{2bn};\frac{5}{4}+\frac{i}{2bn};e^{-2i(a+b\log(cx^n))}\right)\right)}{(2i-3bn)(2i+bn)(2i+3bn)\csc^{\frac{3}{2}}(a+b\log(cx^n))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[a + b*Log[c*x^n]]^(-3/2),x]
```

```
[Out] ((2*I)*x*((2 - I*b*n)*(-2 + 3*b*n*Cot[a + b*Log[c*x^n]]) - (3*b^2*n^2*(-1 +
 E^((2*I)*a)*(c*x^n)^((2*I)*b))*Csc[a + b*Log[c*x^n]]^2*Hypergeometric2F1[1
, 3/4 + (I/2)/(b*n), 5/4 + (I/2)/(b*n), E^((-2*I)*(a + b*Log[c*x^n]))])/(E^
((2*I)*a)*(c*x^n)^((2*I)*b))))/((2*I - 3*b*n)*(2*I + b*n)*(2*I + 3*b*n)*Csc
[a + b*Log[c*x^n]]^(3/2))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\csc\left(a+b\ln\left(c\,x^{n}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/csc(a+b*ln(c*x^n))^(3/2),x)
```

```
[Out] int(1/csc(a+b*ln(c*x^n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csc(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
[Out] integrate(csc(b*log(c*x^n) + a)^(-3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csc(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\csc^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csc(a+b*ln(c*x**n))**(3/2),x)
[Out] Integral(csc(a + b*log(c*x**n))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csc(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
[Out] integrate(csc(b*log(c*x^n) + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{1}{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/sin(a + b*log(c*x^n)))^(3/2),x)
[Out] int(1/(1/sin(a + b*log(c*x^n)))^(3/2), x)
```

3.317 $\int \frac{1}{x \csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=98

$$-\frac{2\cos(a+b\log(cx^n))}{3bn\sqrt{\csc(a+b\log(cx^n))}}+\frac{2\sqrt{\csc(a+b\log(cx^n))}\,F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b\log(cx^n)\right)\middle|2\right)\sqrt{\sin(a+b\log(cx^n))}}{3bn}$$

```
[Out] -2/3*cos(a+b*ln(c*x^n))/b/n/csc(a+b*ln(c*x^n))^(1/2)-2/3*(sin(1/2*a+1/4*Pi+
1/2*b*ln(c*x^n))^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*ln(c*x^n))*EllipticF(cos(1
/2*a+1/4*Pi+1/2*b*ln(c*x^n)),2^(1/2))*csc(a+b*ln(c*x^n))^(1/2)*sin(a+b*ln(c
*x^n))^(1/2)/b/n
```

Rubi [A]
time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3854, 3856, 2720}

$$\frac{2\sqrt{\sin(a+b\log(cx^n))}\sqrt{\csc(a+b\log(cx^n))}\,F\left(\frac{1}{2}\left(a+b\log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{3bn}-\frac{2\cos(a+b\log(cx^n))}{3bn\sqrt{\csc(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Csc[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (-2*Cos[a + b*Log[c*x^n]])/(3*b*n*Sqrt[Csc[a + b*Log[c*x^n]]]) + (2*Sqrt[Cs
c[a + b*Log[c*x^n]]]*EllipticF[(a - Pi/2 + b*Log[c*x^n])/2, 2]*Sqrt[Sin[a +
 b*Log[c*x^n]]])/(3*b*n)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{x \csc^{\frac{3}{2}}(a+b\log(cx^n))}dx = \frac{\text{Subst}\left(\int \frac{1}{\csc^{\frac{3}{2}}(a+bx)}dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2\cos(a+b\log(cx^n))}{3bn\sqrt{\csc(a+b\log(cx^n))}} + \frac{\text{Subst}\left(\int \sqrt{\csc(a+bx)}\,dx, x, \log(cx^n)\right)}{3n}$$

$$= -\frac{2\cos(a+b\log(cx^n))}{3bn\sqrt{\csc(a+b\log(cx^n))}} + \frac{\left(\sqrt{\csc(a+b\log(cx^n))}\sqrt{\sin(a+b\log(cx^n}{3}$$

$$= -\frac{2\cos(a+b\log(cx^n))}{3bn\sqrt{\csc(a+b\log(cx^n))}} + \frac{2\sqrt{\csc(a+b\log(cx^n))}\,F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b\log(c\right.\right.}{3bn}$$

Mathematica [A]
time = 0.20, size = 76, normalized size = 0.78

$$-\frac{\sqrt{\csc(a+b\log(cx^n))}\left(2F\left(\frac{1}{4}(-2a+\pi-2b\log(cx^n))\middle|2\right)\sqrt{\sin(a+b\log(cx^n))}+\sin(2(a+b\log(cx^n)))\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Csc[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] -1/3*(Sqrt[Csc[a + b*Log[c*x^n]]]*(2*EllipticF[(-2*a + Pi - 2*b*Log[c*x^n])
/4, 2]*Sqrt[Sin[a + b*Log[c*x^n]]] + Sin[2*(a + b*Log[c*x^n])]))/(b*n)
```

Maple [A]
time = 0.37, size = 131, normalized size = 1.34

method	result
derivativedivides	$\frac{\frac{\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+2}\,\sqrt{-\sin(a+b\ln(c\,x^n))}\,\text{EllipticF}\left(\right.}{3}}{n\cos(a+b\ln(c\,x^n))\sqrt{\sin(a+b\ln(c\,x}}$
default	$\frac{\frac{\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+2}\,\sqrt{-\sin(a+b\ln(c\,x^n))}\,\text{EllipticF}\left(\right.}{3}}{n\cos(a+b\ln(c\,x^n))\sqrt{\sin(a+b\ln(c\,x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/csc(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(1/3*(sin(a+b*ln(c*x^n))+1)^(1/2)*(-2*sin(a+b*ln(c*x^n))+2)^(1/2)*(-sin
(a+b*ln(c*x^n)))^(1/2)*EllipticF((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2))-
2/3*cos(a+b*ln(c*x^n))^2*sin(a+b*ln(c*x^n)))/cos(a+b*ln(c*x^n))/sin(a+b*ln(
c*x^n))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csc(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*csc(b*log(c*x^n) + a)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.93, size = 107, normalized size = 1.09

$$-\frac{2\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sqrt{\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}+i\sqrt{2i}\,\mathrm{weierstrassPInverse}(4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+i\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right))-i\sqrt{-2i}\,\mathrm{weierstrassPInverse}(4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-i\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right))}{3\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csc(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*cos(b*n*log(x) + b*log(c) + a)*sqrt(sin(b*n*log(x) + b*log(c) + a))
 + I*sqrt(2*I)*weierstrassPInverse(4, 0, cos(b*n*log(x) + b*log(c) + a) + I
*sin(b*n*log(x) + b*log(c) + a)) - I*sqrt(-2*I)*weierstrassPInverse(4, 0, c
os(b*n*log(x) + b*log(c) + a) - I*sin(b*n*log(x) + b*log(c) + a)))/(b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\csc^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csc(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Integral(1/(x*csc(a + b*log(c*x**n))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csc(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*csc(b*log(c*x^n) + a)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(\frac{1}{\sin(a+b\,\ln(c\,x^n))}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1/sin(a + b*log(c*x^n)))^(3/2)),x)
[Out] int(1/(x*(1/sin(a + b*log(c*x^n)))^(3/2)), x)
```

3.318 $\int \frac{1}{\csc^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=110

$$\frac{2x\,{}_2F_1\left(-\frac{5}{2},\frac{1}{4}\left(-5-\frac{2i}{bn}\right);-\frac{2i+bn}{4bn};e^{2ia}(cx^n)^{2ib}\right)}{(2-5ibn)\left(1-e^{2ia}\left(cx^n\right)^{2ib}\right)^{5/2}\csc^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

```
[Out] 2*x*hypergeom([-5/2, -5/4-1/2*I/b/n],[1/4*(-2*I-b*n)/b/n],exp(2*I*a)*(c*x^n
)^(2*I*b))/(2-5*I*b*n)/(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(5/2)/csc(a+b*ln(c*x^
n))^(5/2)
```

Rubi [A]
time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4600, 4604, 371}

$$\frac{2x\,{}_2F_1\left(-\frac{5}{2},\frac{1}{4}\left(-5-\frac{2i}{bn}\right);-\frac{bn+2i}{4bn};e^{2ia}(cx^n)^{2ib}\right)}{(2-5ibn)\left(1-e^{2ia}\left(cx^n\right)^{2ib}\right)^{5/2}\csc^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*Log[c*x^n]]^(-5/2),x]
```

```
[Out] (2*x*Hypergeometric2F1[-5/2, (-5 - (2*I)/(b*n))/4, -1/4*(2*I + b*n)/(b*n),
E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((2 - (5*I)*b*n)*(1 - E^((2*I)*a)*(c*x^n)^(
(2*I)*b))^(5/2)*Csc[a + b*Log[c*x^n]]^(5/2))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4600

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4604

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
```

```
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\csc^{\frac{5}{2}}(a+b\log(cx^n))}\,dx &= \frac{\left(x(cx^n)^{-1/n}\right)\mathrm{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\csc^{\frac{5}{2}}(a+b\log(x))}\,dx, x, cx^n\right)}{n} \\
&= \frac{\left(x(cx^n)^{\frac{5ib}{2}-\frac{1}{n}}\right)\mathrm{Subst}\left(\int x^{-1-\frac{5ib}{2}+\frac{1}{n}}\left(1-e^{2ia}x^{2ib}\right)^{5/2}\,dx, x, cx^n\right)}{n\left(1-e^{2ia}\left(cx^n\right)^{2ib}\right)^{5/2}\csc^{\frac{5}{2}}(a+b\log(cx^n))} \\
&= \frac{2x\,{}_2F_1\left(-\frac{5}{2},\frac{1}{4}\left(-5-\frac{2i}{bn}\right);-\frac{2i+bn}{4bn};e^{2ia}(cx^n)^{2ib}\right)}{(2-5ibn)\left(1-e^{2ia}\left(cx^n\right)^{2ib}\right)^{5/2}\csc^{\frac{5}{2}}(a+b\log(cx^n))}
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 862 vs. $2(110) = 220$.
time = 8.87, size = 862, normalized size = 7.84

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[a + b*Log[c*x^n]]^(-5/2),x]
```

```
[Out] ((-30*I)*Sqrt[2]*b^3*n^3*x^(1 - I*b*n)*Sqrt[(I*E^(I*(a + b*(-(n*Log[x]) + L
og[c*x^n])))*x^(I*b*n))/(-1 + E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n])))*
x^((2*I)*b*n))]*((2*I + b*n)*(-1 + E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n
])))*x^((2*I)*b*n)) + (2*I + b*n + E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n
])))*(-2*I + b*n))*Sqrt[1 - E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n])))*x^
((2*I)*b*n)]*Hypergeometric2F1[1/2, -1/4*(2*I + b*n)/(b*n), 3/4 - (I/2)/(b*
n), E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n])))*x^((2*I)*b*n)]))/(E^(I*(a
+ b*(-(n*Log[x]) + Log[c*x^n])))*(-2 + (5*I)*b*n)*(-2*I + 5*b*n)*(4 + b^2*n
^2)*(2*I + b*n + E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n])))*(-2*I + b*n))
) + Sqrt[Csc[a + b*n*Log[x] + b*(-(n*Log[x]) + Log[c*x^n])]]*(-1/4*(x*Cos[b
*n*Log[x]]*(-12 - 55*b^2*n^2 + 12*Cos[2*(a + b*(-(n*Log[x]) + Log[c*x^n]))]
 + 65*b^2*n^2*Cos[2*(a + b*(-(n*Log[x]) + Log[c*x^n]))] + 4*b*n*Sin[2*(a +
b*(-(n*Log[x]) + Log[c*x^n]))]))/((-2*I + 5*b*n)*(2*I + 5*b*n)*(b*n*Cos[a +
 b*(-(n*Log[x]) + Log[c*x^n])] + 2*Sin[a + b*(-(n*Log[x]) + Log[c*x^n])]))
+ (x*Sin[b*n*Log[x]]*(16*b*n - 4*b*n*Cos[2*(a + b*(-(n*Log[x]) + Log[c*x^n]
))] + 12*Sin[2*(a + b*(-(n*Log[x]) + Log[c*x^n]))] + 65*b^2*n^2*Sin[2*(a +
```

```
b*(-(n*Log[x]) + Log[c*x^n]))]))/(4*(-2*I + 5*b*n)*(2*I + 5*b*n)*(b*n*Cos[a
 + b*(-(n*Log[x]) + Log[c*x^n])] + 2*Sin[a + b*(-(n*Log[x]) + Log[c*x^n])])
) + (x*Cos[3*b*n*Log[x]]*(5*b*n*Cos[3*(a + b*(-(n*Log[x]) + Log[c*x^n]))] -
 2*Sin[3*(a + b*(-(n*Log[x]) + Log[c*x^n]))]))/(2*(-2*I + 5*b*n)*(2*I + 5*b
*n)) - (x*Sin[3*b*n*Log[x]]*(2*Cos[3*(a + b*(-(n*Log[x]) + Log[c*x^n]))] +
5*b*n*Sin[3*(a + b*(-(n*Log[x]) + Log[c*x^n]))]))/(2*(-2*I + 5*b*n)*(2*I +
5*b*n)))
```

Maple [**F**]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\csc \left(a +b \ln \left(c \,x^{n}\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/csc(a+b*ln(c*x^n))^(5/2),x)
```

```
[Out] int(1/csc(a+b*ln(c*x^n))^(5/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csc(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*log(c*x^n) + a)^(-5/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csc(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csc(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csc(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*log(c*x^n) + a)^(-5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\sin(a+b\,\ln(c\,x^n))}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/sin(a + b*log(c*x^n)))^(5/2),x)
```

```
[Out] int(1/(1/sin(a + b*log(c*x^n)))^(5/2), x)
```

3.319 $\int \frac{1}{x\csc^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=98

$$-\frac{2\cos(a+b\log(cx^n))}{5bn\csc^{\frac{3}{2}}(a+b\log(cx^n))}+\frac{6\sqrt{\csc(a+b\log(cx^n))}\,E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b\log(cx^n)\right)\middle|2\right)\sqrt{\sin(a+b\log(cx^n))}}{5bn}$$

```
[Out] -2/5*cos(a+b*ln(c*x^n))/b/n/csc(a+b*ln(c*x^n))^(3/2)-6/5*(sin(1/2*a+1/4*Pi+
1/2*b*ln(c*x^n))^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*ln(c*x^n))*EllipticE(cos(1
/2*a+1/4*Pi+1/2*b*ln(c*x^n)),2^(1/2))*csc(a+b*ln(c*x^n))^(1/2)*sin(a+b*ln(c
*x^n))^(1/2)/b/n
```

Rubi [A]
time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3854, 3856, 2719}

$$\frac{6\sqrt{\sin(a+b\log(cx^n))}\sqrt{\csc(a+b\log(cx^n))}\,E\left(\frac{1}{2}\left(a+b\log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{5bn}-\frac{2\cos(a+b\log(cx^n))}{5bn\csc^{\frac{3}{2}}(a+b\log(cx^n))}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Csc[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (-2*Cos[a + b*Log[c*x^n]])/(5*b*n*Csc[a + b*Log[c*x^n]]^(3/2)) + (6*Sqrt[Cs
c[a + b*Log[c*x^n]]]*EllipticE[(a - Pi/2 + b*Log[c*x^n])/2, 2]*Sqrt[Sin[a +
 b*Log[c*x^n]]])/(5*b*n)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{x\csc^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\csc^{\frac{5}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$
$$= -\frac{2\cos(a+b\log(cx^n))}{5bn\csc^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{\csc(a+bx)}}\,dx, x, \log(cx^n)\right)}{5n}$$
$$= -\frac{2\cos(a+b\log(cx^n))}{5bn\csc^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{\left(3\sqrt{\csc(a+b\log(cx^n))}\,\sqrt{\sin(a+b\log(cx^n)}\right.}{5n}$$
$$= -\frac{2\cos(a+b\log(cx^n))}{5bn\csc^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{6\sqrt{\csc(a+b\log(cx^n))}\,E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b\log(cx\right.\right.}{5bn}$$

Mathematica [A]

time = 0.23, size = 88, normalized size = 0.90

$$-\frac{2\sqrt{\csc(a+b\log(cx^n))}\left(3E\left(\frac{1}{4}(-2a+\pi-2b\log(cx^n))\middle|2\right)\sqrt{\sin(a+b\log(cx^n))}+\cos(a+b\log(cx^n))\sin^2(a+b\log(cx^n))\right)}{5bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Csc[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (-2*Sqrt[Csc[a + b*Log[c*x^n]]]*(3*EllipticE[(-2*a + Pi - 2*b*Log[c*x^n])/4
, 2]*Sqrt[Sin[a + b*Log[c*x^n]]] + Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^
n]]^2))/(5*b*n)
```

Maple [A]

time = 0.32, size = 205, normalized size = 2.09

method	result
derivativedivides	$\frac{\frac{2\left(\sin^4(a+b\ln(c\,x^n))\right)}{5}-\frac{2\left(\sin^2(a+b\ln(c\,x^n))\right)}{5}-\frac{6\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+}}{}}{}$
default	$\frac{\frac{2\left(\sin^4(a+b\ln(c\,x^n))\right)}{5}-\frac{2\left(\sin^2(a+b\ln(c\,x^n))\right)}{5}-\frac{6\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/csc(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(2/5*sin(a+b*ln(c*x^n))^4-2/5*sin(a+b*ln(c*x^n))^2-6/5*(sin(a+b*ln(c*x^
n))+1)^(1/2)*(-2*sin(a+b*ln(c*x^n))+2)^(1/2)*(-sin(a+b*ln(c*x^n)))^(1/2)*El
lipticE((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2))+3/5*(sin(a+b*ln(c*x^n))+1
)^(1/2)*(-2*sin(a+b*ln(c*x^n))+2)^(1/2)*(-sin(a+b*ln(c*x^n)))^(1/2)*Ellipti
cF((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2)))/cos(a+b*ln(c*x^n))/sin(a+b*ln
(c*x^n))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csc(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*csc(b*log(c*x^n) + a)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.26, size = 130, normalized size = 1.33

$$\frac{3\sqrt{2i}\ \mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+i\ \sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)))+3\sqrt{-2i}\ \mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-i\ \sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)))+\frac{2\left(\cos(bn\log(x)+b\log(c)+a)^3-\cos(bn\log(x)+b\log(c)+a)\right)}{\sqrt{\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}}}{5\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csc(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(3*sqrt(2*I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*n*lo
g(x) + b*log(c) + a) + I*sin(b*n*log(x) + b*log(c) + a))) + 3*sqrt(-2*I)*we
ierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*n*log(x) + b*log(c) + a
) - I*sin(b*n*log(x) + b*log(c) + a))) + 2*(cos(b*n*log(x) + b*log(c) + a)^
3 - cos(b*n*log(x) + b*log(c) + a))/sqrt(sin(b*n*log(x) + b*log(c) + a)))/(
b*n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csc(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csc(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*csc(b*log(c*x^n) + a)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(\frac{1}{\sin(a+b\,\ln(c\,x^n))}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1/sin(a + b*log(c*x^n)))^(5/2)),x)
```

```
[Out] int(1/(x*(1/sin(a + b*log(c*x^n)))^(5/2)), x)
```

3.320 $\int (ex)^m \csc^3(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=122

$$-\frac{8e^{3iad}(ex)^{1+m}\left(cx^n\right)^{3ibd}\,{}_2F_1\left(3,-\frac{i(1+m)-3bdn}{2bdn};-\frac{i(1+m)-5bdn}{2bdn};e^{2iad}\left(cx^n\right)^{2ibd}\right)}{e(i(1+m)-3bdn)}$$

```
[Out] -8*exp(3*I*a*d)*(e*x)^(1+m)*(c*x^n)^(3*I*b*d)*hypergeom([3, 1/2*(-I*(1+m)+3
*b*d*n)/b/d/n],[1/2*(-I*(1+m)+5*b*d*n)/b/d/n],exp(2*I*a*d)*(c*x^n)^(2*I*b*d
))/e/(I*(1+m)-3*b*d*n)
```

Rubi [A]
time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{4606, 4602, 371\}$

$$-\frac{8e^{3iad}(ex)^{m+1}\left(cx^n\right)^{3ibd}\,{}_2F_1\left(3,-\frac{i(m+1)-3bdn}{2bdn};-\frac{i(m+1)-5bdn}{2bdn};e^{2iad}\left(cx^n\right)^{2ibd}\right)}{e(-3bdn+i(m+1))}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Csc[d*(a + b*Log[c*x^n])]^3,x]
```

```
[Out] (-8*E^((3*I)*a*d)*(e*x)^(1 + m)*(c*x^n)^((3*I)*b*d)*Hypergeometric2F1[3, -1
/2*(I*(1 + m) - 3*b*d*n)/(b*d*n), -1/2*(I*(1 + m) - 5*b*d*n)/(b*d*n), E^((2
*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(e*(I*(1 + m) - 3*b*d*n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4602

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[(-2*I)^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(
2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4606

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int (ex)^m \csc^3\left(d(a+b\log\left(cx^n\right))\right)\,dx &= \frac{\left((ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{n}}\csc^3(d(a+b\log(x)))\,dx, x, cx\right.}{en} \\
&= \frac{\left(8ie^{3iad}(ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int \frac{x^{-1+3ibd+\frac{1+m}{n}}}{\left(1-e^{2iad}x^{2ibd}\right)^3}\,dx, x, cx^n\right)}{en} \\
&= -\frac{8e^{3iad}(ex)^{1+m}\left(cx^n\right)^{3ibd}\,{}_2F_1\left(3, -\frac{i(1+m)-3bdn}{2bdn}; -\frac{i(1+m)-5bdn}{2bdn}; e^{2iad}\left(cx^n\right)\right.}{i(e+em)-3bden}
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 367 vs. $2(122) = 244$.
time = 2.48, size = 367, normalized size = 3.01

$$
\frac{x(ex)^m\left(-bdn\csc^2\left(\frac{1}{2}d(a+b\log\left(cx^n\right))\right)-4(1+m)\csc\left(d(a-bn\log(x)+b\log\left(cx^n\right))\right)+bdn\sec^2\left(\frac{1}{2}d(a+b\log\left(cx^n\right))\right)+2(1+m)\csc\left(\frac{1}{2}d(a+b\log\left(cx^n\right))\right)\csc\left(\frac{1}{2}d(a-bn\log(x)+b\log\left(cx^n\right))\right)\sin\left(\frac{1}{2}bdn\log(x)\right)-2(1+m)\sec\left(\frac{1}{2}d(a+b\log\left(cx^n\right))\right)\sec\left(\frac{1}{2}d(a-bn\log(x)+b\log\left(cx^n\right))\right)\sin\left(\frac{1}{2}bdn\log(x)\right)+8(1+m-ibdn)x^{ibdn}\,{}_2F_1\left(1,\frac{-i-im+bdn}{2bdn};-\frac{i(1+m+3ibdn)}{2bdn};x^{2ibdn}\left(\cos\left(2d(a-bn\log(x)+b\log\left(cx^n\right))\right)+i\sin\left(2d(a-bn\log(x)+b\log\left(cx^n\right))\right)\right)\right)\left(-i\cos\left(d(a-bn\log(x)+b\log\left(cx^n\right))\right)+\sin\left(d(a-bn\log(x)+b\log\left(cx^n\right))\right)\right)\right)}{8b^2d^2n^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Csc[d*(a + b*Log[c*x^n])]^3,x]
```

```
[Out] (x*(e*x)^m*(-(b*d*n*Csc[(d*(a + b*Log[c*x^n]))/2]^2) - 4*(1 + m)*Csc[d*(a -
 b*n*Log[x] + b*Log[c*x^n])] + b*d*n*Sec[(d*(a + b*Log[c*x^n]))/2]^2 + 2*(1
 + m)*Csc[(d*(a + b*Log[c*x^n]))/2]*Csc[(d*(a - b*n*Log[x] + b*Log[c*x^n]))
/2]*Sin[(b*d*n*Log[x])/2] - 2*(1 + m)*Sec[(d*(a + b*Log[c*x^n]))/2]*Sec[(d*
(a - b*n*Log[x] + b*Log[c*x^n]))/2]*Sin[(b*d*n*Log[x])/2] + 8*(1 + m - I*b*
d*n)*x^(I*b*d*n)*Hypergeometric2F1[1, (-I - I*m + b*d*n)/(2*b*d*n), ((-1/2*
I)*(1 + m + (3*I)*b*d*n))/(b*d*n), x^((2*I)*b*d*n)*(Cos[2*d*(a - b*n*Log[x]
 + b*Log[c*x^n])] + I*Sin[2*d*(a - b*n*Log[x] + b*Log[c*x^n])])]*((-I)*Cos[
d*(a - b*n*Log[x] + b*Log[c*x^n])] + Sin[d*(a - b*n*Log[x] + b*Log[c*x^n])]
)))/(8*b^2*d^2*n^2)
```

Maple [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$
\int (ex)^m\left(\csc^3\left(d(a+b\ln\left(c\,x^n\right))\right)\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*csc(d*(a+b*ln(c*x^n)))^3,x)
```

```
[Out] int((e*x)^m*csc(d*(a+b*ln(c*x^n)))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*csc(d*(a+b*log(c*x^n)))^3,x, algorithm="maxima")
```

```
[Out] -((b*d*n*cos(b*d*log(c))*e^m - (m*sin(b*d*log(c)) + sin(b*d*log(c)))*e^m)*x
*x^m*cos(b*d*log(x^n) + a*d) - (b*d*n*e^m*sin(b*d*log(c)) + (m*cos(b*d*log(
c)) + cos(b*d*log(c)))*e^m)*x*x^m*sin(b*d*log(x^n) + a*d) + (((b*d*cos(4*b*
d*log(c))*cos(3*b*d*log(c)) + b*d*sin(4*b*d*log(c))*sin(3*b*d*log(c)))*n*e^
m - ((cos(3*b*d*log(c))*sin(4*b*d*log(c)) - cos(4*b*d*log(c))*sin(3*b*d*log
(c)))*m + cos(3*b*d*log(c))*sin(4*b*d*log(c)) - cos(4*b*d*log(c))*sin(3*b*d
*log(c)))*e^m)*x*x^m*cos(3*b*d*log(x^n) + 3*a*d) + ((b*d*cos(4*b*d*log(c))*
cos(b*d*log(c)) + b*d*sin(4*b*d*log(c))*sin(b*d*log(c)))*n*e^m + ((cos(b*d*
log(c))*sin(4*b*d*log(c)) - cos(4*b*d*log(c))*sin(b*d*log(c)))*m + cos(b*d*
log(c))*sin(4*b*d*log(c)) - cos(4*b*d*log(c))*sin(b*d*log(c)))*e^m)*x*x^m*c
os(b*d*log(x^n) + a*d) + ((b*d*cos(3*b*d*log(c))*sin(4*b*d*log(c)) - b*d*co
s(4*b*d*log(c))*sin(3*b*d*log(c)))*n*e^m + ((cos(4*b*d*log(c))*cos(3*b*d*lo
g(c)) + sin(4*b*d*log(c))*sin(3*b*d*log(c)))*m + cos(4*b*d*log(c))*cos(3*b*
d*log(c)) + sin(4*b*d*log(c))*sin(3*b*d*log(c)))*e^m)*x*x^m*sin(3*b*d*log(x
^n) + 3*a*d) + ((b*d*cos(b*d*log(c))*sin(4*b*d*log(c)) - b*d*cos(4*b*d*log(
c))*sin(b*d*log(c)))*n*e^m - ((cos(4*b*d*log(c))*cos(b*d*log(c)) + sin(4*b*
d*log(c))*sin(b*d*log(c)))*m + cos(4*b*d*log(c))*cos(b*d*log(c)) + sin(4*b*
d*log(c))*sin(b*d*log(c)))*e^m)*x*x^m*sin(b*d*log(x^n) + a*d))*cos(4*b*d*lo
g(x^n) + 4*a*d) - (2*((b*d*cos(3*b*d*log(c))*cos(2*b*d*log(c)) + b*d*sin(3*
b*d*log(c))*sin(2*b*d*log(c)))*n*e^m + ((cos(2*b*d*log(c))*sin(3*b*d*log(c)
) - cos(3*b*d*log(c))*sin(2*b*d*log(c)))*m + cos(2*b*d*log(c))*sin(3*b*d*lo
g(c)) - cos(3*b*d*log(c))*sin(2*b*d*log(c)))*e^m)*x*x^m*cos(2*b*d*log(x^n)
+ 2*a*d) + 2*((b*d*cos(2*b*d*log(c))*sin(3*b*d*log(c)) - b*d*cos(3*b*d*log(
c))*sin(2*b*d*log(c)))*n*e^m - ((cos(3*b*d*log(c))*cos(2*b*d*log(c)) + sin(
3*b*d*log(c))*sin(2*b*d*log(c)))*m + cos(3*b*d*log(c))*cos(2*b*d*log(c)) +
sin(3*b*d*log(c))*sin(2*b*d*log(c)))*e^m)*x*x^m*sin(2*b*d*log(x^n) + 2*a*d)
 - (b*d*n*cos(3*b*d*log(c))*e^m + (m*sin(3*b*d*log(c)) + sin(3*b*d*log(c)))
*e^m)*x*x^m)*cos(3*b*d*log(x^n) + 3*a*d) - 2*(((b*d*cos(2*b*d*log(c))*cos(b
*d*log(c)) + b*d*sin(2*b*d*log(c))*sin(b*d*log(c)))*n*e^m + ((cos(b*d*log(c
))*sin(2*b*d*log(c)) - cos(2*b*d*log(c))*sin(b*d*log(c)))*m + cos(b*d*log(c
))*sin(2*b*d*log(c)) - cos(2*b*d*log(c))*sin(b*d*log(c)))*e^m)*x*x^m*cos(b*
d*log(x^n) + a*d) + ((b*d*cos(b*d*log(c))*sin(2*b*d*log(c)) - b*d*cos(2*b*d
*log(c))*sin(b*d*log(c)))*n*e^m - ((cos(2*b*d*log(c))*cos(b*d*log(c)) + sin
(2*b*d*log(c))*sin(b*d*log(c)))*m + cos(2*b*d*log(c))*cos(b*d*log(c)) + sin
(2*b*d*log(c))*sin(b*d*log(c)))*e^m)*x*x^m*sin(b*d*log(x^n) + a*d))*cos(2*b
*d*log(x^n) + 2*a*d) + 2*(b^6*d^6*n^6*e^m + (b^4*d^4*m^2 + 2*b^4*d^4*m + b^
4*d^4)*n^4*e^m + ((b^6*d^6*cos(4*b*d*log(c))^2 + b^6*d^6*sin(4*b*d*log(c))^
```

```
2)*n^6*e^m + (b^4*d^4*cos(4*b*d*log(c))^2 + b^4*d^4*sin(4*b*d*log(c))^2 + (
b^4*d^4*cos(4*b*d*log(c))^2 + b^4*d^4*sin(4*b*d*log(c))^2)*m^2 + 2*(b^4*d^4
*cos(4*b*d*log(c))^2 + b^4*d^4*sin(4*b*d*log(c))^2)*m)*n^4*e^m)*cos(4*b*d*l
og(x^n) + 4*a*d)^2 + 4*((b^6*d^6*cos(2*b*d*log(c))^2 + b^6*d^6*sin(2*b*d*lo
g(c))^2)*n^6*e^m + (b^4*d^4*cos(2*b*d*log(c))^2 + b^4*d^4*sin(2*b*d*log(c))
^2 + (b^4*d^4*cos(2*b*d*log(c))^2 + b^4*d^4*sin(2*b*d*log(c))^2)*m^2 + 2*(b
^4*d^4*cos(2*b*d*log(c))^2 + b^4*d^4*sin(2*b*d*log(c))^2)*m)*n^4*e^m)*cos(2
*b*d*log(x^n) + 2*a*d)^2 + ((b^6*d^6*cos(4*b*d*log(c))^2 + b^6*d^6*sin(4*b*
d*log(c))^2)*n^6*e^m + (b^4*d^4*cos(4*b*d*log(c))^2 + b^4*d^4*sin(4*b*d*log
(c))^2 + (b^4*d^4*cos(4*b*d*log(c))^2 + b^4*d^4*sin(4*b*d*log(c))^2)*m^2 +
2*(b^4*d^4*cos(4*b*d*log(c))^2 + b^4*d^4*sin(4*b*d*log(c))^2)*m)*n^4*e^m)*s
in(4*b*d*log(x^n) + 4*a*d)^2 + 4*((b^6*d^6*cos(2*b*d*log(c))^2 + b^6*d^6*si
n(2*b*d*log(c))^2)*n^6*e^m + (b^4*d^4*cos(2*b*d*log(c))^2 + b^4*d^4*sin(2*b
*d*log(c))^2 + (b^4*d^4*cos(2*b*d*log(c))^2 + b^4*d^4*sin(2*b*d*log(c))^2)*
m^2 + 2*(b^4*d^4*cos(2*b*d*log(c))^2 + b^4*d^4*sin(2*b*d*log(c))^2)*m)*n^4*
e^m)*sin(2*b*d*log(x^n) + 2*a*d)^2 + 2*(b^6*d^6*n^6*cos(4*b*d*log(c))*e^m +
 (b^4*d^4*m^2*cos(4*b*d*log(c)) + 2*b^4*d^4*m*cos(4*b*d*log(c)) + b^4*d^4*c
os(4*b*d*log(c)))*n^4*e^m - 2*((b^6*d^6*cos(4*b*d*log(c))*cos(2*b*d*log(c))
 + b^6*d^6*sin(4*b*d*log(c))*sin(2*b*d*log(c)))*n^6*e^m + (b^4*d^4*cos(4*b*
d*log(c))*cos(2*b*d*log(c)) + b^4*d^4*sin(4*b*d*log(c))*sin(2*b*d*log(c)) +
 (b^4*d^4*cos(4*b*d*log(c))*cos(2*b*d*log(c)) + b^4*d^4*sin(4*b*d*log(c))*s
in(2*b*d*log(c)))*m^2 + 2*(b^4*d^4*cos(4*b*d*log(c))*cos(2*b*d*log(c)) + b^
4*d^4*sin(4*b*d*log(c))*sin(2*b*d*log(c)))*m)*n^4*e^m)*cos(2*b*d*log(x^n) +
 2*a*d) - 2*((b^6*d^6*cos(2*b*d*log(c))*sin(4*b*d*log(c)) - b^6*d^6*cos(4*b
*d*log(c))*sin(2*b*d*log(c)))*n^6*e^m + (b^4*d^4*cos(2*b*d*log(c))*sin(4*b*
d*log(c)) - b^4*d^4*cos(4*b*d*log(c))*sin(2*b*d*log(c)) + (b^4*d^4*cos(2*b*
d*log(c))*sin(4*b*d*log(c)) - b^4*d^4*cos(4*b*d*log(c))*sin(2*b*d*log(c)))*
m^2 + 2*(b^4*d^4*cos(2*b*d*log(c))*sin(4*b*d*log(c)) - b^4*d^4*cos(4*b*d*lo
g(c))*sin(2*b*d*log(c)))*m)*n^4*e^m)*sin(2*b*d*log(x^n) + 2*a*d))*cos(4*b*d
*log(x^n) + 4*a*d) - 4*(b^6*d^6*n^6*cos(2*b*d*l...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*csc(d*(a+b*log(c*x^n)))^3,x, algorithm="fricas")
```

```
[Out] integral((x*e)^m*csc(b*d*log(c*x^n) + a*d)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \csc^3 (ad + bd \log (cx^n))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*csc(d*(a+b*ln(c*x**n)))**3,x)
```

```
[Out] Integral((e*x)**m*csc(a*d + b*d*log(c*x**n))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*csc(d*(a+b*log(c*x^n)))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*csc((b*log(c*x^n) + a)*d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m}{\sin\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/sin(d*(a + b*log(c*x^n)))^3,x)
```

```
[Out] int((e*x)^m/sin(d*(a + b*log(c*x^n)))^3, x)
```

3.321 $\int (ex)^m \csc^2 (d(a + b\log(cx^n)))\, dx$

Optimal. Leaf size=119

$$-\frac{4e^{2iad}(ex)^{1+m}\left(cx^n\right)^{2ibd}\, {}_2F_1\left(2,-\frac{i(1+m)-2bdn}{2bdn};-\frac{i(1+m)-4bdn}{2bdn};e^{2iad}\left(cx^n\right)^{2ibd}\right)}{e(1+m+2ibdn)}$$

```
[Out] -4*exp(2*I*a*d)*(e*x)^(1+m)*(c*x^n)^(2*I*b*d)*hypergeom([2, 1/2*(-I*(1+m)+2
*b*d*n)/b/d/n],[1/2*(-I*(1+m)+4*b*d*n)/b/d/n],exp(2*I*a*d)*(c*x^n)^(2*I*b*d
))/e/(1+m+2*I*b*d*n)
```

Rubi [A]
time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4606, 4602, 371}

$$-\frac{4e^{2iad}(ex)^{m+1}\left(cx^n\right)^{2ibd}\, {}_2F_1\left(2,-\frac{i(m+1)-2bdn}{2bdn};-\frac{i(m+1)-4bdn}{2bdn};e^{2iad}\left(cx^n\right)^{2ibd}\right)}{e(2ibdn+m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Csc[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (-4*E^((2*I)*a*d)*(e*x)^(1 + m)*(c*x^n)^((2*I)*b*d)*Hypergeometric2F1[2, -1
/2*(I*(1 + m) - 2*b*d*n)/(b*d*n), -1/2*(I*(1 + m) - 4*b*d*n)/(b*d*n), E^((2
*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(e*(1 + m + (2*I)*b*d*n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4602

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[(-2*I)^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(
2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4606

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int (ex)^m \csc^2\left(d(a+b\log\left(cx^n\right))\right)\,dx &= \frac{\left((ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\mathrm{Subst}\left(\int x^{-1+\frac{1+m}{n}}\csc^2(d(a+b\log(x)))\,dx,x,c\right.}{en}\\
&= -\frac{\left(4e^{2iad}(ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\mathrm{Subst}\left(\int \frac{x^{-1+2ibd+\frac{1+m}{n}}}{\left(1-e^{2iad}x^{2ibd}\right)^2}\,dx,x,cx^n\right)}{en}\\
&= -\frac{4e^{2iad}(ex)^{1+m}\left(cx^n\right)^{2ibd}\ {}_2F_1\left(2,-\frac{i(1+m)-2bdn}{2bdn};-\frac{i(1+m)-4bdn}{2bdn};e^{2iad}(cx^n\right.}{e(1+m+2ibdn)}
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 534 vs. $2(119) = 238$.
time = 6.57, size = 534, normalized size = 4.49

$$
\frac{x(ex)^m\csc\left(d\left(a+b\left(-n\log(x)+\log\left(cx^n\right)\right)\right)\right)\csc\left(bdn\log(x)+d\left(a+b\left(-n\log(x)+\log\left(cx^n\right)\right)\right)\right)\sin(bdn\log(x))}{bdn} - \frac{(1+m)x^{-m}(ex)^m\csc\left(d\left(a+b\left(-n\log(x)+\log\left(cx^n\right)\right)\right)\right)\left(\frac{x^{1+m}\csc(d(a+b\log(cx^n)))\sin(bdn\log(x))}{1+m} - \cdots\right)}{\cdots}
$$

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Csc[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (x*(e*x)^m*Csc[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Csc[b*d*n*Log[x] + d*(
a + b*(-(n*Log[x]) + Log[c*x^n]))]*Sin[b*d*n*Log[x]])/(b*d*n) - ((1 + m)*(e
*x)^m*Csc[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*((x^(1 + m)*Csc[d*(a + b*Lo
g[c*x^n])]*Sin[b*d*n*Log[x]])/(1 + m) - (I*(I*E^((a + 2*a*m + b*(1 + m)*n*L
og[x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n]))/(b*n))*(1 + m + (2*I)*b*d*n
)*Cot[d*(a + b*Log[c*x^n])] - E^((a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2
*m)*(-(n*Log[x]) + Log[c*x^n]))/(b*n))*(1 + m + (2*I)*b*d*n)*Hypergeometric
2F1[1, ((-1/2*I)*(1 + m))/(b*d*n), 1 - ((I/2)*(1 + m))/(b*d*n), E^((2*I)*d*
(a + b*Log[c*x^n]))] - E^((a*(1 + 2*m + (2*I)*b*d*n))/(b*n) + (1 + m + (2*I
)*b*d*n)*Log[x] + ((1 + 2*m + (2*I)*b*d*n)*(-(n*Log[x]) + Log[c*x^n]))/n)*(
1 + m)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m + (2*I)*b*d*n))/(b*d*n), ((-1/
2*I)*(1 + m + (4*I)*b*d*n))/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))])*Sin[d
*(a + b*(-(n*Log[x]) + Log[c*x^n]))])/(E^(((1 + 2*m)*(a + b*(-(n*Log[x]) +
Log[c*x^n])))/(b*n))*(1 + m)*(1 + m + (2*I)*b*d*n))))/(b*d*n*x^m)
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int (ex)^m\left(\csc^2\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*csc(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int((e*x)^m*csc(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*csc(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
[Out] (2*x*cos(2*b*d*log(x^n) + 2*a*d)*e^(m*log(x) + m)*sin(2*b*d*log(c)) + 2*x*c
os(2*b*d*log(c))*e^(m*log(x) + m)*sin(2*b*d*log(x^n) + 2*a*d) + ((b^2*d^2*c
os(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2 + (b^2*d^2*cos(2*b*d*log(c
))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*m)*n^2*cos(2*b*d*log(x^n) + 2*a*d)^2*e^
m + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2 + (b^2*d^2*c
os(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*m)*n^2*e^m*sin(2*b*d*log(
x^n) + 2*a*d)^2 - 2*(b^2*d^2*m*cos(2*b*d*log(c)) + b^2*d^2*cos(2*b*d*log(c)
))*n^2*cos(2*b*d*log(x^n) + 2*a*d)*e^m + 2*(b^2*d^2*m*sin(2*b*d*log(c)) + b
^2*d^2*sin(2*b*d*log(c)))*n^2*e^m*sin(2*b*d*log(x^n) + 2*a*d) + (b^2*d^2*m
+ b^2*d^2)*n^2*e^m)*integrate((x^m*cos(b*d*log(x^n) + a*d)*sin(b*d*log(c))
+ x^m*cos(b*d*log(c))*sin(b*d*log(x^n) + a*d))/(2*b^2*d^2*n^2*cos(b*d*log(c
))*cos(b*d*log(x^n) + a*d) - 2*b^2*d^2*n^2*sin(b*d*log(c))*sin(b*d*log(x^n)
 + a*d) + b^2*d^2*n^2 + (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*sin(b*d*log(c)
)^2)*n^2*cos(b*d*log(x^n) + a*d)^2 + (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*s
in(b*d*log(c))^2)*n^2*sin(b*d*log(x^n) + a*d)^2), x) - ((b^2*d^2*cos(2*b*d*
log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2 + (b^2*d^2*cos(2*b*d*log(c))^2 + b^
2*d^2*sin(2*b*d*log(c))^2)*m)*n^2*cos(2*b*d*log(x^n) + 2*a*d)^2*e^m + (b^2*
d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2 + (b^2*d^2*cos(2*b*d*
log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*m)*n^2*e^m*sin(2*b*d*log(x^n) + 2*
a*d)^2 - 2*(b^2*d^2*m*cos(2*b*d*log(c)) + b^2*d^2*cos(2*b*d*log(c)))*n^2*co
s(2*b*d*log(x^n) + 2*a*d)*e^m + 2*(b^2*d^2*m*sin(2*b*d*log(c)) + b^2*d^2*si
n(2*b*d*log(c)))*n^2*e^m*sin(2*b*d*log(x^n) + 2*a*d) + (b^2*d^2*m + b^2*d^2
)*n^2*e^m)*integrate(-(x^m*cos(b*d*log(x^n) + a*d)*sin(b*d*log(c)) + x^m*co
s(b*d*log(c))*sin(b*d*log(x^n) + a*d))/(2*b^2*d^2*n^2*cos(b*d*log(c))*cos(b
*d*log(x^n) + a*d) - 2*b^2*d^2*n^2*sin(b*d*log(c))*sin(b*d*log(x^n) + a*d)
- b^2*d^2*n^2 - (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*sin(b*d*log(c))^2)*n^2
*cos(b*d*log(x^n) + a*d)^2 - (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*sin(b*d*l
og(c))^2)*n^2*sin(b*d*log(x^n) + a*d)^2), x))/(2*b*d*n*cos(2*b*d*log(c))*co
s(2*b*d*log(x^n) + 2*a*d) - 2*b*d*n*sin(2*b*d*log(c))*sin(2*b*d*log(x^n) +
2*a*d) - (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*cos(2*b*d*lo
g(x^n) + 2*a*d)^2 - (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*s
in(2*b*d*log(x^n) + 2*a*d)^2 - b*d*n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*csc(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral((x*e)^m*csc(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \csc^2\left(ad + bd\log\left(cx^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*csc(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Integral((e*x)**m*csc(a*d + b*d*log(c*x**n))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*csc(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*csc((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m}{\sin\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/sin(d*(a + b*log(c*x^n)))^2,x)
```

```
[Out] int((e*x)^m/sin(d*(a + b*log(c*x^n)))^2, x)
```

3.322 $\int (ex)^m \csc(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=123

$$\frac{2e^{iad}(ex)^{1+m}(cx^n)^{ibd}\,{}_2F_1\left(1,-\frac{i+im-bdn}{2bdn};-\frac{i(1+m)-3bdn}{2bdn};e^{2iad}(cx^n)^{2ibd}\right)}{e(i(1+m)-bdn)}$$

```
[Out] 2*exp(I*a*d)*(e*x)^(1+m)*(c*x^n)^(I*b*d)*hypergeom([1, 1/2*(-I-I*m+b*d*n)/b
/d/n],[1/2*(-I*(1+m)+3*b*d*n)/b/d/n],exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/e/(I*(
1+m)-b*d*n)
```

Rubi [A]
time = 0.06, antiderivative size = 118, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4606, 4602, 371}

$$\frac{2e^{iad}(ex)^{m+1}(cx^n)^{ibd}\,{}_2F_1\left(1,\frac{1}{2}\left(1-\frac{i(m+1)}{bdn}\right);-\frac{i(m+1)-3bdn}{2bdn};e^{2iad}(cx^n)^{2ibd}\right)}{e(-bdn+i(m+1))}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Csc[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (2*E^(I*a*d)*(e*x)^(1 + m)*(c*x^n)^(I*b*d)*Hypergeometric2F1[1, (1 - (I*(1
+ m))/(b*d*n))/2, -1/2*(I*(1 + m) - 3*b*d*n)/(b*d*n), E^((2*I)*a*d)*(c*x^n)
^((2*I)*b*d)])/(e*(I*(1 + m) - b*d*n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4602

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[(-2*I)^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(
2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rule 4606

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int (ex)^m \csc\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx &= \frac{\left((ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{n}}\csc(d(a+b\log(x)))\,dx,x,cx^{n}\right.}{en}\\
&= -\frac{\left(2ie^{iad}(ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int \frac{x^{-1+ibd+\frac{1+m}{n}}}{1-e^{2iad}x^{2ibd}}\,dx,x,cx^n\right)}{en}\\
&= \frac{2e^{iad}(ex)^{1+m}\left(cx^n\right)^{ibd}\,{}_2F_1\left(1,\frac{1}{2}\left(1-\frac{i(1+m)}{bdn}\right);-\frac{i(1+m)-3bdn}{2bdn};e^{2iad}\left(cx^n\right)^{2}\right.}{i(e+em)-bden}
\end{aligned}
$$

Mathematica [A]
time = 0.35, size = 181, normalized size = 1.47

$$
\frac{2x^{1+ibdn}(ex)^m\,{}_2F_1\left(1,\frac{-i-im+bdn}{2bdn};-\frac{i(1+m+3ibdn)}{2bdn};x^{2ibdn}\left(\cos\left(2d\left(a+b\left(-n\log(x)+\log\left(cx^n\right)\right)\right)\right)+i\sin\left(2d\left(a+b\left(-n\log(x)+\log\left(cx^n\right)\right)\right)\right)\right)\right)\left(-i\cos\left(d\left(a+b\left(-n\log(x)+\log\left(cx^n\right)\right)\right)\right)+\sin\left(d\left(a+b\left(-n\log(x)+\log\left(cx^n\right)\right)\right)\right)\right)}{1+m+ibdn}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Csc[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (2*x^(1 + I*b*d*n)*(e*x)^m*Hypergeometric2F1[1, (-I - I*m + b*d*n)/(2*b*d*n
), ((-1/2*I)*(1 + m + (3*I)*b*d*n))/(b*d*n), x^((2*I)*b*d*n)*(Cos[2*d*(a +
b*(-(n*Log[x]) + Log[c*x^n]))] + I*Sin[2*d*(a + b*(-(n*Log[x]) + Log[c*x^n]
))])]*((-I)*Cos[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))] + Sin[d*(a + b*(-(n*L
og[x]) + Log[c*x^n]))]))/(1 + m + I*b*d*n)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$
\int (ex)^m \csc\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*csc(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int((e*x)^m*csc(d*(a+b*ln(c*x^n))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*csc(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate((x*e)^m*csc((b*log(c*x^n) + a)*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*csc(d*(a+b*log(c*x^n))),x, algorithm="fricas")
[Out] integral((x*e)^m*csc(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \csc(ad + bd\log(cx^n))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*csc(d*(a+b*ln(c*x**n))),x)
[Out] Integral((e*x)**m*csc(a*d + b*d*log(c*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*csc(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate((e*x)^m*csc((b*log(c*x^n) + a)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m}{\sin\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/sin(d*(a + b*log(c*x^n))),x)
[Out] int((e*x)^m/sin(d*(a + b*log(c*x^n))), x)
```

3.323 $\int x^m \csc^{\frac{5}{2}}(a+b\log(cx^n))\,dx$

Optimal. Leaf size=130

$$\frac{2x^{1+m}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{5/2}\csc^{\frac{5}{2}}(a+b\log(cx^n))\,{}_2F_1\left(\frac{5}{2},-\frac{2i+2im-5bn}{4bn};-\frac{2i+2im-9bn}{4bn};e^{2ia}(cx^n)^{2ib}\right)}{2+2m+5ibn}$$

```
[Out] 2*x^(1+m)*(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(5/2)*csc(a+b*ln(c*x^n))^(5/2)*hyp
ergeom([5/2, 1/4*(-2*I-2*I*m+5*b*n)/b/n],[1/4*(-2*I-2*I*m+9*b*n)/b/n],exp(2
*I*a)*(c*x^n)^(2*I*b))/(2+2*m+5*I*b*n)
```

Rubi [A]
time = 0.07, antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4606, 4604, 371}

$$\frac{2x^{m+1}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i(m+1)}{bn}\right);-\frac{2im-9bn+2i}{4bn};e^{2ia}(cx^n)^{2ib}\right)\csc^{\frac{5}{2}}(a+b\log(cx^n))}{5ibn+2m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Csc[a + b*Log[c*x^n]]^(5/2),x]
```

```
[Out] (2*x^(1 + m)*(1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^(5/2)*Csc[a + b*Log[c*x^n]
]^(5/2)*Hypergeometric2F1[5/2, (5 - ((2*I)*(1 + m))/(b*n))/4, -1/4*(2*I + (
2*I)*m - 9*b*n)/(b*n), E^((2*I)*a)*(c*x^n)^((2*I)*b)])/(2 + 2*m + (5*I)*b*n
)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4604

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 4606

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
```

```
((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^m \csc^{\frac{5}{2}}(a+b\log(cx^n))\,dx = \frac{\left(x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{n}}\csc^{\frac{5}{2}}(a+b\log(x))\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x^{1+m}(cx^n)^{-\frac{5ib}{2}-\frac{1+m}{n}}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{5/2}\csc^{\frac{5}{2}}(a+b\log(cx^n))\right)\text{Subst}\left(\int\right.}{n}$$

$$= \frac{2x^{1+m}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{5/2}\csc^{\frac{5}{2}}(a+b\log(cx^n))\,{}_2F_1\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i(1+m)}{bn}\right);-\right.}{2+2m+5ibn}$$

Mathematica [A]
time = 6.38, size = 207, normalized size = 1.59

$$\frac{2x^{1+m}\left(-\left((2+2m+bn\cot(a+b\log(cx^n)))\sqrt{\csc(a+b\log(cx^n))}\right)+(2+2m-ibn)\sqrt{2-2e^{2ia}(cx^n)^{2ib}}\sqrt{\frac{ie^{ia}(cx^n)^{ib}}{-1+e^{2ia}(cx^n)^{2ib}}}\,{}_2F_1\left(\frac{1}{2},\frac{-2i-2im+bn}{4bn};-\frac{2i+2im-5bn}{4bn};e^{2ia}(cx^n)^{2ib}\right)\right)}{3b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Csc[a + b*Log[c*x^n]]^(5/2),x]
```

```
[Out] (2*x^(1 + m)*(-((2 + 2*m + b*n*Cot[a + b*Log[c*x^n]])*Sqrt[Csc[a + b*Log[c*
x^n]]]) + (2 + 2*m - I*b*n)*Sqrt[2 - 2*E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[
(I*E^(I*a)*(c*x^n)^(I*b))/(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))]*Hypergeomet
ric2F1[1/2, (-2*I - (2*I)*m + b*n)/(4*b*n), -1/4*(2*I + (2*I)*m - 5*b*n)/(b
*n), E^((2*I)*a)*(c*x^n)^((2*I)*b)]))/(3*b^2*n^2)
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int x^m\left(\csc^{\frac{5}{2}}(a+b\ln(c\,x^n))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*csc(a+b*ln(c*x^n))^(5/2),x)
```

```
[Out] int(x^m*csc(a+b*ln(c*x^n))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*csc(b*log(c*x^n) + a)^(5/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csc(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \left(\frac{1}{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(1/sin(a + b*log(c*x^n)))^(5/2),x)
```

```
[Out] int(x^m*(1/sin(a + b*log(c*x^n)))^(5/2), x)
```

3.324 $\int x^m \csc^{\frac{3}{2}}(a+b\log(cx^n))\,dx$

Optimal. Leaf size=130

$$\frac{2x^{1+m}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}\csc^{\frac{3}{2}}(a+b\log(cx^n))\,{}_2F_1\left(\frac{3}{2},-\frac{2i+2im-3bn}{4bn};-\frac{2i+2im-7bn}{4bn};e^{2ia}(cx^n)^{2ib}\right)}{2+2m+3ibn}$$

```
[Out] 2*x^(1+m)*(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(3/2)*csc(a+b*ln(c*x^n))^(3/2)*hyp
ergeom([3/2, 1/4*(-2*I-2*I*m+3*b*n)/b/n],[1/4*(-2*I-2*I*m+7*b*n)/b/n],exp(2
*I*a)*(c*x^n)^(2*I*b))/(2+2*m+3*I*b*n)
```

Rubi [A]
time = 0.07, antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4606, 4604, 371}

$$\frac{2x^{m+1}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}{}_2F_1\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i(m+1)}{bn}\right);-\frac{2im-7bn+2i}{4bn};e^{2ia}(cx^n)^{2ib}\right)\csc^{\frac{3}{2}}(a+b\log(cx^n))}{3ibn+2m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Csc[a + b*Log[c*x^n]]^(3/2),x]
```

```
[Out] (2*x^(1 + m)*(1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2)*Csc[a + b*Log[c*x^n]
]^(3/2)*Hypergeometric2F1[3/2, (3 - ((2*I)*(1 + m))/(b*n))/4, -1/4*(2*I + (
2*I)*m - 7*b*n)/(b*n), E^((2*I)*a)*(c*x^n)^((2*I)*b)])/(2 + 2*m + (3*I)*b*n
)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4604

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rule 4606

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
```

```
((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^m \csc^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{\left(x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\mathrm{Subst}\left(\int x^{-1+\frac{1+m}{n}}\csc^{\frac{3}{2}}(a+b\log(x))\,dx, x, cx^n\right)}{n}$$
$$= \frac{\left(x^{1+m}(cx^n)^{-\frac{3ib}{2}-\frac{1+m}{n}}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}\csc^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\right)\mathrm{Subst}\Big(}{n}$$
$$= \frac{2x^{1+m}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}\csc^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\,{}_2F_1\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i(1+m)}{bn}\right);\right.}{2+2m+3ibn}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 466 vs. $2(130) = 260$.
time = 10.34, size = 466, normalized size = 3.58

$$\frac{x^{1+m-ibn}\left(\left(4+8m+4m^2+b^2n^2\right)x^{2ibn}\sqrt{2-2e^{2ia}\left(cx^n\right)^{2ib}}\sqrt{\frac{ie^{ia}\left(cx^n\right)^{ib}}{-1+e^{2ia}\left(cx^n\right)^{2ib}}}\,{}_2F_1\left(\frac{1}{2},-\frac{2i+2im-3bn}{4bn};-\frac{2i+2im-7bn}{4bn};e^{2ia}(cx^n)^{2ib}\right)+(-2i-2im+3bn)\left((-2i-2im+bn)\sqrt{2-2e^{2ia}\left(cx^n\right)^{2ib}}\sqrt{\frac{ie^{ia}\left(cx^n\right)^{ib}}{-1+e^{2ia}\left(cx^n\right)^{2ib}}}\,{}_2F_1\left(\frac{1}{2},-\frac{2i+2im+bn}{4bn};-\frac{2i+2im-3bn}{4bn};e^{2ia}(cx^n)^{2ib}\right)-2x^{ibn}\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}\left(bn\cos(bn\log(x))-2(1+m)\sin(bn\log(x))\right)\right)\right)}{bn(-2i-2im+3bn)\left(bn\cos\left(a-bn\log(x)+b\log\left(cx^n\right)\right)+2(1+m)\sin\left(a-bn\log(x)+b\log\left(cx^n\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m*Csc[a + b*Log[c*x^n]]^(3/2),x]
```

```
[Out] (x^(1 + m - I*b*n)*((4 + 8*m + 4*m^2 + b^2*n^2)*x^((2*I)*b*n)*Sqrt[2 - 2*E^
((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[(I*E^(I*a)*(c*x^n)^(I*b))/(-1 + E^((2*I)*
a)*(c*x^n)^((2*I)*b))]*Hypergeometric2F1[1/2, -1/4*(2*I + (2*I)*m - 3*b*n)/
(b*n), -1/4*(2*I + (2*I)*m - 7*b*n)/(b*n), E^((2*I)*a)*(c*x^n)^((2*I)*b)] +
 (-2*I - (2*I)*m + 3*b*n)*((-2*I - (2*I)*m + b*n)*Sqrt[2 - 2*E^((2*I)*a)*(c
*x^n)^((2*I)*b)]*Sqrt[(I*E^(I*a)*(c*x^n)^(I*b))/(-1 + E^((2*I)*a)*(c*x^n)^(
(2*I)*b))]*Hypergeometric2F1[1/2, -1/4*(2*I + (2*I)*m + b*n)/(b*n), -1/4*(2
*I + (2*I)*m - 3*b*n)/(b*n), E^((2*I)*a)*(c*x^n)^((2*I)*b)] - 2*x^(I*b*n)*S
qrt[Csc[a + b*Log[c*x^n]]]*(b*n*Cos[b*n*Log[x]] - 2*(1 + m)*Sin[b*n*Log[x]]
))))/(b*n*(-2*I - (2*I)*m + 3*b*n)*(b*n*Cos[a - b*n*Log[x] + b*Log[c*x^n]]
+ 2*(1 + m)*Sin[a - b*n*Log[x] + b*Log[c*x^n]]))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int x^m\left(\csc^{\frac{3}{2}}\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*csc(a+b*ln(c*x^n))^(3/2),x)
```

```
[Out] int(x^m*csc(a+b*ln(c*x^n))^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*csc(b*log(c*x^n) + a)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csc(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6436 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \left(\frac{1}{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(1/sin(a + b*log(c*x^n)))^(3/2),x)
[Out] int(x^m*(1/sin(a + b*log(c*x^n)))^(3/2), x)
```

3.325 $\int x^m \sqrt{\csc(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=130

$$\frac{2x^{1+m}\sqrt{1-e^{2ia}(cx^n)^{2ib}}\sqrt{\csc(a+b\log(cx^n))}\,{}_2F_1\left(\frac{1}{2},-\frac{2i+2im-bn}{4bn};-\frac{2i+2im-5bn}{4bn};e^{2ia}(cx^n)^{2ib}\right)}{2+2m+ibn}$$

```
[Out] 2*x^(1+m)*hypergeom([1/2, 1/4*(-2*I-2*I*m+b*n)/b/n],[1/4*(-2*I-2*I*m+5*b*n)
/b/n],exp(2*I*a)*(c*x^n)^(2*I*b))*(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(1/2)*csc(
a+b*ln(c*x^n))^(1/2)/(2+2*m+I*b*n)
```

Rubi [A]
time = 0.06, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4606, 4604, 371}

$$\frac{2x^{m+1}\sqrt{1-e^{2ia}(cx^n)^{2ib}}\,{}_2F_1\left(\frac{1}{2},-\frac{2im-bn+2i}{4bn};-\frac{2im-5bn+2i}{4bn};e^{2ia}(cx^n)^{2ib}\right)\sqrt{\csc(a+b\log(cx^n))}}{ibn+2m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sqrt[Csc[a + b*Log[c*x^n]]],x]
```

```
[Out] (2*x^(1 + m)*Sqrt[1 - E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[Csc[a + b*Log[c*x
^n]]]*Hypergeometric2F1[1/2, -1/4*(2*I + (2*I)*m - b*n)/(b*n), -1/4*(2*I +
(2*I)*m - 5*b*n)/(b*n), E^((2*I)*a)*(c*x^n)^((2*I)*b)])/(2 + 2*m + I*b*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4604

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 4606

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
```

```
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^m \sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}\,dx = \frac{\left(x^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{n}}\sqrt{\csc(a+b\log(x))}\,dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x^{1+m}\left(cx^n\right)^{-\frac{ib}{2}-\frac{1+m}{n}}\sqrt{1-e^{2ia}\left(cx^n\right)^{2ib}}\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}\right)\text{Subst}\left(\right.}{n}$$

$$= \frac{2x^{1+m}\sqrt{1-e^{2ia}\left(cx^n\right)^{2ib}}\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}\,{}_2F_1\left(\frac{1}{2}, -\frac{2i+2im-bn}{4bn}; -\frac{2i\cdots}{}\right.}{2+2m+ibn}$$

Mathematica [A]
time = 4.41, size = 163, normalized size = 1.25

$$\frac{2x^{1+m}\sqrt{2-2e^{2ia}\left(cx^n\right)^{2ib}}\sqrt{\frac{ie^{ia}\left(cx^n\right)^{ib}}{-1+e^{2ia}\left(cx^n\right)^{2ib}}}\,{}_2F_1\left(\frac{1}{2}, \frac{-2i-2im+bn}{4bn}; -\frac{2i+2im-5bn}{4bn}; e^{2ia}\left(cx^n\right)^{2ib}\right)}{2+2m+ibn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sqrt[Csc[a + b*Log[c*x^n]]],x]
```

```
[Out] (2*x^(1 + m)*Sqrt[2 - 2*E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[(I*E^(I*a)*(c*x
^n)^(I*b))/(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))]*Hypergeometric2F1[1/2, (-2
*I - (2*I)*m + b*n)/(4*b*n), -1/4*(2*I + (2*I)*m - 5*b*n)/(b*n), E^((2*I)*a
)*(c*x^n)^((2*I)*b)])/(2 + 2*m + I*b*n)
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int x^m\left(\sqrt{\csc}\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*csc(a+b*ln(c*x^n))^(1/2),x)
```

```
[Out] int(x^m*csc(a+b*ln(c*x^n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*sqrt(csc(b*log(c*x^n) + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sqrt{\csc (a + b \log (cx^n))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csc(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(csc(a + b*log(c*x**n))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*sqrt(csc(b*log(c*x^n) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \sqrt{\frac{1}{\sin (a + b\, \ln (c\, x^n))}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(1/sin(a + b*log(c*x^n)))^(1/2),x)
```

```
[Out] int(x^m*(1/sin(a + b*log(c*x^n)))^(1/2), x)
```

3.326 $\int \frac{x^m}{\sqrt{\csc(a+b\log(cx^n))}}\,dx$

Optimal. Leaf size=129

$$\frac{2x^{1+m}\,{}_2F_1\left(-\frac{1}{2},-\frac{2i+2im+bn}{4bn};-\frac{2i+2im-3bn}{4bn};e^{2ia}(cx^n)^{2ib}\right)}{(2+2m-ibn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}\,\sqrt{\csc(a+b\log(cx^n))}}$$

```
[Out] 2*x^(1+m)*hypergeom([-1/2, 1/4*(-2*I-2*I*m-b*n)/b/n],[1/4*(-2*I-2*I*m+3*b*n
)/b/n],exp(2*I*a)*(c*x^n)^(2*I*b))/(2+2*m-I*b*n)/(1-exp(2*I*a)*(c*x^n)^(2*I
*b))^(1/2)/csc(a+b*ln(c*x^n))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 126, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4606, 4604, 371}

$$\frac{2x^{m+1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4}\left(-\frac{2i(m+1)}{bn}-1\right);-\frac{2im-3bn+2i}{4bn};e^{2ia}(cx^n)^{2ib}\right)}{(-ibn+2m+2)\sqrt{1-e^{2ia}(cx^n)^{2ib}}\,\sqrt{\csc(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[Csc[a + b*Log[c*x^n]]],x]
```

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[-1/2, (-1 - ((2*I)*(1 + m))/(b*n))/4, -1/4*(
2*I + (2*I)*m - 3*b*n)/(b*n), E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((2 + 2*m - I
*b*n)*Sqrt[1 - E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[Csc[a + b*Log[c*x^n]]])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4604

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 4606

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
```

```
((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^m}{\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}}\,dx &= \frac{\left(x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}}{\sqrt{\csc(a+b\log(x))}}\,dx, x, cx^n\right)}{n}\\
&= \frac{\left(x^{1+m}(cx^n)^{\frac{ib}{2}-\frac{1+m}{n}}\right)\text{Subst}\left(\int x^{-1-\frac{ib}{2}+\frac{1+m}{n}}\sqrt{1-e^{2ia}x^{2ib}}\,dx, x, cx^n\right)}{n\sqrt{1-e^{2ia}\left(cx^n\right)^{2ib}}\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}}\\
&= \frac{2x^{1+m}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4}\left(-1-\frac{2i(1+m)}{bn}\right);-\frac{2i+2im-3bn}{4bn};e^{2ia}(cx^n)^{2ib}\right)}{(2+2m-ibn)\sqrt{1-e^{2ia}\left(cx^n\right)^{2ib}}\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}}
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 441 vs. $2(129) = 258$.
time = 7.82, size = 441, normalized size = 3.42

$$
-\frac{2be^{ia}nx^{1+m}(cx^n)^{ib}\sqrt{2-2e^{2ia}\left(cx^n\right)^{2ib}}\sqrt{\frac{ie^{ia}\left(cx^n\right)^{ib}}{-1+e^{2ia}\left(cx^n\right)^{2ib}}}\left((2i+2im+bn)x^{2ibn}\,{}_2F_1\left(\frac{1}{2},-\frac{2i+2im-3bn}{4bn};-\frac{2i+2im-7bn}{4bn};e^{2ia}(cx^n)^{2ib}\right)+(-2i-2im+3bn)\,{}_2F_1\left(\frac{1}{2},-\frac{2i+2im+bn}{4bn};-\frac{2i+2im-3bn}{4bn};e^{2ia}(cx^n)^{2ib}\right)\right)}{(2+2m-ibn)(2+2m+3ibn)\left((2i+2im+bn)x^{2ibn}+e^{2ia}(-2i-2im+bn)\left(cx^n\right)^{2ib}\right)}+\frac{2x^{1+m}\sin\left(a-bn\log(x)+b\log\left(cx^n\right)\right)}{\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}\left(bn\cos\left(a-bn\log(x)+b\log\left(cx^n\right)\right)+2(1+m)\sin\left(a-bn\log(x)+b\log\left(cx^n\right)\right)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[Csc[a + b*Log[c*x^n]]],x]
```

```
[Out] (-2*b*E^(I*a)*n*x^(1 + m)*(c*x^n)^(I*b)*Sqrt[2 - 2*E^((2*I)*a)*(c*x^n)^((2*
I)*b)]*Sqrt[(I*E^(I*a)*(c*x^n)^(I*b))/(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))]
*((2*I + (2*I)*m + b*n)*x^((2*I)*b*n)*Hypergeometric2F1[1/2, -1/4*(2*I + (2
*I)*m - 3*b*n)/(b*n), -1/4*(2*I + (2*I)*m - 7*b*n)/(b*n), E^((2*I)*a)*(c*x^
n)^((2*I)*b)] + (-2*I - (2*I)*m + 3*b*n)*Hypergeometric2F1[1/2, -1/4*(2*I +
 (2*I)*m + b*n)/(b*n), -1/4*(2*I + (2*I)*m - 3*b*n)/(b*n), E^((2*I)*a)*(c*x
^n)^((2*I)*b)]))/((2 + 2*m - I*b*n)*(2 + 2*m + (3*I)*b*n)*((2*I + (2*I)*m +
 b*n)*x^((2*I)*b*n) + E^((2*I)*a)*(-2*I - (2*I)*m + b*n)*(c*x^n)^((2*I)*b))
) + (2*x^(1 + m)*Sin[a - b*n*Log[x] + b*Log[c*x^n]])/(Sqrt[Csc[a + b*Log[c*
x^n]]]*(b*n*Cos[a - b*n*Log[x] + b*Log[c*x^n]] + 2*(1 + m)*Sin[a - b*n*Log[
x] + b*Log[c*x^n]]))
```

Maple [**F**]
time = 0.12, size = 0, normalized size = 0.00

$$
\int \frac{x^m}{\sqrt{\csc\left(a+b\ln\left(c\,x^n\right)\right)}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/csc(a+b*ln(c*x^n))^(1/2),x)
```

```
[Out] int(x^m/csc(a+b*ln(c*x^n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/csc(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(csc(b*log(c*x^n) + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/csc(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{\csc(a + b\log(cx^n))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/csc(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(x**m/sqrt(csc(a + b*log(c*x**n))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/csc(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(csc(b*log(c*x^n) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{\sqrt{\frac{1}{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(1/sin(a + b*log(c*x^n)))^(1/2),x)
[Out] int(x^m/(1/sin(a + b*log(c*x^n)))^(1/2), x)
```

3.327 $\int \frac{x^m}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=130

$$\frac{2x^{1+m}\,{}_2F_1\left(-\frac{3}{2},-\frac{2i+2im+3bn}{4bn};-\frac{2i+2im-bn}{4bn};e^{2ia}(cx^n)^{2ib}\right)}{(2+2m-3ibn)\left(1-e^{2ia}\left(cx^n\right)^{2ib}\right)^{3/2}\csc^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

```
[Out] 2*x^(1+m)*hypergeom([-3/2, 1/4*(-2*I-2*I*m-3*b*n)/b/n],[1/4*(-2*I-2*I*m+b*n
)/b/n],exp(2*I*a)*(c*x^n)^(2*I*b))/(2+2*m-3*I*b*n)/(1-exp(2*I*a)*(c*x^n)^(2
*I*b))^(3/2)/csc(a+b*ln(c*x^n))^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4606, 4604, 371}

$$\frac{2x^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(-\frac{2i(m+1)}{bn}-3\right);-\frac{2im-bn+2i}{4bn};e^{2ia}(cx^n)^{2ib}\right)}{(-3ibn+2m+2)\left(1-e^{2ia}\left(cx^n\right)^{2ib}\right)^{3/2}\csc^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Csc[a + b*Log[c*x^n]]^(3/2),x]
```

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[-3/2, (-3 - ((2*I)*(1 + m))/(b*n))/4, -1/4*(
2*I + (2*I)*m - b*n)/(b*n), E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((2 + 2*m - (3*
I)*b*n)*(1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2)*Csc[a + b*Log[c*x^n]]^(3/
2))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4604

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 4606

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx &= \frac{\left(x^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\mathrm{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}}{\csc^{\frac{3}{2}}(a+b\log(x))}\,dx,x,cx^n\right)}{n}\\ &= \frac{\left(x^{1+m}(cx^n)^{\frac{3ib}{2}-\frac{1+m}{n}}\right)\mathrm{Subst}\left(\int x^{-1-\frac{3ib}{2}+\frac{1+m}{n}}\left(1-e^{2ia}x^{2ib}\right)^{3/2}\,dx,x,cx^n\right)}{n\left(1-e^{2ia}\left(cx^n\right)^{2ib}\right)^{3/2}\csc^{\frac{3}{2}}(a+b\log(cx^n))}\\ &= \frac{2x^{1+m}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{2i(1+m)}{bn}\right);-\frac{2i+2im-bn}{4bn};e^{2ia}(cx^n)^{2ib}\right)}{(2+2m-3ibn)\left(1-e^{2ia}\left(cx^n\right)^{2ib}\right)^{3/2}\csc^{\frac{3}{2}}(a+b\log(cx^n))}\end{aligned}$$

Mathematica [A]
time = 6.98, size = 228, normalized size = 1.75

$$\frac{2x^{1+m}\left(\frac{2+2m-3bn\cot(a+b\log(cx^n))}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}+\frac{3b^2n^2\sqrt{2-2e^{2ia}\left(cx^n\right)^{2ib}}\sqrt{\frac{ie^{ia}\left(cx^n\right)^{ib}}{-1+e^{2ia}\left(cx^n\right)^{2ib}}}\,{}_2F_1\left(\frac{1}{2},\frac{-2i-2im+bn}{4bn};-\frac{2i+2im-5bn}{4bn};e^{2ia}(cx^n)^{2ib}\right)}{2+2m+ibn}\right)}{4+8m+4m^2+9b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Csc[a + b*Log[c*x^n]]^(3/2),x]
```

```
[Out] (2*x^(1 + m)*((2 + 2*m - 3*b*n*Cot[a + b*Log[c*x^n]])/Csc[a + b*Log[c*x^n]]
^(3/2) + (3*b^2*n^2*Sqrt[2 - 2*E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[(I*E^(I*
a)*(c*x^n)^(I*b))/(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))]*Hypergeometric2F1[1
/2, (-2*I - (2*I)*m + b*n)/(4*b*n), -1/4*(2*I + (2*I)*m - 5*b*n)/(b*n), E^(
(2*I)*a)*(c*x^n)^((2*I)*b)])/(2 + 2*m + I*b*n)))/(4 + 8*m + 4*m^2 + 9*b^2*n
^2)
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\csc\left(a+b\ln\left(c\,x^n\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/csc(a+b*ln(c*x^n))^(3/2),x)
```

```
[Out] int(x^m/csc(a+b*ln(c*x^n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/csc(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/csc(b*log(c*x^n) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/csc(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\csc^{\frac{3}{2}}\left(a + b\log\left(cx^n\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/csc(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Integral(x**m/csc(a + b*log(c*x**n))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/csc(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/csc(b*log(c*x^n) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{\left(\frac{1}{\sin(a+b\,\ln(c\,x^n))}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(1/sin(a + b*log(c*x^n)))^(3/2),x)
[Out] int(x^m/(1/sin(a + b*log(c*x^n)))^(3/2), x)
```

3.328 $\int (ex)^m \csc^p(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=139

$$\frac{(ex)^{1+m}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^p\csc^p(d(a+b\log(cx^n)))\,{}_2F_1\left(p,-\frac{i+im-bdnp}{2bdn};\frac{1}{2}\left(2-\frac{i(1+m)}{bdn}+p\right);e^{2iad}(cx^n)^{2ibd}\right)}{e(1+m+ibdnp)}$$

```
[Out] (e*x)^(1+m)*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^p*csc(d*(a+b*ln(c*x^n)))^p*h
ypergeom([p, 1/2*(-I-I*m+b*d*n*p)/b/d/n],[1-1/2*I*(1+m)/b/d/n+1/2*p],exp(2*
I*a*d)*(c*x^n)^(2*I*b*d))/e/(1+m+I*b*d*n*p)
```

Rubi [A]
time = 0.08, antiderivative size = 133, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4606, 4604, 371}

$$\frac{(ex)^{m+1}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^p\,{}_2F_1\left(p,\frac{1}{2}\left(p-\frac{i(m+1)}{bdn}\right);\frac{1}{2}\left(-\frac{i(m+1)}{bdn}+p+2\right);e^{2iad}(cx^n)^{2ibd}\right)\csc^p(d(a+b\log(cx^n)))}{e(ibdnp+m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Csc[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] ((e*x)^(1 + m)*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p*Csc[d*(a + b*Log[c
*x^n])]^p*Hypergeometric2F1[p, (((-I)*(1 + m))/(b*d*n) + p)/2, (2 - (I*(1 +
 m))/(b*d*n) + p)/2, E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(e*(1 + m + I*b*d*
n*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4604

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 4606

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
```

```
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int (ex)^m \csc^p(d(a+b\log(cx^n)))\,dx = \frac{\left((ex)^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\mathrm{Subst}\left(\int x^{-1+\frac{1+m}{n}}\csc^p(d(a+b\log(x)))\,dx, x, cx$$
$$\quad \hphantom{=}\frac{}{en}$$
$$= \frac{\left((ex)^{1+m}(cx^n)^{-\frac{1+m}{n}-ibdp}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^p\csc^p(d(a+b\log(cx^n)))\right.}{en}$$
$$= \frac{(ex)^{1+m}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^p\csc^p(d(a+b\log(cx^n)))\ {}_2F_1\left(p, \frac{1}{2}\left(-\frac{i(1+}{bdn}\right.\right.}{e(1+m+ibdnp)}$$

Mathematica [A]
time = 1.57, size = 169, normalized size = 1.22

$$\frac{x(ex)^m\left(2-2e^{2iad}(cx^n)^{2ibd}\right)^p\left(\frac{ie^{iad}(cx^n)^{ibd}}{-1+e^{2iad}(cx^n)^{2ibd}}\right)^p {}_2F_1\left(p,-\frac{i(1+m+ibdnp)}{2bdn};\frac{1}{2}\left(2-\frac{i(1+m)}{bdn}+p\right);e^{2iad}(cx^n)^{2ibd}\right)}{1+m+ibdnp}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Csc[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] (x*(e*x)^m*(2 - 2*E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p*((I*E^(I*a*d)*(c*x^n
)^(I*b*d))/(-1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))^p*Hypergeometric2F1[p,
 ((-1/2*I)*(1 + m + I*b*d*n*p))/(b*d*n), (2 - (I*(1 + m))/(b*d*n) + p)/2, E
^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(1 + m + I*b*d*n*p)
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int (ex)^m\left(\csc^p(d(a+b\ln(c\,x^n)))\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*csc(d*(a+b*ln(c*x^n)))^p,x)
```

```
[Out] int((e*x)^m*csc(d*(a+b*ln(c*x^n)))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*csc(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
[Out] integrate((x*e)^m*csc((b*log(c*x^n) + a)*d)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*csc(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
[Out] integral((x*e)^m*csc(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \csc^p (ad + bd \log (cx^n))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*csc(d*(a+b*ln(c*x**n)))**p,x)
[Out] Integral((e*x)**m*csc(a*d + b*d*log(c*x**n))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*csc(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
[Out] integrate((e*x)^m*csc((b*log(c*x^n) + a)*d)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,x)^m\,{\left(\frac{1}{\sin\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(1/sin(d*(a + b*log(c*x^n))))^p,x)
[Out] int((e*x)^m*(1/sin(d*(a + b*log(c*x^n))))^p, x)
```

3.329 $\int x \csc^p(a+b\log(cx^n))\,dx$

Optimal. Leaf size=106

$$\frac{x^2\left(1-e^{2ia}(cx^n)^{2ib}\right)^p\csc^p(a+b\log(cx^n))\,{}_2F_1\left(p,\frac{1}{2}\left(-\frac{2i}{bn}+p\right);\frac{1}{2}\left(2-\frac{2i}{bn}+p\right);e^{2ia}(cx^n)^{2ib}\right)}{2+ibnp}$$

```
[Out] x^2*(1-exp(2*I*a)*(c*x^n)^(2*I*b))^p*csc(a+b*ln(c*x^n))^p*hypergeom([p, -I/
b/n+1/2*p],[1-I/b/n+1/2*p],exp(2*I*a)*(c*x^n)^(2*I*b))/(2+I*b*n*p)
```

Rubi [A]
time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4606, 4604, 371}

$$\frac{x^2\left(1-e^{2ia}(cx^n)^{2ib}\right)^p\,{}_2F_1\left(p,\frac{1}{2}\left(p-\frac{2i}{bn}\right);\frac{1}{2}\left(p-\frac{2i}{bn}+2\right);e^{2ia}(cx^n)^{2ib}\right)\csc^p(a+b\log(cx^n))}{2+ibnp}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csc[a + b*Log[c*x^n]]^p,x]
```

```
[Out] (x^2*(1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^p*Csc[a + b*Log[c*x^n]]^p*Hypergeo
metric2F1[p, ((-2*I)/(b*n) + p)/2, (2 - (2*I)/(b*n) + p)/2, E^((2*I)*a)*(c*
x^n)^((2*I)*b)])/(2 + I*b*n*p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4604

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 4606

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^
((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x\csc^p(a+b\log(cx^n))\,dx = \frac{\left(x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int x^{-1+\frac{2}{n}}\csc^p(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(x^2(cx^n)^{-\frac{2}{n}-ibp}\left(1-e^{2ia}(cx^n)^{2ib}\right)^p\csc^p(a+b\log(cx^n))\right)\text{Subst}\left(\int x^{-1+\frac{2}{n}+ib}\right.}{n}$$
$$= \frac{x^2\left(1-e^{2ia}(cx^n)^{2ib}\right)^p\csc^p(a+b\log(cx^n))\,{}_2F_1\left(p,\frac{1}{2}\left(-\frac{2i}{bn}+p\right);\frac{1}{2}\left(2-\frac{2i}{bn}+\right.\right.}{2+ibnp}$$

Mathematica [A]
time = 1.19, size = 142, normalized size = 1.34

$$-\frac{ix^2\left(2-2e^{2ia}(cx^n)^{2ib}\right)^p\left(\frac{ie^{ia}(cx^n)^{ib}}{-1+e^{2ia}(cx^n)^{2ib}}\right)^p{}_2F_1\left(-\frac{i}{bn}+\frac{p}{2},p;1-\frac{i}{bn}+\frac{p}{2};e^{2ia}(cx^n)^{2ib}\right)}{-2i+bnp}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csc[a + b*Log[c*x^n]]^p,x]
[Out] ((-I)*x^2*(2 - 2*E^((2*I)*a)*(c*x^n)^((2*I)*b))^p*((I*E^(I*a)*(c*x^n)^(I*b)
)/(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))^p*Hypergeometric2F1[(-I)/(b*n) + p/
2, p, 1 - I/(b*n) + p/2, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/(-2*I + b*n*p)
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int x(\csc^p(a+b\ln(c\,x^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(a+b*ln(c*x^n))^p,x)
[Out] int(x*csc(a+b*ln(c*x^n))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(a+b*log(c*x^n))^p,x, algorithm="maxima")
```

[Out] integrate(x*csc(b*log(c*x^n) + a)^p, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(a+b*log(c*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral(x*csc(b*log(c*x^n) + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\csc^{p}\left(a+b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(a+b*ln(c*x**n))**p,x)
```

```
[Out] Integral(x*csc(a + b*log(c*x**n))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(a+b*log(c*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate(x*csc(b*log(c*x^n) + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(\frac{1}{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1/sin(a + b*log(c*x^n)))^p,x)
```

```
[Out] int(x*(1/sin(a + b*log(c*x^n)))^p, x)
```

3.330 $\int \csc^p(a+b\log(cx^n))\,dx$

Optimal. Leaf size=107

$$\frac{x\left(1-e^{2ia}(cx^n)^{2ib}\right)^p \csc^p(a+b\log(cx^n))\ {}_2F_1\left(p,-\frac{i-bnp}{2bn};\frac{1}{2}\left(2-\frac{i}{bn}+p\right);e^{2ia}(cx^n)^{2ib}\right)}{1+ibnp}$$

```
[Out] x*(1-exp(2*I*a)*(c*x^n)^(2*I*b))^p*csc(a+b*ln(c*x^n))^p*hypergeom([p, 1/2*(
-I+b*n*p)/b/n],[1-1/2*I/b/n+1/2*p],exp(2*I*a)*(c*x^n)^(2*I*b))/(1+I*b*n*p)
```

Rubi [A]
time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4600, 4604, 371}

$$\frac{x\left(1-e^{2ia}(cx^n)^{2ib}\right)^p\ {}_2F_1\left(p,-\frac{i-bnp}{2bn};\frac{1}{2}\left(p-\frac{i}{bn}+2\right);e^{2ia}(cx^n)^{2ib}\right)\csc^p(a+b\log(cx^n))}{1+ibnp}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*Log[c*x^n]]^p,x]
```

```
[Out] (x*(1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^p*Csc[a + b*Log[c*x^n]]^p*Hypergeome
tric2F1[p, -1/2*(I - b*n*p)/(b*n), (2 - I/(b*n) + p)/2, E^((2*I)*a)*(c*x^n)
^((2*I)*b)])/(1 + I*b*n*p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4600

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Di
st[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 4604

```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
)), Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \csc^p\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\csc^p(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(x(cx^n)^{-\frac{1}{n}-ibp}\left(1-e^{2ia}(cx^n)^{2ib}\right)^p\csc^p\left(a+b\log\left(cx^n\right)\right)\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}+ibp}\left(1\right.\right.}{n}$$
$$= \frac{x\left(1-e^{2ia}(cx^n)^{2ib}\right)^p\csc^p\left(a+b\log\left(cx^n\right)\right)\,{}_2F_1\left(p,-\frac{i-bnp}{2bn};\frac{1}{2}\left(2-\frac{i}{bn}+p\right);e^{2ia}(cx\right.}{1+ibnp}$$

Mathematica [**A**]
time = 0.93, size = 142, normalized size = 1.33

$$-\frac{ix\left(2-2e^{2ia}(cx^n)^{2ib}\right)^p\left(\frac{ie^{ia}(cx^n)^{ib}}{-1+e^{2ia}(cx^n)^{2ib}}\right)^p\,{}_2F_1\left(p,\frac{-i+bnp}{2bn};\frac{1}{2}\left(2-\frac{i}{bn}+p\right);e^{2ia}(cx^n)^{2ib}\right)}{-i+bnp}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*Log[c*x^n]]^p,x]
```

```
[Out] ((-I)*x*(2 - 2*E^((2*I)*a)*(c*x^n)^((2*I)*b))^p*((I*E^(I*a)*(c*x^n)^(I*b))/
(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))^p*Hypergeometric2F1[p, (-I + b*n*p)/(
2*b*n), (2 - I/(b*n) + p)/2, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/(-I + b*n*p)
```

Maple [**F**]
time = 0.10, size = 0, normalized size = 0.00

$$\int \csc^p\left(a+b\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(a+b*ln(c*x^n))^p,x)
```

```
[Out] int(csc(a+b*ln(c*x^n))^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate(csc(b*log(c*x^n) + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^p,x, algorithm="fricas")
[Out] integral(csc(b*log(c*x^n) + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^{p}\left(a+b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*ln(c*x**n))**p,x)
[Out] Integral(csc(a + b*log(c*x**n))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^p,x, algorithm="giac")
[Out] integrate(csc(b*log(c*x^n) + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(a + b*log(c*x^n)))^p,x)
[Out] int((1/sin(a + b*log(c*x^n)))^p, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
           'sin','cos','tan','cot','sec','csc',
           'arcsin','arccos','arctan','arccot','arcsec','arccsc',
           'sinh','cosh','tanh','coth','sech','csch',
           'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
       'arctan2','floor','abs'
       ]
   if debug:
      if m:
         print ("func ", func , " is elementary_function")
      else:
         print ("func ", func , " is NOT elementary_function")


   return m

def is_special_function(func):
   #debug=False
   if debug:
      print ("type(func)=", type(func))

   m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
         'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
         'elliptic_pi','exp_integral_e','log_integral']

   if debug:
      print ("m=",m)
      if m:
         print ("func ", func ," is special_function")
      else:
         print ("func ", func ," is NOT special_function")


   return m


def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstar
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```